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MBJ-neutrosophic Ideals of KU-algebras

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Abstract. In this paper, the notion of MBJ-neutrosophic ideal for KU-algebra is introduced and its properties are investigated. Also, a condition for an MBJ-neutrosophic subalgebra to be an MBJ-neutrosophic ideal and converse part of a KU-algebra are discussed.

Keywords and phrases: MBJ-N set, MBJ-NSA, MBJ-NI.

1. Introduction

The fuzzy set was introduced in 1965 by Zadeh [10] and the intuitionistic fuzzy set was introduced by Atanassov in 1983. Prabpayak and Leerawat [5, 6] introduced a algebraic structure called KU algebras and also examined ideals and congruences. In addition, homomorphism, quotients and isomorphisms in KU algebras. The concept of the neutrosophic set was developed by Smarandache [7, 8, 9]. In 2017, Bijan Davvaz et al. [1] investigated the neutrosophic ideal of the KU-neutrosophic algebras.

Mohseni et al. [3] introduced the generalization of neutrosophic set called MBJ-neutrosophic set with $M_L, \tilde{B}_L$ and J_L as the truth, the indeterminate and the false membership functions, respectively. Also, MBJ-neutrosophic subalgebras in BCK/BCI-algebras and investigated related properties.

In [2], the notion of MBJ-neutrosophic sets is introduced and applied to KU-algebras. Also, a characterization of MBJ-neutrosophic subalgebra is provided with KU-algebra and Homomorphic inverse image and translation of MBJ-neutrosophic subalgebra in KU-algebra are discussed.

In this paper, we apply the notion of MBJ-neutrosophic sets to ideals of KU-algebras and investigate several properties. We provide a condition for an MBJ-neutrosophic subalgebra to be an MBJ-neutrosophic ideal and converse part of a KU-algebra are discussed.

2. Preliminaries

We let $L(\tau)$ be the class of all algebras with type $\tau = (2, 0)$. A KU-algebra [5, 6] on a system $P = (P, \diamond, 0) \in L(\tau)$ satisfies

(KU1) $(k_{01} \diamond k_{02}) \diamond ((k_{02} \diamond k_{03}) \diamond (k_{01} \diamond k_{03})) = 0$,

(KU2) $k_{01} \diamond 0 = 0$,

(KU3) $0 \diamond k_{01} = k_{01}$,

(KU4) $k_{01} \diamond k_{02} = 0$ & $k_{02} \diamond k_{01} = 0$ implies $k_{01} = k_{01}$,

(KU5) $k_{01} \diamond k_{01} = 0$, $\forall\ k_{01}, k_{02}, k_{03} \in P$.

Also a binary relation $\leq$ by putting $k_{01} \leq k_{02} \Leftrightarrow k_{02} \diamond k_{01} = 0, \forall\ k_{01}, k_{02} \in P$.
In a KU-algebra P, the following hold:

(KU1′) $(k_{02} \diamond k_{03}) \diamond (k_{01} \diamond k_{03}) \leq (k_{01} \diamond k_{02})$,

(KU2′) $0 \leq k_{01}$,

(KU3′) $k_{01} \leq k_{02}$, $k_{02} \leq k_{01}$ implies $k_{01} = k_{02}$,

(KU4′) $k_{02} \diamond k_{01} \leq k_{01}$.

Theorem 2.1 [4] In a KU-algebra P, the following axioms are satisfied: $\forall\ k_{01}, k_{02}, k_{03} \in P$,

(i) $k_{01} \leq k_{02}$ imply $k_{02} \diamond k_{03} \leq k_{01} \diamond k_{03}$,

(ii) $k_{01} \diamond (k_{02} \diamond k_{03}) = k_{02} \diamond (k_{01} \diamond k_{03})$, $\forall\ k_{01}, k_{02}, k_{03} \in P$,

(iii) $((k_{02} \diamond k_{01}) \diamond k_{01}) \leq k_{02}$,

(iv) $(((k_{02} \diamond k_{01}) \diamond k_{01}) \diamond k_{01}) = (k_{02} \diamond k_{01})$.

Definition 2.1 [5, 6] A non-empty subset S of a KU-algebra P is called a KU-subalgebra of P if $l_{11} \diamond l_{22} \in S\ \forall\ l_{11}, l_{22} \in S$.

Definition 2.2 [5, 6] A subset S of a KU-algebra P is called an ideal of P if it satisfies the following:

(I1) $0 \in S$,

(I2) $(\forall\ k_{01}, k_{02} \in P)\ (k_{01} \diamond k_{02} \in S, k_{01} \in S \Rightarrow k_{01} \in S)$.

Definition 2.3 [5, 6] A subset S of a KU-algebra P is called a closed ideal of P if it is an ideal of P which satisfies:

$$(\forall\ k_{01} \in P)(k_{01} \in S \Rightarrow 0 \diamond k_{01} \in S). \tag{1}$$

A closed subinterval $\tilde{l} = [l^-, l^+]$ of I, where $0 \leq l^- \leq l^+ \leq 1$ and $[I]$ denotes the set of all interval numbers. Consider two interval numbers $\tilde{l}_{01} := [l^-_{01}, l^+_{01}]$ and $\tilde{l}_{02} := [l^-_{02}, l^+_{02}]$. Let us define as refined minimum and refined maximum (briefly, $re\min$ and $re\max$) of two elements in $[I]$, then

$$re\min\{\tilde{l}_{01}, \tilde{l}_{02}\} = [\min\{l^-_{01}, l^-_{02}\}, \min\{l^+_{01}, l^+_{02}\}],$$
$$re\max\{\tilde{l}_{01}, \tilde{l}_{02}\} = [\max\{l^-_{01}, l^-_{02}\}, \max\{l^+_{01}, l^+_{02}\}],$$

Let P be a nonempty set. A function $L : P \to [I]$ is called an interval-valued fuzzy (in short, IVF) set in P. Let $[I]^P$ denotes the set of all IVF sets in P. For every $L \in [I]^P$ and $k \in P, L(k) = [L^-(k), L^+(k)]$ is called the degree of membership of an element k to L, where $L^- : P \to I$ is a a lower fuzzy set in P and $L^+ : P \to I$ an upper fuzzy set in P, respectively. We denote $L = [L^-, L^+]$.

Definition 2.4 [8] Let P be a non-empty set. A neutrosophic set (NS) in P is of the form:

$$L := \{\langle k; L_T(k), L_I(k), L_F(k)\rangle | k \in P\}$$

where $L_T, L_I, L_F : P \to [0, 1]$ is a truth, an indeterminate and a false membership function. We use the symbol $L = (L_T, L_I, L_F)$.

Definition 2.5 [3] Let P be a non-empty set. By an MBJ-neutrosophic (briefly, MBJ-N) set in P is of the form

$$\mathcal{L} := \{\langle k; M_L(k), \tilde{B}_L(k), J_L(k)\rangle | k \in P\}$$

where M_L and J_L are fuzzy sets in P called a truth and a false membership functions and $\tilde{B}_L$ is an IVF set in P called an indeterminate interval-valued membership function. We use the symbol $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$.

Definition 2.6 [2] Let P be a KU algebra. An MBJ-N set $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ in P is called an MBJ-neutrosophic subalgebra of P (briefly, MBJ-$NSA(P)$) if it satisfies:

$$(\forall l_{01}, l_{02} \in P) \begin{pmatrix} M_L(l_{01} \diamond l_{02}) \geq \min\{M_L(l_{01}), M_L(l_{02})\} \\ \tilde{B}_L(l_{01} \diamond l_{02}) \succeq re\min\left\{\tilde{B}_L(l_{01}), \tilde{B}_L(l_{02})\right\} \\ J_L(l_{01} \diamond l_{02}) \leq \max\{J_L(l_{01}), J_L(l_{02})\} \end{pmatrix} \quad (2)$$

3. MBJ-neutrosophic Ideals of KU-algebras

Definition 3.1 Let P be a KU-algebra. An MBJ-N set $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ in P is called an MBJ-neutrosophic ideal of P (briefly, MBJ-$NI(P)$) if it satisfies:

$$(\forall\ l_{01} \in P) \begin{pmatrix} M_L(0) \geq & M_L(l_{01}) \\ \tilde{B}_L(0) \succeq & \tilde{B}_L(l_{01}) \\ J_L(0) \leq & J_L(l_{01}) \end{pmatrix} \quad (3)$$

and

$$(\forall\ l_{01}, l_{02} \in P) \begin{pmatrix} M_L(l_{01}) \geq & \min\{M_L(l_{02} \diamond l_{01}), M_L(l_{02})\} \\ \tilde{B}_L(l_{01}) \succeq & re\min\{\tilde{B}_L(l_{02} \diamond l_{01}), \tilde{B}_L(y)\} \\ J_L(l_{01}) \leq & \max\{J_L(l_{02} \diamond l_{01}), J_L(l_{02})\} \end{pmatrix}. \quad (4)$$

An MBJ-NI $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ of a KU-algebra P is said to be closed if

$$(\forall\ l_{01} \in P) \begin{pmatrix} M_L(l_{01} \diamond 0) \geq & M_L(l_{01}) \\ \tilde{B}_L(l_{01} \diamond 0) \succeq & \tilde{B}_L(l_{01}) \\ J_L(l_{01} \diamond 0) \leq & J_L(l_{01}) \end{pmatrix} \quad (5)$$

Example 3.1 Consider a set $P = \{0_6, a_6, b_6, c_6\}$ with the binary operation $\diamond$. Table 1 defined the following operation. Then

Table 1:

$\diamond$	0_6	a_6	b_6	c_6	d_6
0_6	0_6	a_6	b_6	c_6	d_6
a_6	0_6	0_6	b_6	c_6	d_6
b_6	0_6	a_6	0_6	c_6	c_6
c_6	0_6	0_6	b_6	0_6	b_6
d_6	0_6	0_6	0_6	0_6	0_6

$(P, \diamond, 0)$ is a KU algebra.

Table 2:

P	$M_L(l)$	$\tilde{B}_L(l)$	$J_L(l)$
0_6	0.8	$[0.4, 0.9]$	0.2
a_6	0.7	$[0.3, 0.8]$	0.3
b_6	0.5	$[0.2, 0.6]$	0.7
c_6	0.3	$[0.1, 0.5]$	0.5
d_6	0.3	$[0.1, 0.5]$	0.7

Let $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ be an MBJ-N set in P as in Table 2. It is a closed MBJ-$NI(P)$ of KU-algebra.

Proposition 3.1 Let P be a KU-algebra. Then every MBJ-NI $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ of P satisfies,

$$l_{02} \diamond l_{01} \leq l_{03} \Rightarrow \begin{cases} M_L(l_{01}) \geq \min\{M_L(l_{02}), M_L(l_{03})\} \\ \tilde{B}_L(l_{01}) \succeq r\min\{\tilde{B}_L(l_{02}), \tilde{B}_L(l_{03})\} \\ J_L(l_{01}) \leq \max\{J_L(l_{02}), J_L(l_{03})\} \end{cases} \tag{6}$$

for all $l_{01}, l_{02}, l_{03} \in P$.

Proof. Let $l_{01}, l_{02}, l_{03} \in P$ be such that $l_{02} \diamond l_{01} \leq l_{03}$. Then

$$M_L(l_{02} \diamond l_{01}) \geq \min\{M_L(l_{03} \diamond (l_{02} \diamond l_{01})), M_L(l_{03})\} = \min\{M_L(0), M_L(l_{03})\} = M_L(l_{03}),$$

$$\tilde{B}_L(l_{02} \diamond l_{01}) \succeq re\min\{\tilde{B}_L(l_{03} \diamond (l_{02} \diamond l_{01})), \tilde{B}_L(l_{03})\} = re\min\{\tilde{B}_L(0), \tilde{B}_L(l_{03})\} = \tilde{B}_L(l_{03}),$$

and

$$J_L(l_{02} \diamond l_{01}) \leq \max\{J_L(l_{03} \diamond (l_{02} \diamond l_{01})), J_L(l_{03})\} = \max\{J_L(0), J_L(l_{03})\} = J_L(l_{03}).$$

It follows that

$$M_L(l_{01}) \geq \min\{M_L(l_{02} \diamond l_{01}), M_L(l_{02})\} = \min\{M_L(l_{02}), M_L(l_{03})\},$$

$$\tilde{B}_L(l_{01}) \succeq re\min\{\tilde{B}_L(l_{02} \diamond l_{01}), \tilde{B}_L(l_{02})\} = re\min\{\tilde{B}_L(l_{02}), \tilde{B}_L(l_{03})\},$$

and

$$J_L(l_{01}) \leq \max\{J_L(l_{02} \diamond l_{01}), J_L(l_{02})\} = \max\{J_L(l_{02}), J_L(l_{03})\}.$$

This completes the proof.

Theorem 3.1 Every MBJ-N set in a KU-algebra P satisfying (3) & (6) is an MBJ-$NI(P)$.

Proof. Let $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ be an MBJ-N set in P satisfying (3) & (6). Note that $(l_{02} \diamond l_{01}) \diamond l_{01} \leq l_{02}$ for all $l_{01}, l_{02} \in P$. It follows from (6) that

$$M_L(l_{01}) \geq \min\{M_L(l_{02} \diamond l_{01}), M_L(l_{02})\},$$

$$\tilde{B}_L(l_{01}) \succeq re\min\{\tilde{B}_L(l_{02} \diamond l_{01}), \tilde{B}_L(l_{02})\},$$

and

$$J_L(l_{01}) \leq \max\{J_L(l_{02} \diamond l_{01}), J_L(l_{02})\}.$$

Therefore $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ is an MBJ-$NI(P)$.

Theorem 3.2 Given an MBJ-N set $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ in a KU-algebra P, if (M_L, J_L) is an intuitionistic fuzzy ideal (IFI) of P and B_L^- & B_L^+ are fuzzy ideals of P, then $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ is an MBJ-$NI(P)$.

Proof. It is sufficient t.s.t $\tilde{B}_L$ satisfies

$$(\forall\, l_{01} \in P)(\tilde{B}_L(0) \succeq \tilde{B}_L(l_{01})) \tag{7}$$

and

$$(\forall\, l_{01}, l_{02} \in P)(\tilde{B}_L(l_{01}) \succeq re\min\{\tilde{B}_L(l_{02} \diamond l_{01}), \tilde{B}_L(l_{02})\}). \tag{8}$$

For any $l_{01}, l_{02} \in P$, we get

$$\tilde{B}_L(0) = [B_L^-(0), B_L^+(0)] \succeq [B_L^-(l_{01}), B_L^+(l_{01})] = \tilde{B}_L(l_{01})$$

and

$$\begin{aligned}\tilde{B}_L(l_{01}) &= \left[B_L^-(l_{01}), B_L^+(l_{01})\right] \\ &\succeq \left[\min\left\{B_L^-\left(l_{02} \diamond l_{01}\right), B_L^-(l_{02})\right\}, \min\left\{B_L^+\left(l_{02} \diamond l_{01}\right), B_L^+(l_{02})\right\}\right] \\ &= re\min\left\{\left[B_L^-\left(l_{02} \diamond l_{01}\right), B_L^+\left(l_{02} \diamond l_{01}\right)\right], \left[B_L^-(l_{02}), B_L^+(l_{02})\right]\right. \\ &= re\min\left\{\tilde{B}_L\left(l_{02} \diamond l_{01}\right), \tilde{B}_L(l_{02})\right\}\end{aligned}$$

Therefore $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ is an MBJ-NI of P.

If $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ is an MBJ-NI of a KU-algebra P, then

$$\begin{aligned}\left[B_L^-(l_{01}), B_L^+(l_{01})\right] &= \tilde{B}_L(l_{01}) \succeq re\min\left\{\tilde{B}_L\left(l_{02} \diamond l_{01}\right), \tilde{B}_L(l_{02})\right\} \\ &= re\min\left\{\left[B_L^-\left(l_{02} \diamond l_{01}\right), B_L^+\left(l_{02} \diamond l_{01}\right), \left[B_L^-(l_{02}), B_L^+(l_{02})\right]\right\}\right. \\ &= \left[\min\left\{B_L^-\left(l_{02} \diamond l_{01}\right), B_L^-(l_{02})\right\}, \min\left\{B_L^+\left(l_{02} \diamond l_{01}\right), B_L^+(l_{02})\right\}\right]\end{aligned}$$

for all $l_{01}, l_{02} \in P$. It follows that $B_L^-(l_{01}) \geq \min\{B_L^-(l_{02} \diamond l_{01}), B_L^-(l_{02})\}$ and $B_L^+(l_{01}) \geq \min\{B_L^+(l_{02} \diamond l_{01}), B_L^+(l_{02})\}$. Thus B_L^- and B_L^+ are fuzzy ideals of P. But (M_L, J_L) is not an IFI of P in Example 3.1. Therefore the converse of Theorem 3.2 is not true.

Given an MBJ-N set $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ in a KU-algebra P, we consider the following sets.

$$\begin{aligned}&U\left(M_L; t_1\right) := \left\{l_{01} \in P \mid M_L(l_{01}) \geq t_1\right\}, \\ &U\left(\tilde{B}_L; [\zeta_1, \zeta_2]\right) := \left\{l_{01} \in P \mid \tilde{B}_L(l_{01}) \succeq [\zeta_1, \zeta_2]\right\} \\ &L\left(J_L; t_2\right) := \left\{l_{01} \in P \mid J_L(l_{01}) \leq t_2\right\},\end{aligned}$$

where $t_1, t_2 \in [0, 1]$ and $[\zeta_1, \zeta_2] \in [I]$.

Theorem 3.3 An MBJ-N set $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ in a KU-algebra P is an MBJ-$NI(P)$ iff the non-empty sets $U(M_L; t_1), U(\tilde{B}_L; [\zeta_1, \zeta_2])$ and $L(J_L; t_2)$ are ideals of P for all $t_1, t_2 \in [0, 1]$ and $[\zeta_1, \zeta_2] \in [I]$.

Proof. Suppose that $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ is an MBJ-NI of P. Let $t_1, t_2 \in [0, 1]$ and $[\zeta_1, \zeta_2] \in [I] \ni U(M_L; t_1), U(\tilde{B}_L; [\zeta_1, \zeta_2])$ and $L(J_L; t_2)$ are non-empty. Obviously, $0 \in U(M_L; t_1) \cap U(\tilde{B}_L; [\zeta_1, \zeta_2]) \cap L(J_L; t_2)$. For any $\iota, \kappa, p, q, m, n \in P$, if $\kappa \diamond \iota \in U(M_L; t_1), \kappa \in U(M_L; t_1), q \diamond p \in U(\tilde{B}_L; [\zeta_1, \zeta_2])$, $q \in U\ (\tilde{B}_L; [\zeta_1, \zeta_2])$, $n \diamond m \in L(U_L; t_2)$ and $n \in L(J_L; t_2)$, then

$$\begin{aligned}&M_L(\iota) \geq \min\left\{M_L\left(\kappa \diamond \iota\right), M_L(\kappa)\right\} \geq \min\{t_1, t_1\} = t_1 \\ &\tilde{B}_L(p) \succeq re\min\left\{\tilde{B}_L(q \diamond p), \tilde{B}_L(q)\right\} \succeq re\min\left\{[\zeta_1, \zeta_2], [\zeta_1, \zeta_2]\right\} = [\zeta_1, \zeta_2] \\ &J_L(m) \leq \max\left\{J_L\left(n \diamond m\right), J_L(n)\right\} \leq \min\{t_2, t_2\} = t_2\end{aligned}$$

and so $\iota \in U(M_L; t_1), p \in U(\tilde{B}_L; [\zeta_1, \zeta_2])$ and $m \in L(J_L; t_2)$. Therefore $U(M_L; t_1), U(\tilde{B}_L; [\zeta_1, \zeta_2])$ and $L(J_L; t_2)$ are ideals of P.

Conversely, assume that $U(M_L; t_1), U(\tilde{B}_L; [\zeta_1, \zeta_2])$ and $L(J_L; t_2)$ are ideals of P for all $t_1, t_2 \in [0, 1]$ and $[\zeta_1, \zeta_2] \in [I]$. Assume that $M_L(0) < M_L(p), \tilde{B}_L(0) \prec \tilde{B}_L(p)$ and $J_L(0) > J_L(p)$ for some $p \in P$. Then $0 \notin U(M_L; M_L(p)) \cap U(\tilde{B}_L; \tilde{B}_L(p)) \cap L(J_L; J_L(p))$, which is a contradiction. Hence $M_L(0) \geq M_L(\iota)$, $\tilde{B}_L(0) \succeq \tilde{B}_L(\iota)$ and $J_L(0) \leq J_L(\iota)$ for all $\iota \in P$. If

$$M_L(p_0) < \min\{M_L(q_0 \diamond p_0), M_L(q_0)\}$$

for some $p_0, q_0 \in P$, then $q_0 \diamond p_0 \in U(M_L; t_0)$ and $q_0 \in U(M_L; t_0)$ but $p_0 \notin U(M_L; t_0)$ for $t_0 := \min\{M_L(q_0 \diamond p_0), M_L(q_0)\} \implies$ a contradiction. Thus $M_L(p) \geq \min\{M(q \diamond p), M(q)\}$ for all $p, q \in P$. Similarly, we can show that $J_L(p) \leq \max\{J_L(q \diamond p), J_L(q)\}$ for all $p, q \in P$. Suppose that $\tilde{B}_L(p_0) \prec re\min\{\tilde{B}_L(q_0 \diamond p_0), \tilde{B}_L(q_0)\}$ for some $p_0, q_0 \in P$. Let $\tilde{B}_L(q_0 \diamond p_0) = [\rho_1, \rho_2], \tilde{B}_L(q_0) = [\rho_3, \rho_4]$ and $\tilde{B}_L(p_0) = [\zeta_1, \zeta_2]$. Then

$$[\zeta_1, \zeta_2] \prec re\min\{[\rho_1, \rho_2], [\rho_3, \rho_4]\} = [\min\{\rho_1, \rho_3\}, \min\{\rho_2, \rho_4\}]$$

and so $\zeta_1 < \min\{\rho_1, \rho_3\}$ and $\zeta_2 < \min\{\rho_2, \rho_4\}$. Taking

$$[\gamma_1, \gamma_2] := \frac{1}{2}\left(\tilde{B}_L(p_0) + re\min\left\{\tilde{B}_L(q_0 \diamond p_0), \tilde{B}_L(q_0)\right\}\right)$$

implies that

$$\begin{aligned}[\gamma_1, \gamma_2] &= \frac{1}{2}\left([\zeta_1, \zeta_2] + [\min\{\rho_1, \rho_3\}, \min\{\rho_2, \rho_4\}]\right) \\ &= \left[\frac{1}{2}(\zeta_1 + \min\{\rho_1, \rho_3\}), \frac{1}{2}(\zeta_2 + \min\{\rho_2, \rho_4\})\right].\end{aligned}$$

It follows that

$$\min\{\rho_1, \rho_3\} > \gamma_1 = \frac{1}{2}(\zeta_1 + \min\{\rho_1, \rho_3\}) > \zeta_1$$

and

$$\min\{\rho_2, \rho_4\} > \gamma_2 = \frac{1}{2}(\zeta_2 + \min\{\rho_2, \rho_4\}) > \zeta_2.$$

Hence $[\min\{\rho_1, \rho_3\}, \min\{\rho_2, \rho_4\}] \succ [\gamma_1, \gamma_2] \succ [\zeta_1, \zeta_2] = \tilde{B}_L(p_0)$, and therefore $p_0 \notin U(\tilde{B}_L; [\gamma_1, \gamma_2])$. On the other hand,

$$\tilde{B}_L(q_0 \diamond p_0) = [\rho_1, \rho_2] \succeq [\min\{\rho_1, \rho_3\}, \min\{\rho_2, \rho_4\}] \succ [\gamma_1, \gamma_2]$$

and

$$\tilde{B}_L(q_0) = [\rho_3, \rho_4] \succeq [\min\{\rho_1, \rho_3\}, \min\{\rho_2, \rho_4\}] \succ [\gamma_1, \gamma_2],$$

that is, $q_0 \diamond p_0, q_0 \in U(\tilde{B}_L; [\gamma_1, \gamma_2]) \implies$ a contradiction. Thus $\tilde{B}_L(\iota) \succeq re\min\{\tilde{B}_L(\kappa \diamond \iota), \tilde{B}_L(\kappa)\}$ for all $\iota, \kappa \in P$. Consequently $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ is an $MBJ\text{-}NI(P)$.

Theorem 3.4 Given an ideal H of a KU-algebra P, let $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ be an $MBJ\text{-}N$ set in P,

$$M_L(\iota) = \begin{cases} t_1 & \text{if } \iota \in H, \\ 0 & \text{otherwise} \end{cases}, \quad \tilde{B}_L(\iota) = \begin{cases} [\rho_1, \rho_2] & \text{if } \iota \in H, \\ [0, 0] & \text{otherwise} \end{cases}, \quad J_L(\iota) = \begin{cases} t_2 & \text{if } \iota \in H, \\ 1 & \text{otherwise} \end{cases} \tag{9}$$

where $t_1 \in (0, 1], t_2 \in [0, 1)$ and $\rho_1, \rho_2 \in (0, 1]$ with $\rho_1 < \rho_2$. Then $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ is an $MBJ\text{-}NI$ of $P \ni U(M_L; t_1) = U(\tilde{B}_L; [\rho_1, \rho_2]) = L(J_L; t_2) = H$.

Proof. Let $l_{01}, l_{02} \in P$. If $l_{02} \diamond l_{01} \in H$ and $l_{02} \in H$, then $l_{01} \in H$ and so

$$\begin{aligned} M_L(l_{01}) &= t_1 = \min\left\{M_L\left(l_{02} \diamond l_{01}\right), M_L(l_{02})\right\} \\ \tilde{B}_L(l_{01}) &= [\rho_1, \rho_2] = re\min\left\{[\rho_1, \rho_2], [\rho_1, \rho_2]\right\} = re\min\left\{\tilde{B}_L(l_{02} \diamond l_{01}), \tilde{B}_L(l_{02})\right\} \\ J_L(l_{01}) &= t_2 = \max\left\{J_L\left(l_{02} \diamond l_{01}\right), J_L(l_{02})\right\}. \end{aligned}$$

If any one of $l_{02} \diamond l_{01}$ and l_{02} is contained in H, say $l_{02} \diamond l_{01} \in H$, then $M_L\left(l_{02} \diamond l_{01}\right) = t_1, \tilde{B}_L\left(l_{02} \diamond l_{01}\right) = [\rho_1, \rho_2], J_L\left(l_{02} \diamond l_{01}\right) = t_2$, $M_L(l_{02}) = 0, \tilde{B}_L(l_{02}) = [0, 0]$ and $J_L(l_{02}) = 1$. Hence

$$\begin{aligned} M_L(l_{01}) &\geq 0 = \min\{t_1, 0\} = \min\left\{M_L\left(l_{02} \diamond l_{01}\right), M_L(l_{02})\right\} \\ \tilde{B}_L(l_{01}) &\succeq [0, 0] = re\min\left\{[\rho_1, \rho_2], [0, 0]\right\} = re\min\left\{\tilde{B}_L\left(l_{02} \diamond l_{01}\right), \tilde{B}_L(l_{02})\right\}, \\ J_L(l_{01}) &\leq 1 = \max\{t_2, 1\} = \max\left\{J_L\left(l_{02} \diamond l_{01}\right), J_L(l_{02})\right\}. \end{aligned}$$

If $l_{02} \diamond l_{01}, l_{02} \notin H$, then $M_L\left(l_{02} \diamond l_{01}\right) = 0 = M_L(l_{02}), \tilde{B}_L\left(l_{02} \diamond l_{01}\right) = [0, 0] = \tilde{B}_L(l_{02})$ and $J_L\left(l_{02} \diamond l_{01}\right) = 1 = J_L(l_{02})$. It follows that

$$\begin{aligned} M_L(l_{01}) &\geq 0 = \min\{0, 0\} = \min\left\{M_L\left(l_{02} \diamond l_{01}\right), M_L(l_{02})\right\}, \\ \tilde{B}_L(l_{01}) &\succeq [0, 0] = re\min\{[0, 0], [0, 0]\} = re\min\left\{\tilde{B}_L\left(l_{02} \diamond l_{01}\right), \tilde{B}_L(l_{02})\right\} \\ J_L(l_{01}) &\leq 1 = \max\{1, 1\} = \max\left\{J_L\left(l_{02} \diamond l_{01}\right), J_L(l_{02})\right\} \end{aligned}$$

It is obvious that $M_L(0) \geq M_L(l_{01}), \tilde{B}_L(0) \succeq \tilde{B}_L(l_{01})$ and $J_L(0) \leq J_L(l_{01})$ for all $l_{01} \in P$. Therefore $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ is an MBJ-$NI(P)$. Obviously, we have $U(M_L; t_1) = U(\tilde{B}_L; [\rho_1, \rho_2]) = L(J_L; t_2) = H$.

Theorem 3.5 For any non-empty subset H of P, let $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ be an MBJ-N set in P of (9). If $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ is an MBJ-NI of P, then H is an ideal of P.

Proof. Obviously, $0 \in H$. Let $l_{01}, l_{02} \in P \ni l_{02} \diamond l_{01} \in H$ & $l_{02} \in H$. Then $M_L(l_{02} \diamond l_{01}) = t_1 = M_L(l_{02})$, $\tilde{B}_L(l_{02} \diamond l_{01}) = [\gamma_1, \gamma_2] = \tilde{B}_L(l_{02})$ and $J_L(l_{02} \diamond l_{01}) = t_2 = J_L(l_{02})$. Thus

$$M_L(l_{01}) \geq \min\{M_L(l_{02} \diamond l_{01}), M_L(l_{02})\} = t_1,$$

$$\tilde{B}_L(l_{01}) \succeq re\min\{\tilde{B}_L(l_{02} \diamond l_{01}), \tilde{B}_L(l_{02})\} = [\gamma_1, \gamma_2],$$
$$J_L(l_{01}) \leq \max\{J_L(l_{02} \diamond l_{01}), J_L(l_{02})\} = t_2,$$

and hence $l_{01} \in H$. Therefore H is an ideal of P.

Theorem 3.6 In a KU-algebra, every MBJ-NI is an MBJ-NSA.

Proof. Let $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ be an MBJ-NI of a KU-algebra P. Since $l_{01} \diamond (l_{01} \diamond l_{02}) \leq l_{02}$ for all $l_{01}, l_{02} \in P$, it follows from Proposition 3.1 that

$$M_L(l_{01} \diamond l_{02}) \geq \min\{M_L(l_{01}), M_L(l_{02})\},$$

$$\tilde{B}_L(l_{01} \diamond l_{02}) \succeq re\min\{\tilde{B}_L(l_{01}), \tilde{B}_L(l_{02})\},$$
$$J_L(l_{01} \diamond l_{02}) \leq \max\{J_L(l_{01}), J_L(l_{02})\}$$

for all $l_{01}, l_{02} \in P$. Hence $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ is an MBJ-NSA of a KU-algebra P.

The converse of Theorem 3.6 may not be true as seen in the following example.

Example 3.2 Consider a KU-algebra set $P = \{0_5, a_5, b_5, c_5\}$ with the binary operation $\diamond$. Table 3 defined the following operation. Then

Table 3:

$\diamond$	0_5	a_5	b_5	c_5
0_5	0_5	a_5	b_5	c_5
a_5	0_5	0_5	a_5	c_5
b_5	0_5	0_5	0_5	c_5
c_5	0_5	a_5	b_5	0_5

Table 4:

P	$M_L(l)$	$\tilde{B}_L(l)$	$J_L(l)$
0_5	0.7	$[0.3, 0.8]$	0.2
a_5	0.3	$[0.1, 0.5]$	0.6
b_5	0.1	$[0.3, 0.8]$	0.4
c_5	0.5	$[0.1, 0.5]$	0.7

Let $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ be an MBJ-N set in P as in Table 4. Then it is an MBJ-$NSA(P)$ but it is not an MBJ-$NI(P)$ since

$$\tilde{B}_L(a_5) \not\succeq re\min\{\tilde{B}_L(b_5 \diamond a_5), \tilde{B}_L(b_5)\}.$$

Theorem 3.7 Let $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ be an MBJ-NSA of a KU-algebra P satisfies (6). Then $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ is an MBJ-NI of P.

Proof. For any $l_{01} \in P$, we get

$$\begin{aligned} M_L(0) =& M_L(l_{01} \diamond l_{01}) \geq \min\{M_L(l_{01}), M_L(l_{01})\} = M_L(l_{01}) \\ \tilde{B}_L(0) =& \tilde{B}_L(l_{01} \diamond l_{01}) \succeq re\min\left\{\tilde{B}_L(l_{01}), \tilde{B}_L(l_{01})\right\} \\ =& re\min\left\{\left[B_L^-(l_{01}), B_L^+(l_{01})\right], \left[B_L^-(l_{01}), B_L^+(l_{01})\right]\right\} \\ =& \left[B_L^-(l_{01}), B_L^+(l_{01})\right] = \tilde{B}_L(l_{01}), \end{aligned}$$

and

$$J_L(0) = J_L(l_{01} \diamond l_{01}) \leq \max\{J_L(l_{01}), J_L(l_{01})\} = J_L(l_{01})$$

Since $(l_{02} \diamond l_{01}) \diamond l_{01} \leq l_{02}$ for all $l_{01}, l_{02} \in P$, it follows from (6) that

$$\begin{aligned} & M_L(l_{01}) \geq \min\{M_L(l_{02} \diamond l_{01}), M_L(l_{02})\} \\ & \tilde{B}_L(l_{01}) \succeq re\min\left\{\tilde{B}_L(l_{02} \diamond l_{01}), \tilde{B}_L(l_{02})\right\} \\ & J_L(l_{01}) \leq \max\{J_L(l_{02} \diamond l_{01}), J_L(l_{02})\} \end{aligned}$$

for all $l_{01}, l_{02} \in P$. Therefore $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ is an MBJ-NI of P.

Definition 3.2 An MBJ-NI $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ of a KU-algebra P is said to be closed if

$$(\forall\, l_{01} \in P)(M_L(l_{01} \diamond 0) \geq M_L(l_{01}), \tilde{B}_L(l_{01} \diamond 0) \succeq \tilde{B}_L(l_{01}), J_L(l_{01} \diamond 0) \leq J_L(l_{01})). \quad (10)$$

Theorem 3.8 In a KU-algebra, every closed MBJ-NI is an MBJ-NSA.

Proof. Let $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ be a closed MBJ-NI of a KU-algebra P. Using (KU5), (4), (5) and (2.1), we have

$$M_L(l_{01} \diamond l_{02}) \geq \min\{M_L(l_{02} \diamond (l_{01} \diamond l_{02})), M_L(l_{02})\} = \min\{M_L(l_{01} \diamond 0), M_L(l_{02})\}$$
$$\geq \min\{M_L(l_{01}), M_L(l_{02})\},$$
$$\tilde{B}_L(l_{01} \diamond l_{02}) \succeq re\min\{\tilde{B}_L(l_{02} \diamond (l_{01} \diamond l_{02})), \tilde{B}_L(l_{02})\} = re\min\{\tilde{B}_L(l_{01} \diamond 0), \tilde{B}_L(l_{02})\}$$
$$\succeq re\min\{\tilde{B}_L(l_{01}), \tilde{B}_L(l_{02})\},$$

and

$$J_L(l_{01} \diamond l_{02}) \leq \max\{J_L(l_{02} \diamond (l_{01} \diamond l_{02})), J_L(l_{02})\} = \max\{J_L(l_{01} \diamond 0), J_L(l_{02})\} \leq \max\{J_L(l_{01}), J_L(l_{02})\}$$

for all $l_{01}, l_{02} \in P$. Hence $\mathcal{L} = (M_L, \tilde{B}_L, J_L)$ is an MBJ-$NSA(P)$.

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