Malaya Journal of Matematik, Vol. S, No. 1, 630-642, 2021

https://doi.org/10.26637/MJMS2101/0144

Operations on bipolar interval valued neutrosophic graphs

L. Jagadeeswari[1*] and V. J. Sudhakar[2]

Abstract

In this article, we combine the concept of bipolar neutrosophic set with graph theory, we introduced the bipolar interval valued neutrosophic graphs ,discussed some operations on interval valued neutrosophic graphs and investigate some of their properties with proofs and examples.

Keywords

Bipolar, neutrosophic graphs, interval valued neutrosophic graphs.

AMS Subject Classification

03E72, 05C72, 05C78, 05C99.

[1,2] *Department of Mathematics, Islamiah College (Autonomous), Vaniyambadi–635752, Tamil Nadu, India.*

***Corresponding author**: [1]jaga25maths@gmail.com

Article History: Received **09** January **2021**; Accepted **19** March **2021**

Contents

1. Introduction

Neutosophic sets proposed by smarandache [11,12] is a powerful mathematical tool for dealing with incomplete, indeterminate and inconsistent information in real world. They are a generalization of the theory of fuzzy sets, intuitionistic fuzzy set, interval valued fuzzy set and interval valued intuitionistic fuzzy sets [8]. The neutrosophic sets are characterized by a truth membership function (T) an indeterminacy membership function (I) and a falsity membership function (F) independently, which are within the real standard or nonstandard unit interval $]^{-}0,1^{+}[$. In order to practice NS in real life applications conveniently, Wang et al.[5] introduced the concept of a single valued neutrosophic sets (SVNS), a subclass of the neutrosophic sets. The same author introduced the concept of interval valued neutrosophic sets, which is more precise and flexible than single valued neutrosophic sets. The IVNS is a generalization of single valued neutrosophic sets, in which three membership functions are independent and their value belong to the unit interval $[0,1]$ some more work on single valued neutrosophic sets, interval valued neutrosophic sets and their application may be found on [1,2,4-7,13-17,19-21].

Graph theory has now become a major branch of applied mathematics and it is generally regarded as a branch of combinatorics. Graph is a widely used tool for solving combinational problems in different areas such as geometry, algebra, number theory, topology, and optimization and computer science [9,10]. Most important thing which is to be noted that, when we have uncertainty regarding either the set of vertices or edges or both, the model becomes a fuzzy graph. The extension of fuzzy [3] graph theory have been developed by several researches including intuitionistic fuzzy graphs considered the vertex sets and edge sets as intuitionistic fuzzy sets. Interval value fuzzy graphs considered the vertex sets and edge sets as interval valued intuitionistic fuzzy sets. Bipolar fuzzy graph considered the vertex set and edge sets as bipolar fuzzy sets. M-polar fuzzy graph considered the vertex sets and edge sets as m-polar fuzzy sets. But, when the relations between nodes (or vertices) in problems are indeterminate, the fuzzy graph and their extensions are failed. For this purpose, samarandache have defined four main categories of neutrosophic graphs, two based on literal indeterminancy, which called them; I edge neutrosophic graph and I-vertex neutrosophic graph, these concept ate studied deeply and has gained popularity among the researches due to its applications via real world problems the two other graph based on (T,I,F) components and called them; the (T,I,F)-edge neutrosophic graph and the (T,I,F)-vertex neutrosophic graph, these concept are not developed at all. Later on, Broumi et al. [18] introduced a third neutrosophic graph model this model allow the attachment of truth-membership (T), indeterminacy-membership (I) and falsity-membership degrees (F) both to vertices and edges and investigated some of their properties. The third neutrosophic graph model is called single valued neutrosophic graph (SVNG) the single valued neutrosophic graph model is called single valued graph and intuitionistic fuzzy graph. Also the same authors introduced neighborhood degree of a vertex and closed neighborhood degree of vertex in single valued neutrosophic graph as a generalization of neighborhood degree of a vertex and closed neighborhood degree of vertex in fuzzy graph and intuitionistic fuzzy graph in the literature the study of interval valued neutrosophic graph is still blank we shall focus on the study of interval valued neutrosophic graphs.Then, Sudhakar et al.[22-26] introduced the concept of interval valued signed neutrosophic graph and self-centered interval valued signed neutrosophic graph.In this paper some operations on bipolar interval valued neutrosophic graphs are discussed with examples.

2. Preliminaries

In this section, we recall some notation, related to neutrosophic sets, single valued neutrosophic sets, interval valued neutrosophic sets, single valued neutrosophic graphs and interval valued neutrosophic graphs related to this work.

Definition 2.1 ((Smarandache 2006)). *Let X be a space of points with generic elements in X denoted by x; then the neutrosophic set A (NSA) is an object having the form*

$$A = \{< x : T_A(x), I_A(x), F_A(x) > x \in X\},$$

where the functions $T.I, F : X \to]^{-}0, 1^{+}[$ define respectively a truth membership function, an indeterminacy - membership function, and a falsity membership function of the element $x \in X$ to the set A with the condition:

$$^{-}0 \leq T_A(x)\,|I_A(x)|\,F_A(x) \leq 3^{+}$$

The functions $T_A(x), I_A(x)$ and $F_A(x)$ are real Standard or non standard subsets of $]^{-}0.1^{+}[$.

Definition 2.2 ((Wang et al 2010a)). *Let X be a space of points with generic elements in X denoted by x. As single valued neutrosophic set A is characterized by truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$, For each point x in X, $T_A(x), I_A(x), F_A(x) \in [0,1]$ SVNS can be written as,*

$$A = \{< x, T_A(x), I_A(x), F_A(x) >, x \in X\}.$$

Definition 2.3 ((Wang et al. 2005a)). *Let X be a space of points with generic elements in X denoted by x. An interval valued neutrosophic set (IVNS) A in X is characterized by truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$ and falsity-membership function $F_A(x)$. For each point x in X,*

$$T_A(x) = [T_{AL}(x), T_{AU}(x)]$$
$$I_A(x) = [I_{AL}(x), I_{AU}(x)]$$
$$F_A(x) = [F_{AL}(x), F_{AU}(x)] \subseteq [0,1], \text{ and}$$
$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$$

Definition 2.4 ((Wang et al 2005 a)). *An IVNS A is contained in the IVNS B, $A \subseteq B$, if and only if*

$$T_{AL}(x) \leq T_{BL}(x), T_{AU}(x) \leq T_{BU}(x)$$
$$I_{AL}(x) \geq I_{BL}(x), I_{AU}(x) \geq I_{BU}(x)$$
$$F_{AL}(x) \geq F_{BL}(x), F_{AU}(x) \geq F_{BU}(x), \text{ for any } x \text{ in } X$$

Definition 2.5 ((Wang et al 2005 a)). *The union of two interval valued neutrosophic sets A and B is an interval neutrosophic set C, written as $C = A \cup B$, whose truth-membership, indeterminacy-membership and false membership are related to those A and B by*

$$\begin{aligned}
T_{CL}(x) &= \max(T_{AL}(x), T_{BL}(x)) \\
T_{CU}(x) &= \max(T_{AU}(x), T_{BU}(x)) \\
I_{CL}(x) &= \min(I_{AL}(x), I_{BL}(x)) \\
I_{CU}(x) &= \min(I_{AU}(x), I_{BU}(x)) \\
F_{CL}(x) &= \min(F_{AL}(x), F_{BL}(x)) \\
F_{CU}(x) &= \min(F_{AU}(x), F_{BU}(x)), \text{ for all } x \text{ in } X.
\end{aligned}$$

Definition 2.6 ((Wang et al 2005 a)). *Let X and Y be two non-empty crisp sets. An interval valued neutrosophic relation $R(x,y)$ is a subset of product space $X \times Y$, and is characterized by the truth membership function $T_R(x,y)$, the indeterminacy membership function $I_R(x,y)$, and the falsity membership $F_R(x,y)$, where $x \in X$ and $y \in Y$ and $T_R(x,y), I_R(x,y), F_R(x,y) \subseteq [0,1]$.*

Definition 2.7 ((Mishra and pal 2013)). *An interval valued intuitionistic fuzzy graph (IVIFG) $G = (A,B)$ satisfies the following conditions:*

1. *$V = \{V_1 V_2 \ldots V_n\}$ such that $M_{AL} : V \to [0,1]$, $M_{AU} : V \to [0,1]$ and $N_{AL} : V \to [0,1]$,*
   $N_{AU} : V \to [0,1]$, denote the degree of membership and non-membership of the element $y \in V$, respectively, and

   $$0 \leq M_A(x) + N_A(x) \leq 1$$

   for every $x \in V$.

2. *The functions $M_{BL} : V \times V \to [0,1]$, $M_{BU} : V \times V \to [0,1]$, and $N_{BL} : V \times V \to [0,1]$, $N_{BU} : V \times V \to [0,1]$, are denoted by*

   $$\begin{aligned}
   M_{BL}(xy) &\leq \min[M_{AL}(x), M_{AL}(y)], \\
   M_{BU}(xy) &\leq \min[M_{AU}(x), M_{AU}(y)], \\
   N_{BL}(xy) &\geq \max[N_{BL}(x), N_{BL}(y)], \\
   N_{BU}(xy) &\geq \max[N_{BU}(x), N_{BU}(y)],
   \end{aligned}$$

   such that $0 \leq M_B(xy) + N_B(xy) \leq 1$ for every $xy \in B$.

Definition 2.8 ((Broumi et al 2016 b)). *A single valued neutrosophic graph (SVN-graph) with underlying set V is defined to be a pair $G = (A, B)$ where;*

1. *The functions $T_A : V \to [0,1], I_A : V \to [0,1]$ and $F_A : V \to [0,1]$ denote the degree of truth-membership, degree of indeterminacy membership and falsity-membership of the and element $v_i \in V$, respectively, and*

   $$0 \le T_A(v_i) + I_A(v_i) + F_A(v_i) \le 3. \forall v_i \in V\,(i = 1, 2 \dots n)$$

2. *The functions $T_B : E \subseteq V \times V \to [0,1]$ $I_B : E \subseteq V \times V \to [0,1]$ and $F_B : E \subseteq V \times V \to [0,1]$ are defined by*

   $$\begin{aligned} T_B(\{v_i, v_j\}) &\le \min[T_A(v_i), T_A(v_j)] \\ I_B(\{v_i, v_j\}) &\ge \max[I_A(v_i), I_A(v_j)] \\ F_B(\{v_i, v_j\}) &\ge \max[F_A(v_i), F_A(v_j)] \end{aligned}$$

   and denote the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_i, v_j) \in E$ respectively, where

   $$0 \le T_B(\{v_i, v_j\}) + I_B(\{v_i, v_j\}) + F_B(\{v_i, v_j\}) \le 3$$

   for all $\{v_i, v_j\} \in E\,(i, j = 1, 2, \dots, n)$.

Definition 2.9 ((Broumi et al 2016 b)). *Let $G = (A, B)$ be a single valued neutrosophic graph. Then the degree of vertex V is defined by $d(V) = (d_T(v), d_I(v), d_F(v))$ where*

$$d_T(v) = \sum_{u \ne v} T_B(u, v), d_I(v) = \sum_{u \ne v} I_B(u, v) \text{ and } d_F(v) = \sum_{u \ne v} F_B(u, v)$$

Definition 2.10 ((Broumi et al 2016 b)). *A single valued neutrosophic graph $G = (A, B)$ and G^* is called strong neutrosophic graph*

$$\begin{aligned} T_B(v_i, v_j) &= \min[T_A(v_i), T_A(v_j)] \\ I_B(v_i, v_j) &= \max[I_A(v_i), I_A(v_j)] \\ F_B(v_i, v_j) &= \max[F_A(v_i), F_A(v_j)]\ \forall (v_i, v_j) \in E \end{aligned}$$

Definition 2.11 ((Broumi et al 2016 b)). *The complement of a strong single valued neutrosophic graph G on G^* is strong single Valued neutrosophic graph $\bar{G}$ on G^* where*

1. $\bar{V} = V$
2. $\overline{T_A}(v_i) = T_A(v_i); \overline{I_A}(v_i) = I_A(v_i), \overline{F_A}(v_i) = F_A(v_i)$
3. $\overline{T_B}(v_i, v_j) = \min[T_A(v_i), T_A(v_j)] - T_B(v_i, v_j)$
   $\overline{I_B}(v_i, v_j) = \max[I_A(v_i), I_A(v_j)] - I_B(v_i, v_j)$
   and $\overline{F_B}(v_i, v_j) = \max[F_A(v_i), F_A(v_j)] - F_B(v_i, v_j)\ \forall (v_i, v_j) \in E$

Definition 2.12 ((Broumi et al 2016 b)). *A single valued neutrosophic graph $G = (A, B)$ is a single valued called complete, if*

$$T_B(v_i, v_j) = \min(T_A(v_i), T_A(v_j))$$
$$I_B(v_i, v_j) = \max(I_A(v_i), I_A(v_j))$$
$$\text{and } F_B(v_i, v_j) = \max(F_A(v_i), F_A(v_j)) \quad \forall v_i v_j \in V$$

3. Operations on Bipolar Interval Valued Neutrosophic Graphs

In this section $G_B = (V, E)$ denotes a crisp graph and G-an interval valued neutrosophic graph.

Definition 3.1. *By Bipolar interval valued neutrosophic graph $G_B(A, B)$ where*

$$A = <\left[T^P_{AL} T^P_{AU}\right]\left[I^P_{AL} I^P_{AU}\right]\left[F^P_{AL} F^P_{AU}\right]\left[T^N_{AL} T^N_{AU}\right]\left[I^N_{AL} I^N_{AU}\right]\left[F^N_{AL} F^N_{AU}\right]>$$

is an Bipolar interval valued neutrosophic set on V and

$$B = <\left[T^P_{BL} T^P_{BU}\right]\left[I^P_{BL} I^P_{BU}\right]\left[F^P_{BL} F^P_{BU}\right]\left[T^N_{BL} T^N_{BU}\right]\left[I^N_{BL} I^N_{BU}\right]\left[F^N_{BL} F^N_{BU}\right]>$$

is an Bipolar interval valued neutrosophic relation on E.
Satisfying the following conditions:

1. *$V = \{v_1 v_2 \dots v_n\}$ such that $T^P_{AL} : V \to [0,1]$, $T^P_{AU} : V \to [0,1]$, $I^P_{AL} : V \to [0,1]$, $I^P_{AU} : V \to [0,1]$, $F^P_{AL} : V \to [0,1]$, $F^P_{AU} : V \to [0,1]$, $T^N_{AL} : V \to [-1,0]$, $T^N_{AU} : V \to [-1,0]$, $I^N_{AL} : V \to [-1,0]$, $I^N_{AU} : V \to [-1,0]$, $F^N_{AL} : V \to [-1,0]$, $F^N_{AU} : V \to [-1,0]$ denote the degree of truth membership, the degree of indeterminacy-membership and falsity-membership of the element $v \in V$, respectively, and*

   $$0 \le T^P_A(v_i) + I^P_A(v_i) + F^P_A(v_i) \le 3$$
   $$-3 \le T^N_A(v_i) + I^N_A(v_i) + F^N_A(v_i) \le 0\ \forall v_i \in V.$$

2. *The functions $T^P_{BL} : V \times V \to [0,1] T^P_{BU} : V \times V \to [0,1] I^P_{BL} : V \times V \to [0,1] I^P_{BU} : V \times V \to [0,1] F^P_{BL} : V \times V \to [0,1] F^P_{BU} : V \times V \to [0,1]$,*
   $T^N_{BL} : V \times V \to [-1,0] T^N_{BU} : V \times V \to [-1,0] I^N_{BL} : V \times V \to [-1,0] I^N_{BU} : V \times V \to [-1,0] F^N_{BL} : V \times V \to [-1,0] F^N_{BU} : V \times V \to [-1,0]$ such that

   $$\begin{aligned} T^P_{BL}(v_i, v_j) &\le \min[T^P_{AL}(v_i), T^P_{AL}(v_j)] \\ T^P_{BU}(v_i, v_j) &\le \min[T^P_{AU}(v_i), T^P_{AU}(v_j)] \\ I^P_{BL}(v_i, v_j) &\ge \max[I^P_{AL}(v_i), I^P_{AL}(v_j)] \\ I^P_{BU}(v_i, v_j) &\ge \max[I^P_{AU}(v_i), I^P_{AU}(v_j)] \\ F^P_{BL}(v_i, v_j) &\ge \max[F^P_{AL}(v_i), F^P_{AL}(v_j)] \\ F^P_{BU}(v_i, v_j) &\ge \max[F^P_{AU}(v_i), F^P_{AU}(v_j)] \end{aligned}$$

   $$\begin{aligned} T^N_{BL}(v_i, v_j) &\ge \max[T^N_{AL}(v_i), T^N_{AU}(v_j)] \\ T^N_{BU}(v_i, v_j) &\ge \max[T^N_{AL}(v_i), T^N_{AU}(v_j)] \\ I^N_{BL}(v_i, v_j) &\le \min[I^N_{AL}(v_i), I^N_{AU}(v_j)] \\ I^N_{BU}(v_i, v_j) &\le \min[I^N_{AL}(v_i), I^N_{AU}(v_j)] \\ F^N_{BL}(v_i, v_j) &\le \min[F^N_{AL}(v_i), F^N_{AU}(v_j)] \\ F^N_{BU}(v_i, v_j) &\le \min[F^N_{AL}(v_i), F^N_{AU}(v_j)] \end{aligned}$$

   denote the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_i, v_j) \in E$ respectively, where

   $$0 \le T^P_B(v_i, v_j) + I^P_B(v_i, v_j) + F^P_B(v_i, v_j) \le 3$$
   $$-3 \le T^N_B(v_i, v_j) + I^N_B(v_i, v_j) + F^N_B(v_i, v_j) \le 0\ \forall (v_i, v_j) \in E.$$

Definition 3.2. *Let $G = G_1 \times G_2 = (V,E)$ the cartesian product of two graphs where $V = V_1 \times V_2$ and $E = \{(x,x_2)(x,y_2)\,/\,x \in V_1,\, x_2y_2 \in E_2\} \cup \{(x_1,z)(y_1,z)\,/\,z \in V_2,\, x_1,y_1 \in E_1\}$ then the cartesian product $G_B = G_{B_1} \times G_{B_2} = (A_1 \times A_2 B_1 \times B_2)$ is an Bipolar interval valued neutrosophic graph defined by*
1.

$$\begin{aligned}
\left(T^P_{A_1L} \times T^P_{A_2L}\right)(x,x_2) &= \min\left(T^P_{A_1L}(x), T^P_{A_2L}(x_2)\right)\\
\left(T^P_{A_1U} \times T^P_{A_2U}\right)(x,x_2) &= \min\left(T^P_{A_1U}(x), T^P_{A_2U}(x_2)\right)\\
\left(I^P_{A_1L} \times I^P_{A_2L}\right)(x,x_2) &= \max\left(I^P_{A_1L}(x), I^P_{A_2L}(x_2)\right)\\
\left(I^P_{A_1U} \times I^P_{A_2U}\right)(x,x_2) &= \max\left(I^P_{A_1U}(x), I^P_{A_2U}(x_2)\right)\\
\left(F^P_{A_1L} \times F^P_{A_2L}\right)(x,x_2) &= \max\left(F^P_{A_1L}(x), F^P_{A_2L}(x_2)\right)\\
\left(F^P_{A_1U} \times F^P_{A_2U}\right)(x,x_2) &= \max\left(F^P_{A_1U}(x), F^P_{A_2U}(x_2)\right)
\end{aligned}$$

$$\begin{aligned}
\left(T^N_{A_1L} \times T^N_{A_2L}\right)(x,x_2) &= \max\left(T^N_{A_1L}(x), T^N_{A_2L}(x_2)\right)\\
\left(T^N_{A_1U} \times T^N_{A_2U}\right)(x,x_2) &= \max\left(T^N_{A_1U}(x), T^N_{A_2U}(x_2)\right)\\
\left(I^N_{A_1L} \times I^N_{A_2L}\right)(x,x_2) &= \min\left(I^N_{A_1L}(x), I^N_{A_2L}(x_2)\right)\\
\left(I^N_{A_1U} \times I^N_{A_2U}\right)(x,x_2) &= \min\left(I^N_{A_1U}(x), I^N_{A_2U}(x_2)\right)\\
\left(F^N_{A_1L} \times F^N_{A_2L}\right)(x,x_2) &= \min\left(F^N_{A_1L}(x), F^N_{A_2L}(x_2)\right)\\
\left(F^N_{A_1U} \times F^N_{A_2U}\right)(x,x_2) &= \min\left(F^N_{A_1U}(x), F^N_{A_2U}(x_2)\right)
\end{aligned}$$

for all $(x,x_2) \in V$.
2.

$$\begin{aligned}
\left(T^P_{B_1L} \times T^P_{B_2L}\right)((x,x_2)(x,y_2)) &= \min\left(T^P_{A_1L}(x), T^P_{B_2L}(x_2y_2)\right)\\
\left(T^P_{B_1U} \times T^P_{B_2U}\right)((x,x_2)(x,y_2)) &= \min\left(T^P_{A_1U}(x), T^P_{B_2U}(x_2y_2)\right)\\
\left(I^P_{B_1L} \times I^P_{B_2L}\right)((x,x_2)(x,y_2)) &= \max\left(I^P_{A_1L}(x), I^P_{B_2L}(x_2y_2)\right)\\
\left(I^P_{B_1U} \times I^P_{B_2U}\right)((x,x_2)(x,y_2)) &= \max\left(I^P_{A_1U}(x), I^P_{B_2U}(x_2y_2)\right)\\
\left(F^P_{B_1L} \times F^P_{B_2L}\right)((x,x_2)(x,y_2)) &= \max\left(F^P_{A_1L}(x), F^P_{B_2L}(x_2y_2)\right)\\
\left(F^P_{B_1U} \times F^P_{B_2U}\right)((x,x_2)(x,y_2)) &= \max\left(F^P_{A_1U}(x), F^P_{B_2U}(x_2y_2)\right)
\end{aligned}$$

$\forall x \in V_1, \forall x_2y_2 \in E_2$.

$$\begin{aligned}
\left(T^N_{B_1L} \times T^N_{B_2L}\right)((x,x_2)(x,y_2)) &= \max\left(T^N_{A_1L}(x), T^N_{B_2L}(x_2y_2)\right)\\
\left(T^N_{B_1U} \times T^N_{B_2U}\right)((x,x_2)(x,y_2)) &= \max\left(T^N_{A_1U}(x), T^N_{B_2U}(x_2y_2)\right)\\
\left(I^N_{B_1L} \times I^N_{B_2L}\right)((x,x_2)(x,y_2)) &= \min\left(I^N_{A_1L}(x), I^N_{B_2L}(x_2y_2)\right)\\
\left(I^N_{B_1U} \times I^N_{B_2U}\right)((x,x_2)(x,y_2)) &= \min\left(I^N_{A_1U}(x), I^N_{B_2U}(x_2y_2)\right)\\
\left(F^N_{B_1L} \times F^N_{B_2L}\right)((x,x_2)(x,y_2)) &= \min\left(F^N_{A_1L}(x), F^N_{B_2L}(x_2y_2)\right)\\
\left(F^N_{B_1U} \times F^N_{B_2U}\right)((x,x_2)(x,y_2)) &= \min\left(F^N_{A_1U}(x), F^N_{B_2U}(x_2y_2)\right)
\end{aligned}$$

3.

$$\begin{aligned}
\left(T^P_{B_1L} \times T^P_{B_2L}\right)((x_1,z)(y_1,z)) &= \min\left(T^P_{B_1L}(x_1y_1), T^P_{A_2L}(z)\right)\\
\left(T^P_{B_1U} \times T^P_{B_2U}\right)((x_1,z)(y_1,z)) &= \min\left(T^P_{B_1U}(x_1y_1), T^P_{A_2U}(z)\right)\\
\left(I^P_{B_1L} \times I^P_{B_2L}\right)((x_1,z)(y_1,z)) &= \max\left(I^P_{B_1L}(x_1y_1), I^P_{A_2L}(z)\right)\\
\left(I^P_{B_1U} \times I^P_{B_2U}\right)((x_1,z)(y_1,z)) &= \max\left(I^P_{B_1U}(x_1y_1), I^P_{A_2U}(z)\right)\\
\left(F^P_{B_1L} \times F^P_{B_2L}\right)((x_1,z)(y_1,z)) &= \max\left(F^P_{B_1L}(x_1y_1), F^P_{A_2L}(z)\right)\\
\left(F^P_{B_1U} \times F^P_{B_2U}\right)((x_1,z)(y_1,z)) &= \max\left(F^P_{B_1U}(x_1y_1), F^P_{A_2U}(z)\right)
\end{aligned}$$

$$\begin{aligned}
\left(T^N_{B_1L} \times T^N_{B_2L}\right)((x_1,z)(y_1,z)) &= \min\left(T^N_{B_1L}(x_1y_1), T^N_{A_2L}(z)\right)\\
\left(T^N_{B_1U} \times T^N_{B_2U}\right)((x_1,z)(y_1,z)) &= \min\left(T^N_{B_1U}(x_1y_1), T^N_{A_2U}(z)\right)\\
\left(I^N_{B_1L} \times I^N_{B_2L}\right)((x_1,z)(y_1,z)) &= \max\left(I^N_{B_1L}(x_1y_1), I^N_{A_2L}(z)\right)\\
\left(I^N_{B_1U} \times I^N_{B_2U}\right)((x_1,z)(y_1,z)) &= \max\left(I^N_{B_1U}(x_1y_1), I^N_{A_2U}(z)\right)\\
\left(F^N_{B_1L} \times F^N_{B_2L}\right)((x_1,z)(y_1,z)) &= \max\left(F^N_{B_1L}(x_1y_1), F^N_{A_2L}(z)\right)\\
\left(F^N_{B_1U} \times F^N_{B_2U}\right)((x_1,z)(y_1,z)) &= \max\left(F^N_{B_1U}(x_1y_1), F^N_{A_2U}(z)\right)
\end{aligned}$$

$\forall Z \in V_2, \forall x_1y_1 \in E_1$.

Example 3.3. *Let $G_{B_1} = (A_1B_1)$ and $G_{B_2} = (A_2B_2)$ be two graphs where $V_1 = \{w,x\}$ $V_2 = \{y,z\}$ $E_1 = \{w,x\}$ and $E_2 = \{y,z\}$.*

Consider two Bipolar interval valued neutrosophic graphs.

$$\begin{aligned}
A^P_1 &= \left\{\begin{array}{l} < w\,[0.5,0.7]\,[0.2,0.3]\,[0.2,0.4] > \\ < x\,[0.6,0.7]\,[0.2,0.4]\,[0.1,0.3] > \end{array}\right\}\\
B^p_1 &= \left\{ < wx\,[0,4,0.6]\,[0.2,0.4]\,[0.2,0.4] > \right\}\\
A^P_2 &= \left\{\begin{array}{l} < y\,[0.4,0.6]\,[0.2,0.3]\,[0.1,0.3] > \\ < z\,[0.4,0.7]\,[0.2,0.4]\,[0.1,0.3] > \end{array}\right\}\\
B^P_2 &= \left\{ < yz\,[0.3,0.6]\,[0.2,0.4]\,[0.2,0.5] > \right\}
\end{aligned}$$

$$\begin{aligned}
A^N_1 &= \left\{\begin{array}{l} < w\,[-0.3,-0.2]\,[-0.5,-0.4]\,[-0.3,-0.1] > \\ < x\,[-0.4,-0.2]\,[-0.3,-0.2]\,[-0.2,-0.1] > \end{array}\right\}\\
B^p_1 &= \left\{ < wx\,[-0.3,-0.2]\,[-0.5,-0.4]\,[-0.3,-0.1] > \right\}\\
A^N_2 &= \left\{\begin{array}{l} < y\,[-0.4,-0.2]\,[-0.4,-0.1]\,[-0.2,-0.1] > \\ < z\,[-0.3,-0.1]\,[-0.2,-0.1]\,[-0.3,-0.1] > \end{array}\right\}\\
B^N_2 &= \left\{ < yz\,[-0.3,-0.1]\,[-0.4,-0.1]\,[-0.3,-0.1] > \right\}
\end{aligned}$$

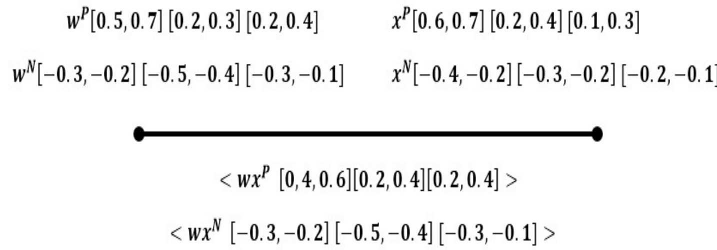

Figure 1. Bipolar interval valued neutrosophic graphs

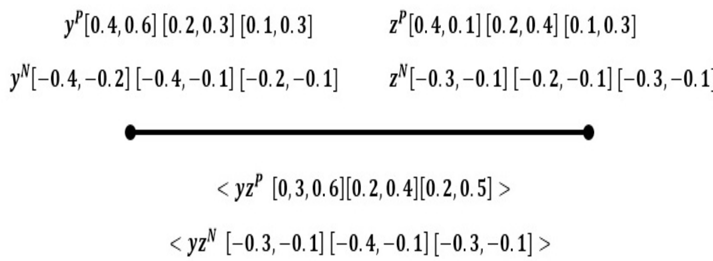

Figure 2. Bipolar interval valued neutrosophic graphs

Proposition 3.4. *The cartesian product $G_{B_1} \times G_{B_2} = (A_1 \times A_2, B_1 \times B_2)$ of two Bipolar interval valued neutrosophic graphs of the graphs $G^*_{B_1}$ and $G^*_{B_2}$ is an interval valued neutrosophic graph of $G^*_{B_1} \times G^*_{B_2}$.*

Proof. Let $E = \{(x,x_2)(x,y_2) \,/\, x \in V,\, x_2y_2 \in E_2\} \cup \{(x,z)(y,z) \,/\, z \in V_2,\, x_1y_1 \in E_1\}$
Let $(x,x_2)(x,y_2) \subset E$

$$\left(T^P_{B_1L} \times T^P_{B_2L}\right)((x,x_2)(x,y_2))$$
$$= \min\left(T^P_{A_1L}(x), T^P_{B_2L}(x_2y_2)\right)$$
$$\leq \min\left(T^P_{A_1L}(x), \min\left(T^P_{A_2L}(x_2), T^P_{A_2L}(y_2)\right)\right)$$
$$= \min\left(\min\left(T^P_{A_1L}(x), T^P_{A_2L}(x_2)\right), \min\left(T^P_{A_1L}(x), T^P_{A_2L}(y_2)\right)\right)$$
$$= \min\left(\left(T^P_{A_1L} \times T^P_{A_2L}\right)(x,x_2), \left(T^P_{A_1L} \times T^P_{A_2L}\right)(x,y_2)\right)$$
$$\left(T^P_{B_1U} \times T^P_{B_2U}\right)((x,x_2)(x,y_2))$$
$$= \min\left(T^P_{A_1U}(x), T^P_{B_2U}(x_2y_2)\right)$$
$$\leq \min\left(T^P_{A_1U}(x), \min\left(T^P_{A_2U}(x_2), T^P_{A_2U}(y_2)\right)\right)$$
$$= \min\left(\min\left(T^P_{A_1U}(x), T^P_{A_2U}(x_2)\right), \min\left(T^P_{A_1U}(x), T^P_{A_2U}(y_2)\right)\right)$$
$$= \min\left(\left(T^P_{A_1U} \times T^P_{A_2U}\right)(x,x_2), \left(T^P_{A_1U} \times T^P_{A_2U}\right)(x,y_2)\right)$$
$$\left(I^P_{B_1L} \times I^P_{B_2L}\right)((x,x_2)(x,y_2))$$
$$= \max\left(I^P_{A_1L}(x), I^P_{B_2L}(x_2y_2)\right)$$
$$\geq \max\left(I^P_{A_1L}(x), \max\left(I^P_{A_2L}(x_2), I^P_{A_2L}(y_2)\right)\right)$$
$$= \max\left(\max\left(I^P_{A_1L}(x), I^P_{A_2L}(x_2)\right), \max\left(I^P_{A_1L}(x), I^P_{A_2L}(y_2)\right)\right)$$
$$= \max\left(\left(I^P_{A_1L} \times I^P_{A_2L}\right)(x,x_2), \left(I^P_{A_1L} \times I^P_{A_2L}\right)(x,y_2)\right)$$
$$\left(I^P_{B_1U} \times I^P_{B_2U}\right)((x,x_2)(x,y_2))$$
$$= \max\left(I^P_{A_1U}(x), I^P_{B_2U}(x_2y_2)\right)$$
$$\geq \max\left(I^P_{A_1U}(x), \max\left(I^P_{A_2U}(x_2), I^P_{A_2U}(y_2)\right)\right)$$
$$= \max\left(\max\left(I^P_{A_1U}(x), I^P_{A_2U}(x_2)\right), \max\left(I^P_{A_1U}(x), I^P_{A_2U}(y_2)\right)\right)$$
$$= \max\left(\left(I^P_{A_1U} \times I^P_{A_2U}\right)(x,x_2), \left(I^P_{A_1U} \times I^P_{A_2U}\right)(x,y_2)\right)$$
$$\left(F^P_{B_1L} \times F^P_{B_2L}\right)((x,x_2)(x,y_2))$$
$$= \max\left(F^P_{A_1L}(x), F^P_{B_2L}(x_2y_2)\right)$$
$$\geq \max\left(F^P_{A_1L}(x), \max\left(F^P_{A_2L}(x_2), F^P_{A_2L}(y_2)\right)\right)$$
$$= \max\left(\max\left(F^P_{A_1L}(x), F^P_{A_2L}(x_2)\right), \max\left(F^P_{A_1L}(x), F^P_{A_2L}(y_2)\right)\right)$$
$$= \max\left(\left(F^P_{A_1L} \times F^P_{A_2L}\right)(x,x_2), \left(F^P_{A_1L} \times F^P_{A_2L}\right)(x,y_2)\right)$$
$$\left(F^P_{B_1U} \times F^P_{B_2U}\right)((x,x_2)(x,y_2))$$
$$= \max\left(F^P_{A_1U}(x), F^P_{B_2U}(x_2y_2)\right)$$
$$\geq \max\left(F^P_{A_1U}(x), \max\left(F^P_{A_2U}(x_2), F^P_{A_2U}(y_2)\right)\right)$$
$$= \max\left(\max\left(F^P_{A_1U}(x), F^P_{A_2U}(x_2)\right), \max\left(F^P_{A_1U}(x), F^P_{A_2U}(y_2)\right)\right)$$
$$= \max\left(\left(F^P_{A_1U} \times F^P_{A_2U}\right)(x,x_2), \left(F^P_{A_1U} \times F^P_{A_2U}\right)(x,y_2)\right)$$
$$\left(T^N_{B_1L} \times T^N_{B_2L}\right)((x,x_2)(x,y_2))$$
$$= \max\left(T^N_{A_1L}(x), T^N_{B_2L}(x_2y_2)\right)$$
$$\geq \max\left(T^N_{A_1L}(x), \max\left(T^N_{A_2L}(x_2), T^N_{A_2L}(y_2)\right)\right)$$

$= \max\left(\max\left(T^N_{A_1L}(x), T^N_{A_2L}(x_2)\right), \max\left(T^N_{A_1L}(x), T^N_{A_2L}(y_2)\right)\right)$

$= \max\left(\left(T^N_{A_1L} \times T^N_{A_2L}\right)(x,x_2), \left(T^N_{A_1L} \times T^N_{A_2L}\right)(x,y_2)\right)$

$\left(T^N_{B_1U} \times T^N_{B_2U}\right)((x,x_2)(x,y_2))$

$= \max\left(T^N_{A_1U}(x), T^N_{B_2U}(x_2y_2)\right)$

$\geq \max\left(T^N_{A_1U}(x), \max\left(T^N_{A_2U}(x_2), T^N_{A_2U}(y_2)\right)\right)$

$= \max\left(\max\left(T^N_{A_1U}(x), T^N_{A_2U}(x_2)\right), \max\left(T^N_{A_1U}(x), T^N_{A_2U}(y_2)\right)\right)$

$= \max\left(\left(T^N_{A_1U} \times T^N_{A_2U}\right)(x,x_2), \left(T^N_{A_1U} \times T^N_{A_2U}\right)(x,y_2)\right)$

$\left(I^N_{B_1L} \times I^N_{B_2L}\right)((x,x_2)(x,y_2))$

$= \min\left(I^N_{A_1L}(x), I^N_{B_2L}(x_2y_2)\right)$

$\leq \min\left(I^N_{A_1L}(x), \min\left(I^N_{A_2L}(x_2), I^N_{A_2L}(y_2)\right)\right)$

$= \min\left(\min\left(I^N_{A_1L}(x), I^N_{A_2L}(x_2)\right), \min\left(I^N_{A_1L}(x), I^N_{A_2L}(y_2)\right)\right)$

$= \min\left(\left(I^N_{A_1L} \times I^N_{A_2L}\right)(x,x_2), \left(I^N_{A_1L} \times I^N_{A_2L}\right)(x,y_2)\right)$

$\left(I^N_{B_1U} \times I^N_{B_2U}\right)((x,x_2)(x,y_2))$

$= \min\left(I^N_{A_1U}(x), I^N_{B_2U}(x_2y_2)\right)$

$\leq \min\left(I^N_{A_1U}(x), \min\left(I^N_{A_2U}(x_2), I^N_{A_2U}(y_2)\right)\right)$

$= \min\left(\min\left(I^N_{A_1U}(x), I^N_{A_2U}(x_2)\right), \min\left(I^N_{A_1U}(x), I^N_{A_2U}(y_2)\right)\right)$

$= \min\left(\left(I^N_{A_1U} \times I^N_{A_2U}\right)(x,x_2), \left(I^N_{A_1U} \times I^N_{A_2U}\right)(x,y_2)\right)$

$\left(F^N_{B_1L} \times F^N_{B_2L}\right)((x,x_2)(x,y_2))$

$= \min\left(F^N_{A_1L}(x), F^N_{B_2L}(x_2y_2)\right)$

$\leq \min\left(F^N_{A_1L}(x), \min\left(F^N_{A_2L}(x_2), F^N_{A_2L}(y_2)\right)\right)$

$= \min\left(\min\left(F^N_{A_1L}(x), F^N_{A_2L}(x_2)\right), \min\left(F^N_{A_1L}(x), F^N_{A_2L}(y_2)\right)\right)$

$= \min\left(\left(F^N_{A_1L} \times F^N_{A_2L}\right)(x,x_2), \left(F^N_{A_1L} \times F^N_{A_2L}\right)(x,y_2)\right)$

$\left(F^N_{B_1U} \times F^N_{B_2U}\right)((x,x_2)(x,y_2))$

$= \min\left(F^N_{A_1U}(x), F^N_{B_2U}(x_2y_2)\right)$

$\leq \min\left(F^N_{A_1U}(x), \max\left(F^N_{A_2U}(x_2), F^N_{A_2U}(y_2)\right)\right)$

$= \min\left(\min\left(F^N_{A_1U}(x), F^N_{A_2U}(x_2)\right), \min\left(F^N_{A_1U}(x), F^N_{A_2U}(y_2)\right)\right)$

$= \min\left(\left(F^N_{A_1U} \times F^N_{A_2U}\right)(x,x_2), \left(F^N_{A_1U} \times F^N_{A_2U}\right)(x,y_2)\right)$

for $(x_1,z)(y_1,z) \in E$.

$\left(T^P_{B_1L} \times T^P_{B_2L}\right)((x_1,z)(y_1,z))$

$= \min\left(T^P_{B_1L}(x_1y_1), T^P_{A_2L}(z)\right)$

$\leq \min\left(\min\left(T^P_{A_1L}(x_1), T^P_{A_1L}(y_1)\right), T^P_{A_2L}(z)\right)$

$= \min\left(\min\left(T^P_{A_1L}(x), T^P_{A_2L}(z)\right), \min\left(T^P_{A_1L}(y_1), T^P_{A_2L}(z)\right)\right)$

$= \min\left(\left(T^P_{A_1L} \times T^P_{A_2L}\right)(x_1,z), \left(T^P_{A_1L} \times T^P_{A_2L}\right)(y_1,z)\right)$

$\left(T^P_{B_1U} \times T^P_{B_2U}\right)((x_1,z)(y_1,z))$

$= \min\left(T^P_{B_1U}(x_1y_1), T^P_{A_2U}(z)\right)$

$\leq \min\left(\min\left(T^P_{A_1U}(x_1), T^P_{A_1U}(y_1)\right), T^P_{A_2U}(z)\right)$

$= \min\left(\min\left(T^P_{A_1U}(x), T^P_{A_2U}(z)\right), \min\left(T^P_{A_1U}(y_1), T^P_{A_2U}(z)\right)\right)$

$= \min\left(\left(T^P_{A_1U} \times T^P_{A_2U}\right)(x_1,z), \left(T^P_{A_1U} \times T^P_{A_2U}\right)(y_1,z)\right)$

$\left(I^P_{B_1L} \times I^P_{B_2L}\right)((x_1,z)(y_1,z)) = \max\left(I^P_{B_1L}(x_1y_1), I^P_{A_2L}(z)\right)$

$\geq \max\left(\max\left(I^P_{A_1L}(x_1), I^P_{A_1L}(y_1)\right), I^P_{A_2L}(z)\right)$

$= \max\left(\max\left(I^P_{A_1L}(x), I^P_{A_2L}(z)\right), \max\left(I^P_{A_1L}(y_1), I^P_{A_2L}(z)\right)\right)$

$= \max\left(\left(I^P_{A_1L} \times I^P_{A_2L}\right)(x_1,z), \left(I^P_{A_1L} \times I^P_{A_2L}\right)(y_1,z)\right)$

$\left(I^P_{B_1U} \times I^P_{B_2U}\right)((x_1,z)(y_1,z)) = \max\left(I^P_{B_1U}(x_1y_1), I^P_{A_2U}(z)\right)$

$\geq \max\left(\max\left(I^P_{A_1U}(x_1), I^P_{A_1U}(y_1)\right), I^P_{A_2U}(z)\right)$

$= \max\left(\max\left(I^P_{A_1U}(x), I^P_{A_2U}(z)\right), \max\left(I^P_{A_1U}(y_1), I^P_{A_2U}(z)\right)\right)$

$= \max\left(\left(I^P_{A_1U} \times I^P_{A_2U}\right)(x_1,z), \left(I^P_{A_1U} \times I^P_{A_2U}\right)(y_1,z)\right)$

$\left(F^P_{B_1L} \times F^P_{B_2L}\right)((x_1,z)(y_1,z)) = \max\left(F^P_{B_1L}(x_1y_1), F^P_{A_2L}(z)\right)$

$\geq \max\left(\max\left(F^P_{A_1L}(x_1), F^P_{A_1L}(y_1)\right), F^P_{A_2L}(z)\right)$

$= \max\left(\max\left(F^P_{A_1L}(x), F^P_{A_2L}(z)\right), \max\left(F^P_{A_1L}(y_1), F^P_{A_2L}(z)\right)\right)$

$= \max\left(\left(F^P_{A_1L} \times F^P_{A_2L}\right)(x_1,z), \left(F^P_{A_1L} \times F^P_{A_2L}\right)(y_1,z)\right)$

$\left(F^P_{B_1U} \times F^P_{B_2U}\right)((x_1,z)(y_1,z)) = \max\left(F^P_{B_1U}(x_1y_1), F^P_{A_2U}(z)\right)$

$\geq \max\left(\max\left(F^P_{A_1U}(x_1), F^P_{A_1U}(y_1)\right), F^P_{A_2U}(z)\right)$

$= \max\left(\max\left(F^P_{A_1U}(x), F^P_{A_2U}(z)\right), \max\left(F^P_{A_1U}(y_1), F^P_{A_2U}(z)\right)\right)$

$= \max\left(\left(F^P_{A_1U} \times F^P_{A_2U}\right)(x_1,z), \left(F^P_{A_1U} \times F^P_{A_2U}\right)(y_1,z)\right)$

$\left(T^N_{B_1L} \times T^N_{B_2L}\right)((x_1,z)(y_1,z)) = \max\left(T^N_{B_1L}(x_1y_1), T^N_{A_2L}(z)\right)$

$\geq \max\left(\max\left(T^N_{A_1L}(x_1), T^N_{A_1L}(y_1)\right), T^N_{A_2L}(z)\right)$

$= \max\left(\max\left(T^N_{A_1L}(x), T^N_{A_2L}(z)\right), \max\left(T^N_{A_1L}(y_1), T^N_{A_2L}(z)\right)\right)$

$= \max\left(\left(T^N_{A_1L} \times T^N_{A_2L}\right)(x_1,z), \left(T^N_{A_1L} \times T^N_{A_2L}\right)(y_1,z)\right)$

$\left(T^N_{B_1U} \times T^N_{B_2U}\right)((x_1,z)(y_1,z)) = \max\left(T^N_{B_1U}(x_1y_1), T^N_{A_2U}(z)\right)$

$\geq \max\left(\max\left(T^N_{A_1U}(x_1), T^N_{A_1U}(y_1)\right), T^N_{A_2U}(z)\right)$

$= \max\left(\max\left(T^N_{A_1U}(x), T^N_{A_2U}(z)\right), \max\left(T^N_{A_1U}(y_1), T^N_{A_2U}(z)\right)\right)$

$= \max\left(\left(T^N_{A_1U} \times T^N_{A_2U}\right)(x_1,z), \left(T^N_{A_1U} \times T^N_{A_2U}\right)(y_1,z)\right)$

$\left(I^N_{B_1L} \times I^N_{B_2L}\right)((x_1,z)(y_1,z)) = \min\left(I^N_{B_1L}(x_1y_1), I^N_{A_2L}(z)\right)$

$\leq \min\left(\min\left(I^N_{A_1L}(x_1), I^N_{A_1L}(y_1)\right), I^N_{A_2L}(z)\right)$

$= \min\left(\min\left(I^N_{A_1L}(x), I^N_{A_2L}(z)\right), \min\left(I^N_{A_1L}(y_1), I^N_{A_2L}(z)\right)\right)$

$= \min\left(\left(I^N_{A_1L} \times I^N_{A_2L}\right)(x_1,z), \left(I^N_{A_1L} \times I^N_{A_2L}\right)(y_1,z)\right)$

$\left(I^N_{B_1U} \times I^N_{B_2U}\right)((x_1,z)(y_1,z)) = \min\left(I^N_{B_1U}(x_1y_1), I^N_{A_2U}(z)\right)$

$\leq \min\left(\min\left(I^N_{A_1U}(x_1), I^N_{A_1U}(y_1)\right), I^N_{A_2U}(z)\right)$

$= \min\left(\min\left(I^N_{A_1U}(x), I^N_{A_2U}(z)\right), \min\left(I^N_{A_1U}(y_1), I^N_{A_2U}(z)\right)\right)$

$= \min\left(\left(I^N_{A_1U} \times I^N_{A_2U}\right)(x_1,z), \left(I^N_{A_1U} \times I^N_{A_2U}\right)(y_1,z)\right)$

$\left(F^N_{B_1L} \times F^N_{B_2L}\right)((x_1,z)(y_1,z)) = \min\left(F^N_{B_1L}(x_1y_1), F^N_{A_2L}(z)\right)$

$\leq \min\left(\min\left(F^N_{A_1L}(x_1), F^N_{A_1L}(y_1)\right), F^N_{A_2L}(z)\right)$

$= \min\left(\min\left(F^N_{A_1L}(x), F^N_{A_2L}(z)\right), \min\left(F^N_{A_1L}(y_1), F^N_{A_2L}(z)\right)\right)$

$= \min\left(\left(F^N_{A_1L} \times F^N_{A_2L}\right)(x_1,z), \left(F^N_{A_1L} \times F^N_{A_2L}\right)(y_1,z)\right)$

$\left(F^N_{B_1U} \times F^N_{B_2U}\right)((x_1,z)(y_1,z)) = \min\left(F^N_{B_1U}(x_1y_1), F^N_{A_2U}(z)\right)$

$\leq \min\left(\min\left(F^N_{A_1U}(x_1), F^N_{A_1U}(y_1)\right), F^N_{A_2U}(z)\right)$

$= \min\left(\min\left(F^N_{A_1U}(x), F^N_{A_2U}(z)\right), \min\left(F^N_{A_1U}(y_1), F^N_{A_2U}(z)\right)\right)$

$= \min\left(\left(F^N_{A_1U} \times F^N_{A_2U}\right)(x_1,z), \left(F^N_{A_1U} \times F^N_{A_2U}\right)(y_1,z)\right)$

Hence the proof. □

Example 3.5. Let $G_1^* = (A_1, B_1)$ and $G_2^* = (A_2, B_2)$ be two graphs, where $V_1 = \{wx\}, V_2 = \{yz\}$ $E_1 = \{wx\}$ and $E_2 = \{yz\}$. Consider the two Bipolar interval valued neutrosophic graphs.

$$A_1 = \left\{ \begin{array}{c} < w[0.5,0.2]\,[0.2,0.3]\,[0.2,0.4] >, \\ < x[0.6,0.4]\,[0.5,0.4]\,[0.3,0.1] > \\ < [-0.3,-0.2]\,[-0.5,-0.4]\,[-0.3,-0.1] >, \\ < [-0.4,-0.2]\,[0.3,0.2]\,[0.2,0.1] > \end{array} \right\}$$

$$B_1 = \left\{ \begin{array}{c} < wx[0.5,0.2]\,[0.2,0.4]\,[0.2,0.4] > \\ < [-0.3,-0.2]\,[-0.5,-0.4]\,[-0.3,-0.1] > \end{array} \right\}$$

$$A_2 = \left\{ \begin{array}{c} < y[0.4,0.3]\,[0.2,0.3]\,[0.1,0.3] >, \\ < z[0.4,0.2]\,[0.2,0.4]\,[0.1,0.3] > \\ < [-0.4,-0.3]\,[-0.4,-0.1]\,[-0.2,-0.1] >, \\ < [-0.3,-0.1]\,[-0.2,-0.1]\,[-0.3,-0.1] > \end{array} \right\}$$

$$B_2 = \left\{ \begin{array}{c} < yz[0.4,0.2]\,[0.2,0.4]\,[0.1,0.3] > \\ < [-0.3,-0.1]\,[-0.4,-0.1]\,[-0.3,-0.1] > \end{array} \right\}$$

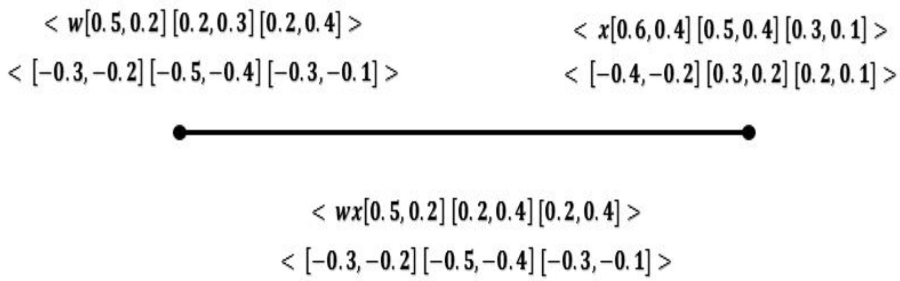

Figure 3. Bipolar interval valued neutrosophic graph G_1

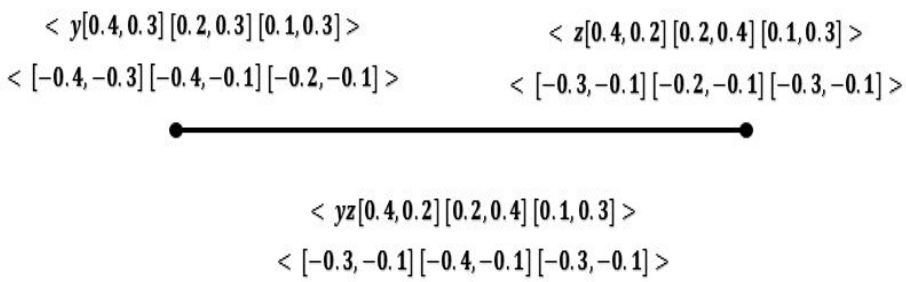

Figure 4. Bipolar interval valued neutrosophic graph G_2

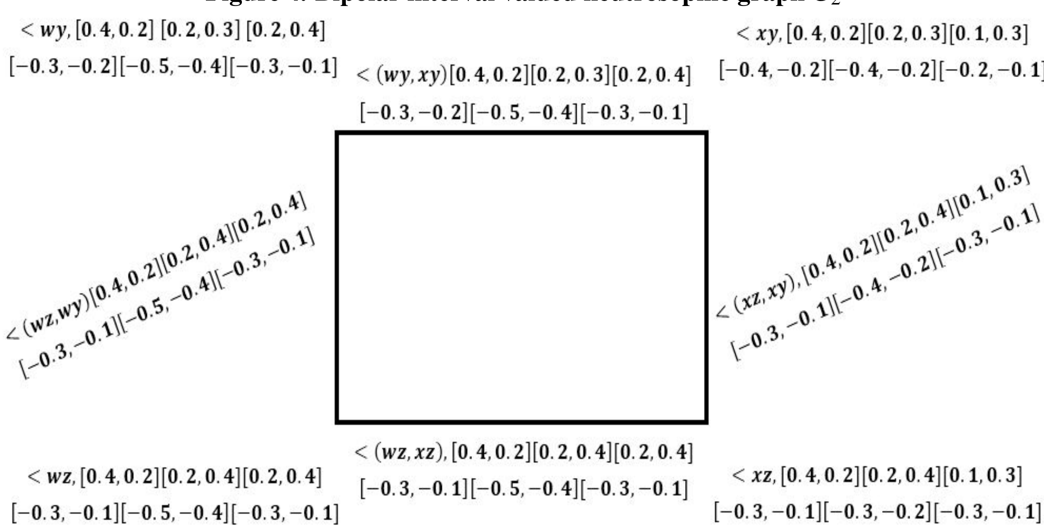

Figure 5. Cartesian product of Bipolar interval valued neutrosophic graph

Definition 3.6. *The union $G_1 \cup G_2 = (A_1 \cup A_2, B_1 \cup B_2)$ of two Bipolar interval valued neutrosophic graphs of the graphs $G^*_{B_1}$ and $G^*_{B_2}$ an Bipolar interval valued neutrosophic graph of $G^*_{B_1} \cup G^*_{B_2}$.*

1. $\left(T^P_{A_1L} \cup T^P_{A_2L}\right)(x) = T^P_{A_1L}(x)$ if $x \in V_1$ and $x \notin V_2$
$\left(T^P_{A_1L} \cup T^P_{A_2L}\right)(x) = T^P_{A_2L}(x)$ if $x \notin V_1$ and $x \in V_2$
$\left(T^P_{A_1L} \cup T^P_{A_2L}\right)(x) = \max\left(T^P_{A_1L}(x), T^P_{A_2L}(x)\right)$ if $x \in V_1 \cap V_2$
2. $\left(T^P_{A_1U} \cup T^P_{A_2U}\right)(x) = T^P_{A_1U}(x)$ if $x \in V_1$ and $x \notin V_2$
$\left(T^P_{A_1U} \cup T^P_{A_2U}\right)(x) = T^P_{A_2U}(x)$ if $x \notin V_1$ and $x \in V_2$
$\left(T^P_{A_1U} \cup T^P_{A_2U}\right)(x) = \max\left(T^P_{A_1U}(x), T^P_{A_2U}(x)\right)$ if $x \in V_1 \cap V_2$
3. $\left(I^P_{A_1L} \cup I^P_{A_2L}\right)(x) = I^P_{A_1L}(x)$ if $x \in V_1$ and $x \notin V_2$
$\left(I^P_{A_1L} \cup I^P_{A_2L}\right)(x) = I^P_{A_2L}(x)$ if $x \notin V_1$ and $x \in V_2$
$\left(I^P_{A_1L} \cup I^P_{A_2L}\right)(x) = \min\left(I^P_{A_1L}(x), I^P_{A_2L}(x)\right)$ if $x \in V_1 \cap V_2$
4. $\left(I^P_{A_1U} \cup I^P_{A_2U}\right)(x) = I^P_{A_1U}(x)$ if $x \in V_1$ and $x \notin V_2$
$\left(I^P_{A_1U} \cup I^P_{A_2U}\right)(x) = I^P_{A_2U}(x)$ if $x \notin V_1$ and $x \in V_2$
$\left(I^P_{A_1U} \cup I^P_{A_2U}\right)(x) = \min\left(I^P_{A_1U}(x), I^P_{A_2U}(x)\right)$ if $x \in V_1 \cap V_2$
5. $\left(F^P_{A_1L} \cup F^P_{A_2L}\right)(x) = F^P_{A_1L}(x)$ if $x \in V_1$ and $x \notin V_2$
$\left(F^P_{A_1L} \cup F^P_{A_2L}\right)(x) = F^P_{A_2L}(x)$ if $x \notin V_1$ and $x \in V_2$
$\left(F^P_{A_1L} \cup F^P_{A_2L}\right)(x) = \min\left(F^P_{A_1L}(x), F^P_{A_2L}(x)\right)$ if $x \in V_1 \cap V_2$
6. $\left(F^P_{A_1U} \cup F^P_{A_2U}\right)(x) = F^P_{A_1U}(x)$ if $x \in V_1$ and $x \notin V_2$
$\left(F^P_{A_1U} \cup F^P_{A_2U}\right)(x) = F^P_{A_2U}(x)$ if $x \notin V_1$ and $x \in V_2$
$\left(F^P_{A_1U} \cup F^P_{A_2U}\right)(x) = \min\left(F^P_{A_1U}(x), F^P_{A_2U}(x)\right)$ if $x \in V_1 \cap V_2$
7. $\left(T^N_{A_1L} \cup T^N_{A_2L}\right)(x) = T^N_{A_1L}(x)$ if $x \in V_1$ and $x \notin V_2$
$\left(T^N_{A_1L} \cup T^N_{A_2L}\right)(x) = T^N_{A_2L}(x)$ if $x \notin V_1$ and $x \in V_2$
$\left(T^N_{A_1L} \cup T^N_{A_2L}\right)(x) = \min\left(T^N_{A_1L}(x), T^N_{A_2L}(x)\right)$ if $x \in V_1 \cap V_2$
8. $\left(T^N_{A_1U} \cup T^N_{A_2U}\right)(x) = T^N_{A_1U}(x)$ if $x \in V_1$ and $x \notin V_2$
$\left(T^N_{A_1U} \cup T^N_{A_2U}\right)(x) = T^N_{A_2U}(x)$ if $x \notin V_1$ and $x \in V_2$
$\left(T^N_{A_1U} \cup T^N_{A_2U}\right)(x) = \min\left(T^N_{A_1U}(x), T^N_{A_2U}(x)\right)$ if $x \in V_1 \cap V_2$
9. $\left(I^N_{A_1L} \cup I^N_{A_2L}\right)(x) = I^N_{A_1L}(x)$ if $x \in V_1$ and $x \notin V_2$
$\left(I^N_{A_1L} \cup I^N_{A_2L}\right)(x) = I^N_{A_2L}(x)$ if $x \notin V_1$ and $x \in V_2$
$\left(I^N_{A_1L} \cup I^N_{A_2L}\right)(x) = \max\left(I^N_{A_1L}(x), I^N_{A_2L}(x)\right)$ if $x \in V_1 \cap V_2$
10. $\left(I^N_{A_1U} \cup I^N_{A_2U}\right)(x) = I^N_{A_1U}(x)$ if $x \in V_1$ and $x \notin V_2$
$\left(I^N_{A_1U} \cup I^N_{A_2U}\right)(x) = I^N_{A_2U}(x)$ if $x \notin V_1$ and $x \in V_2$
$\left(I^N_{A_1U} \cup I^N_{A_2U}\right)(x) = \max\left(I^N_{A_1U}(x), I^N_{A_2U}(x)\right)$ if $x \in V_1 \cap V_2$
11. $\left(F^N_{A_1L} \cup F^N_{A_2L}\right)(x) = F^N_{A_1L}(x)$ if $x \in V_1$ and $x \notin V_2$
$\left(F^N_{A_1L} \cup F^N_{A_2L}\right)(x) = F^N_{A_2L}(x)$ if $x \notin V_1$ and $x \in V_2$
$\left(F^N_{A_1L} \cup F^N_{A_2L}\right)(x) = \max\left(F^N_{A_1L}(x), F^N_{A_2L}(x)\right)$ if $x \in V_1 \cap V_2$
12. $\left(F^N_{A_1U} \cup F^N_{A_2U}\right)(x) = F^N_{A_1U}(x)$ if $x \in V_1$ and $x \notin V_2$
$\left(F^N_{A_1U} \cup F^N_{A_2U}\right)(x) = F^N_{A_2U}(x)$ if $x \notin V_1$ and $x \in V_2$
$\left(F^N_{A_1U} \cup F^N_{A_2U}\right)(x) = \max\left(F^N_{A_1U}(x), F^N_{A_2U}(x)\right)$ if $x \in V_1 \cap V_2$
13. $\left(T^P_{B_1L} \cup T^P_{B_2L}\right)(xy) = T^P_{B_1L}(xy)$ if $xy \in E_1$ and $xy \notin E_2$
$\left(T^P_{B_1L} \cup T^P_{B_2L}\right)(xy) = T^P_{B_2L}(xy)$ if $xy \notin V_1$ and $xy \in E_2$
$\left(T^P_{B_1L} \cup T^P_{B_2L}\right)(xy) = \max\left(T^P_{B_1L}(xy), T^P_{B_2L}(xy)\right)$ if $xy \in E_1 \cap E_2$
14. $\left(T^P_{B_1U} \cup T^P_{B_2U}\right)(xy) = T^P_{B_1U}(xy)$ if $xy \in E_1$ and $xy \notin E_2$
$\left(T^P_{B_1U} \cup T^P_{B_2U}\right)(xy) = T^P_{B_2U}(xy)$ if $xy \notin V_1$ and $xy \in E_2$
$\left(T^P_{B_1U} \cup T^P_{B_2U}\right)(xy) = \max\left(T^P_{B_1U}(xy), T^P_{B_2U}(xy)\right)$ if $xy \in E_1 \cap E_2$
15. $\left(I^P_{B_1L} \cup I^P_{B_2L}\right)(xy) = I^P_{B_1L}(xy)$ if $xy \in E_1$ and $xy \notin E_2$
$\left(I^P_{B_1L} \cup I^P_{B_2L}\right)(xy) = I^P_{B_2L}(xy)$ if $xy \notin V_1$ and $xy \in E_2$
$\left(I^P_{B_1L} \cup I^P_{B_2L}\right)(xy) = \min\left(I^P_{B_1L}(xy), I^P_{B_2L}(xy)\right)$ if $xy \in E_1 \cap E_2$
16. $\left(I^P_{B_1U} \cup I^P_{B_2U}\right)(xy) = I^P_{B_1U}(xy)$ if $xy \in E_1$ and $xy \notin E_2$
$\left(I^P_{B_1U} \cup I^P_{B_2U}\right)(xy) = I^P_{B_2U}(xy)$ if $xy \notin V_1$ and $xy \in E_2$
$\left(I^P_{B_1U} \cup I^P_{B_2U}\right)(xy) = \min\left(I^P_{B_1U}(xy), I^P_{B_2U}(xy)\right)$ if $xy \in E_1 \cap E_2$
17. $\left(F^P_{B_1L} \cup F^P_{B_2L}\right)(xy) = F^P_{B_1L}(xy)$ if $xy \in E_1$ and $xy \notin E_2$
$\left(F^P_{B_1L} \cup F^P_{B_2L}\right)(xy) = F^P_{B_2L}(xy)$ if $xy \notin V_1$ and $xy \in E_2$
$\left(F^P_{B_1L} \cup F^P_{B_2L}\right)(xy) = \min\left(F^P_{B_1L}(xy), F^P_{B_2L}(xy)\right)$ if $xy \in E_1 \cap E_2$
18. $\left(F^P_{B_1U} \cup F^P_{B_2U}\right)(xy) = F^P_{B_1U}(xy)$ if $xy \in E_1$ and $xy \notin E_2$
$\left(F^P_{B_1U} \cup F^P_{B_2U}\right)(xy) = F^P_{B_2U}(xy)$ if $xy \notin V_1$ and $xy \in E_2$
$\left(F^P_{B_1U} \cup F^P_{B_2U}\right)(xy) = \min\left(F^P_{B_1U}(xy), F^P_{B_2U}(xy)\right)$ if $xy \in E_1 \cap E_2$
19. $\left(T^N_{B_1L} \cup T^N_{B_2L}\right)(xy) = T^N_{B_1L}(xy)$ if $xy \in E_1$ and $xy \notin E_2$
$\left(T^N_{B_1L} \cup T^N_{B_2L}\right)(xy) = T^N_{B_2L}(xy)$ if $xy \notin V_1$ and $xy \in E_2$
$\left(T^N_{B_1L} \cup T^N_{B_2L}\right)(xy) = \min\left(T^N_{B_1L}(xy), T^N_{B_2L}(xy)\right)$ if $xy \in E_1 \cap E_2$
20. $\left(T^N_{B_1U} \cup T^N_{B_2U}\right)(xy) = T^N_{B_1U}(xy)$ if $xy \in E_1$ and $xy \notin E_2$
$\left(T^N_{B_1U} \cup T^N_{B_2U}\right)(xy) = T^N_{B_2U}(xy)$ if $xy \notin V_1$ and $xy \in E_2$
$\left(T^N_{B_1U} \cup T^N_{B_2U}\right)(xy) = \min\left(T^N_{B_1U}(xy), T^N_{B_2U}(xy)\right)$ if $xy \in E_1 \cap E_2$
21. $\left(I^N_{B_1L} \cup I^N_{B_2L}\right)(xy) = I^N_{B_1L}(xy)$ if $xy \in E_1$ and $xy \notin E_2$
$\left(I^N_{B_1L} \cup I^N_{B_2L}\right)(xy) = I^N_{B_2L}(xy)$ if $xy \notin V_1$ and $xy \in E_2$
$\left(I^N_{B_1L} \cup I^N_{B_2L}\right)(xy) = \max\left(I^N_{B_1L}(xy), I^N_{B_2L}(xy)\right)$ if $xy \in E_1 \cap E_2$
22. $\left(I^N_{B_1U} \cup I^N_{B_2U}\right)(xy) = I^N_{B_1U}(xy)$ if $xy \in E_1$ and $xy \notin E_2$
$\left(I^N_{B_1U} \cup I^N_{B_2U}\right)(xy) = I^N_{B_2U}(xy)$ if $xy \notin V_1$ and $xy \in E_2$
$\left(I^N_{B_1U} \cup I^N_{B_2U}\right)(xy) = \max\left(I^N_{B_1U}(xy), I^N_{B_2U}(xy)\right)$ if $xy \in E_1 \cap E_2$

23. $\left(F^N_{B_1L} \cup F^N_{B_2L}\right)(xy) = F^N_{B_1L}(xy)$ if $xy \in E_1$ and $xy \notin E_2$

$\left(F^N_{B_1L} \cup F^N_{B_2L}\right)(xy) = F^N_{B_2L}(xy)$ if $xy \notin V_1$ and $xy \in E_2$

$\left(F^N_{B_1L} \cup F^N_{B_2L}\right)(xy) = \max\left(F^N_{B_1L}(xy), F^N_{B_2L}(xy)\right)$ if $xy \in E_1 \cap E_2$

24. $\left(F^N_{B_1U} \cup F^N_{B_2U}\right)(xy) = F^N_{B_1U}(xy)$ if $xy \in E_1$ and $xy \notin E_2$

$\left(F^N_{B_1U} \cup F^N_{B_2U}\right)(xy) = F^N_{B_2U}(xy)$ if $xy \notin V_1$ and $xy \in E_2$

$\left(F^N_{B_1U} \cup F^N_{B_2U}\right)(xy) = \max\left(F^N_{B_1U}(xy), F^N_{B_2U}(xy)\right)$ if $xy \in E_1 \cap E_2$

Proposition 3.7. *Let G_{B_1} and G_{B_2} are two Bipolar interval valued neutrosophic graphs, then $G_{B_1} \cup G_{B_2}$ is also an Bipolar interval valued neutrosophic graph.*

Proof. Let $xy \in E_1 \cap E_2$ and verifying conditions for B_1 and B_2

$$\begin{aligned}
&\left(T^P_{B_1L} \cup T^P_{B_2L}\right)(xy) = \max\left(T^P_{B_1L}(xy), T^P_{B_2L}(xy)\right) \\
&\leq \max\left(\min\left(T^P_{A_1L}(x), T^P_{A_1L}(y)\right), \min\left(T^P_{A_2L}(x), T^P_{A_2L}(y)\right)\right) \\
&= \min\left(\max\left(T^P_{A_1L}(x), T^P_{A_2L}(x)\right), \max\left(T^P_{A_1L}(y), T^P_{A_2L}(y)\right)\right) \\
&= \min\left(\left(T^P_{A_1L} \cup T^P_{A_2L}\right)(x), \left(T^P_{A_1L} \cup T^P_{A_2L}\right)(y)\right) \\
&\left(T^P_{B_1U} \cup T^P_{B_2U}\right)(xy) = \max\left(T^P_{B_1U}(xy), T^P_{B_2U}(xy)\right) \\
&\leq \max\left(\min\left(T^P_{A_1U}(x), T^P_{A_1U}(y)\right), \min\left(T^P_{A_2U}(x), T^P_{A_2U}(y)\right)\right) \\
&= \min\left(\max\left(T^P_{A_1U}(x), T^P_{A_2U}(x)\right), \max\left(T^P_{A_1U}(y), T^P_{A_2U}(y)\right)\right) \\
&= \min\left(\left(T^P_{A_1U} \cup T^P_{A_2U}\right)(x), \left(T^P_{A_1U} \cup T^P_{A_2U}\right)(y)\right) \\
&\left(T^N_{B_1L} \cup T^N_{B_2L}\right)(xy) = \min\left(T^N_{B_1L}(xy), T^N_{B_2L}(xy)\right) \\
&\geq \min\left(\max\left(T^N_{A_1L}(x), T^N_{A_1L}(y)\right), \max\left(T^N_{A_2L}(x), T^N_{A_2L}(y)\right)\right) \\
&= \max\left(\min\left(T^N_{A_1L}(x), T^N_{A_2L}(x)\right), \min\left(T^N_{A_1L}(y), T^N_{A_2L}(y)\right)\right) \\
&= \max\left(\left(T^N_{A_1L} \cup T^N_{A_2L}\right)(x), \left(T^N_{A_1L} \cup T^N_{A_2L}\right)(y)\right) \\
&\left(T^N_{B_1U} \cup T^N_{B_2U}\right)(xy) = \min\left(T^N_{B_i1U}(xy), T^N_{B_i2U}(xy)\right) \\
&\geq \min\left(\max\left(T^N_{A_1U}(x), T^N_{A_1U}(y)\right), \max\left(T^N_{A_2U}(x), T^N_{A_2U}(y)\right)\right) \\
&= \max\left(\min\left(T^N_{A_1U}(x), T^N_{A_2U}(x)\right), \min\left(T^N_{A_1U}(y), T^N_{A_2U}(y)\right)\right) \\
&= \max\left(\left(T^N_{A_1U} \cup T^N_{A_2U}\right)(x), \left(T^N_{A_1U} \cup T^N_{A_2U}\right)(y)\right) \\
&\left(I^P_{B_1L} \cup I^P_{B_2L}\right)(xy) = \min\left(I^P_{B_1L}(xy), I^P_{B_2L}(xy)\right) \\
&\geq \min\left(\max\left(I^P_{A_1L}(x), I^P_{A_1L}(y)\right), \max\left(I^P_{A_2L}(x), I^P_{A_2L}(y)\right)\right) \\
&= \max\left(\min\left(I^P_{A_1L}(x), I^P_{A_2L}(x)\right), \min\left(I^P_{A_1L}(y), I^P_{A_2L}(y)\right)\right) \\
&= \max\left(\left(I^P_{A_1L} \cup I^P_{A_2L}\right)(x), \left(I^P_{A_1L} \cup I^P_{A_2L}\right)(y)\right) \\
&\left(I^P_{B_1U} \cup I^P_{B_2U}\right)(xy) = \min\left(I^P_{B_i1U}(xy), I^P_{B_i2U}(xy)\right) \\
&\geq \min\left(\max\left(I^P_{A_1U}(x), I^P_{A_1U}(y)\right), \max\left(I^P_{A_2U}(x), I^P_{A_2U}(y)\right)\right) \\
&= \max\left(\min\left(I^P_{A_1U}(x), I^P_{A_2U}(x)\right), \min\left(I^P_{A_1U}(y), I^P_{A_2U}(y)\right)\right) \\
&= \max\left(\left(I^P_{A_1U} \cup I^P_{A_2U}\right)(x), \left(I^P_{A_1U} \cup I^P_{A_2U}\right)(y)\right) \\
&\left(I^N_{B_1L} \cup I^N_{B_2L}\right)(xy) = \max\left(I^N_{B_1L}(xy), I^N_{B_2L}(xy)\right) \\
&\leq \max\left(\min\left(I^N_{A_1L}(x), I^N_{A_1L}(y)\right), \min\left(I^N_{A_2L}(x), I^N_{A_2L}(y)\right)\right) \\
&= \min\left(\max\left(I^N_{A_1L}(x), I^N_{A_2L}(x)\right), \max\left(I^N_{A_1L}(y), I^N_{A_2L}(y)\right)\right) \\
&= \min\left(\left(I^N_{A_1L} \cup I^N_{A_2L}\right)(x), \left(I^N_{A_1L} \cup I^N_{A_2L}\right)(y)\right) \\
&\left(I^N_{B_1U} \cup I^N_{B_2U}\right)(xy) = \max\left(I^N_{B_i1U}(xy), I^N_{B_i2U}(xy)\right) \\
&\leq \max\left(\min\left(I^N_{A_1U}(x), I^N_{A_1U}(y)\right), \min\left(I^N_{A_2U}(x), I^N_{A_2U}(y)\right)\right) \\
&= \min\left(\max\left(I^N_{A_1U}(x), I^N_{A_2U}(x)\right), \max\left(I^N_{A_1U}(y), I^N_{A_2U}(y)\right)\right) \\
&= \min\left(\left(I^N_{A_1U} \cup I^N_{A_2U}\right)(x), \left(I^N_{A_1U} \cup I^N_{A_2U}\right)(y)\right) \\
&\left(F^P_{B_1L} \cup F^P_{B_2L}\right)(xy) = \min\left(F^P_{B_1L}(xy), F^P_{B_2L}(xy)\right) \\
&\geq \min\left(\max\left(F^P_{A_1L}(x), F^P_{A_1L}(y)\right), \max\left(F^P_{A_2L}(x), F^P_{A_2L}(y)\right)\right) \\
&= \max\left(\min\left(F^P_{A_1L}(x), F^P_{A_2L}(x)\right), \min\left(F^P_{A_1L}(y), F^P_{A_2L}(y)\right)\right) \\
&= \max\left(\left(F^P_{A_1L} \cup F^P_{A_2L}\right)(x), \left(F^P_{A_1L} \cup F^P_{A_2L}\right)(y)\right) \\
&\left(F^P_{B_1U} \cup F^P_{B_2U}\right)(xy) = \min\left(F^P_{B_i1U}(xy), F^P_{B_i2U}(xy)\right) \\
&\geq \min\left(\max\left(F^P_{A_1U}(x), F^P_{A_1U}(y)\right), \max\left(F^P_{A_2U}(x), F^P_{A_2U}(y)\right)\right) \\
&= \max\left(\min\left(F^P_{A_1U}(x), F^P_{A_2U}(x)\right), \min\left(F^P_{A_1U}(y), F^P_{A_2U}(y)\right)\right) \\
&= \max\left(\left(F^P_{A_1U} \cup F^P_{A_2U}\right)(x), \left(F^P_{A_1U} \cup F^P_{A_2U}\right)(y)\right) \\
&\left(F^N_{B_1L} \cup F^N_{B_2L}\right)(xy) = \max\left(F^N_{B_1L}(xy), F^N_{B_2L}(xy)\right) \\
&\leq \max\left(\min\left(F^N_{A_1L}(x), F^N_{A_1L}(y)\right), \min\left(F^N_{A_2L}(x), F^N_{A_2L}(y)\right)\right) \\
&= \min\left(\max\left(F^N_{A_1L}(x), F^N_{A_2L}(x)\right), \max\left(F^N_{A_1L}(y), F^N_{A_2L}(y)\right)\right) \\
&= \min\left(\left(F^N_{A_1L} \cup F^N_{A_2L}\right)(x), \left(F^N_{A_1L} \cup F^N_{A_2L}\right)(y)\right) \\
&\left(F^N_{B_1U} \cup F^N_{B_2U}\right)(xy) = \max\left(F^N_{B_i1U}(xy), F^N_{B_i2U}(xy)\right) \\
&\leq \max\left(\min\left(F^N_{A_1U}(x), F^N_{A_1U}(y)\right), \min\left(F^N_{A_2U}(x), F^N_{A_2U}(y)\right)\right) \\
&= \min\left(\max\left(F^N_{A_1U}(x), F^N_{A_2U}(x)\right), \max\left(F^N_{A_1U}(y), F^N_{A_2U}(y)\right)\right) \\
&= \min\left(\left(F^N_{A_1U} \cup F^N_{A_2U}\right)(x), \left(F^N_{A_1U} \cup F^N_{A_2U}\right)(y)\right)
\end{aligned}$$

□

Example 3.8. *Let $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ be two graphs such that*
$V_1 = \{v_1, v_2, v_3, v_4, v_5\}$, $V_2 = \{v_1, v_2, v_3, v_4\}$,
$E_1 = \{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_1\}$ *and*
$E_2 = \{v_1v_2, v_1v_4, v_2v_4, v_2v_3, v_3v_4\}$

Consider two Bipolar interval valued neutrosophic graphs $G_1 = (A_1B_1)$ and $G_2 = (A_2B_2)$

$$\begin{aligned}
V_1V_2 =& (0.4,0.2)(0.2,0.3)(0.2,0.4) \\
V_2V_3 =& (0.4,0.2)(0.2,0.4)(0.1,0.3) \\
& (-0.3,-0.2)(-0.5,-0.4)(-0.3,-0.1) \\
& (-0.4,-0.2)(-0.4,-0.1)(-0.2,-0.1) \\
V_3V_4 =& (0.5,0.1)(0.4,0.4)(0.3,0.3) \\
V_4V_5 =& (0.2,0.1)(0.4,0.3)(0.3,0.1) \\
& (-0.4,-0.2)(-0.4,-0.1)(-0.2,-0.1) \\
& (-0.3,-0.1)(-0.2,-0.3)(-0.3,-0.1) \\
V_5V_1 =& (0.2,0.2)(0.4,0.3)(0.3,0.4) \\
& (-0.3,-0.1)(-0.5,-0.4)(-0.3,-0.1)
\end{aligned}$$

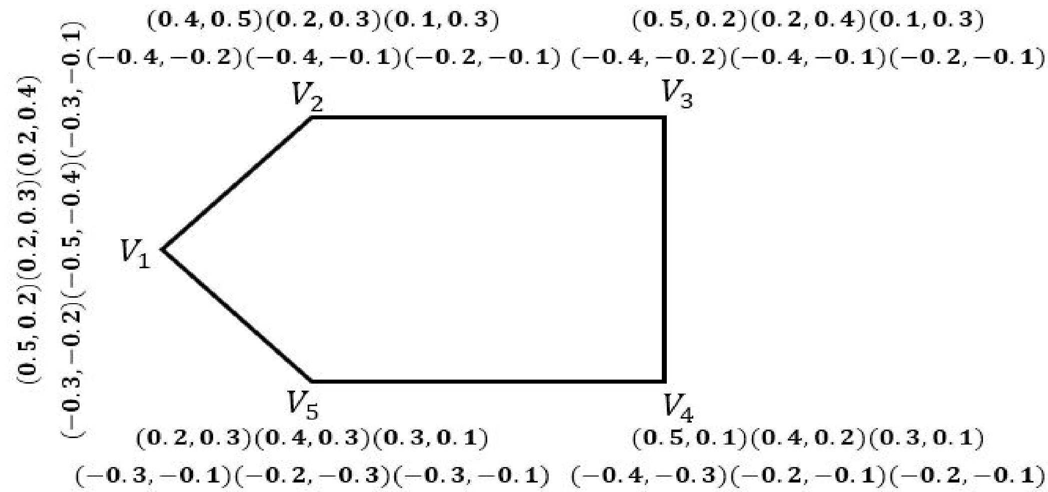

Figure 6. Bipolar interval valued neutrosophic graph G_1

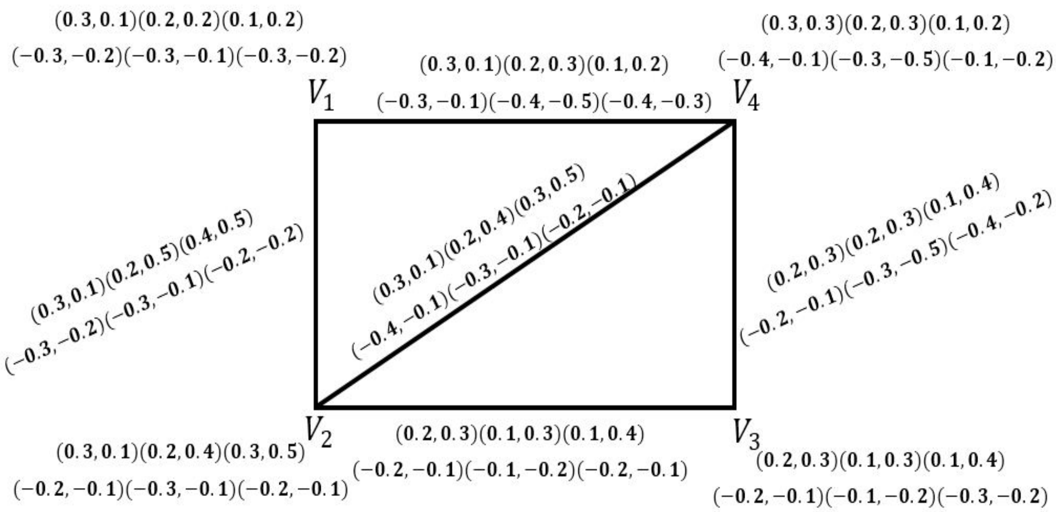

Figure 7. Bipolar interval valued neutrosophic graph G_2

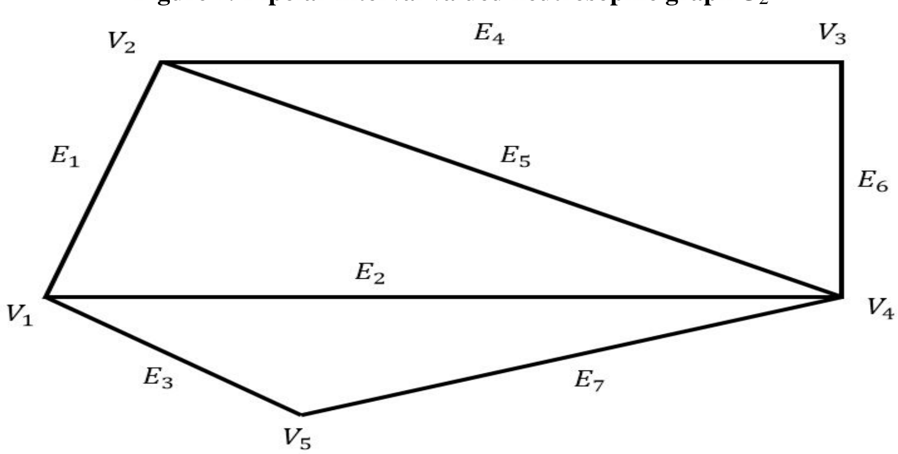

Figure 8. Bipolar interval valued neutrosophic graph $G_1 \cup G_2$

$$
\begin{aligned}
V_1 =&(0.3,0.1)(0.2,0.3)(0.2,0.4)\\
V_2 =&(0.3,0.1)(0.2,0.4)(0.3,0.5)\\
&(-0.3,-0.2)(-0.5,-0.4)(-0.3,-0.2)\\
&(-0.2,-0.1)(-0.4,-0.1)(-0.2,-0.1)\\
V_3 =&(0.2,0.2)(0.2,0.4)(0.1,0.4)\\
V_4 =&(0.3,0.1)(0.4,0.3)(0.3,0.2)\\
&(-0.2,-0.1)(-0.4,-0.1)(-0.3,-0.2)\\
&(-0.4,-0.1)(-0.3,-0.5)(-0.2,-0.2)\\
V_5 =&(0.2,0.3)(0.4,0.3)(0.3,0.1)\\
&(-0.3,-0.1)(-0.2,-0.3)(-0.3,-0.1)\\
E_1 =&(0.3,0.1)(0.2,0.4)(0.3,0.5)\\
E_2 =&(0.2,0.1)(0.2,0.4)(0.2,0.4)\\
&(-0.2,-0.1)(-0.5,-0.4)(-0.3,-0.2)\\
&(-0.2,-0.1)(-0.5,-0.4)(-0.3,-0.2)\\
E_3 =&(0.2,0.1)(0.4,0.3)(0.3,0.4)\\
E_4 =&(0.2,0.1)(0.2,0.4)(0.3,0.5)\\
&(-0.3,-0.1)(-0.5,-0.4)(-0.3,-0.2)\\
&(-0.2,-0.1)(-0.4,-0.1)(-0.3,-0.2)\\
E_5 =&(0.3,0.1)(0.4,0.4)(0.3,0.5)\\
E_6 =&(-0.2,-0.1)(-0.4,-0.5)(-0.3,-0.2)\\
&(-0.2,-0.1)(-0.4,-0.5)(-0.2,-0.2)\\
&(-0.2,-0.1)(-0.4,-0.5)(-0.3,-0.2)\\
E_7 =&(0.2,0.1)(0.4,0.3)(0.3,0.2)\\
&(-0.3,-0.1)(-0.3,-0.5)(-0.3,-0.2)
\end{aligned}
$$

Definition 3.9. *Addition of $G_1+G_2=(A_1+A_2,B_1+B_2)$ Bipolar interval valued neutrosophic graphs G_1 and G_2.*

$$
\begin{aligned}
\left(T^P_{A_1L}+T^P_{A_2L}\right)(x) &= \min\left\{T^P_{A_1L}(x),T^P_{A_2L}(x)\right\}\\
\left(T^P_{A_1U}+T^P_{A_2U}\right)(x) &= \min\left\{T^P_{A_1U}(x),T^P_{A_2U}(x)\right\}\\
\left(I^P_{A_1L}+I^P_{A_2L}\right)(x) &= \max\left\{I^P_{A_1L}(x),I^P_{A_2L}(x)\right\}\\
\left(I^P_{A_1U}+I^P_{A_2U}\right)(x) &= \max\left\{I^P_{A_1U}(x),I^P_{A_2U}(x)\right\}\\
\left(F^P_{A_1L}+F^P_{A_2L}\right)(x) &= \max\left\{F^P_{A_1L}(x),F^P_{A_2L}(x)\right\}\\
\left(F^P_{A_1U}+F^P_{A_2U}\right)(x) &= \max\left\{F^P_{A_1U}(x),F^P_{A_2U}(x)\right\}\\
\left(T^N_{A_1L}+T^N_{A_2L}\right)(x) &= \max\left\{T^N_{A_1L}(x),T^N_{A_2L}(x)\right\}\\
\left(T^N_{A_1U}+T^N_{A_2U}\right)(x) &= \max\left\{T^N_{A_1U}(x),T^N_{A_2U}(x)\right\}\\
\left(I^N_{A_1L}+I^N_{A_2L}\right)(x) &= \min\left\{I^N_{A_1L}(x),I^N_{A_2L}(x)\right\}\\
\left(I^N_{A_1U}+I^N_{A_2U}\right)(x) &= \min\left\{I^N_{A_1U}(x),I^N_{A_2U}(x)\right\}\\
\left(F^N_{A_1L}+F^N_{A_2L}\right)(x) &= \min\left\{F^N_{A_1L}(x),F^N_{A_2L}(x)\right\}\\
\left(F^N_{A_1U}+F^N_{A_2U}\right)(x) &= \min\left\{F^N_{A_1U}(x),F^N_{A_2U}(x)\right\}
\end{aligned}
$$

Similarly for the edges

$$
\begin{aligned}
\left(T^P_{B_1L}+T^P_{B_2L}\right)(xy) &= \min\left\{T^P_{B_1L}(x),T^P_{B_2L}(x)\right\}\\
\left(T^P_{B_1U}+T^P_{B_2U}\right)(xy) &= \min\left\{T^P_{B_1U}(x),T^P_{B_2U}(x)\right\}\\
\left(I^P_{B_1L}+I^P_{B_2L}\right)(xy) &= \max\left\{I^P_{B_1L}(x),I^P_{B_2L}(x)\right\}\\
\left(I^P_{B_1U}+I^P_{B_2U}\right)(xy) &= \max\left\{I^P_{B_1U}(x),I^P_{B_2U}(x)\right\}\\
\left(F^P_{B_1L}+F^P_{B_2L}\right)(xy) &= \max\left\{F^P_{B_1L}(x),F^P_{B_2L}(x)\right\}\\
\left(F^P_{B_1U}+F^P_{B_2U}\right)(xy) &= \max\left\{F^P_{B_1U}(x),F^P_{B_2U}(x)\right\} \text{ if } xy\in E.
\end{aligned}
$$

$$
\begin{aligned}
\left(T^N_{B_1L}+T^N_{B_2L}\right)(xy) &= \max\left\{T^N_{B_1L}(x),T^N_{B_2L}(x)\right\}\\
\left(T^N_{B_1U}+T^N_{B_2U}\right)(xy) &= \max\left\{T^N_{B_1U}(x),T^N_{B_2U}(x)\right\}\\
\left(I^N_{B_1L}+I^N_{B_2L}\right)(xy) &= \min\left\{I^N_{B_1L}(x),I^N_{B_2L}(x)\right\}\\
\left(I^N_{B_1U}+I^N_{B_2U}\right)(xy) &= \min\left\{I^N_{B_1U}(x),I^N_{B_2U}(x)\right\}\\
\left(F^N_{B_1L}+F^N_{B_2L}\right)(xy) &= \min\left\{F^N_{B_1L}(x),F^N_{B_2L}(x)\right\}\\
\left(F^N_{B_1U}+F^N_{B_2U}\right)(xy) &= \min\left\{F^N_{B_1U}(x),F^N_{B_2U}(x)\right\} \text{ if } xy\in E'.
\end{aligned}
$$

where E' is the set of all edges joining of V_1 and V_2 is E' and $V_1\cap V_2=\phi$.

Proposition 3.10. *$G_1=(A_1B_1)$ and $G_2=(A_2B_2)$ be two Bipolar interval valued neutrosophic graphs, then the sum also the Bipolar interval valued neutrosophic graph. We can prove this by an example.*

Example 3.11. $V_1=\{v_1\ v_2\}$, $V_2=\{v_3\ v_4\}$, $E_1=\{v_1\ v_2\}$, $E_2=\{v_3,v_4\}$

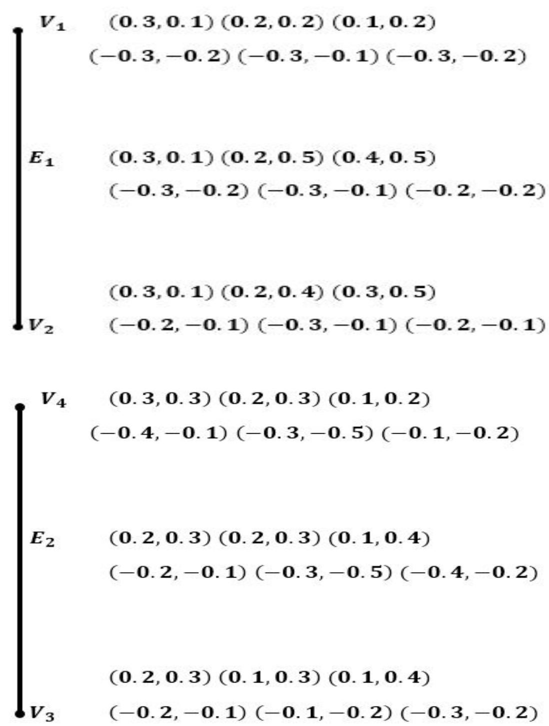

Figure 9. Bipolar interval valued neutrosophic graphs of G_1 and G_2

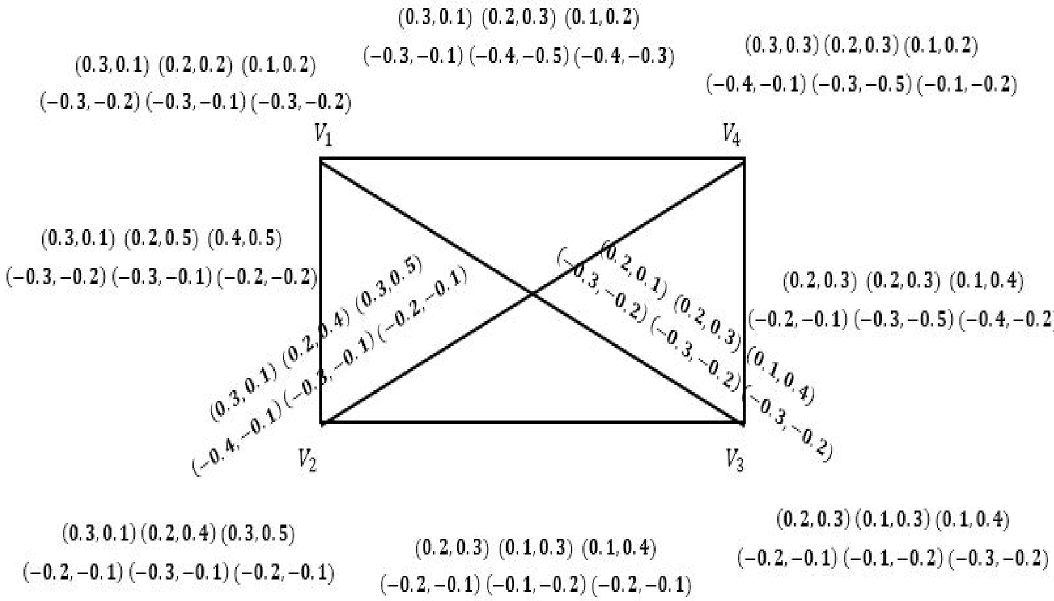

Figure 10. Bipolar interval valued neutrosophic graphs of $G_1 + G_2$

4. Conclusion

In this paper, we introduced some addition operations, cartesian product, union, examples and propositions. In future we plan to introduce other operations like intersection of Bipolar interval valued neutrosophic graphs.

References

[1] A.Q. Ansari, R. Biswas & S. Aggarwal, Neutrosophic classifier: An extension of fuzzy classifier, *Elsevier-Applied soft computing*, 13(2013), 563-573.

[2] A.Q. Ansari, R.Biswas & S.Aggarwal Extension to fuzzy logic representation: moving towards neutrosophic logic-A new laboratory rat, fuzzy systems (FUZZY), *IEEE international conference,* (2013) 1-8, Doi:10.1109/fuzz-IEEE.2013.6622412.

[3] A.Nagoor Gani and S.R latha, On irregular fuzzy graphs, *Applied mathematical science*, 6(11)(2012), 517-523.

[4] F.Smarandache. Refined literal indeterminacy and the multiplication law of. 9(2015), 58-63.

[5] H. Wang, F. Samrandache, Y. Zhang and R. Sunderraman, Single Valued Neutrosophic Sets, *Multispace and Multistructure,* 4(2010), 410-413.

[6] H.Y.Zhang, J.Q.Wang, X.H.chen, Interval neutrosophic sets and their application in multicriteria decision making problems, *The scientific world journal*, (2014), DOI:10.1155/2014/645953.

[7] J. Ye some aggregation operators of interval neutrosophic linguistic numbers for multiple attribute decision making, *Journal of intelligent & FUZZY Systems,* (27)(2014), 2231-2241.

[8] K. Atanassov. and G. gargov, interval valued intutionistic fuzzy sets, *Fuzzy Sets and systems*, 31(1989), 343-349.

[9] R.Ridvan, A. Kucuk, subsethood measure for single valued neutrosophic sets, *journal of intelligent & Fuzzy systems,* 29(2)(2015), 525-530.

[10] S. Nmihra and A.Pal, product of interval valued intuitionistic fuzzy graph, *Analysis of pure and applied mathamatics,* 5(1)(2013), 37-46.

[11] Y.Hai-long, G. She, Yanhonge, L.Ziuwu, On single valued neutrosophic relations, *Journal of intelligent systems*, (2015), 1-12.

[12] W.B. Vasanthkandasamy and F.Smarandache, *Fuzzy cognitive maps and neutrosophiccongtive maps.* 2013.

[13] W.B. Vasanthkandasamy, K. Illanthenral and florentin smarandache, *Neutrosophic Graphic: A New Dimension to Graph Theory*, Kindle Edition, 2015.

[14] Nicolae Pop, Luigu Vladareanu, Ile , ana Nicoieta Popescu, Constantic Ghita, Ionel Alexandru Gal, Shuang Cang, Hongnian Yu, Vasile Bratu, Mingcong Deng, A numerical dynamic behaviour model for 3D contact problems with friction, *Computational Materials Science*, 94(2014), 285-291.

[15] W.D vasanthkandasamy and F.smarandache, Analysis of social aspect of migrant laborers living with IIIV/AIDS using FUZZY Theory and Neutrosophic Cognitive maps, *xiquan*, phoenix (2004).

[16] Interval Valued Neutrosophic Graphs, Said Broumi, Mohamed Talea Assia Bakali, *Florentin Smarandache.*

[17] Aydogdu, on similary and Entropy of single valued Neutrosophic Sets, *Gen. Math. Notes*, 29(1)(2015), 67-74.

[18] Broumi, s., Talea, M., Bakali, A. and Smarandache, F., single Valued Neutrosophic Graphs, *Journal of New Theory,* 10(2016), 86-101.

[19] Gani, N. and Begum, S.S, Degree, Order and size in Intuitionistic Fuzzy graphs. *International journal of Al-*

gorithms, computing and mathematics, 3(3)(2010).

[20] Harary, F On the notion of balance of signed graph, *Michigan Math. J.*, 2(2)(1953), 143-146.

[21] Mishra S.N and Pal. A, Intuitionistic Fuzzy signed graphs. *International journal of pure and applied Mathematics,* 106(6)(2016), 113-122.

[22] Sudhakar .V.J, Mohamed Ali.A, and Vinoth. D., Interval Valued Signed Neutrosophic graphs, *International journal of Mathematical Archive*, 9(9)(2016), 35-43.

[23] Sudhakar .V.J, Navaneethakumar. V and Jayaprakasam.S., Interval Valued Regular Neutrosophic graphs, *International journal of Science and Humanities*, 5(2)(2019), 76-85.

[24] Sudhakar.V.J, Navaneethakumar. V and Yuvaraj., Self-Centered Interval Valued Neutrosophic graphs, *International journal of Science and Humanities*, 5(2)(2019), 86-102.

[25] Sudhakar .V.J, Navaneethakumar. V and L.Jagadeeswari, *Bipolar Interval Valued Neutrosophic Signed graphs.* (Communicated)

⋆⋆⋆⋆⋆⋆⋆⋆⋆
ISSN(P):2319 − 3786
Malaya Journal of Matematik
ISSN(O):2321 − 5666
⋆⋆⋆⋆⋆⋆⋆⋆⋆