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On solving neutrosophic linear complementarity Problem without introducing artificial variables

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Abstract

In this paper, we present a new approach to solve the Neutrosophic linear complementarity problem based on principal pivot method. The special feature of this method is that the linear complementarity problem with Neutrosophic Triangular fuzzy numbers is solved without introducing artificial variables. The effectiveness of the proposed method is illustrated by means of a numerical example. This problem finds many applications in several areas of science, engineering and economics.

Keywords

Linear Complementarity problem, Neutrosophic set, Neutrosophic Triangular fuzzy numbers, Lemke's Algorithm.

AMS Subject Classification

65K05, 90C90, 90C70, 90C29.

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1. Introduction

The linear complementarity problem (LCP) is a well-known problem in mathematical programming and it has been studied by many researchers. LCP is a general problem that unifies linear, quadratic programs and bimatrix games. In 1968, Lemke [7] proposed a complementary pivoting algorithm for solving linear complementarity problem and Katta.G.Murthy [6]. Fuzzy systems and Intuitionistic fuzzy systems cannot successfully deal with a situation where the conclusion is adequate, unacceptable and decision maker declaration is uncertain et al [8, 9]. Therefore, some novel theories are mandatory for solving the problem with uncertainty and see also [1-3]. The neutrosophic sets reflect on the truth membership, indeterminacy membership and falsity membership concurrently, which is more practical and adequate than Fuzzy sets and Intuitionistic fuzzy sets. Single valued neutrosophic sets are an extension of neutrosophic sets which were introduced by Wang et al.[4], Ye[10,11] introduced simplified neutrosophic sets and Peng et al.[5] defined their novel operations as aggregation operators.

Although many researchers and scientists have worked in the recently developed neutrosophic sets and applied it in the field of decision making, there is, however, still some viewpoints regarding defining neutrosophic numbers in different forms, and their corresponding de-impreciseness is very important.This paper provides a new technique for solving linear complementarity problems with fuzzy numbers. The paper is organized as follows. In Section 2, Single Valued Triangular Neutrosophic numbers (SVTN) and the fuzzy arithmetical operators are provided. In Section 3, Fuzzy linear complementarity problem without introducing artificial variables and an algorithm for solving a Principal of pivoting method of Neutrosophic Linear Complementary Problem (q,M) are described. In Section 4, the effectiveness of the proposed method is illustrated by means of an example. Finally, section 5 contains some concluding remarks.

2. Preliminaries

2.1 Triangular Fuzzy Numbers

A Triangular fuzzy number is denoted as $\tilde{A} = (a_1, a_2, a_3)$ and is defined by the membership function as

$$\mu_a(x) = \begin{cases} \frac{(x-a_1)}{(a_2-a_1)} & , a_1 \le x \le a_2 \\ \frac{(a_3-x)}{(a_3-a_2)} & , a_2 \le x \le a_3 \\ 0 & , \text{ otherwise} \end{cases}$$

Definition 2.1. *Let E be a universe. A neutrosophic set A over E is defined by*

$$A = \{\langle x, (T_A(x), I_A(x), F_A(x))\rangle : x \in E\}$$

Where $T_A(x), I_A(x), F_A(x)$ *are called the truth − membership function, indeterminacy membership function and falsity membership function respectively. They are respectively defined by*

$$T_A : E \to]^{\top} 0, 1^+ [, I_A : E \to]^- 0, 1^+ [, F_A : E \to]^- 0, 1^+ [$$

Such that $0^- \le T_A(x) + I_A(x) + F_A(x) \le 3^+$

Definition 2.2. *A Single Valued Triangular Neutrosophic number (SVTN)* $\tilde{A} = \langle (a,b,c); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ *is a special neutrosophic set on the real number set* R, *whose truth - membership, indeterminacy membership and a falsity membership are given as follows:*

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{(x-a)w_{\tilde{a}}}{(b-a)} & , a \le x \le b \\ w_{\tilde{a}} & , x = b \\ \frac{(c-x)w_{\tilde{a}}}{(c-b)} & , b \le x \le c \\ 0 & , \textit{ otherwise} \end{cases}$$

$$v_{\tilde{a}}(x) = \begin{cases} \frac{(b-x+u_{\tilde{a}}(x-a))}{(b-a)} & , a \le x \le b \\ \mu_{\tilde{a}} & , x = b \\ \frac{(x-b+u_{\tilde{a}}(c-x))}{(c-b)} & , b \le x \le c \\ 0 & , \textit{ otherwise} \end{cases}$$

$$\lambda_a(x) = \begin{cases} \frac{(b-x+y_{\tilde{a}}(x-a))}{(b-a)} & , a \le x \le b \\ y_{\tilde{a}} & , x = b \\ \frac{(x-b+y_{\tilde{a}}(c-x))}{(c-b)} & , b \le x \le c \\ 0 & , \textit{ otherwise} \end{cases}$$

2.2 Arithmetic Operations on Triangular Neutrosophic Numbers

Let $\tilde{a} = \langle (a_1, b_1, c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ and $\tilde{b} = \langle (a_2, b_2, c_2); w_{\tilde{b}}, u_{\tilde{b}}, y_{\tilde{b}} \rangle$ be two single valued trapezoidal neutrosophic numbers and $\gamma \ne 0$.

Addition:

$$\tilde{a} + \tilde{b} = \langle (a_1 + a_2, b_1 + b_2, c_1 + c_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle$$

Subtraction:

$$\tilde{a} - \tilde{b} = \langle (a_1 - c_2, b_1 - b_2, c_1 - a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle$$

Multiplication:

$$\tilde{a}\tilde{b} = \begin{cases} \langle (a_1 a_2, b_1 b_2, c_1 c_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle, \\ \quad (c_1 > 0, c_2 > 0) \\ \langle (a_1 c_2, b_1 b_2, c_1 a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle, \\ \quad (c_1 < 0, c_2 > 0) \\ \langle (c_1 c_2, b_1 b_2, a_1 a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle, \\ \quad (c_1 < 0, c_2 < 0) \end{cases}$$

Division:

$$\tilde{a}/\tilde{b} = \begin{cases} \langle (a_1/c_2, b_1/b_2, c_1/a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle, \\ \quad (d_1 > 0, d_2 > 0) \\ \langle (c_1/c_2, b_1/b_2, a_1/a_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle, \\ \quad (d_1 < 0, d_2 > 0) \\ \langle (c_1/a_2, b_1/b_2, a_1/c_2); w_{\tilde{a}} \wedge w_{\tilde{b}}, u_{\tilde{a}} \vee u_{\tilde{b}}, y_{\tilde{a}} \vee y_{\tilde{b}} \rangle, \\ \quad (d_1 < 0, d_2 < 0) \end{cases}$$

Scalar Multiplication:

$$\gamma\tilde{a} = \begin{cases} \langle (\gamma a_1, \gamma b_1, \gamma c_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle, (\gamma > 0) \\ \langle (\gamma c_1, \gamma b_1, \gamma a_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle, (\gamma < 0) \end{cases}$$

Inverse:

$$\tilde{a}^{-1} = \left\langle \left(\frac{1}{c_1}, \frac{1}{b_1}, \frac{1}{a_1} \right); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \right\rangle, (\tilde{a} \ne 0)$$

3. Principal Pivoting Methods for Neutrosophic Linear Complementarity Problem

Consider the Neutrosophic Linear Complementarity Problem $(\tilde{q}, \mathbb{M})$ of order n. The original table for this version of the algorithm is:

$\widetilde{W}$	z	q
$\tilde{i}$	$-\mathbb{M}$	$\tilde{q}$

$$\widetilde{\mathrm{W}}\tilde{\mathrm{Z}} \geqq 0, w^T z = 0 \tag{3.1}$$

This method is most useful for solving Neutrosophic LinearComplementary Problem $(\tilde{q}, \mathbb{M})$ in which M is a P-matrix. It only moves among complementary basic vectors for (6) which are infeasible, and terminates when a complementary feasible basic vector is obtained. It employs only single principal pivot steps. The initial complementary basic vector for starting the method is $\tilde{w} = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n)$.

In this method, the variables may change signs several times during the algorithm, before a complementary solution is obtained in the final step.

Let $\bar{q} = (\bar{q}_1, \ldots\ldots, \bar{q}_n)^T$ be the updated right hand side constants vector in the present canonical tableau of (i).

If $\bar{q} \ge 0$, the present complementary basic vector is feasible and the present BFS of (6) is a solution of the LCP(q, M), terminate.

If $\bar{q} \ngeq 0$. Let

$$r = \text{ Maximum } \{i : i \text{ such that } \bar{q}_i < 0\} \tag{3.2}$$

Make a single principal pivot step in position r, that is, replace the present basic variable in the complementary pair $(\widetilde{w}_r, \tilde{z}_r)$ by its complement.

If this pivot step cannot be carried out because the pivot element is zero, the method is unable to continue further, and it terminates without being able to solve this NLCP. Otherwise the pivot step is carried out and then the method moves to the next step.

4. Conclusion

In this paper, a new approach for solving a linear complementarity problem without introducing artificial variables with neutrosophic fuzzy parameters is suggested. Even though we are considering for solving Linear Complementarity Problem (q, M) Principal of Pivoting method with Single Valued Triangular Neutrosophic numbers (SVTN), this method can also be extended to non-linear and multi objective programming with neutrosophic fuzzy coefficients.

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