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A framework of neutrosophic evidence theory based on probability estimation

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Abstract

Evidence theory can handle both aleatory and epistemic uncertainty. Three important functions in evidence theory are the basic probability assignment function, Belief function and Plausibility function which are used to quantify the given variable. In this paper, we proposed probability estimation in the frame work of Neutrosophic Evidence theory with the help of Dempster Shafer theory. Then the validity of the proposed methodology has been verified with the help of a numerical example.

Keywords

Dempster Shafer Theory (DST), Basic Probability Assignment (BPA), Probability distribution, Neutrosophic belief functions.

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1. Introduction

Neutrosophic set theory was proposed by Smarandache [6] in 2000. The neutrosophic set introduced from a philosophical point of view is difficult to be applied in practical problems since its truth membership function (T), the indeterminacy membership function (I) and the false membership function (F) lie in the non standard interval] 0-, 1 + [.

Probability theory is only exposed for randomness uncertainty and it is inappropriate to represent epistemic uncertainly. To overcome the constraint of probabilistic method, Dempster put forward a mathematical theory of evidence in 1976 and now it is known as Evidence Theory or Dempster Shafer Theory (DST). In a finite discrete space, Dempster Shafer Theory can be interpreted as generalizations of Probability Theory were probabilities assigned to set as opposed to mutually exclusive singletons. In traditional probability theory, evidence is associated with only one possible event. In Dempster Shafer Theory (DST), evidence can be associated with multiple possible events. Further, Evidence theory is based on two dual non additive measures, namely Belief measure and Plausibility measure. Belief and Plausibility measures can conveniently be characterized by a function $m: \rho(X) \rightarrow [0,1]$ such that $m(\emptyset)=0$ and $\sum_{A \in \rho(X)} m(A)=1$.

The function is known as a Basic Probability Assignment (BPA). Every set $A \in \rho(X)$ for which $m(A)>0$ is usually called a focal element of m. The Dempster Rule of combination is critical to the original conception of Dempster-Shafer Theory. The measure of Belief and Plausibility are derived from the combined basic assignments. Dempster's Rule combines multiple belief functions through their basic probability assignment (m). These belief functions are defined on the same frame of discemment, but are based on independent assignment or bodies of evidence. The Dempster Rule of combination is purely a conjunctive operation. The combination rules results in a belief function based on conjunctive pooled evidence.

The standard way of combining evidence is expressed by the formula,

$$m_{1,2}(A)=\frac{\sum_{B \cap C=A} m_1(B) \cdot m_2(C)}{1-\sum_{B \cap C=\varnothing} m_1(B) \cdot m_2(C)},$$

for all $A \neq \emptyset$ and $m_{1,2}(\emptyset)=0$.

Dempster-Shafer evidence theory can be expressed and deal with uncertainty in crisp sets. However, D-S theory itself cannot represent and manage vague information such as "the price is high" or "high age is about 40". To overcome this problem, fuzzy evidence theory was proposed to process imprecise and vague information. But fuzzy systems cannot successfully deal with a situation where the conclusion is adequate, unacceptable and decision-maker declaration is uncertain. Therefore, the neutrosophic set in Evidence theory is used in this research work.

The paper is organized as follows: Section 1 introduces Dempster Shafer Theory and Neutrosophic Set theory. Section 2 deals with the basic definitions about Neutrosophic sets and Intuitionistic Belief function. Definitions of Neutrosophic Belief Functions and Probability Estimation based on Neutrosophic Belief Functions and examples are given in section 3.

2. Preliminaries

Definition 2.1. *Let X be a space of points, with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth membership function T_A, indeterminancy member ship function I_A and falsity membership function $F_A \cdot T_A(x), I_A(x)$ and $F_A(x)$ are real standard or non standard subsets of $]0^-, 1^+[$. That is,*

$$T_A : X \to]0^-, 1^+[$$
$$I_A : X \to]0^-, 1^+[$$
$$F_A : X \to]0^-, 1^+[$$

There is no restriction on the sum of $T_A(x), I_A(x)$ and $F_A(x)$, so $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Definition 2.2. *Let $X = \{x_1, x_2, x_3, \ldots, x_n\}$ be a universe of discourse and let $\mathbb{F}$ be the set of all focal elements. A normalized fuzzy belief function m given in [14] is defined as*

$$\left\{\left\langle A_i^F, m\left(A_i^F\right), \mu_{A_i^F}(x_j)\right\rangle\right\}, A_i^F \in \mathbb{F}, x_j \in X$$

Definition 2.3. *Let $X = \{x_1, x_2, x_3, \ldots, x_n\}$ be a universe of discourse and let $\mathbb{F}$ be the set of all focal elements. An intuitionistic fuzzy belief function m given in [14] is defined as*

$$\left\{\left\langle A_i^F, m\left(A_i^F\right), \mu_{A_i^F}(x_j), \vartheta_{A_i^F}(x_j)\right\rangle\right\}, A_i^F \in \mathbb{F}, x_j \in X$$

3. Probability Estimation based on Neutrosophic Belief Functions

Definition 3.1. *Let $X = \{x_1, x_2, x_3, \ldots, x_n\}$ be a universe of discourse and let $NS(X)$ be the family of all neutrosophic sets in X. A neutrosophic belief function μ is defined as $\{< A_i, m(A_i), T_{A_i}(x_j), I_{A_i}(x_j), F_{A_i}(x_j) >\}, A_i \in NS(X), x_j \in X$, where A_i is neutrosophic set, $T_{A_i}(x_j)$ is the truth membership of neutrosophic set, $F_{A_i}(x_j)$ is the false-membership of neutrosophic set $A_i, I_{A_i}(x_j)$ is the indeterminacy-membership of neutrosophic set A_i and $m(A_i)$ is the BPA of A_i.*

Definition 3.2. *Let the probability of each element in X be $P(x_j)$ $(j = 1, 2, \ldots, n)$ The Basic Probability Assignment (BPA) of Neutrosophic event Ai*

$$A_i = \{< x_j, T_{A_i}(x_j) I_{A_j}(x_j)_3 F_{A_j}(x_j) > / x_j \in X\}$$

is an interval value defined as

$$m(A_i) = [m_{\min}(A_i), m_{\max}((A_i)]$$

where $m_{\min}(A_i)$ is the minimal BPA of A_i. $m_{\max}(A_i)$ is the maximal BPA of A_i are defined as,

$$m_T(A_i) = \sum P(x_i) T_{A_i}(x_j)$$
$$m_I(A_i) = \sum P(x_i)(1 - I_{A_i}(x_j))$$
$$m_F(A_i) = \sum P(x_i)(1 - F_{A_i}(x_j))$$

And

$$m_{\min}(Ai) = \min(m_T(Ai), m_I(A_i), m_F(A_i))$$
$$m_{\max}(Ai) = \max(m_T(Ai), m_I(A_i), m_F(A_i))$$

It is easy to verify that $0 \leq m_{\min}(Ai) \leq m_{\max}(Ai) \leq 1$.

Example 3.3. *Suppose there is a discussion of the amount of money which should be invest for super market. Let us assume that the Neutrosophic event A corresponding to the statement "about fifty lakhs" is given by $A = \{< 30, 0.2, 0.3, 0.4 >, < 40, 0.3, 0.5, 0.4 >, < 50, 1, 0, 0 >, < 60, 0.1, 0.2, 0.4 >, < 70, 0.5, 0.2, 0.3 >\}$.*
The probability distribution of the amount of money is

$$P(30) = P(40) = P(50) = P(60) = P(70) = 0.2$$

Now,

$$m_T(A) = 0.42$$
$$m_I(A) = 0.76$$
$$m_F(A) = 0.70$$
$$m_{\min}(A) = \min(0.42, 0.76, 0.7) = 0.42$$
$$m_{\max}(A) = \max(0.42, 0.76, 0.7) = 0.76$$
$$\therefore m(A) = [0.42, 0.76]$$

It means that the Basic Probability Assignment of the event of "assigning about fifty lakhs for investments" lies in the interval [0.42,0.76]

Definition 3.4. *Let $X = \{x_1, x_2, x_3, \ldots, x_n\}$ be a universe of discourse and let $\mathbb{F}$ be the set of all focal elements. A normalized Neutrosophic belief function μ is given as*

$$\{< A_i, m(A_i), T_{A_i}(x_j), I_{A_i}(x_j), F_{A_i}(x_j) >\}, A_i \in \mathbb{F}, x_j \in X$$

Then, the probability of $x_j (j = 1, 2, \ldots, n)$ can be estimated as

$$\widetilde{P(x_j)} = [\tilde{a}_j, \tilde{b}_j]$$

where $\tilde{a}_1$ and $\tilde{b}_1$ are defined as

$$\tilde{a}_J = \min\left\{\widetilde{T_{A_l}}(x_j), \widetilde{I_{A_l}}(x_j), \widetilde{F_{A_l}}(x_j)\right\}$$
$$\tilde{b}_j = \max\left\{\widetilde{T_{A_l}}(x_j), \widetilde{I_{A_l}}(x_j), \widetilde{F_{A_l}}(x_j)\right\}$$

where

$$T_{A_2}(x_j) = \sum_{A_i \in F} \frac{m(A_i)\, T_{A_i}(x_j)}{\sum_{j=1}^{n}(1 - I_{A_i}(x_j)) + \sum_{j=1}^{n}(1 - F_A(x_j))}$$

$$\widetilde{I_{A_L}}(x_j) = \sum_{A_i \in F} \frac{m(A_i)\,(1 - I_{A_i}(x_j))}{\sum_{j=1}^{n}(T_{A_i}(x_j)) + \sum_{j=1}^{n}(1 - F_A(x_j))}$$

$$\widetilde{F_{A_L}}(x_j) = \sum_{A_i \in F} \frac{m(A_i)\,(1 - F_{A_i}(x_j))}{\sum_{j=1}^{n}(T_{A_i}(x_j)) + \sum_{j=1}^{n}(1 - I_A(x_j))}$$

Example 3.5. *Let the universe of discourse be* $X = \{30, 40, 50, 60, 70\}$. *The assessment result from some experts is that the probability of "assigning about fifty lakhs for advertisement" is* 0.5, *the probability of "assigning a large amount of money for advertisement" is* 0.2, *and the probability of "assigning a small amount of money for advertisement" is* 0.3. *The statements "about fifty lakhs", "a large amount of money", and a "small amount of money" can be expressed by three Neutrosophic events A.B and C respectively, given by* $A = \{< 30, 0.2, 0.3, 0.4 >, < 40, 0.3, 0.5, 0.4 >, < 50, 1, 0, 0 >, < 60, 0.1, 0.2, 0.4 > \leq 70, 0.5, 0.2, 0.3 >\}$ $B = \{< 30, 0, 0, 1 >, < 40, 0.2, 0.4, 0.6 >, < 50, 0.5, 0.4, 0.3 >, < 60, 0.8, 0.6, 0.1 >, < 70, 1, 0, 0 >\}$ $C = \{< 30, 1, 0, 0 >, < 40, 0.8, 0.5, 0.1 >, < 50, 0.3, 0.2, 0.4 >, < 60, 0.1, 0.4, 0.8 >, < 70, 0, 0, 1 >\}$
Experts assessment result can be written as

$$m(A) = 0.5$$
$$m(B) = 0.3$$
$$m(C) = 0.2$$

The Probability distribution in X is given as

$$\tilde{P}(30) = [0.07, 0.30]$$
$$\tilde{P}(40) = [0.11, 0.21]$$
$$\tilde{P}(50) = [0.15, 0.21]$$
$$\tilde{P}(60) = [0.21, 0.23]$$
$$\tilde{P}(70) = [0.12, 0.28]$$

Many methods for comparison of intervals have been proposed, most of which are based on the midpoints of interval numbers. By using this method the following results are obtained.
The midpoint of $\tilde{P}(30) = [0.07, 0.30]$ *is* 0.19.
The midpoint of $\tilde{P}(40) = [0.11, 0.21]$ *is 0.16.*
The midpoint of $\tilde{P}(50) = [0.15, 0.21]$ *is 0.18.*
The midpoint of $\tilde{P}(60) = [0.21, 0.23]$ *is 0.22.*
The midpoint of $\tilde{P}(70) = [0.12, 0.28]$ *is 0.20*

$$\therefore \tilde{P}(60) \geq \tilde{P}(70) \geq \tilde{P}(30) \geq \tilde{P}(50) \geq \tilde{P}(40)$$

Therefore, we can make an obvious decision that assigning sixty thousands for advertisement is acceptable.

4. Conclusion

In this paper, we have defined the probability estimations based on neutrosophic belief functions. We hope that the concept presented here will open new avenue of research in current neutrosophic evidence theory. Finally, an illustrative example is provided to show the effectiveness of the proposed approach. In future, the proposed approach can be used for other real life problems.

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References

[1] Aggarwal .P, Bhatt .D, Devabhaktuni .V, and Bhattacharya .P, "Dempster Shafer neural network algorithm for land vehicle navigation application," Information Sciences, vol. 253, pp. 26-33, 2013.

[2] Benferhat .S, Saffiotti .A, and Smets .P, "Belief functions and default reasoning", Artificial Intelligence, vol. 122, no. 1-2, pp. 1-69, 2000.

[3] Beynon .M, Cosker .D, and Marshall .D, " A expert system for multi-criteria decision making using Dempster-Shafer theory", Experts Systems with Application, vol. 20, no. 4, pp. 357-367, 2001.

[4] Dempster A.P, "Upper and lower probabilities induced by a multivalued mapping", Annals of Mathematical Statistics, vol.38, pp.325-339, 1967.

[5] Florentein Smarandae , Collected papers, vol II, University of Kishinev, Kishinev, 1997.

[6] Florentein Smarandae, Neutrosophic Probability Set and logic, first version, , January 2000.

[7] Jones R.W., Lowe .A, and Harrison M. J., "A framework for intelligent medical diagnosis using the theory of evidence," Knowledge-Based Systems, vol. 15, no. 1-2, pp. 77-84, 2002.

[8] Laha .A, Pal .N.R. and Das.J , " Land cover classification using fuzzy rules and aggregation on contextual information through evidence theory," IEEE Transactions on Geoscience and Remote Sensing, vol. 44, no. 6, pp. 1633-1641, 2006.

[9] Liu Z.G., Pan.Q and Dezert .J, "Evidential classifier for imprecise data based on belief functions," Knowledge-Based Systems, vol. 52, pp. 246-257, 2013.

[10] Shafer. G, A Mathematical Theory of Evidence, Princeton University Press, Princeton, NJ, USA, 1976.

[11] Song .Y, Wang .X, Lei .L and Xue .A, " Combination of interval-valued belief structures based on intuitionstic fuzzy set," Knowledge-Based Systems, vol. 67, pp. 61-70, 2014.

[12] Su Z.G., Wang Y.F. and Wang P.H., "Parametric regression analysis of imprecise and uncertain data in the fuzzy belief function framework," International Journal of Approximate Reasoning, vol. 54, no. 8, pp. 1217-1242, 2013.

[13] Telmoudi. A and Chakhar .S, "Data fusion application from evidential database as a support for decision making," Information and Software Technology, vol. 46, no. 8, pp.547-555, 2004.

[14] Yafei Song and Xiaodan Wang, Probability estimation in the framework of Intuitionistic Fuzzy Evidence theory", Mathematical Problems inEngineering, Hindawi Publishing corporation, Vol 2015.

[15] Yen .J, "Generalizing the Dempster-Shafer theory to fuzzy sets," IEEE Transactions on Systems, Man, and Cybernetics, Vol. 20, no. 3, pp. 559-570, 1990.

[16] Zadeh L.A., "Probability measure of fuzzy events," Jounal of Mathematical Analysis and Applications, vol. 23, pp. 421-427, 1968.

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