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An intelligent multiple-criteria decision-making approach based on sv-neutrosophic hypersoft set with possibility degree setting for investment selection

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ABSTRACT:

The main purpose of this article is to characterize a novel neutrosophic hypersoft set hybrid called possibility single-valued neutrosophic hypersoft set (psv-NHSS) for evaluation of investment projects by using its aggregation operations and decision-support system.

Two approaches: set-theoretic approach and algorithmic approach are employed in this article. The former one is used to characterize the novel notion of psv-NHSS and its some aggregations. The later one is used to construct a decision-support system by using the aggregations like core matrix, maximum valued decision, minimum valued decision and scoring valued decision of psv-NHSS. The adopted algorithm is implemented in real-world scenario of hydro electric power station project evaluation for investment purpose.

The proposed model is more flexible and reliable as it addresses the limitations of literature on neutrosophic set, neutrosophic soft set and other fuzzy set-like models by considering possibility degree, hypersoft setting and neutrosophic setting collectively.

It has limitations for decision-making situations where selection of parameters is of uncertain nature.

The scope of this study may cover a wide range of applications in many fields of mathematical sciences like artificial intelligence, optimization, MCDM, theoretical computer science, soft computing, mathematical statistics etc.

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The proposed model bears the characteristics of most of the relevant existing models collectively and fulfills their insufficiencies by introducing a novel approximate mapping.

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Abstract. **Purpose**-The main purpose of this article is to characterize a novel neutrosophic hypersoft set hybrid called possibility single-valued neutrosophic hypersoft set (psv-NHSS) for evaluation of investment projects by using its aggregation operations and decision-support system.

Design/methodology/approach-Two approaches: set-theoretic approach and algorithmic approach are employed in this article. The former one is used to characterize the novel notion of psv-NHSS and its some aggregations. The later one is used to construct a decision-support system by using the aggregations like core matrix, maximum valued decision, minimum valued decision and scoring valued decision of psv-NHSS. The adopted algorithm is implemented in real-world scenario of hydro electric power station project evaluation for investment purpose.

Findings- The proposed model is more flexible and reliable as it addresses the limitations of literature on neutrosophic set, neutrosophic soft set and other fuzzy set-like models by considering possibility degree, hypersoft setting and neutrosophic setting collectively.

Research limitations/implications- It has limitations for decision-making situations where selection of parameters is of uncertain nature.

Practical implications- The scope of this study may cover a wide range of applications in many fields of mathematical sciences like artificial intelligence, optimization, MCDM, theoretical computer science, soft computing, mathematical statistics etc.

Originality- The proposed model bears the characteristics of most of the relevant existing models collectively and fulfills their insufficiencies by introducing a novel approximate mapping.

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1. **Introduction**

The problem of tackling informational uncertainty has been a challenging task for the researchers. Many models have already been developed to cope with this challenge. Neutrosophic set (NS) [1] is one of them which is the generalization of fuzzy set (FS) [2] and intuitionistic fuzzy set (IFS) [3]. In NS, the uncertain attitude of every entity $\hat{u}$ of initial universe $\hat{\mathcal{U}}$ is assessed by three dimensional membership-graded function $<\zeta_T(\hat{u}), \zeta_I(\hat{u}), \zeta_F(\hat{u})>$. The components $\zeta_T(\hat{u}), \zeta_I(\hat{u})$, and $\zeta_F(\hat{u})$ are known as membership-grade, indeterminate-grade and nonmembership-grade respectively having conditions that $\zeta_T(\hat{u}), \zeta_I(\hat{u}), \zeta_F(\hat{u}) \in]^-0, 1^+[$ such that $^-0 \leq sup\,\zeta_T(\hat{u}) + sup\,\zeta_I(\hat{u}) + sup\,\zeta_F(\hat{u}) \leq 3^+$. The NS is more flexible as compared to IFS in the sense that (i) it considers an extra component $\zeta_I(\hat{u})$ to evaluate the indeterminacy of $\hat{u} \in \hat{\mathcal{U}}$ and (ii) there is no dependency among its uncertain components. In order to ensure the applicability of NS, single-valued NS (sv-NS) [4] has been developed which transforms the non-standard grades of NS into real-valued grades. With the passage of time, it is observed that NS and sv-NS are inadequate with the entitlement of parameterization tool, therefore the concept of soft set (SS) [5] is initiated which utilizes an approximate mapping for judging the vagueness and uncertainty of objects in initial universe corresponding to prescribed attributes. Later on, neutrosophic soft set (NSS) [6] is characterized to equip NS with parameterization mode. It is a matter of common observation that some situations occur in real-world decision-making (DM) scenarios like medical diagnosis, product selection, recruitment process etc., which demand further classification of parameters into their relevant sub-parametric valued non-overlapping sets. The existing literature on NS, SS and NSS is insufficient to manage such kind of situations. Therefore, hypersoft set (HSS) [7] is developed to address such limitations of existing literature. In HSS, a new approximate mapping is utilized which considers the Cartesian product of sub-parametric valued non-overlapping sets as its domain therefore this mapping is also named as multi-argument approximate mapping (MAA-mapping). Since its development, HSS has earned the keen interest of many researchers so far. For the sake of its applicability in other branches and disciplines, its several elementary operational properties and axiomatic operations have been characterized in [8]. The ideas of neutrosophic hypersoft set (NHSS) and single-valued neutrosophic hypersoft set (sv-NHSS) [7] have been developed to adequate NSS with MAA-mapping environment. The aggregation operations of NHSS have been developed by [9] and then applied in DM problem. The concept of NHSS has been applied in DM technique i.e. TOPSIS using its accuracy function by [10]. The tangent similarity measures of sv-NHSS have been computed and applied in DM problem by [11]. Rahman et al. [12–14] discussed the parameterization of NHSS with uncertain settings and applied the proposed concept in real-world DM problems. Saqlain et al. [15] formulated distance and similarity measures of NHSS and then constructed decision-support system based on NHSS-TOPSIS with application in DM. Saeed et al. [16, 17] developed NHSS mappings and applied them in multi-attribute medical diagnostic of hepatitis and brain tumors with the proposal of their appropriate treatment. Ihsan et al. [18] developed sv-NHS expert set (sv-NHSES) by combining NHSS with expert system to consider the multi-decisive opinions of experts in one model. Theory of possibility (poss-theory) [19] is the replacement of probability theory for dealing with uncertain nature of information. Zadeh [20] discussed various aspects of fuzzy set (FS) as the basis for poss-theory. Fedrizzi [21] employed a blended approach of FS and poss-theory to handle with various optimization models. Dubois & Prade [22] emphasized on the clarification of several features of poss-theory and probability theory. The poss-theory uses measures of possibility of any entity between 0 and 1 in the objective space. The poss-theory has already been applied by many authors in fuzzy soft set like models as possibility degree to measure the uncertain nature of approximate components collectively. In the following lines, some relevant literature is reviewed to appraise the research-gap and necessity of proposed study.

Yager [23] and Alkhazaleh et al. [24] introduced novel gluing concept (i.e. possibility fuzzy soft set (pFSS) of FS and SS by assigning a specific grade i.e. possibility grade to each FS-number to assess its uncertainty. Alkhazaleh et al., in addition, worked out the similarity measures between two pFSSs and utilized it in medical-diagnosis based pattern recognition. Bashir et al. [25] introduced the concept of possibility intuitionistic fuzzy soft set (pIFSS) and discussed its some fundamental properties and operations. Karaaslan [26–28] developed possibility neutrosophic soft set (pNSS) with computation of its similarity measures and correlation coefficient. Recently Rahman et al. [29] developed possibility intuitionistic fuzzy hypersoft set (pIFHSS) and computed its aggregation operations and similarity measures then proposed algorithms based on these aggregation operations and similarity measures to solve DM problems.

In order to have reliable decisions, it is observed that the entitlement of attributes only is not sufficient in those situations which require mandatory partitioning of attributes into sets having their respective values. Disregard of such partitioning may lead to biased and dubious decisions. The above described models and other NS-like approaches are not capable to manage such situations. In these models, a single set of parameters is considered as domain of approximate function which is inept for the consideration of attribute-valued classification. This scarcity of existing literature leads to the development of HSS and NHSS. Under above discussion, it is viewed that literature requires an innovative concept which may tackle the situations: (i). The situation which has mandatory partitioning of attributes, (ii). The situation in which membership, non-membership and indeterminate grades are required to be assigned to each alternative for managing its uncertainty corresponding to each attribute and (iii). The situation which requires specific setting for the assessment of the uncertain nature of approximate elements with possibility degree-based setting, collectively in one model. This requisite provides the motivational basis of this study. Therefore instigating from above description in general, and from [24, 25, 29] in particular, a novel structure possibility sv-NHSS (psv-NHSS) is developed along with its utilization in resolving real-world problems based on its aggregations. The major contributions of this study are outlined as:

(1). The notions of a novel model psv-NHSS are characterized that is capable to cope with the situations having demands of: (a). Sub-classes of attributes with attribute-values in the form of disjoint sets, (b). Arrangement for assessment of uncertain attitude of approximate elements with the help of possibility degree-based attachment and (c). Assignment of uncertain components like membership, non-membership and indeterminacy grades to each alternatives with respect to each parameter.

(2). The matrix notation of psv-NHSS is utilized to design its novel aggregation like core matrix, maximum valued decision, minimum valued decision and scoring valued decision. The psv-NHSS numbers are converted to fuzzy values with the help of suitable criteria to get core matrix.

(3). A decision-support system is constructed on the basis of these designed aggregations of psv-NHSS and then an algorithm is proposed to evaluate the particular hydro electric power station project for investment purpose.

(4). The flexibility of the proposed model is discussed with the description on its advantageous aspects. The remaining paper is systemized as in section 2, some relevant definitions like sv-NS,SS, NSS, HSS, NHSS and sv-NHSS are reviewed from literature for the facilitation of the researchers (readers) to understand the main work. In section 3, novel notions of psv-NHSS and its aggregation operations like union, intersection, are characterized and explained with the provision of suitable examples. In section 4, an algorithm is proposed for the optimum selection of a project to invest. The proposed algorithm is further validated by providing a example based real-world scenario of the evaluation for the particular hydro electric power station project for investment purpose. In section 5, a textual description is provided regarding the flexibility of proposed model i.e. psv-NHSS through the depiction of its advantageous features. Lastly the paper is summarized with some future directions.

2. Preliminaries

This section provides a brief review of some essential elementary definitions for the support of main results. Wang et al. [4] put forward the concept of sv-NS which not only generalizes the existing fuzzy set-like models but also increases the applicability of Smarandache's NS. It replaces the non-standard grades of NS with real-valued grades.

Definition 2.1. [4] A sv-NS $\hat{\mathfrak{N}}$ on $\hat{\mathcal{U}}$ is stated as $\hat{\mathfrak{N}} = \{(\hat{u}, < \zeta_T(\hat{u}), \zeta_I(\hat{u}), \zeta_F(\hat{u}) >) : \hat{u} \in \hat{\mathcal{U}}\}$ where $\zeta_T(\hat{u}), \zeta_I(\hat{u})$ and $\zeta_F(\hat{u})$ are membership, indeterminate and non-membership grades respectively for each $\hat{u} \in \hat{\mathcal{U}}$ with conditions that $\zeta_T(\hat{u}), \zeta_I(\hat{u}), \zeta_F(\hat{u}) \in [0,1]$ and $0 \leq \zeta_T(\hat{u}) + \zeta_I(\hat{u}) + \zeta_F(\hat{u}) \leq 3$. The power set of sv-NS is symbolized as $\Omega_{svns}(\hat{\mathcal{U}})$.

In 1999, Molodtsov [5] proposed the concept of soft set which is capable to address the limitations of FS, IFS, NS, sv-NS etc., for the entitlement parameterization tool. This set utilizes a particular mapping that maps the set of parameters to the power set of universal set.

Definition 2.2. [5] Let $\hat{\mathbb{A}} = \{\ddot{a}_1, \ddot{a}_2, ..., \ddot{a}_n\}$ be a set of attributes. A SS $\hat{\mathfrak{S}}$ on $\hat{\mathcal{U}}$ is stated as $\hat{\mathfrak{S}} = \{(\psi_{\hat{\mathfrak{S}}}(\ddot{a}_i), \hat{\mathbb{A}}) : \ddot{a}_i \in \hat{\mathbb{A}}\}$ where $\psi_{\mathfrak{S}}$ is a soft approximate mapping with $\psi_{\hat{\mathfrak{S}}} : \hat{\mathbb{A}} \to P(\hat{\mathcal{U}})$ and $\psi_{\hat{\mathfrak{S}}}(\ddot{a}_i) \subseteq P(\hat{\mathcal{U}})$ is known as $\ddot{a}_i$-approximate element of $\hat{\mathfrak{S}}$.

In 2013, Maji [6] presented the theory of neutrosophic soft set by combining NS with SS. This set has capability to fulfil the shortcoming of both NS and SS for dealing with uncertainties and vagueness.

Definition 2.3. [6] A NSS $\hat{\mathfrak{N}}_{\hat{\mathfrak{S}}}$ on $\hat{\mathcal{U}}$ is characterized by a pair $(\zeta_{\hat{\mathfrak{S}}}(\ddot{a}_i), \hat{\mathbb{A}})$ where $\zeta_{\hat{\mathfrak{S}}} : \hat{\mathbb{A}} \rightarrow PN(\hat{\mathcal{U}})$ is a soft approximate mapping and $\zeta_{\hat{\mathfrak{S}}}(\ddot{a}_i) \subseteq PN(\hat{\mathcal{U}})$ is known as $\ddot{a}_i$-approximate element of $\hat{\mathfrak{N}}_{\hat{\mathfrak{S}}}$. The $PN(\hat{\mathcal{U}})$ is the power set of NSs.

In 2018, Smarandache observed that Molodtsov's soft set considers only single set of attributes for the approximations of objects under consideration whereas it is not sufficient for the situations which demand further categorization of parameters into sub-parametric values in the form of disjoint set that further demand the entitlement of a particular mapping called multi-argument approximate mapping. Consequently he conceptualized hypersoft set to address this limitation of soft set.

Definition 2.4. [7] Let $\hat{\mathbb{A}} = \{\ddot{a}_1, \ddot{a}_2, ..., \ddot{a}_n\}$ be a set of parameters and their relevant sub-parametric values are contained in sets $\hat{\mathbb{A}}_1, \hat{\mathbb{A}}_2, ..., \hat{\mathbb{A}}_n$ such that $\hat{\mathbb{A}}_i \cap \hat{\mathbb{A}}_j$ for $i \neq j$ then a HSS $\hat{\mathfrak{H}}$ is stated as $\hat{\mathfrak{H}} = \{(\psi_{\hat{\mathfrak{H}}}(\ddot{b}_i), \hat{\mathbb{B}}) : \ddot{b}_i \in \hat{\mathbb{B}}\}$ where $\psi_{\hat{\mathfrak{H}}}$ is a MAA-mapping with $\psi_{\hat{\mathfrak{H}}} : \hat{\mathbb{B}} \rightarrow P(\hat{\mathcal{U}})$ and $\psi_{\hat{\mathfrak{H}}}(\ddot{b}_i) \subseteq P(\hat{\mathcal{U}})$ is known as $\ddot{b}_i$-MAA element and $\hat{\mathbb{B}} = \prod_{i=1}^{n} \hat{\mathbb{A}}_i$.

According to arguments provided in [7, 11], a HSS $\hat{\mathfrak{H}}$ (as in Definition 2.4) is claimed to be called NHSS and sv-NHSS on $\hat{\mathcal{U}}$ if its MAA-mapping $\psi_{\hat{\mathfrak{H}}} : \hat{\mathbb{B}} \rightarrow P(\hat{\mathcal{U}})$ is replaces with $\psi_{\hat{\mathfrak{H}}} : \hat{\mathbb{B}} \rightarrow PN(\hat{\mathcal{U}})$ and $\psi_{\hat{\mathfrak{H}}} : \hat{\mathbb{B}} \rightarrow \Omega_{svns}(\hat{\mathcal{U}})$ respectively.

3. Neutrosophic Hypersoft Set wit Possibility-degree Setting

This section explores the characterization of possibility single-valued neutrosophic hypersoft set (psv-NHSS) with development of its some aggregation operations.

Definition 3.1. Let $\hat{\mathbb{A}} = \{\ddot{a}_1, \ddot{a}_2, ..., \ddot{a}_n\}$ be a set of parameters and their relevant sub-parametric values are contained in sets $\hat{\mathbb{A}}_1, \hat{\mathbb{A}}_2, ..., \hat{\mathbb{A}}_n$ such that $\hat{\mathbb{A}}_i \cap \hat{\mathbb{A}}_j$ for $i \neq j$ and $\hat{\mathbb{B}} = \prod_{i=1}^{n} \hat{\mathbb{A}}_i$ then a psv-NHSS Ψ_σ over hypersoft universe $(\hat{\mathcal{U}}, \hat{\mathbb{B}})$ is characterized by the pair $(\hat{\mathbb{B}}, \zeta_{\hat{\mathbb{B}}})$ where

(i) $\zeta_{\hat{\mathbb{B}}} : \hat{\mathbb{B}} \rightarrow \Omega_{svns}(\hat{\mathcal{U}}) \times \mathbb{I}_{\hat{\mathcal{U}}}$ defined by $\zeta_{\hat{\mathbb{B}}}(\hat{\beta}) = \left\{ \left(\frac{\hat{u}_i}{\psi(\hat{\beta})(\hat{u}_i)}, \sigma(\hat{\beta})(\hat{u}_i) \right) : \hat{u}_i \in \hat{\mathcal{U}} \,\&\, \hat{\beta} \in \hat{\mathbb{B}} \right\}$ and $\psi(\hat{\beta})(\hat{u}_i) =< \zeta_T(\hat{\beta})(\hat{u}_i), \zeta_I(\hat{\beta})(\hat{u}_i), \zeta_F(\hat{\beta})(\hat{u}_i) >$ with $0 \leq \zeta_T(\hat{\beta})(\hat{u}_i) + \zeta_I(\hat{\beta})(\hat{u}_i) + \zeta_F(\hat{\beta})(\hat{u}_i) \leq 3$,

(ii) $\mathbb{I}_{\hat{\mathcal{U}}} : \hat{\mathcal{U}} \rightarrow [0, 1]$ and $\sigma(\hat{\beta})(\hat{u}_i) \in \mathbb{I}_{\hat{\mathcal{U}}}$ is the degree of possibility of membership of $\hat{u}_i \in \hat{\mathcal{U}}$ in $\psi(\hat{\beta})$.

Therefore a psv-NHSS Ψ_σ is briefly defined by the set of pairs

$$\Psi_\sigma = \left\{ \left(\hat{\beta}, \left\{ \left(\frac{\hat{u}_i}{\psi(\hat{\beta})(\hat{u}_i)}, \sigma(\hat{\beta})(\hat{u}_i) \right) : \hat{u}_i \in \hat{\mathcal{U}} \right\} \right) : \hat{\beta} \in \hat{\mathbb{B}} \right\}.$$

The class of all psv-NHSSs is represented by $\Theta_{psvnhss}$. For simplicity a psv-NHSS is represented as $\Psi_\sigma = (<\zeta_T(\hat{\beta})(\hat{u}_i), \zeta_I(\hat{\beta})(\hat{u}_i), \zeta_F(\hat{\beta})(\hat{u}_i)>, \sigma(\hat{\beta})(\hat{u}_i))$.

Example 3.2. Mrs. Smith is the head of a technical institution which is providing training services in different technical trades to female individuals of the locality. She wants to purchase some overlock sewing machines to start training classes for learning various techniques of sewing and stitching. She constitutes a committee of some faculty members (having relevant experience) for this purchase. There are four kinds of machines which constitute a set of alternatives $\hat{\mathcal{U}} = \{\tilde{m}_1, \tilde{m}_2, \tilde{m}_3, \tilde{m}_4\}$. The members of the committee consider the attributes i.e. $\ddot{a}_1$ = speed (stitches per minute), $\ddot{a}_2$ = stitch length (millimeter), and $\ddot{a}_3$ = oil lubrication system. The non-overlapping sets having their sub-attribute values are $\hat{\mathbb{A}}_1 = \{\ddot{a}_{11} = 6000, \ddot{a}_{12} = 6500\}$, $\hat{\mathbb{A}}_2 = \{\ddot{a}_{21} = 2, \ddot{a}_{22} = 4\}$, $\hat{\mathbb{A}}_3 = \{\ddot{a}_{31} = automatic\}$ then $\hat{\mathbb{B}} = \hat{\mathbb{A}}_1 \times \hat{\mathbb{A}}_2 \times \hat{\mathbb{A}}_3 = \{\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3, \hat{\beta}_4\}$ where every $\hat{\beta}_i$ is a 3-tuple entity. The approximate elements of psv-NHSS Ψ_σ are given as

$$\Psi_\sigma(\hat{\beta}_1) = \left\{ \left(\frac{\tilde{m}_1}{\prec 0.4,0.2,0.3 \succ}, 0.2\right), \left(\frac{\tilde{m}_2}{\prec 0.5,0.3,0.4 \succ}, 0.3\right), \left(\frac{\tilde{m}_3}{\prec 0.6,0.4,0.5 \succ}, 0.4\right), \left(\frac{\tilde{m}_4}{\prec 0.7,0.5,0.6 \succ}, 0.5\right) \right\},$$
$$\Psi_\sigma(\hat{\beta}_2) = \left\{ \left(\frac{\tilde{m}_1}{\prec 0.6,0.1,0.2 \succ}, 0.8\right), \left(\frac{\tilde{m}_2}{\prec 0.5,0.2,0.3 \succ}, 0.8\right), \left(\frac{\tilde{m}_3}{\prec 0.5,0.3,0.4 \succ}, 0.7\right), \left(\frac{\tilde{m}_4}{\prec 0.4,0.4,0.5 \succ}, 0.6\right) \right\},$$
$$\Psi_\sigma(\hat{\beta}_3) = \left\{ \left(\frac{\tilde{m}_1}{\prec 0.6,0.2,0.2 \succ}, 0.1\right), \left(\frac{\tilde{m}_2}{\prec 0.5,0.2,0.3 \succ}, 0.2\right), \left(\frac{\tilde{m}_3}{\prec 0.6,0.2,0.4 \succ}, 0.3\right), \left(\frac{\tilde{m}_4}{\prec 0.6,0.2,0.4 \succ}, 0.4\right) \right\},$$
$$\Psi_\sigma(\hat{\beta}_4) = \left\{ \left(\frac{\tilde{m}_1}{\prec 0.6,0.1,0.1 \succ}, 0.2\right), \left(\frac{\tilde{m}_2}{\prec 0.4,0.1,0.3 \succ}, 0.3\right), \left(\frac{\tilde{m}_3}{\prec 0.5,0.4,0.4 \succ}, 0.4\right), \left(\frac{\tilde{m}_4}{\prec 0.6,0.2,0.5 \succ}, 0.5\right) \right\}.$$

Its matrix representation is

$$\Psi_\sigma = \begin{pmatrix} \hat{\mathbb{B}} \setminus \hat{\mathcal{U}} & \tilde{m}_1 & \tilde{m}_2 & \tilde{m}_3 & \tilde{m}_4 \\ \hat{\beta}_1 & (\prec 0.4, 0.2, 0.3 \succ, 0.2) & (\prec 0.5, 0.3, 0.4 \succ, 0.3) & (\prec 0.6, 0.4, 0.5 \succ, 0.4) & (\prec 0.7, 0.5, 0.6 \succ, 0.5) \\ \hat{\beta}_2 & (\prec 0.6, 0.1, 0.2 \succ, 0.8) & (\prec 0.5, 0.2, 0.3 \succ, 0.8) & (\prec 0.5, 0.3, 0.4 \succ, 0.7) & (\prec 0.4, 0.4, 0.5 \succ, 0.6) \\ \hat{\beta}_3 & (\prec 0.6, 0.2, 0.2 \succ, 0.1) & (\prec 0.5, 0.2, 0.3 \succ, 0.2) & (\prec 0.6, 0.2, 0.4 \succ, 0.3) & (\prec 0.6, 0.2, 0.4 \succ, 0.4) \\ \hat{\beta}_4 & (\prec 0.6, 0.1, 0.1 \succ, 0.2) & (\prec 0.4, 0.1, 0.3 \succ, 0.3) & (\prec 0.5, 0.4, 0.4 \succ, 0.4) & (\prec 0.6, 0.2, 0.5 \succ, 0.5) \end{pmatrix}.$$

Definition 3.3. Let $\Psi_\sigma^1, \Psi_\sigma^2 \in \Omega_{psvnhss}$ then their union operation $\Psi_\sigma^1 \tilde{\sqcup} \Psi_\sigma^2$ is also a psv-NHSS Ψ_σ^3 such that

$\zeta_T^3(\hat{\beta})(\hat{u}_i) = max\{\zeta_T^1(\hat{\beta})(\hat{u}_i), \zeta_T^2(\hat{\beta})(\hat{u}_i)\}$, $\zeta_I^3(\hat{\beta})(\hat{u}_i) = min\{\zeta_I^1(\hat{\beta})(\hat{u}_i), \zeta_I^2(\hat{\beta})(\hat{u}_i)\}$,
$\zeta_F^3(\hat{\beta})(\hat{u}_i) = min\{\zeta_F^1(\hat{\beta})(\hat{u}_i), \zeta_F^2(\hat{\beta})(\hat{u}_i)\}$, and $\sigma^3(\hat{\beta})(\hat{u}_i) = max\{\sigma^1(\hat{\beta})(\hat{u}_i), \sigma^2(\hat{\beta})(\hat{u}_i)\}$.

Definition 3.4. Let $\Psi_\sigma^1, \Psi_\sigma^2 \in \Omega_{psvnhss}$ then their intersection operation $\Psi_\sigma^1 \tilde{\sqcap} \Psi_\sigma^2$ is also a psv-NHSS Ψ_σ^4 such that

$\zeta_T^4(\hat{\beta})(\hat{u}_i) = min\{\zeta_T^1(\hat{\beta})(\hat{u}_i), \zeta_T^2(\hat{\beta})(\hat{u}_i)\}$, $\zeta_I^4(\hat{\beta})(\hat{u}_i) = max\{\zeta_I^1(\hat{\beta})(\hat{u}_i), \zeta_I^2(\hat{\beta})(\hat{u}_i)\}$,
$\zeta_F^4(\hat{\beta})(\hat{u}_i) = max\{\zeta_F^1(\hat{\beta})(\hat{u}_i), \zeta_F^2(\hat{\beta})(\hat{u}_i)\}$, and $\sigma^4(\hat{\beta})(\hat{u}_i) = min\{\sigma^1(\hat{\beta})(\hat{u}_i), \sigma^2(\hat{\beta})(\hat{u}_i)\}$.

Example 3.5. Taking assumptions from Example 3.2, take psv-NHSS $\Psi_\sigma^1, \Psi_\sigma^2 \in \Omega_{psvnhss}$ with their matrix representations as

$$\Psi_\sigma^1 = \begin{pmatrix} \hat{\mathbb{B}} \setminus \hat{\mathcal{U}} & \tilde{m}_1 & \tilde{m}_2 & \tilde{m}_3 & \tilde{m}_4 \\ \hat{\beta}_1 & (\prec 0.4, 0.2, 0.3 \succ, 0.2) & (\prec 0.5, 0.3, 0.4 \succ, 0.3) & (\prec 0.6, 0.4, 0.5 \succ, 0.4) & (\prec 0.7, 0.5, 0.6 \succ, 0.5) \\ \hat{\beta}_2 & (\prec 0.6, 0.1, 0.2 \succ, 0.8) & (\prec 0.5, 0.2, 0.3 \succ, 0.8) & (\prec 0.5, 0.3, 0.4 \succ, 0.7) & (\prec 0.4, 0.4, 0.5 \succ, 0.6) \\ \hat{\beta}_3 & (\prec 0.6, 0.2, 0.2 \succ, 0.1) & (\prec 0.5, 0.2, 0.3 \succ, 0.2) & (\prec 0.6, 0.2, 0.4 \succ, 0.3) & (\prec 0.6, 0.2, 0.4 \succ, 0.4) \\ \hat{\beta}_4 & (\prec 0.6, 0.1, 0.1 \succ, 0.2) & (\prec 0.4, 0.1, 0.3 \succ, 0.3) & (\prec 0.5, 0.4, 0.4 \succ, 0.4) & (\prec 0.6, 0.2, 0.5 \succ, 0.5) \end{pmatrix}$$

and

$$\Psi_\sigma^2 = \begin{pmatrix} \hat{\mathbb{B}} \setminus \hat{\mathcal{U}} & \tilde{m}_1 & \tilde{m}_2 & \tilde{m}_3 & \tilde{m}_4 \\ \hat{\beta}_1 & (\prec 0.5, 0.1, 0.2 \succ, 0.3) & (\prec 0.6, 0.2, 0.2 \succ, 0.4) & (\prec 0.7, 0.3, 0.4 \succ, 0.5) & (\prec 0.8, 0.4, 0.5 \succ, 0.6) \\ \hat{\beta}_2 & (\prec 0.7, 0.1, 0.1 \succ, 0.9) & (\prec 0.6, 0.1, 0.2 \succ, 0.9) & (\prec 0.6, 0.2, 0.3 \succ, 0.8) & (\prec 0.5, 0.3, 0.4 \succ, 0.7) \\ \hat{\beta}_3 & (\prec 0.7, 0.1, 0.1 \succ, 0.2) & (\prec 0.6, 0.1, 0.2 \succ, 0.3) & (\prec 0.7, 0.1, 0.3 \succ, 0.4) & (\prec 0.7, 0.2, 0.3 \succ, 0.5) \\ \hat{\beta}_4 & (\prec 0.7, 0.1, 0.1 \succ, 0.3) & (\prec 0.5, 0.1, 0.3 \succ, 0.4) & (\prec 0.6, 0.4, 0.4 \succ, 0.5) & (\prec 0.7, 0.2, 0.5 \succ, 0.6) \end{pmatrix}$$

then $\Psi_\sigma^3 = \Psi_\sigma^2$ and $\Psi_\sigma^4 = \Psi_\sigma^1$.

4. Application of psv-NHSS in Investment Selection

This section first proposes an algorithm for real-world DM scenario of investment selection and then authenticates this proposal with the help of real-world application. The steps of algorithm proposed in [30] are followed with partial modifications.

Algorithm 4.1. Let the set $\hat{\mathcal{U}} = \{\hat{\vartheta}_1, \hat{\vartheta}_2, ..., \hat{\vartheta}_s\}$ consists of projects which are likely to be evaluated for investment. A committee of decision-makers is deputed for this evaluation. All the members of the committee have made a mutual consensus on a set of parameters $\hat{\mathcal{E}} = \{\ddot{e}_1, \ddot{e}_2, ..., \ddot{e}_n\}$. In order to ensure reliability of evaluation, the members of the committee further investigate for sub-parametric values of these parameters that are encapsulated in non-overlapping sets $\hat{\mathfrak{W}}_1 = \{\ddot{e}_{11}, \ddot{e}_{12}, ..., \ddot{e}_{1m}\}$, $\hat{\mathfrak{W}}_2 = \{\ddot{e}_{21}, \ddot{e}_{22}, ..., \ddot{e}_{2m}\}$, ..., $\hat{\mathfrak{W}}_n = \{\ddot{e}_{n1}, \ddot{e}_{n2}, ..., \ddot{e}_{nm}\}$. The committee is intended to accomplish the evaluation with the help of HSS environment therefore they compute $\hat{\mathfrak{W}} = \hat{\mathfrak{W}}_1 \times \hat{\mathfrak{W}}_2 \times ... \times \hat{\mathfrak{W}}_n = \{\hat{\omega}_1, \hat{\omega}_2, ..., \hat{\omega}_r\}$ where $r = \prod_{\alpha=1}^{n} |\hat{\mathfrak{W}}_\alpha|$ and the symbol $|\hat{\mathfrak{W}}_\alpha|$ is meant for set cardinality of $\hat{\mathfrak{W}}_\alpha$. Now the committee adopts the following steps to accomplish the evaluation of projects:

Step 1: Construct psv-NHSS $\Psi_\sigma \in \Omega_{psvnhss}$ in accordance with Definition 3.1 as given below:

$$\Psi_\sigma = \left\{ \left(\hat{\omega}, \left\{ \left(\frac{\hat{\vartheta}_i}{\psi(\hat{\omega})(\hat{\vartheta}_i)}, \sigma(\hat{\omega})(\hat{\vartheta}_i) \right) : \hat{\vartheta}_i \in \hat{\mathcal{U}} \right\} \right) : \hat{\omega} \in \hat{\mathfrak{W}} \right\}$$

which is likely to be assessed by n decision-makers for the evaluation of $\hat{\vartheta}_i \in \hat{\mathcal{U}}$ corresponding to attribute-valued tuple $\hat{\omega} \in \hat{\mathfrak{W}}$.

Step 2: Represent constructed psv-NHSS Ψ_σ in matrix notation such that the parametric-tuples are arranged in rows and the alternatives are arranged in columns i.e., $\mathfrak{M}_{r\times s}, s, r \in \mathbb{N}$.

$$\mathfrak{M}_{r\times s} = \begin{pmatrix} \hat{\theta}_{11} & \hat{\theta}_{12} & \cdots & \hat{\theta}_{1s} \\ \hat{\theta}_{21} & \hat{\theta}_{22} & \cdots & \hat{\theta}_{2s} \\ \hat{\theta}_{31} & \hat{\theta}_{32} & \cdots & \hat{\theta}_{3s} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\theta}_{r1} & \hat{\theta}_{r2} & \cdots & \hat{\theta}_{rs} \end{pmatrix}$$

where

$\hat{\theta}_{11} = \left(< \zeta_T(\hat{\omega}_1)(\hat{\vartheta}_1), \zeta_I(\hat{\omega}_1)(\hat{\vartheta}_1), \zeta_F(\hat{\omega}_1)(\hat{\vartheta}_1) >, \sigma(\hat{\omega}_1)(\hat{\vartheta}_1) \right)$,

$\hat{\theta}_{12} = \left(< \zeta_T(\hat{\omega}_1)(\hat{\vartheta}_2), \zeta_I(\hat{\omega}_1)(\hat{\vartheta}_2), \zeta_F(\hat{\omega}_1)(\hat{\vartheta}_2) >, \sigma(\hat{\omega}_1)(\hat{\vartheta}_2) \right)$,

....,

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,$$
$$\hat{\theta}_{1s} = \Big(<\zeta_T(\hat{\omega}_1)(\hat{\vartheta}_s), \zeta_I(\hat{\omega}_1)(\hat{\vartheta}_s), \zeta_F(\hat{\omega}_1)(\hat{\vartheta}_s)>, \sigma(\hat{\omega}_1)(\hat{\vartheta}_s)\Big),$$
$$\hat{\theta}_{21} = \Big(<\zeta_T(\hat{\omega}_2)(\hat{\vartheta}_1), \zeta_I(\hat{\omega}_2)(\hat{\vartheta}_1), \zeta_F(\hat{\omega}_2)(\hat{\vartheta}_1)>, \sigma(\hat{\omega}_2)(\hat{\vartheta}_1)\Big),$$
$$\hat{\theta}_{22} = \Big(<\zeta_T(\hat{\omega}_2)(\hat{\vartheta}_2), \zeta_I(\hat{\omega}_2)(\hat{\vartheta}_2), \zeta_F(\hat{\omega}_2)(\hat{\vartheta}_2)>, \sigma(\hat{\omega}_2)(\hat{\vartheta}_2)\Big),$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,$$
$$\hat{\theta}_{2s} = \Big(<\zeta_T(\hat{\omega}_2)(\hat{\vartheta}_s), \zeta_I(\hat{\omega}_2)(\hat{\vartheta}_s), \zeta_F(\hat{\omega}_2)(\hat{\vartheta}_s)>, \sigma(\hat{\omega}_2)(\hat{\vartheta}_s)\Big),$$
$$\hat{\theta}_{31} = \Big(<\zeta_T(\hat{\omega}_3)(\hat{\vartheta}_1), \zeta_I(\hat{\omega}_3)(\hat{\vartheta}_1), \zeta_F(\hat{\omega}_3)(\hat{\vartheta}_1)>, \sigma(\hat{\omega}_3)(\hat{\vartheta}_1)\Big),$$
$$\hat{\theta}_{32} = \Big(<\zeta_T(\hat{\omega}_3)(\hat{\vartheta}_2), \zeta_I(\hat{\omega}_3)(\hat{\vartheta}_2), \zeta_F(\hat{\omega}_3)(\hat{\vartheta}_2)>, \sigma(\hat{\omega}_3)(\hat{\vartheta}_2)\Big),$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,$$
$$\hat{\theta}_{3s} = \Big(<\zeta_T(\hat{\omega}_3)(\hat{\vartheta}_s), \zeta_I(\hat{\omega}_3)(\hat{\vartheta}_s), \zeta_F(\hat{\omega}_3)(\hat{\vartheta}_s)>, \sigma(\hat{\omega}_3)(\hat{\vartheta}_s)\Big),$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,$$
$$\hat{\theta}_{r1} = \Big(<\zeta_T(\hat{\omega}_r)(\hat{\vartheta}_1), \zeta_I(\hat{\omega}_r)(\hat{\vartheta}_1), \zeta_F(\hat{\omega}_r)(\hat{\vartheta}_1)>, \sigma(\hat{\omega}_r)(\hat{\vartheta}_1)\Big),$$
$$\hat{\theta}_{r2} = \Big(<\zeta_T(\hat{\omega}_r)(\hat{\vartheta}_2), \zeta_I(\hat{\omega}_r)(\hat{\vartheta}_2), \zeta_F(\hat{\omega}_r)(\hat{\vartheta}_2)>, \sigma(\hat{\omega}_r)(\hat{\vartheta}_2)\Big),$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,$$
$$\hat{\theta}_{rs} = \Big(<\zeta_T(\hat{\omega}_r)(\hat{\vartheta}_s), \zeta_I(\hat{\omega}_r)(\hat{\vartheta}_s), \zeta_F(\hat{\omega}_r)(\hat{\vartheta}_s)>, \sigma(\hat{\omega}_r)(\hat{\vartheta}_s)\Big).$$

Step 3: Construct the core matrix $\Xi_{r\times s}$ and compute its all entries with the help of formula i.e., $\hat{\varpi}_{\hat{\omega}_i}(\hat{\vartheta}_j) = \Big(\zeta_T(\hat{\omega}_i)(\hat{\vartheta}_j) + \zeta_I(\hat{\omega}_i)(\hat{\vartheta}_j) + \zeta_F(\hat{\omega}_i)(\hat{\vartheta}_j)\Big) - \sigma(\hat{\omega}_i)(\hat{\vartheta}_j)$ where ${}_1i^r$ and ${}_1j^s$,

$$\Xi_{r\times s} = \begin{pmatrix} \hat{\varpi}_{\hat{\omega}_1}(\hat{\vartheta}_1) & \hat{\varpi}_{\hat{\omega}_1}(\hat{\vartheta}_2) & \ldots & \hat{\varpi}_{\hat{\omega}_1}(\hat{\vartheta}_s) \\ \hat{\varpi}_{\hat{\omega}_2}(\hat{\vartheta}_1) & \hat{\varpi}_{\hat{\omega}_2}(\hat{\vartheta}_2) & \ldots & \hat{\varpi}_{\hat{\omega}_2}(\hat{\vartheta}_s) \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\varpi}_{\hat{\omega}_r}(\hat{\vartheta}_1) & \hat{\varpi}_{\hat{\omega}_r}(\hat{\vartheta}_2) & \ldots & \hat{\varpi}_{\hat{\omega}_r}(\hat{\vartheta}_s) \end{pmatrix}.$$

Step 4: On the basis of computed entries of $\Xi_{r\times s}$, determine $\mathfrak{B}^{\max}\left(\hat{\vartheta}_j\right)$ (Max. limit), $\mathfrak{B}^{\min}\left(\hat{\vartheta}_j\right)$ (Min. limit), and $\mathfrak{Q}\left(\hat{\vartheta}_j\right)$ (scoring value) of all $\hat{\vartheta}_j$:

$$\mathfrak{B}^{\min}\left(\hat{\vartheta}_j\right) = \sum_{i=1}^{r}\Big(1 - \hat{\varpi}_{\hat{\omega}_i}(\hat{\vartheta}_j)\Big)^2, \mathfrak{B}^{\max}\left(\hat{\vartheta}_j\right) = \sum_{i=1}^{r}\Big(\hat{\varpi}_{\hat{\omega}_i}(\hat{\vartheta}_j)\Big)^2,$$

$$\mathfrak{Q}\left(\hat{\vartheta}_j\right) = \mathfrak{B}^{\max}\left(\hat{\vartheta}_j\right) + \mathfrak{B}^{\min}\left(\hat{\vartheta}_j\right).$$

Step 5: Find the maximum score by considering the following formulation and opt it as optimum selection.

$$\hat{\vartheta}_s = Max\left\{\mathfrak{Q}\left(\hat{\vartheta}_1\right), \mathfrak{Q}\left(\hat{\vartheta}_2\right), ..., \mathfrak{Q}\left(\hat{\vartheta}_s\right)\right\}.$$

The flowchart of Algorithm 4.1 is presented in Figure 1. The following example is presented

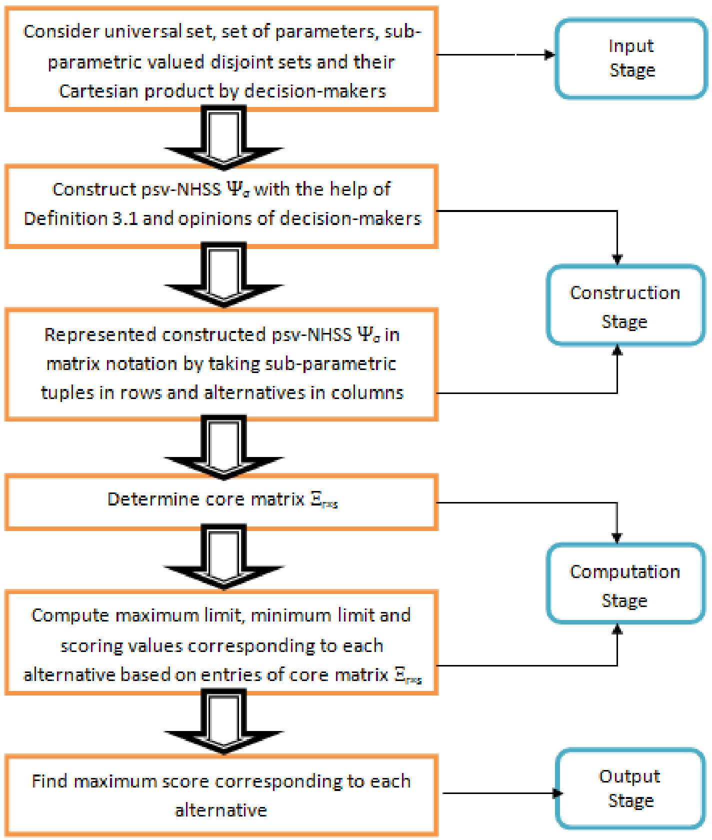

Figure 1. Flowchart of Algorithm 4.1

for the validation and implementation of above Algorithm 4.1. The problem statement, parameters and their sub-parametric values are opted from the numerical example discussed by Ghorabaee et al. [31] (see pg. 11-12) with partial modifications.

Example 4.2. A power company wants to invest a particular capital in a particular hydro-electric power station project. There are five such projects which constitute an initial universe $\hat{\mathcal{U}} = \{\hat{\vartheta}_1, \hat{\vartheta}_2, ..., \hat{\vartheta}_5\}$. The company constitutes a committee consisting of some external experts and company employees with expertise in project evaluation. The committee decides the parameters like $\ddot{e}_1$ = Realization expenditure (d/kW), $\ddot{e}_2$ = Power output (MW), $\ddot{e}_3$ = Life span (years) and $\ddot{e}_4$ = capacity quotient (%-age) for this evaluation. The brief description of prescribed parameters is tabulated in Table 1. After keen observation and investigation, the sub-parametric values corresponding to these parameters are reported in the form of disjoint sets $\hat{\mathfrak{W}}_1 = \{\ddot{e}_{11} = 3100, \ddot{e}_{12} = 3300, \ddot{e}_{13} = 4000\}$, $\hat{\mathfrak{W}}_2 = \{\ddot{e}_{21} = 15, \ddot{e}_{22} = 16, \ddot{e}_{23} = 18\}$, $\hat{\mathfrak{W}}_3 = \{\ddot{e}_{31} = 20, \ddot{e}_{32} = 25, \ddot{e}_{33} = 30\}$ and $\hat{\mathfrak{W}}_4 = \{\ddot{e}_{41} = 35, \ddot{e}_{42} = 40, \ddot{e}_{43} = 45\}$ respectively. For convenience, the committee further evaluates the sub-parametric values and the values $\ddot{e}_{11} = 3100, \ddot{e}_{12} = 3300$ are preferred in $\hat{\mathfrak{W}}_1$, $\ddot{e}_{22} = 16$, $\ddot{e}_{23} = 18$; $\ddot{e}_{33} = 30$ and $\ddot{e}_{43} = 45$ are preferred in $\hat{\mathfrak{W}}_2$, $\hat{\mathfrak{W}}_3$ and $\hat{\mathfrak{W}}_4$ respectively. Therefore $\hat{\mathfrak{W}} = \hat{\mathfrak{W}}_1 \times \hat{\mathfrak{W}}_2 \times \hat{\mathfrak{W}}_4 \times \hat{\mathfrak{W}}_4 = \{\hat{\omega}_1, \hat{\omega}_2, \hat{\omega}_3, \hat{\omega}_4\}$. The members of the committee provides their expert opinions for each project corresponding to these opted sub-parametric tuples $\hat{\omega}_i, i = 1, 2, 3, 4$ in the form of psv-NHSS.

Table 1. Brief description of opted parameters

Parameters	Description
Realization expenditure	It includes all the expenditures that are to meet before the initiation of income returns services. It is usually measured in dollars per kilo watts (d/kW).
Power output	It is power that is being generated by a particular power station. It is usually measured in Mega watts (MW).
Life span	It is an expected period for the utilization of services. It is measured in years.
Capacity quotient	It is the ration of actual output to the potential output of a particular power station. It is calculated in percentage.

Step 1 A psv-NHSS Ψ_σ is constructed by taking into account the expert opinions of decision makers that is provided as

Ψ_σ	$\hat{\vartheta}_1$	$\hat{\vartheta}_2$	$\hat{\vartheta}_3$	$\hat{\vartheta}_4$	$\hat{\vartheta}_5$
$\hat{\omega}_1$	$(\prec 0.7, 0.5, 0.4 \succ, 0.2)$	$(\prec 0.6, 0.4, 0.4 \succ, 0.3)$	$(\prec 0.5, 0.5, 0.5 \succ, 0.4)$	$(\prec 0.4, 0.6, 0.5 \succ, 03)$	$(\prec 0.3, 0.7, 0.6 \succ, 0.2)$
$\hat{\omega}_2$	$(\prec 0.6, 0.4, 0.3 \succ, 0.3)$	$(\prec 0.5, 0.7, 0.4 \succ, 0.4)$	$(\prec 0.4, 0.8, 0.5 \succ, 0.5)$	$(0.5, 0.7, 0.7 \succ, 0.4)$	$(\prec 0.2, 0.8, 0.8 \succ, 0.4)$
$\hat{\omega}_3$	$(\prec 0.5, 0.3, 0.4 \succ, 0.4)$	$(\prec 0.4, 0.8, 0.6 \succ, 0.5)$	$(\prec 0.3, 0.9, 0.6 \succ, 0.6)$	$(\prec 0.4, 0.6, 0.3 \succ, 0.5)$	$(\prec 0.3, 0.3, 0.3 \succ, 0.2)$
$\hat{\omega}_4$	$(\prec 0.3, 0.5, 0.6 \succ, 0.5)$	$(\prec 0.8, 0.3, 0.7 \succ, 0.6)$	$(\prec 0.3, 0.5, 0.8 \succ, 0.7)$	$(\prec 0.9, 0.3, 0.4 \succ, 0.6)$	$(\prec 0.5, 0.2, 0.1 \succ, 0.3)$

Step 2

$$\mathfrak{M}_{5\times 4} = \begin{pmatrix} (\prec 0.7, 0.5, 0.4 \succ, 0.2) & (\prec 0.6, 0.4, 0.3 \succ, 0.3) & (\prec 0.5, 0.3, 0.4 \succ, 0.4) & (\prec 0.3, 0.5, 0.6 \succ, 0.5) \\ (\prec 0.6, 0.4, 0.4 \succ, 0.3) & (\prec 0.5, 0.7, 0.4 \succ, 0.4) & (\prec 0.4, 0.8, 0.6 \succ, 0.5) & (\prec 0.8, 0.3, 0.7 \succ, 0.6) \\ (\prec 0.5, 0.5, 0.5 \succ, 0.4) & (\prec 0.4, 0.8, 0.5 \succ, 0.5) & (\prec 0.3, 0.9, 0.6 \succ, 0.6) & (\prec 0.3, 0.5, 0.8 \succ, 0.7) \\ (\prec 0.4, 0.6, 0.5 \succ, 03) & (\prec 0.5, 0.7, 0.7 \succ, 0.4) & (\prec 0.4, 0.6, 0.3 \succ, 0.5) & (\prec 0.9, 0.3, 0.4 \succ, 0.6) \\ (\prec 0.3, 0.7, 0.6 \succ, 0.2) & (\prec 0.2, 0.8, 0.8 \succ, 0.4) & (\prec 0.3, 0.3, 0.3 \succ, 0.2) & (\prec 0.5, 0.2, 0.1 \succ, 0.3) \end{pmatrix}.$$

Step 3

$$\Xi_{3\times 5} = \begin{pmatrix} 1.4 & 1.0 & 0.8 & 0.9 \\ 1.3 & 1.2 & 1.3 & 1.2 \\ 1.1 & 1.2 & 1.2 & 0.9 \\ 1.2 & 1.5 & 0.8 & 1.0 \\ 1.4 & 1.4 & 0.7 & 0.5 \end{pmatrix}.$$

Step 4 The values of $\mathfrak{B}^{\max}(\hat{\vartheta}_j)$, $\mathfrak{B}^{\min}(\hat{\vartheta}_j)$ and $\mathfrak{Q}(\hat{\vartheta}_j)$ for $j = 1, 2, 3, 4, 5$ are given in the below table

	$\hat{\vartheta}_1$	$\hat{\vartheta}_2$	$\hat{\vartheta}_3$	$\hat{\vartheta}_4$	$\hat{\vartheta}_5$
$\mathfrak{B}^{\min}(\hat{\vartheta}_j)$	0.21	0.26	0.10	0.93	0.66
$\mathfrak{B}^{\max}(\hat{\vartheta}_j)$	4.41	6.26	4.90	5.33	4.66
$\mathfrak{Q}(\hat{\vartheta}_j)$	4.62	6.52	5.00	6.26	5.32

Step 5 According to Figure 2, it is vivid that the maximum score is gained by $\hat{\vartheta}_2$ therefore it is opted as the optimum selection. The ranking of projects is $\hat{\vartheta}_2 > \hat{\vartheta}_4 > \hat{\vartheta}_5 > \hat{\vartheta}_3 > \hat{\vartheta}_1$.

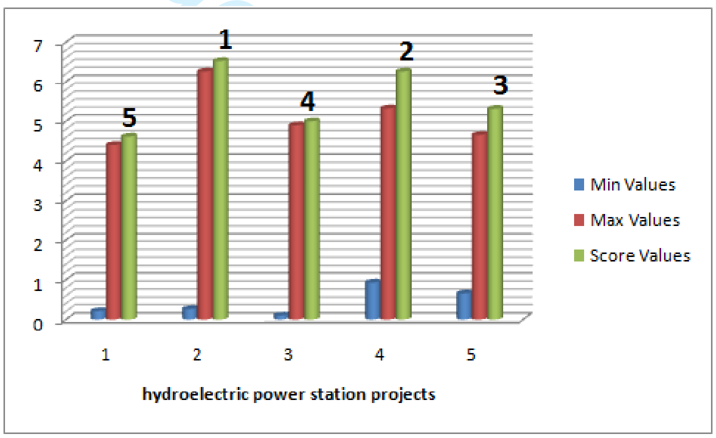

FIGURE 2. Score values corresponding to projects

5. Comparison Analysis

There are many situations where thought of just attributes is not adequate, all opted particular attributes are additionally divided into their relevant sub-attribute valued non-overlapping sets. The DM procedures in view of the majority of existing models are insufficient for such cases. Thusly, our proposed model not just underlines on the due status of such apportioning of attributes but additionally works with the decision-makers to deal with day to day issues without any difficulty. Some advantages of the adopted model i.e. psv-NHSS are outlined as:

(i) The introduced approach took the significance of the idea of possibility alongside the sv-NHSS to deal with current decision-making issues. The considered possibility degree mirrors the possibility of the existence of the level of acknowledgment and dispensation;

along these lines, this association has tremendous potential in the genuine depiction inside the space of computational incursions.

(ii) As the proposed structure emphasizes on in-depth study of attributes (i.e. further partitioning of attributes) rather than focussing on attributes merely therefore it makes the decision-making process better, flexible and more reliable.

(iii) It contains the characteristics and properties of the existing structures i.e. pFSS, pIFSS, pNSS, pIFHSS etc., so it is not unreasonable to call it the generalized form of all these structures.

Moreover, the Table 2 presents the structural comparison of proposed model with relevant existing models to depict its advantageous aspects and flexibility.

TABLE 2. Advantageous aspects of proposed model over relevant existing models

Models	Membership grade	Non-membership grade	Indeterminant type of grade	Single-argument approx-imate mapping	Multi-argument approx-imate mapping	Possibility-degree setting	Project selection ranking
NS [1]	✓	✓	✓	×	×	×	×
FS [2]	✓	×	×	×	×	×	×
IFS [3]	✓	✓	×	×	×	×	×
SS [5]	×	×	×	✓	×	×	×
NSS [6]	✓	✓	✓	✓	×	×	×
HSS [7]	×	×	×	✓	✓	×	×
NHSS [7]	✓	✓	✓	✓	✓	×	×
pFSS [24]	✓	×	×	✓	×	✓	×
pIFSS [25]	✓	✓	×	✓	×	✓	×
pNSS [26]	✓	✓	✓	✓	×	✓	×
pIFHSS [29]	✓	✓	×	✓	✓	✓	×
Proposed Model	✓	✓	✓	✓	✓	✓	✓

6. Conclusions

In this research, authors have developed a novel structure called psv-NHSS that has the ability to address the limitations of existing models like FS, IFS, NS, sv-NS, SS, NSS, sv-NSS, pFSS, pIFSS, pNSS and HSS by introducing its own approximate mapping known as sv-NHS multi-argument approximate mapping. This mapping considers the Cartesian product of sub-classes with respect to attributes as its domain and then provides approximate elements along with attachment of a novel grade called possibility grade. This grade assesses the uncertain nature of sv-NHS numbers as a whole. Additionally, the psv-NHSS is a more flexible as it is adequate with the scenarios: (i) which have mandatory partitioning of attributes, (ii) which assigns membership, non-membership and indeterminate grades to each alternative for managing its uncertainty corresponding to each attribute and (iii) which is enable to assess the uncertain nature of approximate elements with possibility degree-based setting, as a whole. After characterizing the essential aggregations of psv-NHSS, a decision-support system is constructed which is further applied in daily-life problem for the selection of hydroelectric power station project for investment purpose. In psv-NHSS, parameters and their respective sub-parametric tuples are considered without attachment of any uncertain component but in some situations we encounter the uncertain attitudes of parameters and their respective sub-parametric tuples. In such cases, a suitable uncertain grade (membership or non-membership or indeterminacy) is attached with parameters and with sub-parametric tuples as well. Thus proposed model psv-NHSS have limitations regarding these situations therefore in order to address these insufficiencies, the future work may include the development of new structures like fuzzy parameterized psv-NHSS, intuitionistic fuzzy parameterized psv-NHSS and neutrosophic parameterized psv-NHSS. Moreover, some algebraic structures of linear algebra and functional analysis may also be developed with the help of this proposed model.

Conflicts of Interest: The authors declare no conflict of interest.

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