Mapping on Interval Complex Neutrosophic Soft Sets

Article · December 2022

DOI: 10.54216/IJNS.190406

Faisal Al-Sharqi
University of Anbar

Ashraf Al-Quran
King Faisal University

Mapping on Interval Complex Neutrosophic Soft Sets

Faisal Al-Sharqi[1,2,*], Abd Ghafur Ahmad[1], Ashraf Al-Quran[3]

[1] Department of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, Bangi 43600, Selangor, Malaysia;

[2]Department of Mathematics, Faculty of Education for Pure Sciences, University of Anbar, Ramadi, Anbar, Iraq

[3] Preparatory Year Deanship, King Faisal University, Hofuf 31982, Al Ahsa, Saudi Arabia

Emails: faisal.ghazi@uoanbar.edu.iq ; ghafur@ukm.edu.my ; aalquran@kfu.edu.sa

Abstract

The neutrosophic idea is a fertile environment adaptable to different mathematical tools. This paper aims to introduce the mapping of complex interval neutrosophic soft sets (MI-CNSSs). Further, the images and inverse images of complex interval neutrosophic soft sets and their properties are explored. These will be supported by concrete examples, and this paper will present some theorems about complex interval neutrosophic soft images and inverse images.

Keywords: Soft set; Neutrosophic set; Complex neutrosophic set; Interval complex neutrosophic soft set;

1. Introduction

Smarandache [1] defined a neutrophic set (NS) as a generalization of [2,3] to be more suitable in dealing with various mathematical models such as topology, graphs, and algebra, so several researchers investigated the generalisation of the concept of a neutrosophic set with these mathematical models. This theory can cope with ambiguous, indeterminate, and discordant information when the indeterminacy is quantified clearly and truth, indeterminacy, and falsity membership are all entirely independent. So it has the ability to solve many real-life problems in the fields of medicine, economics, engineering, the environment, and so on. In 1999, Molodtsov [4] introduced the concept of the soft set (SS) as a mathematical tool for dealing with uncertainties. The notion of the neutrosophic soft set has been proposed by Maji [5] to blend the forces of both neutrosophic sets and soft sets. The researchers [6-8] made numerous contributions based on ideas NS and SS that contributed to the development of a mechanism for dealing with uncertain data in a variety of life applications. After Ramot [9] presented a complex fuzzy set, many concepts were based on this concept [10–12]. Recently, Al-Sharqi et al. [13–15] developed the above concepts and presented some of the works on interval complex neutrosophic soft sets (ICNSS). The concept of mapping has been widely applied to fuzzy theory and its extensions, beginning with the SS proposed by Kharal and Ahmad. Dependent on it, other concepts emerged based on this idea [16,17], and as a follow-up to these studies in this work, we will apply the mapping idea on I-CNSS by define the images and inverse images of interval complex neutrosophic soft sets (ICNSSs) and their fundamental operations, which will then lead to the concept of mapping on classes of interval complex neutrosophic soft sets. We also present the fulfilment of several of the concept's basic functions and features.

This article is organized as follows: In Section 2, we have briefly redefined each of NS, I-CNSS, and some properties of I-CNSS. In Section 3, we presented in detail each of the images and inverse images of complex interval neutrosophic soft sets, and their properties are explored. We also gave some numerical

Doi: https://doi.org/10.54216/IJNS.190406
Received: May 21, 2022 Accepted: October 15, 2022

examples to illustrate the work of these concepts, as well as some theories with their proofs. Section 4 contains this work's conclusion and recommendations for future studies.

2.Preliminaries

We will retrieve some fundamental ideas of the NS, I-CNSS, and some of their properties.

Definition 1.[1]. A N is a neutrosophic set (NS) on universe of a non-empty universe U and defined as $N=\{\langle u, T_N(u), I_N(u), F_N(u)\rangle\}$, where $T_N(u), I_N(u), F_N(u)$ are truth ,indeterminacy and falsity memberships respectively, such that $0 \leq T_N(u) + I_N(u) + F_N(u) \leq 3$.

Definition 2. [13] Let U be a non-empty universe with generic elements denoted u and $A \subseteq E$ when E be a set of all parameters .Thus an interval-complexneutrosophic soft set (shortly I-CNSS) (N, A) in U is defined by the following membership functons which are an interval-membership function $T_{N_a}(u)$, indeterminacy interval-membership function $I_{N_a}(u)$, and an interval non membership function $F_{N_a}(u)$ as a follows:

$$(N,A)=\left\{\left(a, \frac{T_{N_a}(u)=t_{N_a}(u).e^{j\mu_{N_a}(u)}, I_{N_a}(u)=i_{N_a}(u).e^{j\omega_{N_a}(u)}, F_{N_a}(u)=f_{N_a}(u).e^{j\varphi_{N_a}(u)}}{u}\right); u \in U, a \in A\right\}$$

where the interval-amplitude terms $t_{N_a}(u), i_{N_a}(u), f_{N_a}(u)$ can be written as $t_{N_a}(u), = [t^L_{N_a}(u), t^U_{N_a}(u)]$, $i_{N_a}(u) = [i^L_{N_a}(u), i^U_{N_a}(u)]$, and $f_{N_a}(u) = [f^L_{N_a}(u), f^U_{N_a}(u)]$ where $t^L_{N_a}(u)$, $i^L_{N_a}(u)$,$f^L_{N_a}(u)$ denote the lower-bounds, while $t^U_{N_a}(u)$, $i^U_{N_a}(u)$, $f^U_{N_a}(u)$ indicate the upper-bounds.
And in like manner, for the interval-terms phases $\mu_N(u) = [\mu^L_{N_a}(u), \mu^U_{N_a}(u)]$, $\omega_{N_a}(u) = [\omega^L_{N_a}(u), \omega^U_{N_a}(u)]$ and $\varphi_{N_a}(u) = [\varphi^L_{N_a}(u), \varphi^U_{N_a}(u)]$.

Definition 3. [14] Let (N, A) and (M, B) two I-CNSSs over a non-empty universe U. Then we present a basic operations complement, intersection, andunion as follows:

i. A complement of an I-CNSS (N, A) is indicate by $(N, A)^c$ and is defined as

$$(N,A)^c=\left\{\left(a, \frac{T_{N^c_a}(u)=t_{N^c_a}(u).e^{j\mu_{N^c_a}(u)}, I_{N^c_a}(u)=i_{N^c_a}(u).e^{j\omega_{N^c_a}(u)}, F_{N^c_a}(u)=f_{N^c_a}(u).e^{j\varphi_{N^c_a}(u)}}{u}\right); u \in U, a \in A\right\}$$

where

$T_{N^c_a}(u) = F_{N_a}(u)$ and $\mu_{N^c_a}(u) = 2\pi - \mu_{N_a}(u)$. Similarly,$i_{N^c_a}(u) = \left(i^L_{N^c_a}(u), i^U_{N^c_a}(u)\right)$where $i^L_{N^c_a}(u) = 1 - i^U_{N^c_a}(u)$ and $i^U_{N^c_a}(u) = 1 - i^L_{N^c_a}(u)$ with the phase term $\omega_{N^c}(u) = 2\pi - \omega_{N_a}(u)$. Also $f_{N^c}(u) = t_{N_a}(u)$, while the phase term $\varphi_{N^c_a}(u) = 2\pi - \varphi_{N_a}(u)$.

ii. For union and intersection of two I-CNSSs (M, A) and (N, B) are an I-CNSSs indicate as $(M, A) \cup (\cap)(N, B)$ and all interval-membeship functions of the I-CNSSs defined as

$$T_{M_a \cup(\cap) N_b}(u) = \left[t^L_{M_a \cup(\cap) N_b}(u), t^U_{M_a \cup(\cap) N_b}(u)\right].\, e^{j2\pi\mu_{M_a \cup(\cap) N_b}(u)}$$
$$I_{M_a \cup(\cap) N_b}(u) = \left[i^L_{M_a \cup(\cap) N_b}(u), i^U_{M_a \cup(\cap) N_b}(u)\right].\, e^{j2\pi\omega_{M_a \cup(\cap) N_b}(u)}$$
$$F_{M_a \cup(\cap) N_b}(u) = \left[f^L_{M_a \cup(\cap) N_b}(u), f^U_{M_a \cup(\cap) N_b}(u)\right].\, e^{j2\pi\varphi_{M_a \cup(\cap) N_b}(u)}$$

where

$$t^L_{M_a \cup(\cap) N_b}(u) = \vee(\wedge)\left(t^L_{M_a}(u), t^L_{N_a}(u)\right), t^U_{M_a \cup(\cap) N_b}(u) = \vee(\wedge)\left(t^U_{M_a}(u), t^U_{N_a}(u)\right)$$
$$i^L_{M_a \cup(\cap) N_b}(u) = \wedge(\vee)\left(i^L_{M_a}(u), i^L_{N_a}(u)\right), i^U_{M_a \cup(\cap) N_b}(u) = \wedge(\vee)\left(i^U_{M_a}(u), i^U_{N_a}(u)\right)$$
$$f^L_{M_a \cup(\cap) N_b}(u) = \wedge(\vee)\left(f^L_{M_a}(u), f^L_{N_a}(u)\right), f^U_{M_a \cup(\cap) N_b}(u) = \wedge(\vee)\left(f^U_{M_a}(u), f^U_{N_a}(u)\right).$$

For phase interval-valued terms union and intersection are the same as defined for the amplitude interval-valued terms union and intersection and two symbols ∨, ∧ indicate respectively max, min operators.

Doi: https://doi.org/10.54216/IJNS.190406

Received: May 21, 2022 Accepted: October 15, 2022

3. Mapping on Interval-Complex Neutrosophic Soft Sets (MI-CNSS)

Definition 4. Let $(\widetilde{U,E})$ and $(\widetilde{V,E^{`}})$ be an ICNS-classes. Let $r{:}U \longrightarrow V$ and $s{:}E \longrightarrow E^{`}$ be mappings. Then a mapping $F{:}\,(\widetilde{U,E}) \longrightarrow (\widetilde{V,E^{`}})$ is defined as:
For an ICNSS (H,A) in $(\widetilde{U,E})$ and $(F(H,A),M)$where $M = s(E) \subseteq E^{`}$is an ICNSS in $(V,E^{`})$, then we get the following:

$$F(H,A)(\beta)(v) = \begin{cases} \cup_{x\in r^{-1}(v)} \left(\cup_{\alpha\in s^{-1}(\beta)\cap A} H(\alpha)\right) \; if \; r^{-1}(y) \text{ and } s^{-1}(\beta) \cap A \neq \emptyset \\ ([0,0],[1,1],[1,1]) \qquad other\ wise \end{cases}$$

For $x \in r^{-1}(v)$, $\beta \in M \subseteq E^{`}$,$v \in V$ and $\forall \alpha \in s^{-1}(\beta) \cap A$. Then $(F(H,A),M)$
is called an ICNS-image of the ICNSS (H,A). If $E^{`} = M$ then we will write $(F(H,A),M)$ as $F(H,A)$.

Definition 5. Let $(\widetilde{U,E})$ and $(\widetilde{V,E^{`}})$ be an interval complex neutrosophic soft classes in short (ICNSS-Classes). Let $r{:}U \longrightarrow Y$ and $s{:}E \longrightarrow E^{`}$ be mappings. Then a mapping $F^{-1}: (\widetilde{V,E^{`}}) \longrightarrow (\widetilde{U,E})$ is defined as follows:
For an ICNSS (G,A) in $(V,E^{`})$,$(f^{-1}(G,A),N)$,where $N = s^{-1}(A)$, is an ICNSS in (U,E), and given by:

$$F^{-1}(G,A)(\alpha)(u) = \begin{cases} G(s(\alpha))(r(u)) & if\ s(\alpha) \in A \\ ([0,0],[1,1],[1,1]) & other\ wise \end{cases}$$

for $\alpha \in N \subseteq E$ and $u \in U$ then $(F^{-1}(G,A),N)$ is called an ICNS-inverse image of the ICNSS(G,A). If $E = N$ then we will write $(F^{-1}(G,A),M)$ as $F^{-1}(G,A)$.

The two Definitions above are illustrated as follows in the next example.

Example 1: Let $U = \{u_1,u_2,u_3\}$,$V = \{v_1,v_2,v_3\}$ and let $A \subseteq E = \{e_1,e_2,e_3\}$ and $A^{`} \subseteq E^{`} = \{e^{`}_1,e^{`}_2,e^{`}_3\}$.

Suppose that $(\widetilde{U,A})$ and $(\widetilde{V,A^{`}})$ are interval complex neutrosophic soft classes defined as: $r{:}U \longrightarrow V$,$s{:}A \longrightarrow A^{`}$ as follows:
$r(u_1) = y_1$,$r(u_2) = y_3$, $r(u_3) = y_2$ and
$s(e_1) = \acute{e}_2$, $s(e_2) = \acute{e}_1$, $s(e_3) = \acute{e}_1$.

Let (H,A) and $(G,A^{`})$ be two I-CNSSs over U and Vrespectively such that.

$$(H,A) =$$

$$\left\{\left\{e_1,\left(\frac{\left\langle [0.3,0.4].e^{j2\pi\left[\frac{2}{12},\frac{3}{12}\right]},[0.2,0.3].e^{j2\pi\left[\frac{3}{12},\frac{5}{12}\right]},[0,6,0.7].e^{j2\pi\left[\frac{9}{12},\frac{11}{12}\right]}\right\rangle}{u_1}\right),\right.\right.$$

$$\left(\frac{\left\langle [0.6,0.9].e^{j2\pi\left[\frac{7}{12},\frac{9}{12}\right]},[0.5,0.6].e^{j2\pi\left[\frac{5}{12},\frac{7}{12}\right]},[0,3,0.4].e^{j2\pi\left[0,\frac{1}{12}\right]}\right\rangle}{u_2}\right)$$

$$\left.,\left(\frac{\left\langle [0.5,0.9].e^{j2\pi\left[\frac{7}{12},\frac{10}{12}\right]},[0.2,0.2].e^{j2\pi\left[\frac{1}{12},\frac{2}{12}\right]},[0.2,0.3].e^{j2\pi\left[0,\frac{1}{12}\right]}\right\rangle}{u_3}\right)\right\},$$

$$\left\{e_2,\left(\frac{\left\langle [0.1,0.3].e^{j2\pi\left[\frac{1}{12},\frac{3}{12}\right]},[0.4\ ,0.8].e^{j2\pi\left[\frac{7}{12},\frac{11}{12}\right]},[0.8\ ,0.9].e^{j2\pi\left[\frac{8}{12},\frac{12}{12}\right]}\right\rangle}{u_1}\right),\right.$$

Doi: https://doi.org/10.54216/IJNS.190406
Received: May 21, 2022 Accepted: October 15, 2022

$$\left(\frac{\left\langle [0.4,0.7].\, e^{j2\pi\left[\frac{7}{12},\frac{9}{12}\right]}, [0.3,0.7].\, e^{j2\pi\left[\frac{4}{12},\frac{9}{12}\right]}, [0.2,0.6].\, e^{j2\pi\left[\frac{6}{12},\frac{9}{12}\right]} \right\rangle}{u_2}\right)$$

$$,\left(\frac{\left\langle [0.3,0.6].\, e^{j2\pi\left[\frac{5}{12},\frac{10}{12}\right]}, [0.5,0.8].\, e^{j2\pi\left[\frac{7}{12},\frac{9}{12}\right]}, [0.1,0.3].\, e^{j2\pi\left[\frac{1}{12},\frac{4}{12}\right]} \right\rangle}{u_3}\right)\Bigg\},$$

$$\Bigg\{e_3, \left(\frac{\left\langle [0.2,0.3].\, e^{j2\pi\left[\frac{2}{12},\frac{5}{12}\right]}, [0.2\,,0.7].\, e^{j2\pi\left[\frac{7}{12},\frac{10}{12}\right]}, [0\,,0.1].\, e^{j2\pi\left[\frac{9}{12},\frac{11}{12}\right]} \right\rangle}{u_1}\right),$$

$$\left(\frac{\left\langle [0.3,0.8].\, e^{j2\pi\left[\frac{4}{12},\frac{7}{12}\right]}, [0.2,0.6].\, e^{j2\pi\left[\frac{5}{12},\frac{7}{12}\right]}, [0.2,0.7].\, e^{j2\pi\left[\frac{2}{12},\frac{4}{12}\right]} \right\rangle}{u_2}\right)$$

$$,\left(\frac{\left\langle [0.2,0.3].\, e^{j2\pi\left[\frac{3}{12},\frac{4}{12}\right]}, [0.5,0.8].\, e^{j2\pi\left[\frac{4}{12},\frac{6}{12}\right]}, [0.3,0.9].\, e^{j2\pi\left[\frac{7}{12},\frac{11}{12}\right]} \right\rangle}{u_3}\right)\Bigg\}\Bigg\}$$

$$(G, A^{`}) =$$

$$\Bigg\{\Bigg\{\acute{e}_1, \left(\frac{\left\langle [0.6,0.8].\, e^{j2\pi\left[\frac{6}{12},\frac{10}{12}\right]}, [0\,,0.1].\, e^{j2\pi\left[\frac{2}{12},\frac{3}{12}\right]}, [0.2\,,0.4].\, e^{j2\pi\left[\frac{1}{12},\frac{2}{12}\right]} \right\rangle}{v_1}\right),$$

$$\left(\frac{\left\langle [0.5,0.7].\, e^{j2\pi\left[\frac{2}{12},\frac{4}{12}\right]}, [0.3,0.4].\, e^{j2\pi\left[\frac{2}{12},\frac{4}{12}\right]}, [0.1,0.2].\, e^{j2\pi\left[\frac{3}{12},\frac{6}{12}\right]} \right\rangle}{v_2}\right)$$

$$,\left(\frac{\left\langle [0.3,0.6].\, e^{j2\pi\left[\frac{5}{12},\frac{7}{12}\right]}, [0.2,0.4].\, e^{j2\pi\left[\frac{7}{12},\frac{9}{12}\right]}, [0.5,0.9].\, e^{j2\pi\left[\frac{2}{12},\frac{6}{12}\right]} \right\rangle}{v_3}\right)\Bigg\},$$

$$\Bigg\{\acute{e}_2, \left(\frac{\left\langle [0.4,0.7].\, e^{j2\pi\left[\frac{3}{12},\frac{7}{12}\right]}, [0.2\,,0.5].\, e^{j2\pi\left[\frac{4}{12},\frac{5}{12}\right]}, [0.1\,,0.2].\, e^{j2\pi\left[\frac{2}{12},\frac{4}{12}\right]} \right\rangle}{v_1}\right),$$

$$\left(\frac{\left\langle [0.2,0.4].\, e^{j2\pi\left[\frac{3}{12},\frac{7}{12}\right]}, [0.1,0.3].\, e^{j2\pi\left[\frac{6}{12},\frac{8}{12}\right]}, [0.3,0.5].\, e^{j2\pi\left[\frac{4}{12},\frac{6}{12}\right]} \right\rangle}{v_2}\right)$$

$$,\left(\frac{\left\langle [0.3,0.8].\, e^{j2\pi\left[\frac{6}{12},\frac{9}{12}\right]}, [0.2,0.6].\, e^{j2\pi\left[\frac{1}{12},\frac{4}{12}\right]}, [0.3,0.5].\, e^{j2\pi\left[0,\frac{2}{12}\right]} \right\rangle}{v_3}\right)\Bigg\},$$

$$\Bigg\{\acute{e}_3, \left(\frac{\left\langle [0,0.3].\, e^{j2\pi\left[\frac{1}{12},\frac{3}{12}\right]}, [0.4,0.6].\, e^{j2\pi\left[\frac{6}{12},\frac{8}{12}\right]}, [0.5,0.9].\, e^{j2\pi\left[\frac{3}{12},\frac{7}{12}\right]} \right\rangle}{v_1}\right),$$

$$\left(\frac{\left\langle [0.2,0.4].\, e^{j2\pi\left[\frac{5}{12},\frac{7}{12}\right]}, [0.3,0.7].\, e^{j2\pi\left[\frac{3}{12},\frac{5}{12}\right]}, [0.3,0.8].\, e^{j2\pi\left[\frac{3}{12},\frac{7}{12}\right]} \right\rangle}{v_2}\right)$$

Doi: https://doi.org/10.54216/IJNS.190406
Received: May 21, 2022 Accepted: October 15, 2022

$$,\left(\frac{\langle[0.5,0.9].e^{j2\pi\left[\frac{7}{12},\frac{10}{12}\right]},[0.2,0.2].e^{j2\pi\left[\frac{1}{12},\frac{2}{12}\right]},[0.2,0.3].e^{j2\pi\left[0,\frac{1}{12}\right]}\rangle}{v_3}\right)\Bigg\}$$

By above definition, we have the mapping $F:\widetilde{(U,E)} \longrightarrow \widetilde{(V,E^{`})}$.

For an I-CNSS (H,A) in (U,E), $(F(H,A),M)$ is an I-CNSS in $(V,E^{`})$ such that $M = s(E) = \{e_1^{`}, e_2^{`}, é_3\}$ and is obtained as follows:

$F(H,A)(e_1^{`})(v_1) = \vee_{u\in r^{-1}(v_1)}(\vee_{\alpha\in s^{-1}(e_1^{`})\cap A} H(\alpha)) = \vee_{u\in\{u_1\}} (\vee_{\alpha\in\{e_2,e_3\}} H(\alpha))$
$=\langle[0.1,0.3].e^{j2\pi\left[\frac{1}{12},\frac{3}{12}\right]},[0.4\ ,0.8].e^{j2\pi\left[\frac{7}{12},\frac{11}{12}\right]},[0.8\ ,0.9].e^{j2\pi\left[\frac{8}{12},\frac{12}{12}\right]}\rangle \cup$
$\langle[0.2,0.3].e^{j2\pi\left[\frac{2}{12},\frac{5}{12}\right]},[0.2\ ,0.7].e^{j2\pi\left[\frac{7}{12},\frac{10}{12}\right]},[0\ ,0.1].e^{j2\pi\left[\frac{9}{12},\frac{11}{12}\right]}\rangle$
$=\langle[0.2,0.3].e^{j2\pi\left[\frac{2}{12},\frac{5}{12}\right]},[0.2\ ,0.7].e^{j2\pi\left[\frac{2}{12},\frac{5}{12}\right]},[0\ ,0.1].e^{j2\pi\left[\frac{8}{12},\frac{11}{12}\right]}\rangle.$

$F(H,A)(e_1^{`})(v_2) = \vee_{u\in r^{-1}(v_2)}(\vee_{\alpha\in s^{-1}(e_1^{`})\cap A} H(\alpha)) = \vee_{u\in\{u_3\}} (\vee_{\alpha\in\{e_2,e_3\}} H(\alpha))$
$=\langle[0.3,0.6].e^{j2\pi\left[\frac{5}{12},\frac{10}{12}\right]},[0.5,0.8].e^{j2\pi\left[\frac{7}{12},\frac{9}{12}\right]},[0.1,0.3].e^{j2\pi\left[\frac{1}{12},\frac{4}{12}\right]}\rangle \cup$
$\langle[0.2,0.3].e^{j2\pi\left[\frac{3}{12},\frac{4}{12}\right]},[0.5,0.8].e^{j2\pi\left[\frac{4}{12},\frac{6}{12}\right]},[0.3,0.9].e^{j2\pi\left[\frac{7}{12},\frac{11}{12}\right]}\rangle$
$=\langle[0.3,0.6].e^{j2\pi\left[\frac{5}{12},\frac{10}{12}\right]},[0.5\ ,0.8].e^{j2\pi\left[\frac{4}{12},\frac{6}{12}\right]},[0.1\ ,0.3].e^{j2\pi\left[\frac{8}{12},\frac{11}{12}\right]}\rangle.$

$F(H,A)(e_1^{`})(v_3) = \vee_{u\in r^{-1}(v_3)}(\vee_{\alpha\in s^{-1}(e_1^{`})\cap A} H(\alpha)) = \vee_{u\in\{u_2\}} (\vee_{\alpha\in\{e_2,e_3\}} H(\alpha))$
$=\langle[0.4,0.7].e^{j2\pi\left[\frac{7}{12},\frac{9}{12}\right]},[0.3,0.7].e^{j2\pi\left[\frac{4}{12},\frac{9}{12}\right]},[0.2,0.6].e^{j2\pi\left[\frac{6}{12},\frac{9}{12}\right]}\rangle \cup$
$\langle[0.3,0.8].e^{j2\pi\left[\frac{4}{12},\frac{7}{12}\right]},[0.2,0.6].e^{j2\pi\left[\frac{5}{12},\frac{7}{12}\right]},[0.2,0.7].e^{j2\pi\left[\frac{2}{12},\frac{4}{12}\right]}\rangle$
$=\langle[0.4,0.8].e^{j2\pi\left[\frac{7}{12},\frac{9}{12}\right]},[0.2\ ,0.6].e^{j2\pi\left[\frac{4}{12},\frac{7}{12}\right]},[0.2\ ,0.6].e^{j2\pi\left[\frac{2}{12},\frac{4}{12}\right]}\rangle.$

Then,

$F(H,A)(e_1^{`}) =$

$$\Bigg\{\left(\frac{\langle[0.2,0.3].e^{j2\pi\left[\frac{2}{12},\frac{5}{12}\right]},[0.2\ ,0.7].e^{j2\pi\left[\frac{2}{12},\frac{5}{12}\right]},[0\ ,0.1].e^{j2\pi\left[\frac{8}{12},\frac{11}{12}\right]}\rangle}{v_1}\right),$$
$$\left(\frac{\langle[0.3,0.6].e^{j2\pi\left[\frac{5}{12},\frac{10}{12}\right]},[0.5\ ,0.8].e^{j2\pi\left[\frac{4}{12},\frac{6}{12}\right]},[0.1\ ,0.3].e^{j2\pi\left[\frac{8}{12},\frac{11}{12}\right]}\rangle}{v_2}\right)$$
$$,\left(\frac{\langle[0.4,0.8].e^{j2\pi\left[\frac{7}{12},\frac{9}{12}\right]},[0.2\ ,0.6].e^{j2\pi\left[\frac{4}{12},\frac{7}{12}\right]},[0.2\ ,0.6].e^{j2\pi\left[\frac{2}{12},\frac{4}{12}\right]}\rangle}{v_3}\right)\Bigg\}$$

Using the same arithmetic method as above, we get

$F(H,A)(e_2^{`}) =$

$$\Bigg\{\left(\frac{\langle[0.3,0.4].e^{j2\pi\left[\frac{2}{12},\frac{3}{12}\right]},[0.2,0.3].e^{j2\pi\left[\frac{3}{12},\frac{5}{12}\right]},[0,6,0.7].e^{j2\pi\left[\frac{9}{12},\frac{11}{12}\right]}\rangle}{v_1}\right),$$
$$\left(\frac{\langle[0.5,0.9].e^{j2\pi\left[\frac{7}{12},\frac{10}{12}\right]},[0.2,0.2].e^{j2\pi\left[\frac{1}{12},\frac{2}{12}\right]},[0.2,0.3].e^{j2\pi\left[0,\frac{1}{12}\right]}\rangle}{v_2}\right)$$
$$,\left(\frac{\langle[0.6,0.9].e^{j2\pi\left[\frac{7}{12},\frac{9}{12}\right]},[0.5,0.6].e^{j2\pi\left[\frac{5}{12},\frac{7}{12}\right]},[0,3,0.4].e^{j2\pi\left[0,\frac{1}{12}\right]}\rangle}{v_3}\right)\Bigg\}$$

Doi: https://doi.org/10.54216/IJNS.190406
Received: May 21, 2022 Accepted: October 15, 2022

$F(H,A)(e_3^{`}) =$

$$\left\{\left(\frac{\langle[0.2,0.3].e^{j2\pi\left[\frac{2}{12},\frac{5}{12}\right]},[0.2\,,0.7].e^{j2\pi\left[\frac{7}{12},\frac{10}{12}\right]},[0\,,0.1].e^{j2\pi\left[\frac{9}{12},\frac{11}{12}\right]}\rangle}{v_1}\right),\right.$$

$$\left(\frac{\langle[0.2,0.3].e^{j2\pi\left[\frac{3}{12},\frac{4}{12}\right]},[0.5,0.8].e^{j2\pi\left[\frac{4}{12},\frac{6}{12}\right]},[0.3,0.9].e^{j2\pi\left[\frac{7}{12},\frac{11}{12}\right]}\rangle}{v_2}\right)$$

$$\left.,\left(\frac{\langle[0.3,0.8].e^{j2\pi\left[\frac{4}{12},\frac{7}{12}\right]},[0.2,0.6].e^{j2\pi\left[\frac{5}{12},\frac{7}{12}\right]},[0.2,0.7].e^{j2\pi\left[\frac{2}{12},\frac{4}{12}\right]}\rangle}{v_3}\right)\right\}.$$

Hence,

$(F(H,A),M) =$

$$\left\{\left\{\acute{e}_1,\left(\frac{[0.6,0.8].e^{j2\pi\left[\frac{6}{12},\frac{10}{12}\right]},[0\,,0.1].e^{j2\pi\left[\frac{2}{12},\frac{3}{12}\right]},[0.2\,,0.4].e^{j2\pi\left[\frac{1}{12},\frac{2}{12}\right]}}{v_1}\right),\right.\right.$$

$$\left(\frac{[0.5,0.7].e^{j2\pi\left[\frac{2}{12},\frac{4}{12}\right]},[0.3,0.4].e^{j2\pi\left[\frac{2}{12},\frac{4}{12}\right]},[0.1,0.2].e^{j2\pi\left[\frac{3}{12},\frac{6}{12}\right]}}{v_2}\right)$$

$$\left.,\left(\frac{[0.3,0.6].e^{j2\pi\left[\frac{5}{12},\frac{7}{12}\right]},[0.2,0.4].e^{j2\pi\left[\frac{7}{12},\frac{9}{12}\right]},[0.5,0.9].e^{j2\pi\left[\frac{2}{12},\frac{6}{12}\right]}}{v_3}\right)\right\},$$

$$\left\{\acute{e}_2,\left(\frac{[0.4,0.7].e^{j2\pi\left[\frac{3}{12},\frac{7}{12}\right]},[0.2\,,0.5].e^{j2\pi\left[\frac{4}{12},\frac{5}{12}\right]},[0.1\,,0.2].e^{j2\pi\left[\frac{2}{12},\frac{4}{12}\right]}}{v_1}\right),\right.$$

$$\left(\frac{[0.2,0.4].e^{j2\pi\left[\frac{3}{12},\frac{7}{12}\right]},[0.1,0.3].e^{j2\pi\left[\frac{6}{12},\frac{8}{12}\right]},[0.3,0.5].e^{j2\pi\left[\frac{4}{12},\frac{6}{12}\right]}}{v_2}\right)$$

$$\left.,\left(\frac{[0.3,0.8].e^{j2\pi\left[\frac{6}{12},\frac{9}{12}\right]},[0.2,0.6].e^{j2\pi\left[\frac{1}{12},\frac{4}{12}\right]},[0.3,0.5].e^{j2\pi\left[0,\frac{2}{12}\right]}}{v_3}\right)\right\},$$

$$\left\{\acute{e}_3,\left(\frac{[0,0.3].e^{j2\pi\left[\frac{1}{12},\frac{3}{12}\right]},[0.4,0.6].e^{j2\pi\left[\frac{6}{12},\frac{8}{12}\right]},[0.5,0.9].e^{j2\pi\left[\frac{3}{12},\frac{7}{12}\right]}}{v_1}\right),\right.$$

$$\left(\frac{[0.2,0.4].e^{j2\pi\left[\frac{5}{12},\frac{7}{12}\right]},[0.3,0.7].e^{j2\pi\left[\frac{3}{12},\frac{5}{12}\right]},[0.3,0.8].e^{j2\pi\left[\frac{3}{12},\frac{7}{12}\right]}}{v_2}\right)$$

$$\left.\left.,\left(\frac{\langle[0.5,0.9].e^{j2\pi\left[\frac{7}{12},\frac{10}{12}\right]},[0.2,0.2].e^{j2\pi\left[\frac{1}{12},\frac{2}{12}\right]},[0.2,0.3].e^{j2\pi\left[0,\frac{1}{12}\right]}\rangle}{v_3}\right)\right\}\right\}$$

Definition 6. Let $f:\widetilde{(U,E)} \longrightarrow \widetilde{(V,E^{`})}$ be a mapping and $(H,A),(G,A^{`})$ I-CNSSs in $\widetilde{(U,E)}$. Then for $\beta \in E^{`}, v \in V$ , the union and intersection of I-CNS-images $(H,A)$ and $(G,A^{`})$ are defined as follows:

1. $(F((H,A)\vee F(G,A^{`}))(\beta)(v) = F(H,A)(\beta)(v)\vee F(G,A^{`})(\beta)(v)$.

2. $(F(H,A)F(G,A^{`}))(\beta)(v) = F(H,A)(\beta)(v)\wedge F(G,A^{`})(\beta)(v)$.

Definition 7. Let $f:\widetilde{(U,E)} \longrightarrow \widetilde{(V,E`)}$ be a mapping and $(H,A),(G,A`)$ I-CNSSs in $\widetilde{(U,E)}$ then for $\alpha \in E, u \in U$, the union and intersection of I-CNS-images (H,A) and $(G,A`)$ are defined as follows:

1. $(F^{-1}(H,A) \vee F^{-1}(G,A`))(\alpha)(u) = F^{-1}(H,A)(\alpha)(u) \vee F^{-1}(G,A`)(\alpha)(u)$.

2. $(F^{-1}(H,A) \wedge F^{-1}(G,A`))(\alpha)(u) = F^{-1}(H,A)(\alpha)(u) \wedge F^{-1}(G,A`)(\alpha)(u)$.

Theorem 1. Let $f:\widetilde{(U,E)} \longrightarrow \widetilde{(V,E`)}$ be a mapping. Then for two I-CNSSs $(H,A)$ and $(G,A`)$ in the I-CNS-class $\widetilde{(U,E)}$, we have:

1. $F(\emptyset) = \emptyset$
2. $F(\Omega) = \Omega$
3. $F((H,A) \vee (G,A`))$=$F(H,A)\ ) \vee F(G,A`)$
4. $F((H,A) \wedge (G,A`))$=$F(H,A)\ ) \wedge F(G,A`)$
5. If $(H,A) \subseteq (G,A`)$, then $F(H,A) \subseteq F(G,A`)$

Proof: For (1), (2), and (3) the proof is trivial by from the above definitions. So, we just prove (3) and (4).

For $\beta \in M \subseteq É$ and $v \in V$, we try to prove that $F((H,A) \vee (G,A`)\ )$=$F(H,A)\ ) \vee f(G,A`)$
For left side consider $F((H,A) \vee (G,A`)\ )(\beta)(v) = F(K, A \cup B)(\beta)(v)$.

Then,

$$F(K,A \cup B)(\beta)(v) = \begin{cases} \cup_{x \in r^{-1}(y)} (\cup_{\alpha} K(\alpha))\ if\ r^{-1}(y) and\ v^{-1}(\beta) \cap (A \cup B) \neq \emptyset \\ ([0,0],[1,1],[1,1]) \qquad other\ wise \end{cases} \quad (1)$$

such that $K(\alpha) = F(\alpha) \cup G(\alpha)$.
Considering only the non-trivial case. Then the equation (1) becomes:

$$F(K,A \cup B)(\beta)(v) = \cup_{u \in r^{-1}(v)} (\cup (F(\alpha) \cup G(\alpha))) \quad (2)$$

For the right-hand side and by means of definition 3.3, we have

$$F((H,A) \cup (G,B))(\beta)(v) = F(H,A)(\beta)(v) \cup F(G,B)(\beta)(v)$$

$$= (\cup_{u \in r^{-1}(v)} (\cup_{\alpha \in v^{-1}(\beta) \cap A} H(\alpha))(u)) \cup (\cup_{u \in r^{-1}(v)} (\cup_{\alpha \in s^{-1}(\beta) \cap B} G(\alpha))(u))$$
$$= (\cup_{u \in r^{-1}(v)} \cup_{\alpha \in r^{-1}(\beta) \cap (A \cup B)} (H(\alpha) \cup G(\alpha)(u))$$
$$= \cup_{u \in r^{-1}(v)} (H(\alpha) \cup G(\alpha)). \quad (3)$$

From above three equations, we get
$F((H,A) \vee (G,A`)\ )$=$F(H,A)\ ) \vee F(G,A`)$.

Proof (5): Let $\beta \in M \subseteq É$ and $v \in V$. By Definition 7, we have

$$F((H,A) \wedge (G,A`)\ )(\beta)(v)$$
$$= f(K, A \cup B)(\beta)(v)$$
$$= \cup_{u \in r^{-1}(v)} (\cup_{u \in v^{-1}(\beta) \cap A \cup B)} K(\alpha))(u)$$
$$= \cup_{u \in r^{-1}(v)} (\cup_{u \in s^{-1}(\beta) \cap A \cup B)} H(\alpha) \cup G(\alpha))(u)$$
$$\subseteq (\cup_{u \in r^{-1}(v)} (\cup_{\alpha \in s^{-1}(\beta) \cap A} H(\alpha))) \cap (\cup_{u \in r^{-1}(v)} (\cup_{\alpha \in s^{-1}(\beta) \cap B} G(\alpha)))$$
$$= (F(H,A)(\beta)(v) \wedge F(G,A`)\ )(\beta)(v))$$

Its completes proof (5).

Theorem 2. Let $f:\widetilde{(U,E)} \longrightarrow \widetilde{(V,E`)}$ be a mapping. Then for two I-CNSSs $H,A)$ and $(G,A`)$in the I-CNS-class $\widetilde{(Y,E`)}$, we have:

1. $F^{-1}(\emptyset) = \emptyset$

Doi: https://doi.org/10.54216/IJNS.190406
Received: May 21, 2022 Accepted: October 15, 2022

2. $F^{-1}(\Omega) \subseteq \Omega$
3. $F^{-1}((H,A) \vee (G,A`))= F^{-1}(G,A)) \vee F^{-1}(G,A`)$
4. $F^{-1}((H,A) \wedge (G,A`))= F^{-1}(H,A)) \wedge F^{-1}(G,A`)$
5. If $(H,A) \subseteq (G,A`)$, then $F^{-1}(H,A) \subseteq F^{-1}(G,A`)$

Proof: We can prove using the same method as in the previous proof.

4. CONCLUSION

In this work, we studied mappings on I-CNSSs and their basic properties. We also give some illustrative examples of mapping on I-CNSS. We hope by these essential results will help several researchers to enhance the research on ICNSS theory and its application in our daily life. Moreover, in next future work we planning and recurment to apply this idea with another some mathmaticl structures like hypersoft set [18].

Acknowledgements: We would like to thank the unknown reviewers for their comments, helpful and

suggestions.

Funding: "This research received no external funding"

Conflicts of Interest: "The authors declare no conflict of interest."

References

[1] Smarandache, F. A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability. American Research Press, Rehoboth, NM, 1999.

[2] Zadeh, L. A., Fuzzy Sets. Inform. Control 8 (1965).

[3] Atanassov, K., Intuitionistic fuzzy sets. In V. Sgurev, ed., ITKRS Session, Sofia, June 1983, Central Sci. and Techn. Library, Bulg. Academy of Sciences, 1984.

[4] Molodtsov, D. (1999). Soft set theory—first results. Computers & mathematics with applications, 37(4-5), 19-31.

[5] Maji, P. K. (2013). Neutrosophic soft set. Infinite Study.

[6] Asaad, B. A., & Musa, S. Y. (2022). Continuity and Compactness via Hypersoft Open Sets. International Journal of Neutrosophic Science, 19(2).

[7] Abuqamar, M., & Hassan, N. (2022). The Algebraic Structure of Normal Groups Associated with Q-Neutrosophic Soft Sets. Neutrosophic Sets and Systems, 48, 328-338.

[8] Deli, I., & Broumi, S. (2015). Neutrosophic soft matrices and NSM-decision making. Journal of Intelligent & Fuzzy Systems, 28(5), 2233-2241.

[9] Ramot, D., Milo, R., Friedman, M., & Kandel, A. (2002). Complex fuzzy sets. IEEE Transactions on Fuzzy Systems, 10(2), 171-186.

[10] F. Al-Sharqi, A. G. Ahmad and A. Al-Quran, Interval-Valued Neutrosophic Soft Expert Set from Real Space to Complex Space. CMES-Computer Modeling in Engineering and Sciences 132(1) (2022), 267–293.

[11] Ali, M., & Smarandache, F. (2017). Complex neutrosophic set. Neural computing and applications, 28(7), 1817-1834.

[12] Ali, M., Dat, L. Q., Son, L. H., & Smarandache, F. (2018). Interval complex neutrosophic set: Formulation and applications in decision-making. International Journal of Fuzzy Systems, 20(3), 986-999.

[13] F. Al-Sharqi, A. Al-Quran, A. G. Ahmad and S. Broumi, Interval-valued complex neutrosophic soft set and its applications in decision-making. Neutrosophic Sets and Systems, 40(1) (2021), 149–168.

Doi: https://doi.org/10.54216/IJNS.190406
Received: May 21, 2022 Accepted: October 15, 2022

[14] F. Al-Sharqi, A. G. Ahmad and A. Al-Quran, Interval complex neutrosophic soft relations and their application in decision-making. Journal of Intelligent and Fuzzy Systems, 43(1) (2022), 745-771.
[15] F. Al-Sharqi, A. G. Ahmad and A. Al-Quran, Similarity Measures on Interval-Complex Neutrosophic Soft Sets with Applications to Decision Making and Medical Diagnosis under Uncertainty. Neutrosophic Sets and Systems, 51(1) (2022), 495-515.
[16] Kharal, A., & Ahmad, B. (2011). Mappings on soft classes. New Mathematics and Natural Computation, 7(03), 471-481.
[17] Alkhazaleh, S., & Marei, E. (2014). Mappings on neutrosophic soft classes. Neutrosophic Sets and Systems, 2(1), 3-8.
[18] Smarandache, F. (2018). Extension of soft set to hypersoft set, and then to plithogenic hypersoft set. Neutrosophic Sets and Systems, 22(1), 168-170.

Doi: https://doi.org/10.54216/IJNS.190406
Received: May 21, 2022 Accepted: October 15, 2022