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Mohammad Abobala
Tishreen University

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Research Article

Solutions of Some Kandasamy–Smarandache Problems about Neutrosophic Complex Numbers and Group of Units' Problem

Mohammad Abobala,[1] **Ahmed Hatip**,[2] **and Necati Olgun**[2]

[1]*Faculty of Science, Tishreen University, Latakia, Syria*
[2]*Department of Mathematics, Gaziantep University, Gaziantep, Turkey*

Correspondence should be addressed to Mohammad Abobala; mohammadabobala777@gmail.com

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Neutrosophic complex numbers are a novel interesting generalization of classical complex numbers. This study aims to study the neutrosophic complex infinite rings, where we classify those rings as direct products by using classical complex field C. Also, this work introduces full solutions for 18 Kandasamy–Smarandache open problems concerning these structures, as well as the group of unit's famous problem, where the group of units (invertible elements) of a neutrosophic complex ring will be classified as a direct product of two classical groups of units.

1. Introduction

Neutrosophy, as a new kind of generalized logic that deals with indeterminacy in nature, reality, and the human mind, found its way into pure mathematics. A lot of neutrosophic algebraic structures were defined and studied in a wide range such as neutrosophic topological groups and sets [1, 2], neutrosophic rings, neutrosophic graphs and homomorphisms [3, 4], and neutrosophic vector spaces [5, 6].

In [7], Smarandache and Kandasamy introduced the neutrosophic complex numbers as an interesting generalized structure of the classical complex numbers, by adding an element with logical property (Indeterminate I) to the field C.

Their work has generalized the concept of classical complex numbers into neutrosophic rings [8], and they proposed 150 open problems concerning substructures and factorization properties in these complex neutrosophic structures.

In this study, we aim to continue their efforts and to suggest a classification of neutrosophic complex infinite rings by finding isomorphisms between these neutrosophic structures and the corresponding classical structures. Also, we use these isomorphisms to suggest solutions for 18 problems of Kandasamy–Smarandache problems introduced in [7].

2. Preliminaries

In this section, we present the basic definitions that are useful in this research.

Definition 1 (see [7]). Let $C(\langle Z \cup I \rangle) = \{a + bI + ci + idI \mid a, b, c, d \in Z\}$; then,

(1) An integer complex neutrosophic group under addition
(2) An integer neutrosophic complex monoid commutative monoid under multiplication
(3) An integer neutrosophic complex commutative ring with a unit of infinite order under addition + and multiplication ×

Definition 2 (see [7]). Let $S = \{(a_{ij}) \mid a_{ij} \in C(\langle Z \cup I \rangle); 1 \leq i, j \leq n\}$ be a collection of $n \times n$ complex neutrosophic integer matrices. S is a ring of $n \times n$ integer complex neutrosophic ring of infinite order and is noncommutative. S has zero divisors, units, idempotents, subrings, and ideals.

Definition 3 (see [7]). Let $C(\langle Q\cup I\rangle)=\{a+bi+cI+idI\mid a,b,c,d\in Q\}$; then,

(1) $C(\langle Q\cup I\rangle)$ is a rational complex neutrosophic ring that has no zero divisors

(2) $C(\langle Q\cup I\rangle)$ is a rational complex neutrosophic field

Definition 4 (see [7]). Let $C(Z_n)=\{a+bi_F|a,b\in Z_n, i_F$ be the finite complex modulo number such that $i_F^2=n-1$, $n<\infty\}$; we define i_F as the finite complex modulo number. $C(Z_n)$ is the finite complex modulo integer numbers.

Definition 5 (see [8]). A Smarandache ring (S-ring) is defined to be a ring A, such that a proper subset of A is a field with respect to the operations induced. By proper subset, we understand a set included in A different from the empty set, from the unit element if they were existed and from A.

3. Results

3.1. Problem 1 and Problem 2

Problem 1. Can $C(\langle Z\cup I\rangle)$ be a Smarandache ring?

The answer is no. We give a proof.

We suppose that $C(\langle Z\cup I\rangle)$ is a Smarandache ring; then, there is a proper subset $A\subsetneq C(Z\cup I)$, such that A has a field structure with respect to multiplication on $C(\langle Z\cup I\rangle)$. Consider an arbitrary element $n=a+bI+(c+dI)i$, $a,b,c,d\in Z$, in A; since A is an Abelian group under addition, we can see that $r\cdot x\in A$ for every $r\in Z$; thus, A has infinite cardinality.

It is well known that the minimal field of infinite cardinality is the field of rationales Q; hence, the field A has a characteristic zero (A contains an isomorphic image of Q).

The field A has only two principal ideals $\{0\}$ and A; hence, $A=\langle a+bI+(c+dI)i\rangle$. It is probable that A has a unity different from 1, and we will prove that the identity of A must be 1.

For this goal, we suppose that $m=x+yI+(z+tI)i$ is the identity of A and different from 1.

We have $m\cdot n=n$; thus, $[a+bI+(c+dI)i][x+yI+(z+tI)i]=a+bI+(c+dI)i$, and we get $(a\cdot x-c\cdot z)+I(a\cdot y+b\cdot x+b\cdot y)+a\cdot zi+Ii(a\cdot t+b\cdot z+b\cdot t+c\cdot xi+Ii(c\cdot y+d\cdot x+d\cdot y)+I\quad(-c\cdot t-d\cdot z-d\cdot t)=a+bI+(c+dI)i$; this implies $(a\cdot x-c\cdot z)+(a\cdot z+c\cdot x)i+I(a\cdot y+b\cdot x+b\cdot y-c\cdot t-d\cdot z-d\cdot t)+Ii(a\cdot t+b\cdot z+b\cdot t+c\cdot y+d\cdot x+d\cdot y)=a+ci+bI+dI\ i$; hence,

(1) $a+ci=(a\cdot x-c\cdot z)+(a\cdot z+c\cdot x)i$

(2) $a\cdot y+b\cdot x+b\cdot y-c\cdot t-d\cdot t-d\cdot z=b$

(3) $a\cdot t+b\cdot z+b\cdot t+c\cdot y+d\cdot x+d\cdot y=d$

From equation (1), we get the following two equivalent equations.

(i) $a=a\cdot x-c\cdot z$

(ii) $c=a\cdot z+c\cdot x$

We multiply (i) by c and (ii) by a to obtain

(i) $a\cdot c=a\cdot c\cdot x-c^2z$.

(ii) $a\cdot c=a^2z+a\cdot c\cdot x$; we compute (ii)−(i); $0=a^2z+c^2z=z(a^2+c^2)$; thus, $z=0$, that is because $a^2+c^2\neq 0$ so that $x=1$. By putting $z=0$ and $x=1$ in equations (2) and (3), we obtain the result.

(iii) $a\cdot y+b+b\cdot y-c\cdot t-d\cdot t=b$; hence, $y(a+b)-t(c+d)=0$.

(iv) $a\cdot t+b\cdot t+c\cdot y+d+d\cdot y=d$; hence, $y(c+d)+t(a+b)=0$.

We multiply equation (iii) by $(a+b)$ and equation (iv) by $(c+d)$, and then, we add them to obtain $y\cdot[(a+b)^2+(c+d)^2]=0$; hence, $y=0$, and then, $t=0$ so that $m=1$.

The previous discussion implies that $Q\subseteq A$; thus, $Q\subseteq C(Z\cup I)$ which is a contradiction; thus, A cannot be a field and $C(\langle Z\cup I\rangle)$ is not a Smarandache ring.

Problem 2. Is $M=\{(a_1,a_2);\ a_1,a_2\in C(\langle Z\cup I\rangle)\}$ under $(\times)$ a Smarandache semigroup?

The answer is yes, and we give a proof.

We have to search for a proper subset A of M, where A has a group structure.

It is easy to see that $Z(I)\subseteq C(\langle Z\cup I\rangle)$, so if we take the group of units in the ring $Z(I)$, which is equal to $U(Z(I))=\{1,-1,1-2I,-1+2I\}$, it will be a subgroup of the semigroup $C(\langle Z\cup I\rangle)$; hence, the direct product $U(Z(I))\times U(Z(I))$ is a subgroup contained in the semigroup M; thus, M is a Smarandache semigroup.

4. Neutrosophic Complex Rings as Direct Products

This section is devoted to solve Problems 3–11.

Problem 3. Obtain some interesting results enjoyed by

(a) Neutrosophic complex reals

(b) Neutrosophic complex modulo integers

(c) neutrosophic complex rationales

The best answer we can obtain is to classify those rings as direct products of classical algebraic rings. This desired classification can help in many other problems.

Theorem 1. *Let $C(\langle R\cup I\rangle)$ be the neutrosophic complex ring of reals; then, $C(\langle R\cup I\rangle)=C(I)$.*

Proof. It is clear that the neutrosophic ring $C(I)$ is contained in $C(\langle R\cup I\rangle)$. Conversely, suppose that $x=a+bI+ci+di\ I\in C(\langle R\cup I\rangle)$; then, $x=(a+ci)+I(b+di)\in C(I)$.

Thus, our proof is complete. □

Theorem 2. $C(\langle R\cup I\rangle)\cong C\times C$.

Proof. We shall prove that there is a ring isomorphism between $C(I)$ and $C\times C$.

We define $f\colon C(I)\longrightarrow C\times C$, $f(x+yI)=(x,x+y)$, $x,y\in C$.

(a) f is well defined: suppose that $a+bI=x+yI$; $a,b,x,y\in C$; this implies $a=x$ and $b=y$. Hence, $(a,a+b)=(x,x+y)$, i.e., $f(a+bI)=f(x+yI)$.

(b) f is bijective: it is clear that f is a surjective function. On the contrary, we assume that $f(x+yI)=f(a+bI)$; this means $(a,a+b)=(x,x+y)$; thus, $a=x$ and $b=y$.

(c) f is a ring homomorphism: we take $m=a+bI$, $n=c+dI\in C(I)$; then, $m+n=(a+c)+(b+d)I$ and $m\cdot n=a\cdot c+I(a\cdot d+b\cdot c+b\cdot d)$ so that $f(m+n)=(a+c,a+c+b+d)=(a,a+b)+(c,c+d)=f(m)+f(n)$ and $f(m\cdot n)=(a\cdot c,a\cdot c+a\cdot d+b\cdot c+b\cdot d)=(a\cdot c,(a+b)(c+d))=(a,a+b)\cdot(c,c+d)=f(m)\cdot f(n)$.

Thus, f is an isomorphism and the proof holds. □

Problem 4. Can $C(\langle R\cup I\rangle)$ have irreducible polynomials?

The answer is no, and we give a proof.

Since $C(\langle R\cup I\rangle)\cong C\times C$, then $C(\langle R\cup I\rangle)[x]\cong C\times C[x]$, i.e., for each polynomial $P(x)$ in $C(\langle R\cup I\rangle)[x]$, there is a corresponding polynomial with form $(g(x),h(x))$ in $C\times C[x]$; if $p(x)$ is not irreducible in $C(\langle R\cup I\rangle)[x]$, then one of $g(x),h(x)$ at least is irreducible over the field of complex numbers C, which is not possible, that is because C is an algebraically closed field.

The following theorem classifies $C(\langle Q\cup I\rangle)$.

Theorem 3. *Let $C(\langle Q\cup I\rangle)$ be the neutrosophic complex ring of rationales; then, $C(\langle Q\cup I\rangle)\cong Q(i)\times Q(i)$, where $Q(i)=\{a+bi;\ a,b\in Q\}$ is the algebraic extension of the field Q by i.*

Proof. We define $f\colon C(\langle Q\cup I\rangle)\longrightarrow Q(i)\times Q(i)$; $f(a+bi+(c+di)I=(a+bi,(a+bi)+(c+di)))$, where $a,c,b,d\in Q$. We have

(a) f is well defined: suppose that $a+bi+(c+di)I=x+yi+(z+ti)I$; then, $a=x,b=y,c=z$, and $d=t$.

(b) f is a bijective map: it is similar to that of Theorem 2.

(c) f is a ring homomorphism: consider two arbitrary elements $m=a+bi+(c+di)I$ and $n=x+yi+(z+ti)I$, and we have

$$\begin{aligned}
f(m+n)&=([a+bi+a+yi],[(c+di)+(z+ti)+(a+bi)+(x+yi)])=f(m)+f(n)\\
f(m\cdot n)&=f([(a+bi)\cdot(x+yi)]+I[(a+bi)(z+ti)+(c+di)(x+yi)+(c+di)(z+ti)])\\
&\quad\cdot([(a+bi)(x+yi)],[(a+bi)\cdot(x+yi)+(a+bi)(z+ti)+(c+di)(x+yi)+(c+di)(z+ti)])=f(m)\cdot f(n)
\end{aligned}\tag{1}$$

□

Remark 1. By the same argument, we can write $C(\langle Z\cup I\rangle)\cong Z(i)\times Z(i)$, i.e., $C(\langle Z\cup I\rangle)$ can be classified as a direct product of the ring $Z(i)=\{a+bi;\ a,b\in Z\}$ with itself.

By our classification results, we can answer many other open problems.

Problem 5. Is $C(\langle Q\cup I\rangle)$ a field? Is it a prime field?

The answer is no, that is because, since $C(\langle Q\cup I\rangle)\cong Q(i)\times Q(i)$, we can find that it is not a field since the element $x=(1,0)\in Q(i)\times Q(i)$ and it is not invertible; hence, its inverse isomorphic image $1-I$ is not invertible in $C(\langle Q\cup I\rangle)$. Thus, $C(\langle Q\cup I\rangle)$ cannot be a field.

Problem 6. Determine the irreducible polynomials over $C(\langle Q\cup I\rangle)$.

It is really a hard problem, but by using the fact $C(\langle Q\cup I\rangle)\cong Q(i)\times Q(i)$, we can find all irreducible polynomial over $C(\langle Q\cup I\rangle)$.

Let $P(x)$ be any polynomial defined over $C(\langle Q\cup I\rangle)$; then, it has a corresponding polynomial (g,h) in $(Q(i)\times Q(i))[x]$. It is sufficient to compute its isomorphic image (g,h).

If one of g,h is irreducible at least over $Q(i)$; then, $p(x)$ is irreducible over $C(\langle Q\cup I\rangle)$.

Example 1. Let $p(x)=X^2+(1+(1+i)I)X+1+(5-i)I$, where $X=x_1+x_2I$; $x_1,x_2\in Q(i)$, be a polynomial defined over $C(\langle Q\cup I\rangle)$.

The corresponding isomorphic polynomial of $p(x)$ is

$$\begin{aligned}
n(x_1,x_2)&=f(p(x))=f\left(X^2\right)+f(1+(1+i)I)\cdot f(X)+f(1+(5-i)I)\\
&=\left(x_1^2,(x_1+x_2)^2\right)+(1,2+i)\cdot(x_1,x_1+x_2)+(1,6-i)\\
&=\left(x_1^2+x_1+1,(x_1+x_2)^2+(2+i)(x_1+x_2)+6-i\right).
\end{aligned}\tag{2}$$

We get the following two equivalent polynomials: $g(x_1) = x_1^2 + x_1 + 1, h(x_1, x_2) = (x_1 + x_2)^2 + (2+i)(x_1 + x_2) + 6 - i$; it is clear that g is irreducible over $Q(i)$; thus, $p(x)$ is irreducible over $C(\langle Q \cup I \rangle)$.

Problem 7. Find irreducible polynomials in $C(\langle Z \cup I \rangle)[x]$? Is every ideal in $C(\langle Z \cup I \rangle)$ principal?

The first part of Problem 7 can be solved in a similar way of Problem 6, just by taking the isomorphic corresponding polynomial since $C(\langle Z \cup I \rangle) \cong Z(i) \times Z(i)$.

Example 2. Let $p(x) = X^2 + (1 + (1+i)I)X + 1 + (5-i)I$, where $X = x_1 + x_2 I, x_1, x_2 \in Z(i)$, be a polynomial defined over $C(\langle Z \cup I \rangle)$.

The corresponding isomorphic polynomial of $p(x)$ is

$$\begin{aligned} n(x_1, x_2) &= f(p(x)) = f(X^2) + f(1 + (1+i)I) \cdot f(X) \\ &\quad + f(1 + (5-i)I) \\ &\quad \cdot \left(x_1^2, (x_1 + x_2)^2\right) + (1, 2+i) \cdot (x_1, x_1 + x_2) \\ &\quad + (1, 6-i) = \left(x_1^2 + x_1 + 1, (x_1 + x_2)^2\right. \\ &\quad \left. + (2+i)(x_1 + x_2) + 6 - i\right). \end{aligned} \tag{3}$$

We get the following two equivalent polynomials:

$$\begin{aligned} g(x_1) &= x_1^2 + x_1 + 1, \ h(x_1, x_2) \\ &= (x_1 + x_2)^2 + (2+i)(x_1 + x_2) + 6 - i. \end{aligned} \tag{4}$$

It is clear that g is irreducible over $Z(i)$; thus, $p(x)$ is irreducible over $C(\langle Z \cup I \rangle)$.

Problem 8. Can one say for all polynomials with complex neutrosophic coefficients $C(\langle R \cup I \rangle)$ is algebraically closed?

The answer is yes. That is because $C(\langle R \cup I \rangle) \cong C \times C$, and C is algebraically closed; thus, $C(\langle R \cup I \rangle)$ is an algebraically closed ring, i.e., each root of any polynomial with coefficients from $C(\langle R \cup I \rangle)$ is from $C(\langle R \cup I \rangle)$.

Problem 9. Is $G = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a \cdot d - c \cdot b \neq 0 \text{ and } a, b, c, d \in C(\langle Z \cup I \rangle) \right\}$ a group? Is G simple?

The answer is no. G is not even a group; we take $\begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix} \in G$; its inverse is not in G; thus, G is not a group.

Problem 10. Is $C(\langle Z \cup I \rangle)$ a unique factorization domain?

The answer is no. We clarify our claim in the following discussion.

We have $C(\langle Z \cup I \rangle) \cong Z(i) \times Z(i)$; it is easy to see that $(1,0) = (1,2)(1,0) = (1,3) \cdot (1 \cdot 0)$, i.e., $Z(i) \times Z(i)$ is not a unique factorization domain; hence, $C(\langle Z \cup I \rangle)$ is not a unique factorization domain.

Problem 11. Can $C(\langle R \cup I \rangle)$ be a principal ideal domain?

The answer is no. It is sufficient to prove that $C(\langle R \cup I \rangle)$ has zero divisors.

We have $C(\langle R \cup I \rangle) \cong C \times C$ and $(1,0) \cdot (0,1) = (0,0)$ so that $C \times C$ is not a principal ideal domain because it has zero divisors; thus, $C(\langle R \cup I \rangle)$ is not a principal ideal domain.

5. Other Open Problems

This section is devoted to study Problems 12–18.

Problem 12. Can any geometrical interpretation be given to the field of neutrosophic complex numbers $C(\langle Q \cup I \rangle)$?

The answer is no. However, there is an algebraic interpretation of $C(\langle Q \cup I \rangle)$ We describe it by the following theorem.

Theorem 4. *Let $C(\langle Q \cup I \rangle)$ be the complex neutrosophic ring of rationales. Then, it can be considered as an algebraic extension of the neutrosophic ring $Q(I)$ with degree two.*

Proof. We have $P(x) = x^2 + 1$ is a monic polynomial over $Q(I)$; we shall prove that it is irreducible over $Q(I)$.

Suppose that $p(x) = (x + a + bI)(x + c + dI), a, b, c, d \in Q$; then, $P(x) = x^2 + x(a + bI + c + dI) + a \cdot c + I(a \cdot d + b \cdot c + b \cdot d)$; hence, $a + bI + c + dI = 0$ and $a \cdot d + b \cdot c + b \cdot d = 0$ and $a \cdot c = 1$ so that $a + c = 0\ (*)$ and $b + d = 0$.

We get from $(*)$ $a = -c$; thus, $-a^2 = 1$, which is a contradiction since $a \in Q$. Thus, $p(x)$ is irreducible. $P(x)$ has a root $m = i$; hence, the ring $[Q(I)](i)$ is an algebraic extension of $Q(I)\ Q(I)$ with a degree equal to deg $(P) = 2$. It is clear that $[Q(I)](i) = \{x + yi; x, y \in Q(I)\} = \{a + bI + ci + di\ I; a, b, c, d \in Q\} = C(\langle Q \cup I \rangle)$. □

Problem 13. Let $V = \{(a_1, a_2, a_3, a_4); a_i \in C(\langle Q \cup I \rangle), +\}$ be a group.

(i) Define an automorphism $\eta: V \longrightarrow V$ so that kerη is a nontrivial subgroup.

(ii) Is $V \cong C(\langle Q \cup I \rangle) \times C(\langle Q \cup I \rangle) \times C(\langle Q \cup I \rangle) \times C(\langle Q \cup I \rangle)$?

(i) is not possible since every group automorphism needs to be a bijective map; hence, its kernel will be trivial.

Question (ii) is easy and clear, that is because V is defined to be the direct product of $C(\langle Q \cup I \rangle)$ with itself four times.

Problem 14. Let $M = \left\{ \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}; a_i \in C(\langle Q \cup I \rangle) \right\}$ be a semigroup under multiplication.

(i) Prove M is an S-semigroup.

(ii) Is M commutative?

(iii) Find at least three zero divisors in M.

(iiv) Does M have ideals?

(v) Give sub-semigroups in M which are not ideals.

(a) We define $A = \left\{ \begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix}; x \in C(\langle Q \cup I \rangle) \right\}$, where A is a proper subset of M, and it is, clearly, an abelian group. Thus, M is an S-semigroup.

(b) No, it is not. Since matrices over Q do not commute, and Q is contained in $C(\langle Q\cup I\rangle)$.

(c) Take $x=\begin{pmatrix}0&0\\0&1\end{pmatrix}$, $y=\begin{pmatrix}0&0\\0&2\end{pmatrix}$, and $z=\begin{pmatrix}2&3\\0&0\end{pmatrix}$. It is easy to see that $x\cdot z=y\cdot z=0$; thus, x, y, and z are zero divisors.

(d) Take $S=\left\{\begin{pmatrix}aI&bI\\cI&dI\end{pmatrix};\ a,b,c,d\in C(\langle Q\cup I\rangle)\right\}$, where S is a subgroup with respect to addition. Let $m=\begin{pmatrix}x&y\\z&t\end{pmatrix}\in M$ and $=\begin{pmatrix}aI&bI\\cI&dI\end{pmatrix}\in S$; we have $m\cdot n\in S$; thus, S is an ideal and M has ideals.

(e) We define $S_1=\left\{\begin{pmatrix}a&b\\c&d\end{pmatrix};\ a,b,c,d\in Q(i)\right\}$, $S_2=\left\{\begin{pmatrix}a&b\\c&d\end{pmatrix};\ a,b,c,d\in Q\right\}$. These two sets are semigroups with respect to multiplication, but they are not ideals clearly.

Problem 15. Let

$$S=\left\{\begin{pmatrix}a_1&a_2&a_3\\a_4&a_5&a_6\\a_7&a_8&a_9\\a_{10}&a_{11}&a_{12}\end{pmatrix};\ a_i\in C(\langle Q\cup I\rangle),+\right\}.$$

(i) Find subgroups of S.

(ii) Can S have ideals?

(iii) Can S have idempotents?

(iv) Can S have zero divisors?

We summarize the answer as follows:

(i) Consider that $(H_i,+)$ is a subgroup of $(C(\langle Q\cup I\rangle),+)$ and $M=\left\{\begin{pmatrix}a_1&a_2&a_3\\a_4&a_5&a_6\\a_7&a_8&a_9\\a_{10}&a_{11}&a_{12}\end{pmatrix};\ a_i\in H_i;\ 1\le i\le 12\right\}$; then, S has the following property:

For every $x,y\in M$, we have $x-y\in M$; hence, M is a subgroup of S. All subgroups will have the same form.

(ii) The answer is no. That is because the multiplication is not defined on S; thus, it is not even a ring.

(iii) No, for the same reason.

(iv) No, for the same reason.

Problem 16. Let $V=\left\{\begin{pmatrix}a_1&a_2\\a_3&a_4\end{pmatrix};\ a_i\in C(\langle Q\cup I\rangle)\right\}$ be a semigroup under product.

(i) Is V commutative?

(ii) Can V have idempotents?

(iii) Does V have a semigroup which is not an ideal?

(iv) Can V have zero divisors?

(v) Give an ideal in V.

(vi) Is V a Smarandache semigroup?

(vii) Is V a Smarandache semigroup?

The answer is

(i) No, it is not, since matrices over Q do not commute and Q is contained in $C(\langle Q\cup I\rangle)$

(ii) Yes, for example, take $x=\begin{pmatrix}I&0\\0&I\end{pmatrix}$

(iii) Yes, the set $S=\left\{\begin{pmatrix}a&b\\c&d\end{pmatrix};\ a,b,c,d\in Q\right\}$ is a semigroup that is not an ideal

(iv) Yes, we have $\begin{pmatrix}I&I\\I&I\end{pmatrix}\cdot\begin{pmatrix}-I&-I\\I&I\end{pmatrix}=\begin{pmatrix}0&0\\0&0\end{pmatrix}$

(v) Consider $S=\left\{\begin{pmatrix}aI&bI\\cI&dI\end{pmatrix};\ a,b,c,d\in C(\langle Q\cup I\rangle)\right\}$, and it is an ideal in V

(vi) Yes, V contains the set $S=\left\{\begin{pmatrix}a&b\\c&d\end{pmatrix};\ a\cdot d-c\cdot b\neq 0 \text{ and } a,b,c,d\in Q\right\}$, which is a group with respect to multiplication

(vii) Yes, V contains the set $S=\left\{\begin{pmatrix}a&0\\0&a\end{pmatrix};\ a\neq 0 \text{ and } a\in Q\right\}$, which is an Abelian group with respect to multiplication

Problem 17. What are the advantages of using the algebraic structure $C(\langle R\cup I\rangle)$?

Problem 18. Give some uses of this complete algebraic structure $C(\langle R\cup I\rangle)$.

Problems 17 and 18 are solved partially, that is because $C(\langle R\cup I\rangle)$ is being classified as a direct product of the complex field C with itself according to Theorem 3.

6. Group of Units' Problem

A well-known problem in the theory of rings is to describe the group of units under multiplication for a ring R. We will solve this famous problem in the case of $C(\langle R\cup I\rangle)$, $C(\langle Q\cup I\rangle)$, $C(\langle Z\cup I\rangle)$, by using classification properties.

Theorem 5. *The group of units of the ring $C(\langle R\cup I\rangle)$ is $U=C^*\times C^*$.*

Proof. Since $C(\langle R\cup I\rangle)\cong C\times C$, then $U=U(C)\times U(C)$, but C is a field; hence, $U(C)=C^*$. Thus, the proof is complete. □

Theorem 6. *The group of units of the ring $C(\langle Q\cup I\rangle)$ is $U=(Q(i))^*\times(Q(i))^*$.*

Proof. Since $C(\langle Q\cup I\rangle)\cong Q(i)\times Q(i)$, then $U=U(Q(i))\times U(Q(i))$, but $Q(i)$ is a field; hence, $U(Q(i))=(Q(i))^*$. Thus, the proof is complete. □

Theorem 7. *The group of units of the ring* $C(\langle Z \cup I \rangle)$ *is* $U = Z_2 \times Z_2 \times Z_2 \times Z_2$.

Proof. Since $C(\langle Z \cup I \rangle) \cong Z(i) \times Z(i)$, then $U = U(Z(i)) \times U(Z(i))$, but $U(Z(i)) = \{1, -1, i, -i\} \cong Z_2 \times Z_2$; thus, $U = Z_2 \times Z_2 \times Z_2 \times Z_2$. □

7. Conclusions

In this study, we have built some algebraic isomorphisms between neutrosophic complex structures and their corresponding classical structures. Also, we have used these isomorphisms to present a full solution to 18 open problems proposed by Smarandache and Kandasamy concerning the algebraic structure of infinite rings of neutrosophic complex numbers.

On the contrary, we have applied these isomorphisms to determine the classification of the group with units of these rings as a direct product of well-known classical groups.

As a future research direction, we aim to solve many other Smarandache–Kandasamy open problems about neutrosophic complex rings.

Data Availability

The data are available on request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] R. Alhamido, "Neutrosophic bi-topological group," *International Journal of Neutrosophic Science*, vol. 17, 2021.
[2] R. Suresh and S. Palaniammal, "Neutrosophic weakly generalized open and closed sets," *Neutrosophic Sets and Systems*, vol. 33, 2020.
[3] M. Sahin, "Neutrosophic multigroup homomorphism and some of its properties," *International Journal of Neutrosophic Science*, vol. 17, 2021.
[4] P. K. Singh, "Dark data analysis using intuitionistic plithogenic graphs," *International Journal of Neutrosophic Science*, vol. 16, 2021.
[5] A. A. Agboola, A. D. Akwu, and Y. T. Oyebo, "Neutrosophic groups and subgroups," *International J. Math*, vol. 3, pp. 1–9, 2012.
[6] W. B. V Kandasamy and F. Smarandache, "Neutrosophic Rings," 2006, https://arxiv.org/ftp/math/papers/0607/0607765.pdf.
[7] W. B. V. Kandasamy and F. Smarandache, "Finite neutrosophic complex numbers," 2011, https://arxiv.org/abs/1111.0210.
[8] W. B. V. Kandasamy, *Smarandache Rings*, Infinite Study, Chennai, India, 2002.