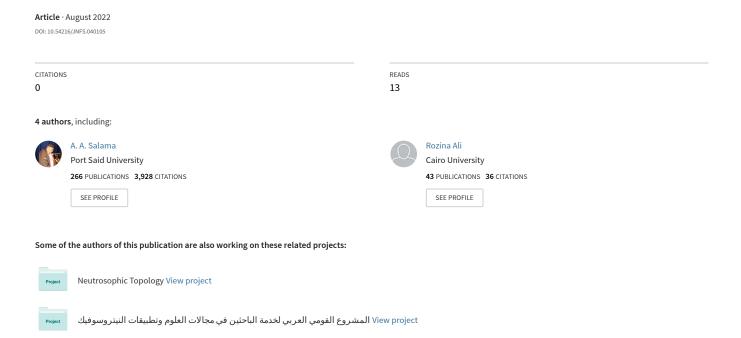
Some Results About 2-Cyclic Refined Neutrosophic Complex Numbers





Some Results About 2-Cyclic Refined Neutrosophic Complex Numbers

Ahmad Salama¹, Rasha Dalla², Malath Al Aswad³, Rozina Ali⁴

¹Department of Mathematics, port said, Egypt

²Departement of Mathematics, Albaath University, Homs, Syria

³PHD of Mathematics, GaziAntep, Turkey

⁴Department Of Mathematics, Cairo University, Egypt

Email:drsalama44@gmail.com; rasha.dallah20@gmail.com; Malaz.Aswad@yahoo.com; rozyyy123n@gmail.com

Abstract

This paper is dedicated to define for the first time the concept of 2-cyclic complex refined neutrosophic numbers as a direct application of 2-cyclic refined neutrosophic sets. Also, it presents some of their elementary properties such as conjugates, absolute values, invertibility, and algebraic operations.

Also, we illustrate many examples to clarify the validity of our discussion.

Keywords: refined neutrosophic complex number ;cyclic refined neutrosophic complex number; refined neutrosophic real number, invertible number.

1.Introduction

Neutrosophy is a new branch of philosophy founded by Smarandache [6,36] to study the indeterminacy in the real world problems and science. It has a master effect in many areas such as topology [7,27,29], equations [3,30], decision making [8], abstract algebra [25,26,39,41], and number theory [35].

Neutrosophic algebra began with the definitions of neutrosophic groups [9,17], and rings [13]. The neutrosophic rings and their generalizations such as refined neutrosophic rings [19], and n-refined neutrosophic rings [11,12], were very useful in the study of neutrosophic algebraic structures.

Neutrosophic algebraic structures were defined as new generalizations of classical ones based on neutrosophic rings and fields, where we find many concepts from linear algebra were generalized into neutrosophic systems such as neutrosophic matrices and spaces over neutrosophic fields [1,42], refined neutrosophic spaces and matrices over refined neutrosophic fields [24], n-refined neutrosophic spaces over n-refined neutrosophic fields [21,32], linear modules and ideals [4,5,20,22].

DOI: https://doi.org/10.54216/JNFS.040105

Neutrosophic complex numbers were firstly studied in [43]. Recently, many of their properties were discussed in [44], especially their invertibility, absolute values, and complex functions.

Through this paper, we define refined neutrosophic complex numbers for the first time. On the other hand, we study many related properties of these numbers such as the inverses, conjugates, and absolute values.

2. Neutrosophic complex number

Definition: Neutrosophic Real Number:

Suppose that w is a neutrosophic number, then it takes the following standard form: w = a + bI where a, b are real coefficients, and I represents the indeterminacy, where 0. I = 0 and $I^n = I$ for all positive integers n.

For example:

$$w = 1 + 2I, w = 3 = 3 + 0I.$$

Definition: Neutrosophic Complex Number:

Suppose that z is a neutrosophic complex number, then it takes the following standard form: z = a + bI + i(c + dI) where a, b, c, d are real coefficients, and I is the indeterminacy element, where $i^2 = -1$ i.e. $i = \sqrt{-1}$.

We recall a + bI the real part, then it takes the following standard form Re(z) = a + bI.

We recall c + dI the imagine part, then it takes the following standard form Im(z) = c + dI.

For example:

$$z = 4 + I + i(2 + 2I)$$

Note: we can say that any real number can be considered a neutrosophic number.

For example: z = 3 = 3 + 0.I + i(0 + 0.I)

2- cyclic refined neutrosophic complex numbers.

Definition:

We define a 2-cyclic refined neutrosophic complex number by the following form:

$$z = (a_o + a_1 I_1 + a_2 I_2) + i(b_o + b_1 I_1 + b_2 I_2)$$
, where $a_o, a_1, a_2, b_0, b_1, b_2$ are real coefficients. For example:

$$z = (1 - I_1 + 2I_2) + i(3 + 2I_1 - I_2).$$

We recall $a_0 + a_1 I_1 + a_2 I_2$ the real part, then it takes the following standard form $Re(z) = a_0 + a_1 I_1 + a_2 I_2$.

We recall $b_0 + b_1 I_1 + b_2 I_2$ the image part, then it takes the following standard form $Im(z) = b_0 + b_1 I_1 + b_2 I_2$.

Remark: A 2-cyclic refined neutrosophic complex number can be defined as follows:

 $z = a + bI_1 + cI_2$ where a, b, c are complex numbers. For example:

$$z = (1-i) + (2+i)I_1 + (3-2i)I_2$$
.

Remark:

 $I_i \times I_j = I_{(i+j \bmod 2)}.$

Definition:

Let $z = (a_o + a_1I_1 + a_2I_2) + i(b_o + b_1I_1 + b_2I_2)$ a 2-cyclic refined neutrosophic complex number. We denote the conjugate of a 2-cyclic refined neutrosophic complex number by \bar{z} and define it by the following form:

$$\bar{z} = (a_0 + a_1 I_1 + a_2 I_2) - i(b_0 + b_1 I_1 + b_2 I_2)$$

For example:

$$z = (-1 + I_1 + 2I_2) + i(1 + 2I_1 - I_2)$$
, Then $\bar{z} = (-1 + I_1 + 2I_2) - i(1 + 2I_1 - I_2)$.

Definition:

Suppose that $z = (a_o + a_1I_1 + a_2I_2) + i(b_o + b_1I_1 + b_2I_2)$ is a 2-cyclic refined neutrosophic complex number. The absolute value of z can be defined by the following form:

$$|z| = \sqrt{(a_0 + a_1I_1 + a_2I_2)^2 + (b_0 + b_1I_1 + b_2I_2)^2}$$

Remark:

$$(1).\overline{(\overline{z})}=z.$$

Proof: Let $\mathbf{z} = (a_o + a_1 I_1 + a_2 I_2) + i(b_o + b_1 I_1 + b_2 I_2)$, then $\overline{\mathbf{z}} = (a_o + a_1 I_1 + a_2 I_2) - i(b_o + b_1 I_1 + b_2 I_2)$.

Now.

$$\overline{(\overline{z})} = \overline{((a_0 + a_1I_1 + a_2I_2) - \iota(b_0 + b_1I_1 + b_2I_2))} = (a_0 + a_1I_1 + a_2I_2) + \iota(b_0 + b_1I_1 + b_2I_2) = z$$

$$(2). z + \overline{z} = 2Re(z)$$

Proof: Let $\mathbf{z} = (a_o + a_1 I_1 + a_2 I_2) + i(b_o + b_1 I_1 + b_2 I_2)$, then $\overline{\mathbf{z}} = (a_o + a_1 I_1 + a_2 I_2) - i(b_o + b_1 I_1 + b_2 I_2)$.

Now.

$$z + \overline{z} = (a_0 + a_1I_1 + a_2I_2) + i(b_0 + b_1I_1 + b_2I_2) + (a_0 + a_1I_1 + a_2I_2) - i(b_0 + b_1I_1 + b_2I_2)$$

$$z + \bar{z} = 2[(a_0 + a_1I_1 + a_2I_2)] = 2Re(z)$$

$$(3). z - \bar{z} = 2Im(z)$$

Let
$$z = (a_0 + a_1I_1 + a_2I_2) + i(b_0 + b_1I_1 + b_2I_2)$$
, then $\bar{z} = (a_0 + a_1I_1 + a_2I_2) - i(b_0 + b_1I_1 + b_2I_2)$.

Now.

$$z - \bar{z} = (a_0 + a_1 I_1 + a_2 I_2) + i(b_0 + b_1 I_1 + b_2 I_2) - [(a_0 + a_1 I_1 + a_2 I_2) - i(b_0 + b_1 I_1 + b_2 I_2)]$$

$$z - \bar{z} = (a_0 + a_1 I_1 + a_2 I_2) + i(b_0 + b_1 I_1 + b_2 I_2) - (a_0 + a_1 I_1 + a_2 I_2) + i(b_0 + b_1 I_1 + b_2 I_2)$$

$$z - \bar{z} = 2i(b_0 + b_1I_1 + b_2I_2) = 2Im(z)$$

$$(4).\,\overline{z_1+z_2}=\bar{z_1}+\bar{z_2}$$

Proof:

Let
$$z_1 = (a_o + a_1I_1 + a_2I_2) + i(b_o + b_1I_1 + b_2I_2), z_2 = (c_o + c_1I_1 + c_2I_2) + i(d_o + d_1I_1 + d_2I_2).$$

Now.

DOI: https://doi.org/10.54216/JNFS.040104

Received: April 15, 2022 Accepted: August 16, 2022

$$z_1 + z_2 = [(a_0 + c_0) + (a_1 + c_1)I_1 + (a_2 + c_2)I_2] + i[(b_0 + d_0) + (b_1 + d_1)I_1 + (b_2 + d_2)I_2]$$

Then.

$$\overline{z_1 + z_2} = [(a_o + c_o) + (a_1 + c_1)I_1 + (a_2 + c_2)I_2] - i[(b_o + d_o) + (b_1 + d_1)I_1 + (b_2 + d_2)I_2]$$

$$\overline{z_1 + z_2} = [(a_o + a_1I_1 + a_2I_2) - i(b_o + b_1I_1 + b_2I_2)] + [(c_o + c_1I_1 + c_2I_2) - i(d_o + d_1I_1 + d_2I_2)] = \overline{z_1} + \overline{z_2}$$

Definition:

Let $w_1 = a_o + a_1 I_1 + a_2 I_2$, $w_2 = b_o + b_1 I_1 + b_2 I_2$ are two cyclic refined neutrosophic real numbers, we define the multiplication w_1 . w_2 as follows:

$$w_1 \times w_2 = (a_o + a_1 I_1 + a_2 I_2) \times (b_o + b_1 I_1 + b_2 I_2) = (a_o \times b_o) + (a_o \times b_1) I_1 + (a_o \times b_2) I_2 + (a_1 \times b_0) I_1 + (a_1 \times b_1) I_1 \times I_1 + (a_1 \times b_2) I_1 \times I_2 + (a_2 \times b_0) I_2 + (a_2 \times b_1) I_2 \times I_1 + (a_2 \times b_2) I_2 \times I_2.$$

By remark 3.2, we have.

$$I_1 \times I_1 = I_2, I_1 \times I_2 = I_1, I_2 \times I_1 = I_1, I_2 \times I_2 = I_2.$$

Then

$$w_1 \times w_2 = (a_o \times b_o) + [(a_o \times b_1) + (a_1 \times b_o) + (a_1 \times b_2) + (a_2 \times b_1)]I_1 + [(a_1 \times b_1) + (a_o \times b_2) + (a_2 \times b_0) + (a_2 \times b_2)]I_2.$$

Definition:

Let
$$z_1 = (a_0 + a_1I_1 + a_2I_2) + i(b_0 + b_1I_1 + b_2I_2), z_2 = (c_0 + c_1I_1 + c_2I_2) + i(d_0 + d_1I_1 + d_2I_2).$$

A product $z_1 \times z_2$ is defined by form:

$$z_1 \times z_2 = [(a_0 + a_1 I_1 + a_2 I_2) + i(b_0 + b_1 I_1 + b_2 I_2)] \times [(c_0 + c_1 I_1 + c_2 I_2) + i(d_0 + d_1 I_1 + d_2 I_2)]$$

$$z_1 \times z_2 = (a_o + a_1 I_1 + a_2 I_2) \times (c_o + c_1 I_1 + c_2 I_2) - (b_o + b_1 I_1 + b_2 I_2) \times (d_o + d_1 I_1 + d_2 I_2) + i[(a_o + a_1 I_1 + a_2 I_2) \times (d_o + d_1 I_1 + d_2 I_2) + (c_o + c_1 I_1 + c_2 I_2) \times (b_o + b_1 I_1 + b_2 I_2)].$$

By using Definition 3.4, we get the product $z_1 \times z_2$.

Remark:

$$(1).\,\overline{z_1\times z_2}=\overline{z_1}\times\overline{z_2}$$

$$(1). z \times \bar{z} = |z|^2.$$

Definition:

Let $w = a_0 + a_1 I_1 + a_2 I_2$ be a 2-cyclic refined neutrosophic real number, then the inverse of w is defined as follows:

$$w^{-1} = \frac{1}{w} = \frac{1}{a_0 + a_1 I_1 + a_2 I_2} = (a_0 + a_1 I_1 + a_2 I_2)^{-1}$$

$$\frac{1}{w} = (a_0)^{-1} + [(a_0 + a_1 + a_2)^{-1} - (a_0 + a_2)^{-1}]I_1 + [(a_0 + a_2)^{-1} - (a_0)^{-1}]I_2$$

Where $a_0 + a_1 + a_2$, $a_0 + a_2$, a_0 are not zero elements.

Definition:

Let $z = (a_0 + a_1 I_1 + a_2 I_2) + i(b_0 + b_1 I_1 + b_2 I_2)$, then the inverse of z is defined as follows:

$$z^{-1} = \frac{1}{z} = \frac{1}{(a_0 + a_1 I_1 + a_2 I_2) + i(b_0 + b_1 I_1 + b_2 I_2)} = \frac{\bar{z}}{z \times \bar{z}} = \frac{\bar{z}}{|z|^2}$$

Example:

Let
$$z = (1 + I_1 - I_2) + i(2 + 2I_1 - I_2)$$
, $\bar{z} = (1 + I_1 - I_2) - i(2 + 2I_1 - I_2)$.

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{(1 + I_1 - I_2) - i(2 + 2I_1 - I_2)}{(1 + I_1 - I_2)^2 + (2 + 2I_1 - I_2)^2} =$$

Now we have.

$$(1+I_1-I_2)^2=(1+I_1)^2-2(1+I_1)\times I_2+(I_2\times I_2)=1+2I_1+(I_1\times I_1)-2I_2-2I_1\times I_2+I_2\times I_2$$

$$(1 + I_1 - I_2)^2 = 1 + 2I_1 + I_2 - 2I_2 - 2I_1 + I_2 = 1$$

$$(2 + 2I_1 - I_2)^2 = (2 + 2I_1)^2 - 2(2 + 2I_1) \times I_2 + (I_2 \times I_2) = 4 + 8I_1 + 4(I_1 \times I_1) - 4I_2 - 4I_1 \times I_2 + I_2 \times I_2$$

$$(2 + 2I_1 - I_2)^2 = 4 + 8I_1 + 4I_2 - 4I_2 - 4I_1 + I_2 = 4 + 4I_1 + I_2$$

Then

$$z^{-1} = \frac{(1 + I_1 - I_2) - i(2 + 2I_1 - I_2)}{1 + 4 + 4I_1 + I_2} = \frac{(1 + I_1 - I_2) - i(2 + 2I_1 - I_2)}{5 + 4I_1 + I_2}$$

$$z^{-1} = [(1 + I_1 - I_2) - i(2 + 2I_1 - I_2)] \times (5 + 4I_1 + I_2)^{-1}$$

$$z^{-1} = (1 + I_1 - I_2) \times (5 + 4I_1 + I_2)^{-1} - i(2 + 2I_1 - I_2) \times (5 + 4I_1 + I_2)^{-1}$$

$$z^{-1} = \left(1 + I_1 - I_2\right) \times \left(\frac{1}{5} - \frac{1}{15}I_1 - \frac{1}{30}I_2\right) - i\left(2 + 2I_1 - I_2\right) \times \left(\frac{1}{5} - \frac{1}{15}I_1 - \frac{1}{30}I_2\right)$$

$$(1 + I_1 - I_2) \times \left(\frac{1}{5} - \frac{1}{15}I_1 - \frac{1}{30}I_2\right) = \frac{1}{5} + \frac{1}{6}I_1 - \frac{4}{15}I_2$$

$$(2 + 2I_1 - I_2) \times \left(\frac{1}{5} - \frac{1}{15}I_1 - \frac{1}{30}I_2\right) = \frac{2}{5} + \frac{4}{15}I_1 - \frac{11}{30}I_2$$

Hence,

$$z^{-1} = \left(\frac{1}{5} + \frac{1}{6}I_1 - \frac{4}{15}I_2\right) - i\left(\frac{2}{5} + \frac{4}{15}I_1 - \frac{11}{30}I_2\right).$$

Conclusion

In this paper, we have defined for the first time the concept of 2-cyclic refined neutrosophic complex numbers. Also, we have discussed some of their elementary properties such as the conjugate, the multiplication, absolute values and other related topics.

As a future research direction, we aim to study the natural generalization of those numbers by n-cyclic refined neutrosophic complex numbers.

References

- [1] Abobala, M., "AH-Subspaces in Neutrosophic Vector Spaces", International Journal of Neutrosophic Science, Vol. 6, pp. 80-86. 2020.
- [2] Abobala, M.,. "A Study of AH-Substructures in *n*-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science", Vol. 9, pp.74-85. 2020.
- [3] Sankari, H., and Abobala, M., "Neutrosophic Linear Diophantine Equations With two Variables", Neutrosophic Sets and Systems, Vol. 38, pp. 22-30, 2020.
- [4] Sankari, H., and Abobala, M." *n*-Refined Neutrosophic Modules", Neutrosophic Sets and Systems, Vol. 36, pp. 1-11. 2020.
- [5] Alhamido, R., and Abobala, M., "AH-Substructures in Neutrosophic Modules", International Journal of Neutrosophic Science, Vol. 7, pp. 79-86. 2020.

DOI: https://doi.org/10.54216/JNFS.040104

- [6]Smarandache, F., " A Unifying Field in Logics: Neutrosophic Logic, Neutrosophy, Neutrosophic Set, Neutrosophic Probability", American Research Press. Rehoboth, 2003.
- [7]Suresh, R., and S. Palaniammal,. "Neutrosophic Weakly Generalized open and Closed Sets", Neutrosophic Sets and Systems, Vol. 33, pp. 67-77,. 2020.
- [8]Olgun, N., and Hatip, A., "The Effect Of The Neutrosophic Logic On The Decision Making, in Quadruple Neutrosophic Theory And Applications", Belgium, EU, Pons Editions Brussels,pp. 238-253. 2020.
- [9] Hatip, A., Alhamido, R., and Abobala, M., "A Contribution to Neutrosophic Groups", International Journal of Neutrosophic Science", Vol. 0, pp. 67-76. 2019.
- [10] Abobala, M., " *n*-Refined Neutrosophic Groups I", International Journal of Neutrosophic Science, Vol. 0, pp. 27-34. 2020.
- [11] Abobala, M., "Classical Homomorphisms Between n-refined Neutrosophic Rings", International Journal of Neutrosophic Science", Vol. 7, pp. 74-78. 2020.
- [12]Smarandache, F., and Abobala, M., n-Refined neutrosophic Rings, International Journal of Neutrosophic Science, Vol. 5, pp. 83-90, 2020.
- [13] Abobala, M., On Some Special Substructures of Neutrosophic Rings and Their Properties, International Journal of Neutrosophic Science", Vol. 4, pp. 72-81, 2020.
- [14] Abobala, M., "On Some Special Substructures of Refined Neutrosophic Rings", International Journal of Neutrosophic Science, Vol. 5, pp. 59-66. 2020.
- [15]Sankari, H., and Abobala, M.," AH-Homomorphisms In neutrosophic Rings and Refined Neutrosophic Rings", Neutrosophic Sets and Systems, Vol. 38, pp. 101-112, 2020.
- [16] Smarandache, F., and Kandasamy, V.W.B., "Finite Neutrosophic Complex Numbers", Source: arXiv. 2011.
- [17] Agboola, A.A.A,. Akwu, A.D,. and Oyebo, Y.T., "Neutrosophic Groups and Subgroups", International .J .Math. Combin, Vol. 3, pp. 1-9. 2012.
- [18]Smarandache, F., "n-Valued Refined Neutrosophic Logic and Its Applications in Physics", Progress in Physics, 143-146, Vol. 4, 2013.
- [19] Adeleke, E.O., Agboola, A.A.A., and Smarandache, F., "Refined Neutrosophic Rings I", International Journal of Neutrosophic Science, Vol. 2(2), pp. 77-81. 2020.
- [20]Hatip, A., and Abobala, M., "AH-Substructures In Strong Refined Neutrosophic Modules", International Journal of Neutrosophic Science, Vol. 9, pp. 110-116 . 2020.
- [21]Smarandache F., and Abobala, M., "n-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science, Vol. 7, pp. 47-54. 2020.
- [22] Sankari, H., and Abobala, M., "Solving Three Conjectures About Neutrosophic Quadruple Vector Spaces", Neutrosophic Sets and Systems, Vol. 38, pp. 70-77. 2020.
- [23] Adeleke, E.O., Agboola, A.A.A., and Smarandache, F., "Refined Neutrosophic Rings II", International Journal of Neutrosophic Science, Vol. 2(2), pp. 89-94. 2020.
- [24] Abobala, M., On Refined Neutrosophic Matrices and Their Applications In Refined Neutrosophic Algebraic Equations, Journal Of Mathematics, Hindawi, 2021
- [25] Abobala, M., A Study of Maximal and Minimal Ideals of n-Refined Neutrosophic Rings, Journal of Fuzzy Extension and Applications, Vol. 2, pp. 16-22, 2021.
- [26] Abobala, M., "Semi Homomorphisms and Algebraic Relations Between Strong Refined Neutrosophic Modules and Strong Neutrosophic Modules", Neutrosophic Sets and Systems, Vol. 39, 2021.
- [27] Giorgio, N, Mehmood, A., and Broumi, S.," Single Valued neutrosophic Filter", International Journal of Neutrosophic Science, Vol. 6, 2020.

- [28] Chellamani, P., and Ajay, D., "Pythagorean neutrosophic Fuzzy Graphs", International Journal of Neutrosophic Science, Vol. 11, 2021.
- [29]Milles, S, Barakat, M, and Latrech, A., "Completeness and Compactness In Standard Single Valued neutrosophic Metric Spaces", International Journal of Neutrosophic Science, Vol.12, 2021.
- [30] Abobala, M., "On Some Neutrosophic Algebraic Equations", Journal of New Theory, Vol. 33, 2020.
- [31] Abobala, M., On The Representation of Neutrosophic Matrices by Neutrosophic Linear Transformations, Journal of Mathematics, Hindawi, 2021.
- [32] Abobala, M., "On Some Algebraic Properties of n-Refined Neutrosophic Elements and n-Refined Neutrosophic Linear Equations", Mathematical Problems in Engineering, Hindawi, 2021
- [33] Kandasamy V, Smarandache F., and Kandasamy I., Special Fuzzy Matrices for Social Scientists . Printed in the United States of America, 2007, book, 99 pages.
- [34] Khaled, H., and Younus, A., and Mohammad, A., "The Rectangle Neutrosophic Fuzzy Matrices", Faculty of Education Journal Vol. 15, 2019. (Arabic version).
- [35] Abobala, M., Partial Foundation of Neutrosophic Number Theory, Neutrosophic Sets and Systems, Vol. 39, 2021.
- [36] F. Smarandache, *Neutrosophic Theory and Applications*, Le Quy Don Technical University, Faculty of Information technology, Hanoi, Vietnam, 17th May 2016.
- [37] Ibrahim, M.A., Agboola, A.A.A, Badmus, B.S. and Akinleye, S.A., "On refined Neutrosophic Vector Spaces I", International Journal of Neutrosophic Science, Vol. 7, pp. 97-109. 2020.
- [38] Ibrahim, M.A., Agboola, A.A.A, Badmus, B.S., and Akinleye, S.A., "On refined Neutrosophic Vector Spaces II", International Journal of Neutrosophic Science, Vol. 9, pp. 22-36. 2020.
- [39] Abobala, M, "n-Cyclic Refined Neutrosophic Algebraic Systems Of Sub-Indeterminacies, An Application To Rings and Modules", International Journal of Neutrosophic Science, Vol. 12, pp. 81-95. 2020.
- [40]Smarandache, F., "Neutrosophic Set a Generalization of the Intuitionistic Fuzzy Sets", Inter. J. Pure Appl. Math., pp. 287-297. 2005.
- [41] Abobala, M., "On Some Special Elements In Neutrosophic Rings and Refined Neutrosophic Rings", Journal of New Theory, vol. 33, 2020.
- [42] Abobala, M., Hatip, A., Olgun, N., Broumi, S., Salama, A,A., and Khaled, E, H., The algebraic creativity In The Neutrosophic Square Matrices, Neutrosophic Sets and Systems, Vol. 40, pp. 1-11, 2021.
- [43] Smarandache, F., and Kandasamy, V.W.B., "Finite Neutrosophic Complex Numbers", Source: arXiv. 2011.
- [44] Aswad, F, M., " A Study of A Neutrosophic Complex Numbers and Applications", Neutrosophic Knowledge, Vol. 1, 2020.
- [45] Abobala, M., Hatip, A., Bal, M., "A Study Of Some Neutrosophic Clean Rings", International journal of neutrosophic science, 2022.
- [46] Abobala, M., "On The Characterization of Maximal and Minimal Ideals In Several Neutrosophic Rings", Neutrosophic Sets and Systems, Vol. 45, 2021.
- [47] Olgun, N., Hatip, A., Bal, M., and Abobala, M., "A Novel Approach To Necessary and Sufficient Conditions For The Diagonalization of Refined Neutrosophic Matrices", International Journal of neutrosophic Science, Vol. 16, pp. 72-79, 2021.
- [48] Abobala, M., Hatip, A., and Bal, M., "A Review On Recent Advantages In Algebraic Theory Of Neutrosophic Matrices", International Journal of Neutrosophic Science, Vol.17, 2021.
- [49] Ibrahim, M., and Abobala, M., "An Introduction To Refined Neutrosophic Number Theory", Neutrosophic sets and systems, Vol. 45, 2021.
- [50] Abobala, M., Bal, M., Aswad, M., "A Short Note On Some Novel Applications of Semi Module Homomorphisms", International journal of neutrosophic science, 2022.

- [51] Singh, P,K., " Data With Turiyam Set for Fourth Dimension Quantum Information Processing", Journal of Neutrosophic and Fuzzy Systems, vol.1, 2022.
- [52] Singh, P, K., Ahmad, K., Bal, M., Aswad, M.," On The Symbolic Turiyam Rings", Journal of Neutrosophic and Fuzzy Systems, 2022.
- [53] Prem Kumar Singh, Three-way n-valued neutrosophic concept lattice at different granulation, International Journal of Machine Learning and Cybernetics, *Springer*, November 2018, Vol 9, Issue 11, pp. 1839-1855.