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On Some Open Problems About n-Cyclic Refined Neutrosophic Rings And Number Theory

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Abstract:

The objective of this paper is to present 40 open problems about the n-cyclic refined neutrosophic rings. These questions concern the n-cyclic refined neutrosophic rings, n-cyclic refined neutrosophic number theory, and n-cyclic refined neutrosophic analysis. They will represent the future of the study of neutrosophic n-cyclic refined rings and the related structures.

Keywords: n-cyclic refined neutrosophic ring; n-cyclic refined group of units, n-cyclic refined neutrosophic equation

Introduction

Neutrosophic algebra began with Smarandache and Kandasamy [1], were they defined neutrosophic rings, groups, and spaces.

In the literature, we find many generalizations of classical algebraic structures using many types of neutrosophic sets such as refined neutrosophic algebraic structures [2-10], n-refined neutrosophic structures [11-19], and n-cyclic refined neutrosophic structures [20]. The n-valued NSS [21-28] and its refined given a new way to deal with data set based on human Turiyam awareness [29]. The data sets with non-Euclidean geometry [30-32] given a new dimension to revisit basic proofs of several mathematical theorems and its orientations [33].

The concept of n-cyclic refined neutrosophic ring and module was released in [20], by using a multiplication structure similar to the cyclic group of residues module n. Also, n-cyclic refined neutrosophic groups laterally.

In this work, we provide 40 open problems about n-cyclic refined neutrosophic rings, spaces, and their corresponding substructures such as congruencies, matrices, and equations.

These questions may reflect the future of the study on n-cyclic refined neutrosophic algebra.

Main Discussion

Definition [20]

Let (R,+,×) be a ring and $I_k; 1 \leq k \leq n$ be n sub-indeterminacies. We define R_n(I)=$\{a_0 + a_1 I + \cdots + a_n I_n \; ; \; a_i \in R\}$ to be n-cyclic refined neutrosophic ring.

Operations on R_n(I) are defined as:

$\sum_{i=0}^{n} x_i I_i + \sum_{i=0}^{n} y_i I_i = \sum_{i=0}^{n} (x_i + y_i) I_i$, $\sum_{i=0}^{n} x_i I_i \times \sum_{i=0}^{n} y_i I_i = \sum_{i,j=0}^{n} (x_i \times y_j) I_i I_j = \sum_{i,j=0}^{n} (x_i \times y_j) I_{(i+j \; mod n)}$.

× is the multiplication on the ring R.

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Example :

(a) The 2-cyclic refined neutrosophic ring of integers is defined as follows:

$Z_2(I) = \{t_0 + t_1I_1 + t_2I_2;\ t_i \in Z\}$.

(b) Addition on $Z_2(I)$ can be clarified as follows:

$(a + bI_1 + cI_2) + (m + nI_1 + tI_2) = (a + m) + I_1(b + n) + I_2(c + t)$.

(c) Multiplication on $Z_2(I)$ can be clarified as follows:

$(a + bI_1 + cI_2)(m + nI_1 + tI_2) = am + anI_1 + atI_2 + bmI_1 + bnI_2 + btI_1 + cmI_2 + cnI_1 + ctI_2$

$= am + I_1(an + bm + bt + cn) + I_2(at + bn + cm + ct)$.

Where $I_1I_1 = I_{(1+1\ mod 2)} = I_2, I_2I_2 = I_{(2+2\ mod\ 2)} = I_2, I_1I_2 = I_{(1+2\ mod\ 2)} = I_1$.

The elements of Z_2 are taken by the form $\{1,2\}$ instead of $\{0,1\}$ in the definition of indices I in sub-indeterminacies I_i. See [20].

Now, we show the open problems as follows:

Open Problem 1:

Find an algorithm to solve a linear n-cyclic refined neutrosophic equation with one variable AX+B=0.

Example:

Consider the following 3-cyclic refined neutrosophic linear equation:

$(2 + I_1 - 3I_2 + I_3)X + I_1 - I_2 - I_3 = 0$.

The solution will depend on the invertibility of $2 + I_1 - 3I_2 + I_3$.

Open Problem 2:

Find an algorithm to solve n-cyclic refined quadratic equations.

Example:

Consider the following 4-cyclic refined neutrosophic quadratic equation:

$(I_1 + I_2 - 5I_3 + 2I_4)X^2 + (1 - I_1 - 2I_2 + I_3)X + (I_3 + I_4) = 0$.

Open Problem 3:

Find an algorithm to compute the inverse of an n-cyclic refined neutrosophic matrix

Example:

Consider the following 2-cyclic refined real neutrosophic matrix

$A = \begin{pmatrix} 1 - I_1 + I_2 & 2 - I_1 + 3I_2 \\ 1 - 5I_1 + 4I_2 & 6 - 2I_1 + 2I_2 \end{pmatrix}$, is it invertible?. How can we find its inverse.

Open Problem 4:

How can we find the eigen values\vectors of an n-cyclic refined neutrosophic matrix especially in the real case.

Open Problem 5:

Find the necessary and sufficient conditions for the diagonalization of an n-cyclic refined neutrosophic matrix.

Open Problem 6:

Find the classification of the group of units of an n-cyclic refined neutrosophic ring.

This problem is very important, that is because the invertibility of the elements in an n-cyclic refined neutrosophic ring will be very effective in finding invertible matrices and linear equations.

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Open Problem 7:

Find an algorithm to find idempotent and nilpotent elements in an n-cyclic refined neutrosophic ring.

Open Problem 8:

If R is clean ring, then does the corresponding n-cyclic refined neutrosophic ring have the same property?

Open Problem 9:

If Kothe's conjecture is true in the ring R, then is it true in the corresponding n-cyclic refined neutrosophic ring?.

Open Problem 10:

Find an easy algorithm to solve linear Diophantine equations in the n-cyclic refined neutrosophic ring of integers?.

Open Problem 11:

Is Euler's theorem still true in the n-cyclic refined neutrosophic ring of integers?.

Open Problem 12:

Find an algorithm to solve congruencies\linear congruencies in the n-cyclic refined neutrosophic ring of integers?.

Open Problem 13:

How can we find maximal and minimal ideals in the n-cyclic refined neutrosophic ring?

Open Problem 14:

Find an algorithm to compute imperfect duplets and triplets in the n-cyclic refined neutrosophic rings?

Open Problem 15:

Find an algorithm to check zero divisors in the n-cyclic refined neutrosophic rings

Open Problem 16:

How can we find units in the n-cyclic refined neutrosophic rings?

Open Problem 17:

How can we find algebraic\transcendental elements in the n-cyclic refined neutrosophic ring

Open Problem 18:

Find an algorithm to compute exponents in an n-cyclic refined neutrosophic field (especially the complex field case).

Open Problem 19:

Find an algorithm to compute GCD in the n-cyclic refined neutrosophic ring of integers?

Open Problem 20:

Find an algorithm to solve Pell's Diophantine equation in the n-cyclic refined neutrosophic ring of integers?

Open Problem 21:

Find an algorithm to find Fermat's triples (especially Pythagoras triples) in the n-cyclic refined neutrosophic rings?

Open Problem 22:

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In the case of n-cyclic refined neutrosophic real field, define continuous, differentiable, integrable functions on it .

Open Problem 23:

Can we define a geometrical system of n-cyclic refined neutrosophic points in a similar way of neutrosophic points?. Is it Euclidean?

Open Problem 24:

Describe the equations of famous geometrical shapes such as circle, sphere, line,..

In the system of n-cyclic refined neutrosophic real numbers.

Open Problem 25:

Define inner products on an n-cyclic refined neutrosophic vector space.

Open Problem 27:

Define Banach and Hillbert n-cyclic refined neutrosophic vector spaces.

Open Problem 28:

Does Caushy-Schwartz inequality still true in n-cyclic refined neutrosophic vector spaces?.

Open Problem 29:

Does Parseval's identity still true in n-cyclic refined neutrosophic vector spaces.?

Open Problem 30:

Can we generalize the representation theory of groups from a classical vector space V to the corresponding n-cyclic refined neutrosophic vector space ?

Open Problem 31:

How can we represent an n-cyclic refined neutrosophic matrix by n-cyclic refined neutrosophic linear transformations?

Open Problem 32:

Define complex inner product n-cyclic refined neutrosophic vector spaces.

Open Problem 33:

Define convexity in n-cyclic refined neutrosophic vector spaces

Open Problem 34:

Find an algorithm to solve a system of linear n-cyclic refined neutrosophic algebraic equations.

Open Problem 35:

Find an algorithm for division in the n-cyclic refined neutrosophic ring of integers

Open Problem 36:

Describe n-cyclic refined neutrosophic real differential equations

Open Problem 37:

Describe n-cyclic refined neutrosophic integral equations

Open Problem 38:

Define the continuous density functions of n-cyclic refined neutrosophic random variables

Open Problem 39:

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Find irreducible polynomials in an n-cyclic refined neutrosophic ring .

Open Problem 40:

Define n-cyclic refined neutrosophic elliptic curves. What are their properties?.

Conclusion

In this paper, we have released 40 open problems concern the n-cyclic refined neutrosophic algebra and analysis. These questions may represent the future of the studies in this field, especially in n-cyclic refined neutrosophic algebraic structures, n-cyclic refined neutrosophic probability, and n-cyclic refined neutrosophic number theory. Also, we aim to solve some these questions in the future works.

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