A Short Contribution To Von Shtawzen's Abelian Group In n-Cyclic Refined Neutrosophic Rings

A .Alrida Basheer [*1], Katy D. Ahmad[2], Rozina Ali[3]

[1] Imam Kadhum College, Iraq
[2] Islamic University Of Gaza, Palestine
[3] Cairo University, Egypt

Emails: basheerabdalrida66n@gmail.com ; katyon765@gmail.com ; rozyyy123n@gmail.com

Abstract

This paper is dedicated to study the group of units (Von Shtawzen's group) of some numerical n-cyclic refined neutrosophic rings such as integer, rational, and real case. Where we write the elements of these abelian groups by using equations derived from the values of some circulant numerical determinants.

Keywords: n-cyclic refined neutrosophic ring; n-cyclic refined integer; group of units; Von Shtawzen's abelian group.

Introduction

The group of units is an important group which is related to every ring with unity [1]. In neutrosophic algebra, the concept of n-cyclic refined neutrosophic ring was defined by Abobala [3] to generalize any classical ring by a bigger ring. This ring is a part of many neutrosophic algebraic structures, see [5-10,17-22,26-33]. Recently, Von Shtawzen [16] studied invertible elements in 4-cyclic refined Ring and Turiyam Rings which motivated this study.

The group of units problem of n-cyclic refined neutrosophic rings was studied for the first time in [4] by Sadiq, where he has suggested some interesting open problems.

Recently, Von Shtawzen [16] has presented a necessary and sufficient condition for the invertibility of a 3-cyclic\4-cyclic refined neutrosophic integer in terms of some Diophantine equations.

The motivation of this work was the work of Von Shtawzen [16] about two conjectures concerning the structure of the group of units of 3-cyclic and 4-cyclic refined neutrosophic rings of integers.

In this work, we define the concept of Von Shtawzen's abelian group by using the circulant determinants. Also, we present an interesting open problem at the end of this paper.

For the definitions and properties of n-cyclic refined neutrosophic rings, see [3].

Main Discussion

Definition:

Let $R_n(I) = \{a_0 + a_1 I_1 + a_2 I_2 + .. + a_n I_n;\ a_i \in Z \text{ or } R \text{ or } Q\}$ be the n-cyclic refined neutrosophic ring of integers, reals, or rationales, then $A = a_0 + a_1 I_1 + a_2 I_2 + .. + a_n I_n$ is called invertible (unit) if and only if there exists $B = b_0 + b_1 I_1 + b_2 I_2 + .. + b_n I_n$ such that AB=1.

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Definition:

Let $R_n(I) = \{a_0 + a_1 I_1 + a_2 I_2 + .. + a_n I_n;\ a_i \in Z\ or\ R\ or\ Q\}$ be the n-cyclic refined neutrosophic ring of integers, reals, or rationales, then we call the group of units of this ring by Von Shtawzen's abelian group.

The Condition Of Invertibility

From the equation AB=1, we get a system of n linear equations with n variables. The unique solution of this system is related with the fact that the determinant of coefficients matrix should be invertible in the desired ring\field.

For example, if R is equal to real field or the rational field Q, then the corresponding determinant should not be zero. If R is equal to integer ring Z, then the determinant should be 1 or -1 [16].

Now, we will show that the corresponding determinant is a circulant determinant.

From the equation AB=1, we can see that the coefficients of I_t is $\sum_{i+j\equiv t\ mod\ n} a_i b_j$.

This implies that the corresponding linear system of n equations is equivalent to:

$\sum_{i+j\equiv 1\ mod\ n} a_i b_j = 0, \sum_{i+j\equiv 2\ mod\ n} a_i b_j = 0, \dots, \sum_{i+j\equiv n\ mod\ n} a_i b_j$=0.

The coefficients matrix is circulant with the following form:

$$T = \begin{pmatrix} a_0 + a_n & a_{n-1} & a_{n-2} \cdots & a_1 \\ \vdots & & \ddots & \vdots \\ a_1 & & \cdots & a_0 + a_n \end{pmatrix}.$$

The value of the determinant of the the previous matrix can be computed by the following formula:

$\det(T) = p(\alpha_1)p(\alpha_2)..p(\alpha_n)$, where α_i are the roots of unity of order n in the complex field C, and

$p(x) = (a_0 + a_n) + (a_{n-1})x + (a_{n-2})x^2 + \cdots + (a_1)x^{n-1}$.

From the discussion above, we get the following results:

Result 1:

If R is the real or rational field, then A is an element of Von Shtawzen's group if and only if $(\alpha_1)p(\alpha_2)..p(\alpha_n) \neq 0$.

Result 2:

If R is the integer ring, then A is an element of Von Shtawzen's group if and only if $p(\alpha_1)p(\alpha_2)..p(\alpha_n) = 1\ or - 1$.

Now, we can generalize the Von Shtawzen's conjectures in [16], to this novel version.

Von Shtawzen's Generalized Conjecture:

If $R_n(I) = \{a_0 + a_1 I_1 + a_2 I_2 + .. + a_n I_n;\ a_i \in Z\ \}$, then the order of Von Shtawzen's group is finite and divisible by 2n. (The original case was about n=3, n=4).

There exists an equivalent formula of the previous conjecture, we can write it as the following:

The Diophantine equation $p(\alpha_1)p(\alpha_2)..p(\alpha_n) = 1\ or - 1, where$ $T = \begin{pmatrix} a_0 + a_n & a_{n-1} & a_{n-2} \cdots & a_1 \\ \vdots & & \ddots & \vdots \\ a_1 & & \cdots & a_0 + a_n \end{pmatrix}$ $and\ a_0 = 1\ or - 1$ has a finite number of solutions. These number must be divisible by 2n.

Open Problem:

Classify the infinite Von Shtawzen's abelian group as a direct product of some familiar abelian infinite groups in the case of real field or rational field for an arbitrary integer n. (Remark that this problem is still open even in the case n=2, see [4]).

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Conclusion

In this paper, we have discussed the invertibility of a 3-cyclic\ 4-cyclic refined neutrosophic integer. Also, we have presented two conjectures concerning the order of the group of units of 3-cyclic\ 4-cyclic refined neutrosophic ring of integers.

Also, we presented the condition for the invertibility of a symbolic Turiyam integer.

In the future, we aim to look for the solutions of these conjectures and to classify the group of units of these rings.

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