A Review On Recent Developments In Neutrosophic Linear Diophantine Equations

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A Review On Recent Developments In Neutrosophic Linear Diophantine Equations

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Abstract:

This paper is dedicated to present a wide review study on the recent advantages of neutrosophic linear diophantine equations and number theory.

We revise the neutrosophic linear diophantine equations, refined neutrosophic Diophantine equations, and n-refined neutrosophic linear Diophantine equations.

Key words: Neutrosophic number theory, neutrosophic linear diophantine equation, n-refined neutrosophic integer.

1. Introduction

Neutrosophy is a new generalization of fuzzy logic presented by Smarandache [1-5]. Neutrosophic sets were very useful in the algebraic strudies such as rings [6-12], modules [13-30], matrices [31-45], and spaces [56-75]. Recently, it is used to deal with Non-Euclidean data sets using Neutrogeometry [76-90] as well as Turiyam set [91-92]. In this way the applications of Neutrosophic Number and its equations is indeed required for multi-decision process.

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Neutrosophic number theory is a new research field released by many authors. Where we find a study of the foundations of neutrosophic number theory [59,73], and refined number theory [80].

In the literature, the linear Diophantine equations in the neutrosophic systems was studied firstly by Sankari et. al [4], and then they were generalized to refined and n-refined neutrosophic integers [79] and its extensive properties [76-80, 90-92].

In this work, we give the interested reader a wide review on recent developments in the field of neutrosophic number theory, especially the subject of neutrosophic linear Diophantine equations.

This work may be very useful in future, especially in defining the Turiyam number theory approaches.

Main Discussion

Definition:

Let $Z(I) = \{a + bI;\ a, b \in Z\}$ be the neutrosophic ring of integers. The neutrosophic linear Diophantine equation with two variables is defined as follows:

$AX + BY = C; A, B, C \in Z(I)$.

Theorem :

Let $Z(I) = \{a + bI;\ a, b \in Z\}$ be the neutrosophic ring of integers. The neutrosophic linear Diophantine equation $AX + BY = C$ with two variables $X = x_1 + x_2 I, Y = y_1 + y_2 I$, where

$A = a_1 + a_2 I, B = b_1 + b_2 I$ is equivalent to the following two classical Diophantine equations:

(1) $a_1 x_1 + b_1 y_1 = c_1$.

(2) $(a_1 + a_2)(x_1 + x_2) + (b_1 + b_2)(y_1 + y_2) = c_1 + c_2$.

Proof:

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It is sufficient to show that $AX + BY = C$ implies (1) and (2).

$AX + BY = C$ is equivalent to:

$(a_1 + a_2I)(x_1 + x_2I) + (b_1 + b_2I)(y_1 + y_2I) = c_1 + c_2I$, by easy computing we find

$[a_1x_1 + b_1y_1] + [a_1x_2 + a_2x_1 + a_2x_2 + b_1y_2 + b_2y_1 + b_2y_2]I = c_1 + c_2I$, hence

$a_1x_1 + b_1y_1 = c_1, and\ a_1x_2 + a_2x_1 + a_2x_2 + b_1y_2 + b_2y_1 + b_2y_2 = c_2$. We can see that we get equation (1). For equation (2) we take

$a_1x_2 + a_2x_1 + a_2x_2 + b_1y_2 + b_2y_1 + b_2y_2 = c_2$, by adding equation (1) to the two sides we obtain

$a_1x_1 + b_1y_1 + a_1x_2 + a_2x_1 + a_2x_2 + b_1y_2 + b_2y_1 + b_2y_2 = c_1 + c_2$, which implies equation (2)

$(a_1 + a_2)(x_1 + x_2) + (b_1 + b_2)(y_1 + y_2) = c_1 + c_2$.

The following theorem determines the criteria for the solvability of neutrosophic linear Diophantine equation.

Theorem:

Let $Z(I) = \{a + bI;\ a, b \in Z\}$ be the neutrosophic ring of integers. The neutrosophic linear Diophantine equation $AX + BY = C$ with two variables $X = x_1 + x_2I, Y = y_1 + y_2I$ and $A = a_1 + a_2I, B = b_1 + b_2I$ is solvable if and only if $\gcd(a_1, b_1)\,|c_1$, $\gcd(a_1 + a_2, b_1 + b_2)\,|c_1 + c_2$.

Example:

(a) The neutrosophic Diophantine equation $(2 + 2I)X + (3 + 4I)Y = 5 + 5I$ is solvable, that is because

$\gcd(2,3)\,|5, and\ \gcd(4,7)|10$.

(b) The neutrosophic Diophantine equation $(2 + 3I)X + (4 + 5I)Y = 5 + I$ is not solvable, since

$\gcd(2,4) = 2$ does not divide 5.

Now, we describe an algorithm to solve a neutrosophic linear Diophantine equation $AX + BY = C$.

Remark:

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Let $Z(I) = \{a + bI;\ a, b \in Z\}$ be the neutrosophic ring of integers. Consider a neutrosophic linear Diophantine equation $AX + BY = C$ with two variables $X = x_1 + x_2 I, Y = y_1 + y_2 I$ and $A = a_1 + a_2 I, B = b_1 + b_2 I$. To solve this equation follow these steps:

(a) Check the solvability of $AX + BY = C$ by Theorem 3.3 .

(b) Solve $a_1 x_1 + b_1 y_1 = c_1$.

(c) Solve $(a_1 + a_2)(x_1 + x_2) + (b_1 + b_2)(y_1 + y_2) = c_1 + c_2$.

(d) Compute x_2, y_2.

Example:

The neutrosophic Diophantine equation $(2 + 2I)X + (3 + 4I)Y = 5 + 5I$ is solvable.

$2x_1 + 3y_1 = 5$ is a classical linear Diophantine equation. It has a solution $x_1 = 4, y_1 = -1$.

$(2 + 2)(x_1 + x_2) + (3 + 4)(y_1 + y_2) = 5 + 5$, i.e $4M + 7N = 10$; $M = x_1 + x_2, N = y_1 + y_2$. It is a classical linear Diophantine equation with M, N as variables. It has a solution $M = -1, N = 2$.

$x_2 = M - x_1 = -5, y_2 = N - y_1 = 3$, thus the equation $(2 + 2I)X + (3 + 4I)Y = 5 + 5I$ has a solution $X = 4 - 5I, Y = -1 + 3I$.

Definition:

Let $Z(I_1, I_2) = \{(a, bI_1, cI_2);\ a, b, c \in Z\}$ be the refined neutrosophic ring of integers. The refined neutrosophic linear Diophantine equation with two variables is defined as follows:

$AX + BY = C; A, B, C \in Z(I_1, I_2)$.

Theorem:

Let $Z(I_1, I_2) = \{(a, bI_1, cI_2);\ a, b, c \in Z\}$ be the refined neutrosophic ring of integers,

$AX + BY = C; A, B, C \in Z(I_1, I_2)$ be a refined neutrosophic linear Diophantine equation, where

$X = (x_0, x_1 I_1, x_2 I_2), Y = (y_0, y_1 I_1, y_2 I_2), A = (a_0, a_1 I_1, a_2 I_2)$,

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$B = (b_0, b_1I_1, b_2I_2), C = (c_0, c_1I_1, c_2I_2)$. Then $AX + BY = C$ is equivalent to the following three Diophantine equations:

(1) $a_0x_0 + b_0y_0 = c_0$.

(2) $(a_0 + a_2)(x_0 + x_2) + (b_0 + b_2)(y_0 + y_2) = c_0 + c_2$.

(3) $(a_0 + a_1 + a_2)(x_0 + x_1 + x_2) + (b_0 + b_1 + b_2)(y_0 + y_1 + y_2) = c_0 + c_1 + c_2$.

Proof:

By replacing A, B, C, X, Y we find

$AX = (a_0, a_1I_1, a_2I_2)(x_0, x_1I_1, x_2I_2) =$

$(a_0x_0, [a_0x_1 + a_1x_0 + a_1x_1 + a_1x_2 + a_2x_1]I_1, [a_0x_2 + a_2x_0 + a_2x_2]I_2)$,

$BY = (b_0, b_1I_1, b_2I_2)(y_0, y_1I_1, y_2I_2)=$

$(b_0y_0, [b_0y_1 + b_1y_0 + b_1y_1 + b_1y_2 + b_2y_1]I_1, [b_0y_2 + b_2y_0 + b_2y_2]I_2)$, thus the equation

$AX + BY = C$ implies

(*) $a_0x_0 + b_0y_0 = c_0$. (Equation (1)).

(**) $a_0x_2 + a_2x_0 + a_2x_2 + b_0y_2 + b_2y_0 + b_2y_2 = c_2$.

(***) $a_0x_1 + a_1x_0 + a_1x_1 + a_1x_2 + a_2x_1 + b_0y_1 + b_1y_0 + b_1y_1 + b_1y_2 + b_2y_1 = c_1$.

By adding (*) to (**) we get $(a_0 + a_2)(x_0 + x_2) + (b_0 + b_2)(y_0 + y_2) = c_0 + c_2$. (Equation (2)).

By adding (2) to (***) we get $(a_0 + a_1 + a_2)(x_0 + x_1 + x_2) + (b_0 + b_1 + b_2)(y_0 + y_1 + y_2) = c_0 + c_1 + c_2$. (Equation (3)).

Theorem:

Let $Z(I_1, I_2) = \{(a, bI_1, cI_2);\ a, b, c \in Z\}$ be the refined neutrosophic ring of integers,

$AX + BY = C; A, B, C \in Z(I_1, I_2)$ be a refined neutrosophic linear Diophantine equation, where

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$X = (x_0, x_1 I_1, x_2 I_2), Y = (y_0, y_1 I_1, y_2 I_2), A = (a_0, a_1 I_1, a_2 I_2),$

$B = (b_0, b_1 I_1, b_2 I_2), C = (c_0, c_1 I_1, c_2 I_2)$. Then $AX + BY = C$ is solvable if and only if:

(a) $\gcd(a_0, b_0)\,|c_0$.

(b) $\gcd(a_0 + a_2, b_0 + b_2)\,|c_0 + c_2$.

(c) $\gcd(a_0 + a_1 + a_2, b_0 + b_1 + b_2)\,|c_0 + c_1 + c_2$.

Example:

(a) Consider the refined neutrosophic linear Diophantine equation

$(1,2I_1, 3I_2).X + (3,3I_1, 8I_2)Y = (2, 4I_1, I_2)$, we have

$\gcd(1,3) = 1|2$, $\gcd(1 + 3,3 + 8) = \gcd(4,11) = 1|(2 + 1 = 3)$,

$\gcd(1 + 2 + 3,3 + 3 + 8) = \gcd(6,14) = 2$ which does not divide $2 + 4 + 1 = 7$, thus it is not solvable.

(b) Consider the refined neutrosophic linear Diophantine equation

$(1,2I_1, 3I_2).X + (3,3I_1, 8I_2)Y = (2, 4I_1, 2I_2)$, we have

$\gcd(1,3) = 1|2$, $\gcd(1 + 3,3 + 8) = \gcd(4,11) = 1|(2 + 2 = 4)$,

$\gcd(1 + 2 + 3,3 + 3 + 8) = \gcd(6,14) = 2|\,(2 + 4 + 2 = 8)$. Thus it is solvable.

Remark:

Let $Z(I_1, I_2) = \{(a, bI_1, cI_2);\ a, b, c \in Z\}$ be the refined neutrosophic ring of integers,

$AX + BY = C; A, B, C \in Z(I_1, I_2)$ be a refined neutrosophic linear Diophantine equation, where

$X = (x_0, x_1 I_1, x_2 I_2), Y = (y_0, y_1 I_1, y_2 I_2), A = (a_0, a_1 I_1, a_2 I_2),$

$B = (b_0, b_1 I_1, b_2 I_2), C = (c_0, c_1 I_1, c_2 I_2)$, we summarize the algorithm of solution as follows:

(a) Check the solvability condition.

(b) Solve $a_0 x_0 + b_0 y_0 = c_0$.

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(c) Solve $(a_0 + a_2)(x_0 + x_2) + (b_0 + b_2)(y_0 + y_2) = c_0 + c_2$.

(d) Compute x_2, y_2.

(e) Solve $(a_0 + a_1 + a_2)(x_0 + x_1 + x_2) + (b_0 + b_1 + b_2)(y_0 + y_1 + y_2) = c_0 + c_1 + c_2$.

(f) Compute x_1, y_1.

Example:

we found that $(1,2I_1, 3I_2).X + (3,3I_1, 8I_2)Y = (2, 4I_1, 2I_2)$ is solvable.

We consider $x_0 + 3y_0 = 2$. It has a solution $x_0 = -1, y_0 = 1$.

We take $(1 + 3)(x_0 + x_2) + (3 + 8)(y_0 + y_2) = 2 + 2,$ i.e $4M + 11N = 4; M = x_0 + x_2, and$

$N = y_0 + y_2,$ it has a solution $M = 1, N = 0,$ thus $x_2 = M - x_0 = 2, y_2 = N - y_0 = -1$.

The third equation is $(1 + 2 + 3)(x_0 + x_1 + x_2) + (3 + 3 + 8)(y_0 + y_1 + y_2) = 2 + 4 + 2,$ i.e

$6S + 14T = 8; S = x_0 + x_1 + x_2, T = y_0 + y_1 + y_2$. It has a solution $S = -1, T = 1,$ thus

$x_1 = S - x_0 - x_2 = -2, y_1 = T - y_0 - y_2 = 1$. The solution of

$(1,2I_1, 3I_2).X + (3,3I_1, 8I_2)Y = (2, 4I_1, 2I_2)$ is $X = (-1, -2I_1, 2I_2),$ $Y = (1, I_1, -I_2)$.

Definition:

Let $Z_n(I)$={$t_0 + t_1 I_1 + \cdots + t_n I_n;\ t_i \in Z$} be the n-refined neutrosophic ring of integers. The following equation

$AX + B = C; A, B, X, C \in Z_n(I)$. Where $A = a_0 + a_1 I_1 + \cdots + a_n I_n$, B=$b_0 + b_1 I_1 + \cdots + b_n I_n, X = x_0 + x_1 I_1 + \cdots + x_n I_n,$ $C = c_0 + c_1 I_1 + \cdots + c_n I_n$

is called an n-refined neutrosophic linear Diophantine equation.

Example :

Let n=3, the following equation is an 3-refined neutrosophic linear Diophantine equation

$(1 - I_1 - I_2)X + (2 + 3I_2 - 4I_3) = I_2 + 2I_3$.

Theorem:

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Let $AX + B = C\ (*);\ A, B, X, C \in Z_n(I)$ be an n-refined neutrosophic linear Diophantine equation. It is solvable if and only if the following system of classical linear Diophantine equations is solvable.

(1-) $a_0x_0 + b_0 = c_0$.

(2-) $(a_0 + a_n)(x_0 + x_n) + (b_0 + b_n) = c_0 + c_n$.

(3-) $(a_0 + a_n + a_{n-1})(x_0 + x_n + x_{n-1}) + (b_0 + b_n + b_{n-1}) = c_0 + c_n + c_{n-1}$.

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(n+1-) $(a_0 + a_1 + \cdots + a_n)(x_0 + x_1 + \cdots + x_n) + (b_0 + b_1 + \cdots + b_n) = c_0 + c_1 + \cdots + c_n$.

Theorem:

The sufficient and necessary condition of the solvability of Diophantine equation (*) is:

$\gcd(a_0, b_0)\,|c_0, \gcd\,(a_0 + a_n, b_0 + b_n)|(c_0 + c_n), \gcd\,(a_0 + a_n + a_{n-1}, b_0 + b_n + b_{n-1})|(c_0 + c_n + c_{n-1}), \ldots \gcd(a_0 + a_1 + \cdots + a_n, b_0 + b_1 + \cdots + b_n)\,, |(c_0 + c_1 + \cdots + c_{\boldsymbol{n}})$.

Example:

Consider the following 3-refined neutrosophic linear Diophantine equation:

$(1 - I_2 + I_3)X + (I_2 + 2I_3) = 2 - I_1 + 4I_3$.

We have: $A = 1 - I_2 + I_3, B = I_2 + 2I_3, C = I_1 + 4I_3, i.e. a_0 = 1, a_1 = 0, a_2 = -1, a_3 = 1$.

$b_0 = 0, b_1 = 0, b_2 = 1, b_3 = 2, c_0 = 0, c_1 = 1, c_2 = 0, c_3 = 4$.

We have : $\gcd(a_0, b_0) = 1|0, gc\,d(a_0 + a_3, b_0 + b_3) = \gcd(2,2) = 2|4\,, gc\,d(a_0 + a_3 + a_2, b_0 + b_3 + b_2) = \gcd(1,3) = 1|4\,, \gcd(a_0 + a_1 + a_2 + a_3, b_0 + b_1 + b_2 + b_3) = \gcd(1,3) = 1|5$. This implies that the previous linear Diophantine equation is solvable.

Now, we find the solution.

The equivalent system is:

$a_0x_0 + b_0 = c_0, thus\ x_0 = 0.$*(1)*

$(a_0 + a_3)(x_0 + x_3) + (b_0 + b_3) = c_0 + c_3, thus\ 2(x_0 + x_3) + 2 = 4.$(2)

$(a_0 + a_3 + a_2)(x_0 + x_3 + x_2) + (b_0 + b_3 + b_2) = c_0 + c_3 + c_2$, thus $(x_0 + x_3 + x_2) + 3 = 4.$ (3)

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$(a_0 + a_3 + a_1 + a_2)(x_0 + x_3 + x_1 + x_2) + (b_0 + b_3 + b_1 + b_2) = c_0 + c_3 + c_1 + c_2$, thus $(x_0 + x_1 + x_3 + x_2) + 3 = 5$. (4)

The equation (1) has a solution $x_0 = 0$. The equation (2) has a solution $x_0 + x_3 = 1, hence\ x_3 = 1$.

The equation (2), has a solution $x_0 + x_3 + x_2 = 1, hence\ x_2 = 0$. The equation (4) has a solution $x_0 + x_1 + x_3 + x_2 = 2, hence\ x_1 = 1$.

The previous discussion means that the solution of the first n-refined neutrosophic linear Diophantine equation is $X = I_1 + I_3$.

Conclusion

In this work, we gave the interested reader a wide review on recent developments in the field of neutrosophic number theory, especially the subject of neutrosophic linear Diophantine equations.

As a future research direction, we aim to generalize the previous efforts to define and study Turiyam linear Diophantine equations.

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References

[1] Smarandache, F., " A Unifying Field in Logics: Neutrosophic Logic, Neutrosophy, Neutrosophic Set, Neutrosophic Probability", American Research Press. Rehoboth, 2003.

[2] Alhamido, R., and Gharibah, T., "Neutrosophic Crisp Tri-Topological Spaces", Journal of New Theory, Vol. 23 , pp.13-21. 2018.

[3] Edalatpanah. S.A., "Systems of Neutrosophic Linear Equations", Neutrosophic Sets and Systems, Vol. 33, pp. 92-104. 2020.

[4] Sankari, H., and Abobala, M., "Neutrosophic Linear Diophantine Equations With two Variables", Neutrosophic Sets and Systems, Vol. 38, pp. 22-30, 2020.

Doi : https://doi.org/10.54216/JNFS.020106
Received June 15, 2021 Accepted: Jan 11, 2022

[5] Sankari, H., and Abobala, M." *n*-Refined Neutrosophic Modules", Neutrosophic Sets and Systems, Vol. 36, pp. 1-11. 2020.

[6] Alhamido, R., and Abobala, M., "AH-Substructures in Neutrosophic Modules", International Journal of Neutrosophic Science, Vol. 7, pp. 79-86 . 2020.

[7] Abobala, M., "AH-Subspaces in Neutrosophic Vector Spaces", International Journal of Neutrosophic Science, Vol. 6 , pp. 80-86. 2020.

[8] Abobala, M.,. "A Study of AH-Substructures in *n*-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science", Vol. 9, pp.74-85. 2020.

[9] Hatip, A., Alhamido, R., and Abobala, M., "A Contribution to Neutrosophic Groups", International Journal of Neutrosophic Science", Vol. 0, pp. 67-76 . 2019.

[10] Abobala, M., " *n*-Refined Neutrosophic Groups I", International Journal of Neutrosophic Science, Vol. 0, pp. 27-34. 2020.

[11] Kandasamy, V.W.B., and Smarandache, F., "Some Neutrosophic Algebraic Structures and Neutrosophic N-Algebraic Structures", Hexis, Phonex, Arizona, 2006.

[12] Agboola, A.A.A., Akinola, A.D., and Oyebola, O.Y., " Neutrosophic Rings I" , International J.Mathcombin, Vol 4,pp 1-14. 2011

[13] Agboola, A.A.A., "On Refined Neutrosophic Algebraic Structures," Neutrosophic Sets and Systems,Vol.10, pp. 99-101. 2015.

[14] Abobala, M., "Classical Homomorphisms Between Refined Neutrosophic Rings and Neutrosophic Rings", International Journal of Neutrosophic Science, Vol. 5, pp. 72-75. 2020.

[15] Smarandache, F., and Abobala, M., n-Refined neutrosophic Rings, International Journal of Neutrosophic Science, Vol. 5 , pp. 83-90, 2020.

[16] Kandasamy, I., Kandasamy, V., and Smarandache, F., "Algebraic structure of Neutrosophic Duplets in Neutrosophic Rings", Neutrosophic Sets and Systems, Vol. 18, pp. 85-95. 2018.

[17] Yingcang, Ma., Xiaohong Zhang ., Smarandache, F., and Juanjuan, Z., "The Structure of Idempotents in Neutrosophic Rings and Neutrosophic Quadruple Rings", Symmetry Journal (MDPI), Vol. 11. 2019.

[18] Kandasamy, V. W. B,. Ilanthenral, K., and Smarandache, F., "Semi-Idempotents in Neutrosophic Rings", Mathematics Journal (MDPI), Vol. 7. 2019.

[19] Abobala, M., On Some Special Substructures of Neutrosophic Rings and Their Properties, International Journal of Neutrosophic Science", Vol. 4 , pp. 72-81, 2020.

[20] Smarandache, F., " An Introduction To neutrosophic Genetics", International Journal of neutrosophic Science, Vol.13, 2021.

[21] Martin, N, Smarandache, F, and Broumi, S., " Covid 19 Decision Making using Extended Plithogenic hypersoft Sets With Dual Dominent Attributes", International Journal of neutrosophic Science, Vol. 13, 2021.

Doi : https://doi.org/10.54216/JNFS.020106
Received June 15, 2021 Accepted: Jan 11, 2022

[22]Agboola, A.A., "Introduction To Neutro groups", International Journal of neutrosophic Science, Vol. 6, 2020.

[23] Abobala, M., "On Some Special Substructures of Refined Neutrosophic Rings*"*, International Journal of Neutrosophic Science, Vol. 5, pp. 59-66. 2020.

[24] Smarandache, F., and Ali, M., "Neutrosophic Triplet Group", Neural. Compute. Appl. 2019.

[25] Sankari, H., and Abobala, M.," AH-Homomorphisms In neutrosophic Rings and Refined Neutrosophic Rings", Neutrosophic Sets and Systems, Vol. 38, pp. 101-112, 2020.

[26] Smarandache, F., and Kandasamy, V.W.B., " Finite Neutrosophic Complex Numbers",·Source: arXiv. 2011.

[27]. Abobala, M., " *n*-Refined Neutrosophic Groups II", International Journal of Neutrosophic Science, Vol. 0, 2020.

[28] Ali, Rozina., " On the Concept of Algebraic Actions In Neutrosophic Groups", Resaechgate.net, 2021.

[29] Ali, Rozina., " Neutrosophic Matrices and Their Properties", researchgat.net, 2021.

[30] Ali, Rozina., " Recent Advantages In Neutrosophic Module Theory", researchgate.net, 2021.

[31] Abobala, M, "*n*-Cyclic Refined Neutrosophic Algebraic Systems Of Sub-Indeterminacies, An Application To Rings and Modules", International Journal of Neutrosophic Science, Vol. 12, pp. 81-95 . 2020.

[32] Smarandache, F., "Neutrosophic Set a Generalization of the Intuitionistic Fuzzy Sets", Inter. J. Pure Appl. Math., pp. 287-297. 2005.

[33] M. Ali, F. Smarandache, M. Shabir and L. Vladareanu., "Generalization of Neutrosophic Rings and Neutrosophic Fields", Neutrosophic Sets and Systems, vol. 5, pp. 9-14, 2014.

[34] Anuradha V. S., "Neutrosophic Fuzzy Hierarchical Clustering for Dengue Analysis in Sri Lanka", Neutrosophic Sets and Systems, vol. 31, pp. 179-199. 2020.

[35] Olgun, N., and Hatip, A., "The Effect Of The Neutrosophic Logic On The Decision Making, in Quadruple Neutrosophic Theory And Applications", Belgium, EU, Pons Editions Brussels,pp. 238-253. 2020.

[36] Abobala, M., Bal, M., and Hatip, A.," A Review On Recent Advantages In Algebraic Theory Of Neutrosophic Matrices**",** International Journal of Neutrosophic Science, Vol. 17, 2021.

[37] Turksen, I., "Interval valued fuzzy sets based on normal forms", Fuzzy Sets and Systems, 20, pp.191-210, 1986. 1986.

[38] Chalapathi, T., and Madhavi, L., "Neutrosophic Boolean Rings", Neutrosophic Sets and Systems, Vol. 33, pp. 57-66. 2020.

[39] Abobala, M., "Classical Homomorphisms Between n-refined Neutrosophic Rings", International Journal of Neutrosophic Science", Vol. 7, pp. 74-78. 2020.

[40] Agboola, A.A.A,. Akwu, A.D,. and Oyebo, Y.T., " Neutrosophic Groups and Subgroups", International .J .Math. Combin, Vol. 3, pp. 1-9. 2012.

Doi : https://doi.org/10.54216/JNFS.020106
Received June 15, 2021 Accepted: Jan 11, 2022

[41] Smarandache, F., " *n*-Valued Refined Neutrosophic Logic and Its Applications in Physics", Progress in Physics, 143-146, Vol. 4, 2013.

[42] Adeleke, E.O., Agboola, A.A.A.,and Smarandache, F., "Refined Neutrosophic Rings I", International Journal of Neutrosophic Science, Vol. 2(2), pp. 77-81. 2020.

[43] Hatip, A., and Abobala, M., "AH-Substructures In Strong Refined Neutrosophic Modules", International Journal of Neutrosophic Science, Vol. 9, pp. 110-116 . 2020.

[44] Hatip, A., and Olgun, N., "On Refined Neutrosophic R-Module", International Journal of Neutrosophic Science, Vol. 7, pp.87-96. 2020.

[45] Bal, M., and Abobala, M., "On The Representation Of Winning Strategies Of Finite Games By Groups and Neutrosophic Groups", Journal Of Neutrosophic and Fuzzy Systems, 2022.

[46] Smarandache F., and Abobala, M., *"n*-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science, Vol. 7, pp. 47-54. 2020.

[47] Sankari, H., and Abobala, M., "Solving Three Conjectures About Neutrosophic Quadruple Vector Spaces", Neutrosophic Sets and Systems, Vol. 38, pp. 70-77. 2020.

[48] Adeleke, E.O., Agboola, A.A.A., and Smarandache, F., "Refined Neutrosophic Rings II", International Journal of Neutrosophic Science, Vol. 2(2), pp. 89-94. 2020.

[49] Abobala, M., On Refined Neutrosophic Matrices and Their Applications In Refined Neutrosophic Algebraic Equations, Journal Of Mathematics, Hindawi, 2021

[50] Abobala, M., A Study of Maximal and Minimal Ideals of n-Refined Neutrosophic Rings, Journal of Fuzzy Extension and Applications, Vol. 2, pp. 16-22, 2021.

[51] Abobala, M., " Semi Homomorphisms and Algebraic Relations Between Strong Refined Neutrosophic Modules and Strong Neutrosophic Modules", Neutrosophic Sets and Systems, Vol. 39, 2021.

[52] Abobala, M., "On Some Neutrosophic Algebraic Equations", Journal of New Theory, Vol. 33, 2020.

[53] Abobala, M., On The Representation of Neutrosophic Matrices by Neutrosophic Linear Transformations, Journal of Mathematics, Hindawi, 2021.

[54] Abobala, M., "On Some Algebraic Properties of n-Refined Neutrosophic Elements and n-Refined Neutrosophic Linear Equations", Mathematical Problems in Engineering, Hindawi, 2021

[55] Kandasamy V, Smarandache F., and Kandasamy I., Special Fuzzy Matrices for Social Scientists . Printed in the United States of America,2007, book, 99 pages.

[56] Khaled, H., and Younus, A., and Mohammad, A., " The Rectangle Neutrosophic Fuzzy Matrices", Faculty of Education Journal Vol. 15, 2019. (Arabic version).

[57] Abobala, M., Partial Foundation of Neutrosophic Number Theory, Neutrosophic Sets and Systems, Vol. 39 , 2021.

Doi : https://doi.org/10.54216/JNFS.020106
Received June 15, 2021 Accepted: Jan 11, 2022

[58] F. Smarandache, *Neutrosophic Theory and Applications*, Le Quy Don Technical University, Faculty of Information technology, Hanoi, Vietnam, 17th May 2016.

[59] Sankari, H, and Abobala, M., " On A New Criterion For The Solvability of non Simple Finite Groups and m-Abelian Solvability, Journal of Mathematics, Hindawi, 2021.

[60] Giorgio, N, Mehmood, A., and Broumi, S.," Single Valued neutrosophic Filter", International Journal of Neutrosophic Science, Vol. 6, 2020.

[61] Abobala, M., "A Study Of Nil Ideals and Kothe's Conjecture In Neutrosophic Rings", International Journal of Mathematics and Mathematical Sciences, hindawi, 2021

[62] Abobala, M., Hatip, A., Olgun, N., Broumi, S., Salama, A,A., and Khaled, E, H., The algebraic creativity In The Neutrosophic Square Matrices, Neutrosophic Sets and Systems, Vol. 40, pp. 1-11, 2021.

[63]Alhamido, K., R., "A New Approach of neutrosophic Topological Spaces", International Journal of neutrosophic Science, Vol.7, 2020.

[64] Abobala, M., "Neutrosophic Real Inner Product Spaces", Neutrosophic Sets and Systems, Vol. 43, 2021.

[65] Abobala, M., "On Some Special Elements In Neutrosophic Rings and Refined Neutrosophic Rings", Journal of New Theory, vol. 33, 2020.

[66] Abobala, M., and Hatip, A., "An Algebraic Approch To Neutrosophic Euclidean Geometry", Neutrosophic Sets and Systems, Vol. 43, 2021.

[67] Sundar, J., Vadivel, A., " New operators Using Neutrosophic ϑ $-$Open Sets", Journal Of Neutrosophic and Fuzzy Systems, 2022.

[68] Sankari, H, and Abobala, M, " A Contribution to m-Power Closed Groups", UMM-Alqura University Journal for Applied Sciences, KSA, 2020.

[69] Abobala, M., "On The Characterization of Maximal and Minimal Ideals In Several Neutrosophic Rings", Neutrosophic Sets and Systems, Vol. 45, 2021.

[70] Chellamani, P., and Ajay, D., "Pythagorean neutrosophic Fuzzy Graphs", International Journal of Neutrosophic Science, Vol. 11, 2021.

[71] Milles, S, Barakat, M, and Latrech, A., " Completeness and Compactness In Standard Single Valued neutrosophic Metric Spaces", International Journal of Neutrosophic Science, Vol.12 , 2021.

[72] Es, Haydar, A., "A Note On neutrosophic Soft Menger Topological Spaces", International Journal of Neutrosophic Science, Vol.7, 2020.

[73] Ceven, Y., and Tekin, S., " Some Properties of Neutrosophic Integers", Kırklareli University Journal of Engineering and Science, Vol. 6, pp.50-59, 2020.

[74] Abobala, M., Hatip, A., Bal,M.," A Study Of Some Neutrosophic Clean Rings", International journal of neutrosophic science, 2022.

Doi : https://doi.org/10.54216/JNFS.020106
Received June 15, 2021 Accepted: Jan 11, 2022

[75] Ahmad, K., Bal, M., Hajjari, A., Ali, R.," On Imperfect Duplets In Some refined Neutrosophic Rings", Journal of Neutrosophic and Fuzzy Systems, 2022.

[76] Singh, P,K., " Anti-geometry and NeutroGeometry Characterization of Non-Euclidean Data", Journal of Neutrosophic and Fuzzy Systems, Vol 1, Issue 1, pp. 24-33, 2021, DOI: https://doi.org/10.54216/JNFS.0101012

[77] Singh, P,K., " Data With Turiyam Set for Fourth Dimension Quantum Information Processing", Journal of Neutrosophic and Fuzzy Systems, Vol 1, Issue 1, pp. 9-23, DOI: https://doi.org/10.54216/JNFS.010101

[78] Singh, P, K., Ahmad, K., Bal, M., Aswad, M.," On The Symbolic Turiyam Rings", Journal of Neutrosophic and Fuzzy Systems**,** Vol. 1 , No. 2 , pp. 80-88 , 2021, Doi **:** https://doi.org/10.54216/JNFS.010204

[79] Ahmad, K., Bal, M., and Aswad, M.," A Short Note on The Solution Of Fermat's Diophantine Equation In Some Neutrosophic Rings", Journal of Neutrosophic and Fuzzy Systems, Vol. 1, 2022.

[80] Ibrahim, M., and Abobala, M., "An Introduction To Refined Neutrosophic Number Theory", Neutrosophic Sets and Systems, Vol. 45, 2021.

[81] Abobala, M., Bal, M., Aswad, M., "A Short Note On Some Novel Applications of Semi Module Homomorphisms", International journal of neutrosophic science, 2022.

[82] Ahmad, K., Bal, M., and Aswad, M.," The kernel of Fuzzy and Anti Fuzzy Groups",Journal of Neutrosophic and Fuzzy Systems, Vol.1, 2022.

[83] Smarandache, F., and Broumi,M., "Neutro-Intelligent Set is a particular case of the refined neutrosophic set", Journal of Neutrosophic and Fuzzy Systems, Vol. 1, 2022.

[84] Olgun, N., Hatip, A., Bal, M., and Abobala, M., " A Novel Approach To Necessary and Sufficient Conditions For The Diagonalization of Refined Neutrosophic Matrices", International Journal of Neutrosophic Science, Vol. 16, pp. 72-79, 2021.

[85] Singh, P.K., “NeutroAlgebra and NeutroGeometry for Dealing Heteroclinic Patterns”, In: Theory and Applications of NeutroAlgebras as Generalizations of Classical Algebras, IGI Global Publishers, April 2022, Chapter 6, DOI: 10.4018/978-1-6684-3495-6, ISBN13: 9781668434956

[86] Singh, P,K, “Data with Non-Euclidean Geometry and its Characterization”, Journal of Artificial Intelligence and Technology, Vol 2, Issue 1, pp. 3-8, 2022, DOI: 10.37965/jait.2021.12001

[87] Singh, P.,K., “Three-way n-valued neutrosophic concept lattice at different granulation”, International Journal of Machine Learning and Cybernetics, Vol 9, Issue 11, pp. 1839-1855, 2019.

[88] Bal, M., Ahmad, K., Hajjari, A., Ali, R.," A Short Note On The Kernel Subgroup Of Intuitionistic Fuzzy groups" Journal of Neutrosophic and Fuzzy Systems, 2022.

Doi : https://doi.org/10.54216/JNFS.020106
Received June 15, 2021 Accepted: Jan 11, 2022

[89] Bal, M., Ahmad, K., Hajjari, A., Ali, R.," The Structure Of Imperfect Triplets In Several Refined Neutrosophic Rings" Journal of Neutrosophic and Fuzzy Systems, 2022.

[90] F. Smarandache, S. Broumi, Singh, P,K, CF Liu, V Venkateswara Rao, H.L. Yang, I. Patrascu, A. Elhasounny, Introduction to Neutrosophy and Neutrosophic Environment, In: Y Guo and Amira S ed., Neutrosophic Set in Medical Image Analysis, ISBN: 9780128181485, Nov 2019, pp. 3-29, Elsevier, https://doi.org/10.1016/B978-0-12-818148-5.00001-1

[91] Singh, P.,K., "Turiyam set a fourth dimension data representation", Journal of Applied Mathematics and Physics,Vol.9,Issue7,pp.1821-1828, 2021, DOI: 10.4236/jamp.2021.97116,

[92] Singh, P,K, Fourth dimension data representation and its analysis using Turiyam Context, Journal of Computer and Communications, vol. 9, no. 6, pp. 222-229, 2021, DOI: 10.4236/jcc.2021.96014

Doi : https://doi.org/10.54216/JNFS.020106
Received June 15, 2021 Accepted: Jan 11, 2022