Neutrosophic Crisp minimal Structure

Riad K. Al-Hamido

Department of Mathematics, College of Science, AlFurat University, Deir-ez-Zor, Syria.
Email: riad-hamido1983@hotmail.com Tel.: (+963988619783)

Abstract

In this paper, the neutrosophic crisp minimal structure which is a more general structure than the neutrosophic minimal structure is built on neutrosophic crisp sets. The necessary arguments which are neutrosophic minimal crisp open set, neutrosophic minimal crisp closed set, neutrosophic crisp minimal closure, and neutrosophic crisp minimal interior are defined and their basic properties are presented. Also, the neutrosophic crisp minimal structure subspace of neutrosophic crisp minimal structure is defined and studied some of its properties. Finally, many examples are presented.

Keywords: Neutrosophic crisp minimal structure, neutrosophic minimal structure, neutrosophic crisp minimal open set, neutrosophic crisp minimal closed set.

1.Introduction

The concept of neutrosophy is a new branch of science introduced by F.smarandache [1,2], and has many applications in different fields of science such as topology. As a generalization of the concept of topological spaces, Also A.A. Salama and et al. [3] defined neutrosophic crisp topological spaces and many kinds of neutrosophic crisp open and closed sets in 2014. A.A. Salama and F. Smarandache [4] defined neutrosophic topological spaces and its neutrosophic sets. in 2012.

supra topological space was introduced by A.S. Mashhour et al. [5] in 1983, as a generalization of the concept of topological space.

G.Jayaparthasarathy, et al. generalised this concept and introduced the concept of neutrosophic supra topological space [6] in 2019, by using the neutrosophic sets.

Furthermore, Mony reserchers extended some of the topololgy to bitopology as, The concept of supra bi-topological spaces which was introduced by R. Gowri, A.K.R. Rajayal [8] in 2017. On the other hand, The concept of bi-topological spaces was introduced by Kelly [8] as an extension of topological spaces in 1963.

Also, R.K.Al-Hamido [10] extended neutrosophic topological spaces to neutrosophic bi-topological spaces in 2019. This concept has been studied in [11].

DOI: https://doi.org/10.54216/JNFS.030103
Received: February 10, 2022 Accepted: May 24, 2022

Also, The concept of neutrosophic crisp bi-topological spaces was introduced by R.K.Al-Hamido [12] as an extension of neutrosophic crisp topological spaces in 2018.

M. Parimala, et al. introduced the concept of neutrosophic minimal structure [14] in 2020, by using the neutrosophic sets.

In this paper, we use the neutrosophic crisp sets to introduce neutrosophic crisp minimal structure. Also, we introduce new neutrosophic crisp minimal open (closed) sets in this neutrosophic crisp minimal structure, and we study some basic properties of this new neutrosophic crisp minimal open (closed) sets.

2. Preliminaries

In this part, we recall some basic definitions and properties which are useful in this paper.

Definition 2.1. [3]

Let $X \neq \emptyset$ be a fixed set. A neutrosophic crisp set (NCS) U is an object having the form $U = < U_1, U_2, U_3 >$; U_1, U_2 and U_3 are subsets of X.

Definition 2.2. [3]

A neutrosophic crisp topology (NCT) on a non-empty set X is a family T of neutrosophic crisp subsets in X satisfying the following axioms:

1. X_N and $\emptyset_N$ belong to T.
2. T is closed under finite intersection.
3. T is closed under arbitrary union.

The pair (X,T) is said to be a neutrosophic crisp topological space (NCTS) in X. Moreover, The elements in T are said to be neutrosophic crisp open sets (NCOS). A neutrosophic crisp set F is closed (NCCS) if and only if its complement F^c is an open neutrosophic crisp set.

Definition 2.3. [13]

A neutrosophic crisp supra topology (NCT) on a non-empty set X is a family T of neutrosophic crisp supra subsets in X satisfying the following axioms:

1. X_N and $\emptyset_N$ belong to T.
2. T is closed under arbitrary union.

The pair(X,T) is said to be a neutrosophic crisp supra topological space (NCTS) in X. Moreover, The elements in T are said to be neutrosophic crisp supra open sets (NCSOS), A neutrosophic crisp set F is neutrosophic crisp supra closed (NCSCS) if and only if its complement F^c is an neutrosophic crisp supra open.

Definition 2.4. [3]

Let X be a non-empty fixed set. A neutrosophic crisp set (NCS for short) B is an object having the form $B=<B_1,B_2,B_3>$ where B_1,B_2 and B_3 are subsets of X.

DOI: https://doi.org/10.54216/JNFS.030103
Received: February 10, 2022 Accepted: May 24, 2022

3. Neutrosophic crisp minimal structure

In this section, we introduce the neutrosophic crisp minimal structure. Moreover, we introduce new types of neutrosophic crisp minimal open (closed) sets in this space, and study their properties, we examine the relationship between them in details.

Definition 3.1.

A neutrosophic crisp minimal structure (NCMS) on a non-empty set X is a family M of neutrosophic crisp subsets in X satisfying the following axioms:

1. $\emptyset_N$ belong to M.
2. X_N belong to M.

The pair(X,M) is said to be a neutrosophic crisp minimal structure (NCMS) in X. Moreover, The elements in M are said to be neutrosophic crisp minimal open sets (NCMOS), A neutrosophic crisp set F is neutrosophic crisp minimal closed (NCMCS) if and only if its complement F^c is an neutrosophic crisp minimal open.

Example 3.2.

Let X={a,b}, M={$\emptyset_N$, X_N, A, B, E}, A={<{a},∅,∅>}, B={<{b},∅,∅>}, E={<{a},{b},∅>}neutrosophic sets over X. Then (X, M) is neutrosophic crisp minimal structure.

Remark 3.3.

- the family of all neutrosophic crisp minimal open sets is denoted by (NCMOS(X)).
- the family of all neutrosophic crisp minimal closed sets is denoted by (NCMCS(X)).

Example 3.4.
In Example 3.2.

the neutrosophic minimal crisp open sets is denoted by are :

NCMOS(X) ={$\emptyset_N$, X_N, A, B, E}.

Remark 3.5.

Let (X, M) be NCMS then

the union of two neutrosophic crisp minimal open sets is not necessary neutrosophic crisp minimal open set.

as the following example:

Example 3.6.

Let X={a,b}, M={$\emptyset_N$, X_N, A, B, E}, A={<{a},∅,∅>}, B={<{b},∅,{a,b}>}, E={<{a},{b},∅>}neutrosophic sets over X. Then (X, M) is neutrosophic crisp minimal structure.
A, B are two neutrosophic crisp minimal open sets but $A_1 \cup A_3 = \{< \{a, b\}, \emptyset, \emptyset >\}$, is not neutrosophic crisp minimal open set.

Remark 3.7.

Let (X, M) be NCMS then the intersection of two neutrosophic minimal open sets is not necessary neutrosophic minimal open set.

as the following example:
Example 3.8.
In Example 3.6, A, B are two neutrosophic crisp minimal open sets but $A_1 \cap A_3 = \{< \emptyset, \emptyset, \{a, b\} >\}$, is not neutrosophic crisp minimal open set.

DOI: https://doi.org/10.54216/JNFS.030103
Received: February 10, 2022 Accepted: May 24, 2022

Remark 3.9.

Let (X, M) be NCMS then

the union of two neutrosophic crisp minimal closed sets is not necessary neutrosophic crisp minimal closed set.

as the following example:

Example 3.10.

Let X={a,b}, M={$\emptyset_N$, X_N, A, B, E}, A={<{a},∅,∅>}, B={<{b},∅,{a}>}, E={<{a},{b},∅>}neutrosophic crisp sets over X. Then (X, M) is neutrosophic crisp minimal structure.
$A^c = \{< \{b\}, X, X >\}, B^c = \{< \{a\}, X, \{b\} >\}$ are two neutrosophic crisp minimal closed sets but $A^c \cup B^c = \{< \{X, X, \{b\} >\}$, is not neutrosophic crisp minimal closed set.

Remark 3.11.

Let (X, M) be NCMS then the intersection of two neutrosophic minimal closed sets is not necessary neutrosophic minimal closed set.

as the following example:
Example 3.12.

In Example 3.10
$B^c = \{< \{a\}, X, \{b\} >\}, E^c = \{< \{b\}, \{a\}, X >\}$ are two neutrosophic crisp minimal closed sets but $B^c \cap E^c = \{< \{\emptyset, \{a\}, X >\}$, is not neutrosophic crisp minimal closed set.

Remark 3.13.

Every NCS is NCMS but the converse is not true.

Example 3.14.

In Example 3.6.

(X, M) is NCMS is. But (X, M) is not NCS.

Remark 3.15.

Every neutrosophic supra topological space is NCMS but the converse is not true.

Example 3.16

In Example 3.6.

(X, M) is NCMS is. But (X, M) is not a neutrosophic supra topological space.

4. The interior and the closure via neutrosophic crisp Minimal open (closed) sets

In this section we define the closure and interior Neutrosophic crisp minimal set based on these new varieties of Neutrosophic crisp minimal open and closed sets. Also we introduce the basic properties of closure and the interior.

Definition 4.1.

Let (X, M) be NCMS, and A a neutrosophic crisp minimal set then :

The union of any neutrosophic crisp minimal open sets, contained in A is called neutrosophic crisp minimal interior of A (NCMint(A)).

NCMint(A) =$\cup$\{B ;B$\subseteq$A; B$\in$NMOS(X)\}.

Theorem 4.2.

DOI: https://doi.org/10.54216/JNFS.030103
Received: February 10, 2022 Accepted: May 24, 2022

Let (X, M) be NCMS then, A, B are neutrosophic crisp sets then :

1. NCMint(A) $\subseteq$ A.
2. NCMint(A) is not necessary neutrosophic minimal open set.
3. A$\subseteq$B $\Rightarrow$ NCMint(A) $\subseteq$ NCMint(B)·

Proof :

1. Follow from the defintion of NCMint(A) as a union of any neutrosophic crisp minimal open sets ,contained in A**.**
2. Follow from remark 3.8.
3. The Proof is obvious.

Definition 4.3.

Let (X, M) be NCMS then, A is neutrosophic crisp sets then :

The intersection of any neutrosophic crisp minimal open sets ,containing A is called neutrosophic crisp minimal closure of A ((NCMcl(A)).

NCMcl(A)=$\cap\{B \ ;B\supseteq A;\ B\in NCMCS(X)\}$

Theorem 4.4.

Let (X, M) be NCMS then, A is neutrosophic crisp sets then :

1. A$\subseteq$ NCMcl(A)·
2. NCMcl(A) is not necessary neutrosophic crisp minimal closed set.

Proof :

1. Follow from the defintion of NCMcl(A) as a intersection of any neutrosophic crisp minimal closed set containing A**.**
2. Follow from remark 3.10.

5. Neutrosophic crisp minimal structure subspace:

In this part, we intruduced the neutrosophic crisp minimal structure subspace of neutrosophic crisp minimal structure, and studied some of its properties.

Theorem 5.1: If (X, M) is a neutrosophic crisp minimal structure , $\emptyset_N \neq G$ neutrosophic crisp subset over X,

$Y_G = \{G\cap A :\ A\in M\}$, then (G, Y_G) is neutrosophic crisp minimal structure.

proof:

DOI: https://doi.org/10.54216/JNFS.030103
Received: February 10, 2022 Accepted: May 24, 2022

since $\emptyset_N \in M$, then $\emptyset_N = G \cap \emptyset_N \in Y_G$, $X_N \in M$, then $G_N = G \cap X_N \in Y_G$ therefore (G, Y_G) is a neutrosophic crisp minimal structure.

Definition5.2: If (X, M) is a neutrosophic crisp minimal strucre , $\emptyset_N \neq G$ neutrosophic crisp subset over X,

$Y_G = \{G \cap A : A \in M\}$, then (G, Y_G) is called a neutrosophic crisp minimal structure subspace of (X, M).

Example 5.3:

Let X={a,b}, M={$\emptyset_N$, X_N, A, B, E}, A={<{a},∅,∅>}, B={<{b},∅,∅>}, E={<{a},{b},∅>}neutrosophic crisp sets over X. Then (X, M) is neutrosophic crisp minimal structure. Let G={<{a},{b},X>},
$Y_G = \{ \emptyset_N, G_N, A \cap G, B \cap G, E \cap G\}$.
(G, Y_G) is a neutrosophic crisp minimal structure subspace of (X, M).

Definition5.4,
Let (X, M) is a neutrosophic crisp minimal strucre, then
- Arbitrary union of neutrosophic minimal open sets in (X, M) is neutrosophic minimal open. (Union Property)
- Finite intersection of neutrosophic minimal open sets in $(X, M)$is neutrosophic minimal open (intersection Property)

Theorem 5.6.
let (X, M) is a neutrosophic crisp minimal structure, then
1-If the neutrosophic minimal structure space (X, M) has the union property, then
the subspace (G, Y_G) also has union property.
2-If the neutrosophic minimal structure space (X, M) has the intersection property, then the subspace (G, Y_G) also has union property.
Proof. Suppose the family of open set $\{ A_i : i \in I\}$in neutrosophic minimal subspace(G, Y_G)
then there exist a family of open sets $\{ U_i : i \in I\}$ in neutrosophic minimal structure space
(X, M) such that $A_i = U_i \cap G$ then $\cup_{i\in I} A_i = \cup_{i \in I}(A_i \cap G) = (\cup_{i\in I} A_i) \cap G$; since the neutrosophic minimal structure space (X, M) has the union property then $\cup_{i\in I} A_i \in M$ therefore the neutrosophic minimal structure space (X, M) also has the union propert.
-The proof of (2) is similarly to (1).

4. Conclusion

In this paper, we have defined a new topological space by using neutrosophic crisp sets. This new space called neutrosophic crisp minimal structure space.Then we have introduced new neutrosophic crisp open(closed) sets in neutrosophic crisp minimal structure space. Also we studied some of their basic properties and their relationship with each other. This paper is just a beginning of a new structure and we have studied a few ideas only, it will be necessary to carry out more theoretical research to establish a general framework for the practical application. In the future, using these notions, various classes of mappings on neutrosophic crisp minimal structure space, separation axioms on the neutrosophic crisp minimal structure space, Neutrosophic crisp minimal α-open sets , Neutrosophic crisp minimal β-open sets , Neutrosophic crisp minimal pre-open sets , Neutrosophic crisp minimal semi-open sets and many researchers can be studied.

References

[1] F. Smarandache. *Neutrosophy / Neutrosophic probability, set, and logic*, American Research Press, 1998. See also: http://gallup.unm.edu/~smarandache/NeutLog.txt.
[2] F. Smarandache, "Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics" University of New Mexico, Gallup, NM 87301, USA (2002).

DOI: https://doi.org/10.54216/JNFS.030103
Received: February 10, 2022 Accepted: May 24, 2022

[3] A Salama; F.Smarandache and Kroumov. "Neutrosophic crisp Sets and Neutrosophic crisp Topological Spaces" Neutrosophic Sets and Systems, Vlo.2, (2014), pp.25-30.

[4] A. Salama and S. A. Alblowi. Neutrosophic set and neutrosophic topological space. ISORJ, Mathematics, Vol. (3), Issue 4, (2012), 31-35.

[5] Mashhour, S.; Allam, A.A.; Mahmoud F.S.; and Khedr, F.H.; On Supra topological spaces", Indian Jr.Pure and Appl.Math.No.4(14)(1983),502-510.

[6] G.Jayaparthasarathy; V.F.Little Flower; M.Arockia Dasan. "Neutrosophic Supra Topological Applications in Data Mining Process", Neutrosophic Sets and Systems, ,(2019) ,vol. 27, pp. 80-97.

[7] B.AL-Nafee; R.K. Al-Hamido; F.Smarandache. "Separation Axioms In Neutrosophic Crisp Topological Spaces",Journal of newtheory, (2018),vol. 22, pp.

[8] R. Gowri, A.K.R. Rajayal, "On Supra Bi-topological spaces", IOSR Journal of Mathematics (IOSR-JM) .No.5(113)(2017),55-58.

[9] J. C. Kelly. "Bi-topological spaces". Proceedings of the London Mathematical Society, 3(1) ,(1963), 71-89.

[10] R.K. Al-Hamido. "A study of multi-Topological Spaces", PhD Theses ,AlBaath university , Syria(2019).

[11] T.Y. Ozturk , A. Ozkan "Neutrosophic Bi-topological Spaces", Neutrosophic Sets and Systems, vol. 30,(2019) ,pp. 88-97.

[12] R.K. Al-Hamido. "Neutrosophic Crisp Bi-Topological Spaces", Neutrosophic Sets and Systems, vol. 21, (2018), pp. 66-73.

[13] R.K. Al-Hamido. "Neutrosophic Crisp Supra Bi-Topological Spaces", Neutrosophic Sets and Systems, Communicated, (2020).

[14] M. Parimala, M. Karthika ,F. Smarandache. "Introduction to neutrosophic minimal structure", Neutrosophic Sets and Systems, vol. 36, (2020) , pp. 376-388.

DOI: https://doi.org/10.54216/JNFS.030103
Received: February 10, 2022 Accepted: May 24, 2022