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Introducing Grubbs’s test for Detecting Outliers under Neutrosophic Statistics- an Application to Medical Data

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Objective

In this paper, we will introduce the designing of Grubbs's test under neutrosophic statistics. The proposed test will be a generalization of Grubbs's test under classical statistics.

Method

We will present the designing and the operational procedure of the proposed test under the neutrosophic statistical interval method.

Results

The application of the proposed test will be given with the aid of real data from the medical field.

Conclusion

From the comparisons, it is concluded that the proposed test is more effective, informative and flexible than the existing Grubbs's tests.

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1 Introduction

The statistical techniques have been playing an important role in analyzing the data, forecasting and estimation purposes in medical sciences and business. In these fields, the practitioners screen out the data for effective forecasting. To achieve, the target of effective forecasting, it is necessary to identify the outliers in the data. The presence of the outliers can affect the effective forecasting and estimation. Therefore, several tests have been applied for detecting and removing the outliers from the data. Among these tests, Grubbs's test which is introduced by (Grubbs, 1950) and recommended by ISO and has been widely applied for the detecting of outliers in the data. Grubbs's test is easy to apply and operated using the mean and standard deviation of the data. This test is used to test the null hypothesis that a suspected value is an outlier versus the alternative hypothesis that the suspected value is not an outlier. The Grubbs's test value is compared with the tabulated value at a fixed value of the level of significance. The null hypothesis that the suspected value is an outlier is accepted if the Grubbs's test value is smaller than the tabulated value. (Cohn et al., 2013) applied Grubbs's test for detecting outlier in flood series. (Urvoy & Autrusseau, 2014) used this test to detect watermarks. (Ryan, Parnell, & Mahoney, 2019) used Grubbs's test for detecting an advanced manufacturing system. (Nguyen, Renault, & Milocco, 2019) gave the application of this test for detecting outliers using smartphones. More applications of Grubbs's test and other tests can be seen in (Bellolio, Serrano, & Stead, 2008), (Urvoy & Autrusseau, 2014) and (Adikaram, Hussein, Effenberger, & Becker, 2015), (Stylianou, Fackrell, & Vasilakis, 2017), (Gandhi et al., 2018) and (Jäntschi, 2019).

Usually, the measurements in the data are not precise and reported in fuzzy or in interval forms. (Kacprzyk, Szmidt, Zadrożny, Atanassov, & Krawczak, 2017) and (Taheri & Hesamian, 2017) pointed out the lifetime of an item, water level, and alloy melting data cannot be recorded in precise form. To analyze such data, the statistical tests based on a fuzzy approach are applied for detecting outliers in the data. (Cateni, Colla, & Vannucci, 2009) developed and discussed the application of Grubbs's test using fuzzy logic. (D'Errico & Murru, 2012) proposed Grubbs's test for measurement data. For more details, the reader may refer to (Van Cutsem & Gath, 1993), (Montenegro, Casals, Lubiano, & Gil, 2001), (Mohanty & AnnanNaidu, 2013), (Moradnezhadi, 2014) and (Choi, Lee, & Irani, 2018).

The neutrosophic logic introduced by (Smarandache, 1998) is one of the generalized forms of the fuzzy logic and provides the measures of truthiness, falseness, and indeterminacy. (Wang, Smarandache, Sunderraman, & Zhang, 2005) proved the efficiency of the neutrosophic logic over the fuzzy approach and interval approach. More information and application of this logic can be seen in (Hanafy, Salama, & Mahfouz, 2013), (Broumi & Smarandache, 2013), (Guo & Sengur, 2015a), (Guo & Sengur, 2015b), (Guo, Şengür, & Tian, 2016), (Patro & Smarandache, 2016), (Broumi, Bakali, Talea, & Smarandache, 2018), (Peng & Dai, 2018), (Abdel-Baset, Chang, & Gamal, 2019), (Abdel-Basset, Mohamed, Elhoseny, Chiclana, & Zaied, 2019), (Abdel-Basset, Nabeeh, El-Ghareeb, & Aboelfetouh, 2019) and (Nabeeh, Smarandache, Abdel-Basset, El-Ghareeb, & Aboelfetouh, 2019). Based on neutrosophic, the neutrosophic statistics (NS) as the generalization of classical statistics (CS) was introduced by (Smarandache, 2014). (Chen, Ye, & Du, 2017) and (Chen, Ye, Du, & Yong, 2017) used the neutrosophic numbers to analyze the rock data. Recently, (Aslam, 2020) introduced tests of homogeneity of variance using the NS. (Aslam, 2018) discussed NS in the sampling plan the first time. More applications of NS can be seen in (Aslam & Albassam, 2019) and (Aslam, 2019).

Although, Grubbs's test under the fuzzy approach and CS are available in the literature. The existing Grubbs's test under CS can be applied when all the observations in the data are determined which is not always true in complex systems. Similarly, Grubbs's test under the fuzzy approach can be applied when measurement is fuzzy but did not provide any information about the measure of indeterminacy in the data. By exploring the literature and best of our knowledge, there is no work on Grubbs's test under NS. In this paper, we will design Grubbs's test under NS. We will present the designing and implantation of the test using real medical data. We will discuss the efficiency of the proposed test over Grubbs's test under CS and Grubbs's test under interval analysis. We expect that the proposed Grubbs's test under NS will be quite effective to be applied as an alternative of the existing tests under an uncertainty environment.

2 Design of Grubbs's test under NS

We present the design of the Grubbs's test under NS. The main objective of designing of this test is to detect outliers in the data measured from the complex system or data having the uncertainty or indeterminacy. The proposed test is applied to test the neutrosophic null hypothesis H_{0N}: the sample has outlier versus the alternative hypothesis H_{1N}: the sample has no outlier. Suppose that neutrosophic random variable is $z_{1N} = a_{1N} + b_{1N}I_N$,…, $z_{nN} = a_{n_N} + b_{n_N}I_N$; where a_{1N},…, a_{nN} and $b_{1N}I_N, \ldots, b_{n_N}I_N$ are the determined part and indeterminate part of the neutrosophic random variable, respectively and $n_N \epsilon [n_L, n_U]$ be a neutrosophic sample size. Suppose we have three variables under study. The necessary steps of the proposed test are given as follows

Step-1: Calculate the means of the lower values of indeterminacy intervals as

$$\bar{a}_{1N} = \frac{1}{n_{1N}} \sum_{i=1}^{n_{1N}} a_i;\ \bar{a}_{2N} = \frac{1}{n_{2N}} \sum_{i=1}^{n_{2N}} a_i;\ \bar{a}_{3N} = \frac{1}{n_{2N}} \sum_{i=1}^{n_{3N}} a_i$$

Step-2: Calculate the means of the upper values of indeterminacy intervals as follows

$$\bar{b}_{1N} = \frac{1}{n_{1N}} \sum_{i=1}^{n_{1N}} b_i;\ \bar{b}_{2N} = \frac{1}{n_{2N}} \sum_{i=1}^{n_{2N}} b_i;\ \bar{b}_{3N} = \frac{1}{n_{2N}} \sum_{i=1}^{n_{3N}} ab_i$$

Step-3: The neutrosophic average is given as

$$\bar{z}_{1N} = \bar{a}_{1N} + \bar{b}_{1N}I_N;\ \bar{z}_{2N} = \bar{a}_{2N} + \bar{b}_{2N}I_N; \ldots;\ I_N \epsilon [I_L, I_U]$$

Step-4: By following (Chen, Ye, & Du, 2017), calculate the sum of squares of as follows

$$(z_1 - \bar{z}_{1N})^2 =$$
$$\begin{bmatrix} \min\left((a_{11} - \bar{a}_1)^2, \left((a_{11} - \bar{a}_1) \left((a_{11} - \bar{a}_1) + 1 \times \left(b_{11} - \bar{b}_1 \right) \right), (a_{11} - \bar{a}_1) + 1 \times \left(b_{11} - \bar{b}_1 \right)^2 \right) \right) \\ \max\left((a_{11} - \bar{a}_1)^2, \left((a_{11} - \bar{a}_1) \left((a_{11} - \bar{a}_1) + 1 \times \left(b_{11} - \bar{b}_1 \right) \right), (a_{11} - \bar{a}_1) + 1 \times \left(b_{11} - \bar{b}_1 \right)^2 \right) \right) \end{bmatrix}$$

$$(z_2 - \bar{z}_{2N})^2 =$$
$$\begin{bmatrix} \min\left((a_{11} - \bar{a}_1)^2, \left((a_{11} - \bar{a}_1) \left((a_{11} - \bar{a}_1) + 1 \times \left(b_{11} - \bar{b}_1 \right) \right), (a_{11} - \bar{a}_1) + 1 \times \left(b_{11} - \bar{b}_1 \right)^2 \right) \right) \\ \max\left((a_{11} - \bar{a}_1)^2, \left((a_{11} - \bar{a}_1) \left((a_{11} - \bar{a}_1) + 1 \times \left(b_{11} - \bar{b}_1 \right) \right), (a_{11} - \bar{a}_1) + 1 \times \left(b_{11} - \bar{b}_1 \right)^2 \right) \right) \end{bmatrix}$$

$(z_3-\bar{z}_{3N})^2 =$

$$\begin{bmatrix} \min\left((a_{11}-\bar{a}_1)^2, \left((a_{11}-\bar{a}_1)\left((a_{11}-\bar{a}_1)+1\times\left(b_{11}-\bar{b}_1\right)\right), (a_{11}-\bar{a}_1)+1\times\left(b_{11}-\bar{b}_1\right)^2\right)\right) \\ \max\left((a_{11}-\bar{a}_1)^2, \left((a_{11}-\bar{a}_1)\left((a_{11}-\bar{a}_1)+1\times\left(b_{11}-\bar{b}_1\right)\right), (a_{11}-\bar{a}_1)+1\times\left(b_{11}-\bar{b}_1\right)^2\right)\right) \end{bmatrix}$$

Step-5: The neutrosophic standard deviation (NSD) of is calculated as follows

$$s_{Nz_1} = \sqrt{\frac{1}{n_{1N}}\sum_{i=1}^{n_{1N}}(z_1-\bar{z}_{1N})^2}, \quad s_{Nz_2} = \sqrt{\frac{1}{n_{1N}}\sum_{i=1}^{n_{2N}}(z_2-\bar{z}_{2N})^2}, \quad s_{Nz_2} = \sqrt{\frac{1}{n_{3N}}\sum_{i=1}^{n_{3N}}(z_3-\bar{z}_{3N})^2}$$

Step-6: The values of Grubbs's test under is given as

$$G_{N_1} = \frac{\left|z_{N_1}^*-\bar{z}_{1N}\right|}{s_{Nz_1}}, \; G_{N_2} = \frac{\left|z_{N_2}^*-\bar{z}_{2N}\right|}{s_{Nz_2}}, \; G_{N_3} = \frac{\left|z_{N_3}^*-\bar{z}_{3N}\right|}{s_{Nz_3}}$$

where $z_{N_1}^*$, $z_{N_2}^*$ and $z_{N_3}^*$ are suspected outliers in the data.

Step-7: Select **t**he Grubbs's test tabulated value at α=0.05. The null hypothesis H_{0N} is accepted if the calculated values of the test are smaller than the tabulated value.

3 **Real Example**

A medical practitioner wants to either know that the patient is at the risk of heart disease or stroke issue. To study and analyze this risk, the medical practitioner is checking the patient's heart conditions such as pulse rate (PR), the blood systolic pressure (BSP) and diastolic pressure (DP). The medical practitioner selected 11 patients to study the risk of heart disease. During examine, he recorded the PR, BSP, and DP of 11 patients. The interval data of these patients having three variables is selected from (Gioia & Lauro, 2005) and shown in Table 1. For the effective heart disease prediction of the patients, it is necessary to detect the outlier in the data and remove it from the data. The presence of the outlier in the data may mislead the medical practitioner for predicting the risk of heart disease. The medical practitioner is interested to apply the proposed Grubbs's test to analyze the data. As the data is measured in intervals and have the indeterminacy intervals. Therefore, the classical Grubbs's test under CS and test based on interval analysis by (Gioia

& Lauro, 2005) cannot be applied for detecting the outlier in the data. Therefore, the medical practitioner decides to apply the proposed Grubbs's test under NS. The necessary computational steps to perform the proposed Grubbs's test under NS are given as follows

Step-1: Calculate the means of the lower values of indeterminacy intervals of PR, BSP, and DP as follows

$$\bar{a}_{1N} = \frac{1}{10}\sum_{i=1}^{11} a_i = \frac{(44+62+\cdots+86)}{10} = 67.18$$

$$\bar{a}_{2N} = \frac{1}{10}\sum_{i=1}^{11} a_i = \frac{(90+90+\cdots+110)}{10} = 108.90$$

$$\bar{a}_{3N} = \frac{1}{10}\sum_{i=1}^{11} a_i = \frac{(50+70+\cdots+78)}{10} = 79.45$$

Step-2: Calculate the means of the upper values of indeterminacy intervals of PR, BSP, and DP as follows

$$\bar{b}_{1N} = \frac{1}{10}\sum_{i=1}^{11} a_i = \frac{(68+72+\cdots+100)}{10} = 89.36$$

$$\bar{b}_{2N} = \frac{1}{10}\sum_{i=1}^{11} a_i = \frac{(100+130+\cdots+150)}{10} = 145.45$$

$$\bar{b}_{3N} = \frac{1}{10}\sum_{i=1}^{11} a_i = \frac{(70+90+\cdots+100)}{10} = 100.72$$

Step-3: The neutrosophic average for PR, BSP and DP is given as

$$\bar{z}_{1N} = \bar{a}_{1N} + \bar{b}_{1N} I_N = 67.18 + 89.36 I_N; I_N \epsilon [0,1]$$

$$\bar{z}_{2N} = \bar{a}_{2N} + \bar{b}_{2N} I_N = 108.90 + 145.45 I_N; I_N \epsilon [0,1]$$

$$\bar{z}_{3N} = \bar{a}_{3N} + \bar{b}_{3N} I_N = 79.45 + 100.72 I_N; I_N \epsilon [0,1]$$

Step-4: By following (Chen, Ye, & Du, 2017), we calculate the sum of squares of PR, BSP and DP as follows

$(z_1 - \overline{z}_{1N})^2 =$

$$\begin{bmatrix} \min\left((a_{11} - \bar{a}_1)^2, \left((a_{11} - \bar{a}_1) \left((a_{11} - \bar{a}_1) + 1 \times \left(b_{11} - \bar{b}_1 \right) \right), (a_{11} - \bar{a}_1) + 1 \times \left(b_{11} - \bar{b}_1 \right)^2 \right) \right) \\ \max\left((a_{11} - \bar{a}_1)^2, \left((a_{11} - \bar{a}_1) \left((a_{11} - \bar{a}_1) + 1 \times \left(b_{11} - \bar{b}_1 \right) \right), (a_{11} - \bar{a}_1) + 1 \times \left(b_{11} - \bar{b}_1 \right)^2 \right) \right) \end{bmatrix}$$

=[46859.17,249816.7]

$(z_2 - \overline{z}_{2N})^2 =$

$$\begin{bmatrix} \min\left((a_{11} - \bar{a}_1)^2, \left((a_{11} - \bar{a}_1) \left((a_{11} - \bar{a}_1) + 1 \times \left(b_{11} - \bar{b}_1 \right) \right), (a_{11} - \bar{a}_1) + 1 \times \left(b_{11} - \bar{b}_1 \right)^2 \right) \right) \\ \max\left((a_{11} - \bar{a}_1)^2, \left((a_{11} - \bar{a}_1) \left((a_{11} - \bar{a}_1) + 1 \times \left(b_{11} - \bar{b}_1 \right) \right), (a_{11} - \bar{a}_1) + 1 \times \left(b_{11} - \bar{b}_1 \right)^2 \right) \right) \end{bmatrix}$$

=[1198,1600]

$(z_3 - \overline{z}_{3N})^2 =$

$$\begin{bmatrix} \min\left((a_{11} - \bar{a}_1)^2, \left((a_{11} - \bar{a}_1) \left((a_{11} - \bar{a}_1) + 1 \times \left(b_{11} - \bar{b}_1 \right) \right), (a_{11} - \bar{a}_1) + 1 \times \left(b_{11} - \bar{b}_1 \right)^2 \right) \right) \\ \max\left((a_{11} - \bar{a}_1)^2, \left((a_{11} - \bar{a}_1) \left((a_{11} - \bar{a}_1) + 1 \times \left(b_{11} - \bar{b}_1 \right) \right), (a_{11} - \bar{a}_1) + 1 \times \left(b_{11} - \bar{b}_1 \right)^2 \right) \right) \end{bmatrix}$$

=[69671.73,344819.1]

Step-5: The neutrosophic standard deviation (NSD) of PR, BSP and DP is calculated as follows

$$s_{Nz_1} = \sqrt{\frac{1}{11}\sum_{i=1}^{11}(z_2 - \bar{z}_{2N})^2} = [68.45,158.05]$$

$$s_{Nz_2} = \sqrt{\frac{1}{11}\sum_{i=1}^{11}(z_2 - \bar{z}_{2N})^2} = [10.94,12.64]$$

$$s_{Nz_3} = \sqrt{\frac{1}{11}\sum_{i=1}^{11}(z_3 - \bar{z}_{3N})^2} = [83.46,185.69]$$

Step-6: The values of Grubbs's test under NS for PR, BSP and DP is calculated as follows

$G_{N_1} = \frac{\left|z_{N_1}^* - \bar{z}_{1N}\right|}{s_{Nz_1}} = [0.3366,0.5602]$, $G_{N_2} = \frac{\left|z_{N_2}^* - \bar{z}_{2N}\right|}{s_{Nz_2}} = [2.6575,5.2465]$, $G_{N_3} = \frac{\left|z_{N_3}^* - \bar{z}_{3N}\right|}{s_{Nz_3}} = [0.3528,0.5933]$.

Step-7: The Grubbs's test tabulated value at α=0.05 is [2.23, 2.23]. By comparing the results of the proposed test with the tabulated values, it can be seen that for PR $G_{N_1} < [2.23, 2.23]$, therefore, we conclude that no outlier in PR data. Similarly, $G_{N_2} > [2.23, 2.23]$, we conclude that outlier the neutrosophic value [138,188] is detecting as outlier in BSP data. As for DP data,$G_{N_1} < [2.23, 2.23]$, therefore, we conclude that no outlier in PR data.

Table 1: The real data

Pulse Rate		Systolic pressure		diastolic pressure	
a_i	b_i	a_i	b_i	a_i	b_i
44	**68**	90	100	**50**	**70**
62	72	90	130	70	90
56	90	140	180	90	100
70	112	110	142	80	108
54	72	90	100	50	70
70	100	130	160	80	110
63	75	60	100	140	150
72	100	130	160	76	90
76	98	110	190	70	110
86	96	**138**	**188**	90	110
86	100	110	150	78	100

Note: the bold values show the suspected outliers

4 Comparative Study

In this section, we will compare Grubbs's test under NS with the existing Grubbs's test under classical statistics and test under interval data analysis. For the fair comparison, the same values of α=0.05,$z^*_{N_1}$, $z^*_{N_2}$ and $z^*_{N_3}$ are used. The proposed test in neutrosophic forms of three variables can be written as: $G_{N_1} = 0.3366 + 0.5602I_N; I_N\epsilon[0,0.4934]$, $G_{N_1} = 2.6575 + 5.2465I_N; I_N\epsilon[0,0.3991]$ and $G_{N_1} = 0.3528 + 0.5933I_N; I_N\epsilon[0,0.4053]$. The proposed test has two parts, the first part when I_N=0 denotes the determined part. Note here that the determined part is also known as the value of Grubbs's test statistic under CS. Table 2 is presented the results of three methods for three variables under study. From Table 2, we noted that the test under CS does not detect an outlier in three variables. While the proposed Grubbs's test and existing interval based Grubbs's test indicate that variable BSP has an outlier. By comparing the three Grubbs's tests, it can be noted that the proposed test provided the measure of indeterminacy has advantages over the existing tests. For example, at the level of significance 0.05, the null hypothesis will be accepted with probability 0.95, do not accepted with the probability 0.05 and test will be performed with the measure of indeterminacy is 0.4934 for variable PR. From this comparison, it can be noted that the proposed test has the statistic values in indeterminacy interval while the existing test has the determined values of statistic. It shows that the proposed test is effective, flexible and adequate to be applied in an uncertainty environment.

Table 2: The results of three tests

Tests	Variables	Test Values	Indeterminacy	Tabulated value	Conclusion
Proposed Test	PR	[0.3366,0.5602]	$I_N \epsilon [0,0.4934]$	2.23	Do not reject H_{0N}
	BSP	[2.6575,5.2465]	$I_N \epsilon [0,0.3991]$	2.23	reject H_{0N}
	DP	[0.3528,0.5933]	$I_N \epsilon [0,0.4053]$	2.23	Do not reject H_{0N}
CS	PR	1.7750	No	2.23	Do not reject H_{0N}
	BSP	1.2282	No	2.23	Do not reject H_{0N}
	DP	1.2212	No	2.23	Do not reject H_{0N}
Interval Based Test	PR	[0.3386,0.1351]	No	2.23	Do not reject H_{0N}
	BSP	[2.6578,3.3635]	No	2.23	reject H_{0N}
	DP	[0.3528,0.1654]	No	2.23	Do not reject H_{0N}

5 Concluding Remarks

Grubbs's test under NS was presented in this paper. The proposed test can be applied for the testing the null hypothesis that the sample has the outlier versus that alternative hypothesis that sample has no outlier. The design of the proposed test under NS was given and operational process of the proposed test was discussed with the help of medial data. We presented the comparisons of the proposed test with the existing tests and found that the proposed test is efficient, effective and adequate to applied in an indeterminacy environment. We recommend applying the proposed test in medical sciences and biostatistics for the analysis of the data obtained in an uncertainty environment. The proposed test can be design for big data as future research.

References

Abdel-Baset, M., Chang, V., & Gamal, A. (2019). Evaluation of the green supply chain management practices: A novel neutrosophic approach. *Computers in Industry, 108*, 210-220.

Abdel-Basset, M., Mohamed, M., Elhoseny, M., Chiclana, F., & Zaied, A. E.-N. H. (2019). Cosine similarity measures of bipolar neutrosophic set for diagnosis of bipolar disorder diseases. *Artificial Intelligence in Medicine, 101*, 101735.

Abdel-Basset, M., Nabeeh, N. A., El-Ghareeb, H. A., & Aboelfetouh, A. (2019). Utilising neutrosophic theory to solve transition difficulties of IoT-based enterprises. *Enterprise Information Systems*, 1-21.

Adikaram, K. L. B., Hussein, M. A., Effenberger, M., & Becker, T. (2015). Data transformation technique to improve the outlier detection power of grubbs' test for data expected to follow linear relation. *Journal of Applied Mathematics, 2015*.

Aslam, M. (2018). A New Sampling Plan Using Neutrosophic Process Loss Consideration. *Symmetry, 10*(5), 132.

Aslam, M. (2019). Neutrosophic analysis of variance: application to university students. *Complex & Intelligent Systems*, 1-5.

Aslam, M. (2020). Design of the Bartlett and Hartley tests for homogeneity of variances under indeterminacy environment. *Journal of Taibah University for Science, 14*(1), 6-10.

Aslam, M., & Albassam, M. (2019). Application of Neutrosophic Logic to Evaluate Correlation between Prostate Cancer Mortality and Dietary Fat Assumption. *Symmetry, 11*(3), 330.

Bellolio, M. F., Serrano, L. A., & Stead, L. G. (2008). Understanding statistical tests in the medical literature: which test should I use? *International journal of emergency medicine, 1*(3), 197.

Broumi, S., Bakali, A., Talea, M., & Smarandache, F. (2018). *Bipolar neutrosophic minimum spanning tree*: Infinite Study.

Broumi, S., & Smarandache, F. (2013). *Correlation coefficient of interval neutrosophic set.* Paper presented at the Applied Mechanics and Materials.

Cateni, S., Colla, V., & Vannucci, M. (2009). *A fuzzy system for combining different outliers detection methods.* Paper presented at the Proceedings of the IASTED international conference on artificial intelligence and applications (AIA 2009), February.

Chen, J., Ye, J., & Du, S. (2017). Scale effect and anisotropy analyzed for neutrosophic numbers of rock joint roughness coefficient based on neutrosophic statistics. *Symmetry, 9*(10), 208.

Chen, J., Ye, J., Du, S., & Yong, R. (2017). Expressions of rock joint roughness coefficient using neutrosophic interval statistical numbers. *Symmetry, 9*(7), 123.

Choi, Y., Lee, H., & Irani, Z. (2018). Big data-driven fuzzy cognitive map for prioritising IT service procurement in the public sector. *Annals of Operations Research, 270*(1-2), 75-104.

Cohn, T., England, J., Berenbrock, C., Mason, R., Stedinger, J., & Lamontagne, J. (2013). A generalized Grubbs-Beck test statistic for detecting multiple potentially influential low outliers in flood series. *Water Resources Research, 49*(8), 5047-5058.

D'Errico, G. E., & Murru, N. (2012). Fuzzy treatment of candidate outliers in measurements. *Advances in Fuzzy Systems, 2012*.

Gandhi, M., Assam, P. N., Turner, E. L., Morisky, D. E., Chan, E., Jafar, T. H., & Group, C.-B. S. (2018). Statistical analysis plan for the control of blood pressure and risk attenuation-rural Bangladesh, Pakistan, Sri Lanka (COBRA-BPS) trial: a cluster randomized trial for a multicomponent intervention versus usual care in hypertensive patients. *Trials, 19*(1), 658.

Gioia, F., & Lauro, C. N. (2005). Basic statistical methods for interval data. *Statistica applicata, 17*(1), 75-104.

Grubbs, F. E. (1950). Sample criteria for testing outlying observations. *The Annals of mathematical statistics, 21*(1), 27-58.

Guo, Y., & Sengur, A. (2015a). NCM: Neutrosophic c-means clustering algorithm. *Pattern Recognition, 48*(8), 2710-2724.

Guo, Y., & Sengur, A. (2015b). NECM: Neutrosophic evidential c-means clustering algorithm. *Neural Computing and Applications, 26*(3), 561-571.

Guo, Y., Şengür, A., & Tian, J.-W. (2016). A novel breast ultrasound image segmentation algorithm based on neutrosophic similarity score and level set. *Computer methods and programs in biomedicine, 123*, 43-53.

Hanafy, I., Salama, A., & Mahfouz, M. (2013). *Correlation coefficients of neutrosophic sets by centroid method*: Infinite Study.

Jäntschi, L. (2019). A Test Detecting the Outliers for Continuous Distributions Based on the Cumulative Distribution Function of the Data Being Tested. *Symmetry, 11*(6), 835.

Kacprzyk, J., Szmidt, E., Zadrożny, S., Atanassov, K. T., & Krawczak, M. (2017). *Advances in Fuzzy Logic and Technology 2017: Proceedings of: EUSFLAT-2017–The 10th Conference of the European Society for Fuzzy Logic and Technology, September 11-15, 2017, Warsaw, Poland IWIFSGN'2017–The Sixteenth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets, September 13-15, 2017, Warsaw, Poland* (Vol. 2): Springer.

Mohanty, V., & AnnanNaidu, P. (2013). Fraud detection using outlier analysis: A survey. *International Journal of Engineering Sciences and Research Technology, 2*(6).

Montenegro, M., Casals, M. a. R., Lubiano, M. a. A., & Gil, M. a. A. (2001). Two-sample hypothesis tests of means of a fuzzy random variable. *Information Sciences, 133*(1-2), 89-100.

Moradnezhadi, Y. M. (2014). Determination of a some simple methods for outlier detection in maximum daily rainfall (case study: Baliglichay Watershed Basin–Ardebil Province–Iran). *Bull. Env. Pharmacol. Life Sci, 3*(3), 110-117.

Nabeeh, N. A., Smarandache, F., Abdel-Basset, M., El-Ghareeb, H. A., & Aboelfetouh, A. (2019). An integrated neutrosophic-topsis approach and its application to personnel selection: A new trend in brain processing and analysis. *IEEE Access, 7*, 29734-29744.

Nguyen, V. K., Renault, É., & Milocco, R. (2019). Environment monitoring for anomaly detection system using smartphones. *Sensors, 19*(18), 3834.

Patro, S., & Smarandache, F. (2016). *The Neutrosophic Statistical Distribution, More Problems, More Solutions*: Infinite Study.

Peng, X., & Dai, J. (2018). Approaches to single-valued neutrosophic MADM based on MABAC, TOPSIS and new similarity measure with score function. *Neural Computing and Applications, 29*(10), 939-954.

Ryan, C. M., Parnell, A., & Mahoney, C. (2019). Real-Time Anomaly Detection for Advanced Manufacturing: Improving on Twitter's State of the Art. *arXiv preprint arXiv:1911.05376*.

Smarandache, F. (1998). Neutrosophy. Neutrosophic Probability, Set, and Logic, ProQuest Information & Learning. *Ann Arbor, Michigan, USA, 105*, 118-123.

Smarandache, F. (2014). *Introduction to neutrosophic statistics*: Infinite Study.

Stylianou, N., Fackrell, R., & Vasilakis, C. (2017). Are medical outliers associated with worse patient outcomes? A retrospective study within a regional NHS hospital using routine data. *BMJ open, 7*(5), e015676.

Taheri, S. M., & Hesamian, G. (2017). Non-parametric statistical tests for fuzzy observations: fuzzy test statistic approach. *International Journal of Fuzzy Logic and Intelligent Systems, 17*(3), 145-153.

Urvoy, M., & Autrusseau, F. (2014). *Application of Grubbs' test for outliers to the detection of watermarks.* Paper presented at the Proceedings of the 2nd ACM workshop on Information hiding and multimedia security.

Van Cutsem, B., & Gath, I. (1993). Detection of outliers and robust estimation using fuzzy clustering. *Computational Statistics & Data Analysis, 15*(1), 47-61.

Wang, H., Smarandache, F., Sunderraman, R., & Zhang, Y.-Q. (2005). *interval neutrosophic sets and logic: theory and applications in computing: Theory and applications in computing* (Vol. 5): Infinite Study.