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On neutro-H_v-semigroups

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Abstract. In this paper, we extend the notion of Hv-semigroups to neutro-Hv-semigroups and anti-Hv-semigroups and investigate many of their properties. We show that these new concepts are different from the classical concept of Hv-semigroups by presenting several examples. In general, the neutro-algebras and anti-algebras are generalizations and alternatives of classical algebras. The goal and benefits of our proposed extension of this study is to explore not only the hyperoperations and axioms that are totally true as in previous algebraic hyperstructures, but also the cases when they have degrees of truth, indeterminacy and falsehood. Therefore, we enlarge the field of research.

Keywords: Hyperoperation, neutro-hyperoperation, semihypergroup, neutro-semihypergroup, anti-semihypergroup, Hv-semigroup, neutro-Hv-semigroup, anti-Hv-semigroup, neutro-algebra, anti-algebra

1. Introduction

A hypergroup as a generalization of the notion of a group, was introduced by F. Marty [26] in 1934. Many authors have developed the discussion of hyperstructures and weak hyperstructures, such as P. Corsini [10] and T. Vougiouklis [42]. We can find well-written books for the introduction to hyperstructures, P. Corssini [10], P. Corsini and V. Leoreanu [13], B. Davvaz [14, 15], B. Davvaz and V. Leoreanu-Fotea [17], B. Davvaz and I. Cristea [16]. Another topic which has aroused the interest of several mathematicians, is that one of weak hyperstructure or Hv-structure, introduced by T. Vougiouklis [42]. Hv-structures are a special kind of hyperstructures, for which the weak associativity holds. Recently, Davvaz and Vougiouklis published a book on Hv-structures and their applications [18].

P. Corsini has developed hyperstructure by investigating the relationship between hypergraphs and hypergroups [11]. Corsini and Leoreanu described hypergroups associated with trees and in [12] and [13] some applications of hyperstructures in rough sets were given. hyperalgebraic systems, such as hyperrings, fuzzy hyperideals, fuzzy hypermodules and hyperlattice was introduced by R. Ameri et al. [6–8]. M. Tarnauceanu showed that the set of all subhypergroups of a hypergroup H is not a lattice in general. This is caused mainly by the fact that the intersection fails to be an operation on the set of all subhypergroups [39]. Moreover, for more results about cyclic hypergroups, transposition hypergroups and mimic fuzzy subhypergroups see [19, 22–23, 27–29, 41]. Nowadays, hypergroups have found applications to many subjects of pure and applied mathematics. For example: in geometry, topology, cryptography and coding theory, graphs and hypergraphs, probability theory, binary relations, theory of fuzzy and rough sets and automata theory, physics and also in biological inheritance. Recently, M. AL-Tahan et al. introduced the Corsini hypergroup, topological hypergroupoids and Fuzzy Multi-Hv-Ideals [2, 4, 5]. They investigated a necessary and sufficient condition for the productional hypergroup to be a Corsini hypergroup and they characterized all Corsini hypergroups of orders 2 and 3 up to isomorphism [2]. M.K. Sen et al. introduced the notion of hyperset and studied some algebraic structures on it [32] and then G. Chowdhury derived a hypergroup from a

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hyperset and studied some properties of hyperset in the light of associate hypergroup [9]. Some equivalence relations on a canonical hypergroup to construct a quotient of such hyperstructures were introduced in [31]. S. Hoskova-Mayerova et al. used the fuzzy multisets to introduce the concept of fuzzy multi-hypergroups as a generalization of fuzzy hypergroups and defined the different operations on fuzzy multi-hypergroups and extended the fuzzy hypergroups to fuzzy multi-hypergroups [21]. Recently, D. Heidari et al. considered some classes of semihypergroup such as regular semihypergroup, hypergroups, regular hypergroups and polygroups and investigated their factorization property [20]. Also, V. Vahedi et al. obtained hyperstructures from hyperconics [40] and Mahboob et al. studied hyperideals in semihypergroups [24, 25].

In 2019 and 2020, within the field of neutrosophy, Smarandache [33–38] generalized the classical algebraic Structures to neutro-algebraic structures (or neutro-algebras) {whose operations and axioms are partially true, partially indeterminate, and partially false} as extensions of partial algebra, and to anti-algebraic structures (or anti-algebras) {whose operations and axioms are totally false}. And in general, he extended any classical structure, in no matter what field of knowledge, to a neutro-structure and an anti-structure. These are new fields of research within neutrosophy. Smarandache in [35] revisited the notions of neutro-algebras and anti-algebras, where he studied partial algebras, universal algebras, effect algebras and Boole's partial algebras, and showed that neutro-algebras are generalization of partial algebras. Also, with respect to the classical hypergraph (that contains hyperedges), Smarandache added the supervertices (a group of vertices put all together form a supervertex), in order to form a superhypergraph (SHG). Then he extended the superhypergraph to n-superhypergraph, by extending the power set $P(V)$ to $P^n(V)$ that is the n-power set of the set V (the n-superhypergraph, through its n-SHG-vertices and n-SHG-edges that belong to $P^n(V)$, can the best (so far) to model our complex and sophisticated reality). Further, he extended the classical hyperalgebra to n-ary hyperalgebra and its alternatives n-ary neutro-hyperalgebra and n-ary anti-hyperalgebra [35]. Also, the neutrosophy was used in studying the canonical hypergroups and hyperrings by Agboola and Davvaz [1]. Also AL-Tahan et al. obtained some results in neutrohyperstructures [3]. In this paper, we extend the notion of *Hv*-semigroups to neutro-*Hv*-semigroups and anti-*Hv*-semigroups which are studied and some properties are investigated. We show that these definitions are different from classical definitions by providing several examples. These are particular cases of the classical algebraic structures generalized to neutro-algebraic structures and anti-algebraic structures (Smarandache, 2019).

2. Preliminaries

In this section we recall some basic notions and results regarding hyperstructures.

Definition 1. ([10]) A hypergroupoid $(H, \circ)$ is a non-empty set H together with a map $\circ : H \times H \to \mathcal{P}^*(H)$ called (binary) hyperoperation, where $\mathcal{P}^*(H)$ denotes the set of all non-empty subsets of H. The image of the pair (x, y) is denoted by $x \circ y$.

If A and B are non-empty subsets of H and $x \in H$, then by $A \circ B$, $A \circ x$, and $x \circ B$ we mean
$A \circ B = \bigcup_{a\in A, b\in B} a \circ b$, $A \circ x = A \circ \{x\}$ and $x \circ B = \{x\} \circ B$.

Definition 2. ([10]) A hyperoperation on a set H is called associative if it satisfies the associative law:

(A) $a \circ (b \circ c) = (a \circ b) \circ c$, for all $a, b, c \in H$.

$(H, \circ)$ is called semihypergroup if the hyperoperation $\circ$ is associative.

Definition 3. ([42]) A hyperoperation $\circ$ on a set H is called weak associative if it satisfies the weak associative law:

(WA) $a \circ (b \circ c) \cap (a \circ b) \circ c \neq \emptyset$, for all $a, b, c \in H$.

$(H, \circ)$ is called H_v-semigroup if the hyperoperation $\circ$ is weak associative.

A hyperoperation $\circ$ on a set H is called commutative if

(C) $a \circ b = b \circ a$, for all $a, b \in H$,

and a hyperoperation $\circ$ on a set H is called weak commutative if

(WC) $a \circ b \cap b \circ a \neq \emptyset$, for all $a, b \in H$.

It clear that every semihypergroup is an H_v-semigroup. Also, every commutative hypergroupoid is a weak commutative hypergroupoid.

Similar to finite semigroups, we can describe the hyperoperation on finite semihypergroups and H_v-semigroups by means Cayley's tables.

Example 1. Let $H = \{a, b, c, d\}$. Define the hyperoperation $(\circ_1)$ on H in Table 1.

Table 1
Cayley table for the semihypergroup $(H, \circ_1)$

$\circ_1$	a	b	c	d
a	a	$\{a, b\}$	$\{a, c\}$	$\{a, d\}$
b	a	$\{a, b\}$	$\{a, c\}$	$\{a, d\}$
c	a	b	c	d
d	a	b	c	d

It is not difficult to see that $(H, \circ_1)$ is a semihypergroup.

Example 2. Let $H = \{a, b, c, d\}$. Define the hyperoperation $(\circ_2)$ on H in Table 2.

Table 2
Cayley table for the H_v-semigroup $(H, \circ_2)$

$\circ_2$	a	b	c	d
a	a	$\{a, b\}$	$\{a, c\}$	$\{a, d\}$
b	a	$\{a, b\}$	$\{a, c\}$	$\{a, d\}$
c	a	$\{b, c\}$	c	d
d	a	$\{a, b\}$	c	d

It is not difficult to see that the (WA) law is true, but

$$(d \circ_2 c) \circ_2 b = c \circ_2 b = \{b, c\} \neq \{a, b, c\} = d \circ_2 \{b, c\} = d \circ_2 (c \circ_2 b).$$

Then $(H, \circ_2)$ is an H_v-semigroup and is not a semihypergroup.

Example 3. Let H be the unit interval $[0, 1]$. For every $x, y \in H$, we define $x \circ_3 y = \left[0, \frac{xy}{2}\right]$. Then, $(H, \circ_3)$ is a semihypergroup, because for every $x, y, z \in H$, we have

$$\begin{aligned}(x \circ_3 y) \circ_3 z &= \left[0, \frac{xy}{2}\right] \circ_3 z = \bigcup_{u \in \left[0, \frac{xy}{2}\right]} u \circ_3 z \\ &= \bigcup_{u \in \left[0, \frac{xy}{2}\right]} \left[0, \frac{uz}{2}\right] = \left[0, \frac{\left(\frac{xy}{2}\right) z}{2}\right] \\ &= \left[0, \frac{x\left(\frac{yz}{2}\right)}{2}\right] = \bigcup_{v \in \left[0, \frac{yz}{2}\right]} \left[0, \frac{xv}{2}\right] \\ &= x \circ_3 \left[0, \frac{yz}{2}\right] = x \circ_3 (y \circ_3 z)\end{aligned}$$

Example 4. Let H be the unit interval $[0, 1]$. For every $x, y \in H$, we define $x \circ_4 y = \left\{x, \frac{y}{2}, \frac{x}{4}\right\}$. For every $x, y, z \in H$ we have $x \in ((x \circ_4 y) \circ_4 z) \cap (x \circ_4 (y \circ_4 z))$. Then the hyperoperation $\circ_4$ is weak associative and so $(H, \circ_4)$ is an H_v-semigroup, but $(x \circ_4 y) \circ_4 z = \left\{x, \frac{y}{2}, \frac{x}{4}\right\} \circ_4 z = \left\{x, \frac{z}{2}, \frac{x}{4}, \frac{y}{2}, \frac{y}{8}, \frac{x}{16}\right\}$ and $x \circ_4 (y \circ_4 z) = x \circ_4 \left\{y, \frac{z}{2}, \frac{y}{4}\right\} = \left\{x, \frac{y}{2}, \frac{x}{4}, \frac{z}{4}, \frac{y}{8}\right\}$. So for some $0 \neq x \in H$, $(x \circ_4 x) \circ_4 x \neq x \circ_4 (x \circ_4 x)$. Then $(H, \circ_4)$ is not a semihypergroup.

3. On Neutro-semihypergroups, Anti-semihypergroups, Neutro-H_v-semigroups and Anti-H_v-semigroups

F. Smarandache generalized the classical algebraic structures to the neutro-algebraic structurers and anti-algebraic structures (see Smarandache [33–36]). In this section, we define neutro-semihypergroups, neutro-H_v-semigroups, anti-semihypergroups and anti-H_v-semigroup. Throughout this section, let $\mathcal{P}^*(H) = \mathcal{P}(H) - \{\emptyset\}$ and $\circ : H \times H \to \mathcal{P}^*(\mathcal{U})$ where $\mathcal{U}$ is a universe of discourse that contains H. The map $(\circ)$ is called neutro-hyperoperation. If $\mathcal{U} = H$ then the neutro-hyperoperation $(\circ)$ is a hyperoperation.

Note that (1,0,0) means that T = 1 (100% true), I = 0, F = 0 and this case corresponds to the classical Hv-semigroup and (0,0,1) means that T = 0, I = 0, F = 1 (100% false) and this corresponds to the anti-Hv-semigroup.

Neutrosophication of an Axiom on a given set X, means to split the set X into three regions such that:

On one region the axiom is true (we say degree of truth T of the axiom), on another region the axiom is indeterminate (we say degree of indeterminacy I of the axiom), and on the third region the axiom is false (we say degree of falsehood F of the axiom), such that the union of the regions covers the whole set, while the regions may or may not be disjoint, where (T, I, F) is different from $(1, 0, 0)$ and from $(0, 0, 1)$.

Antisophication of an Axiom on a given set X, means to have the axiom false on the whole set X (we say total degree of falsehood F of the axiom), or $(0, 0, 1)$.

Neutrosophication of an operation on a given set X, means to split the set X into three regions such that on one region the operation is well-defined (or inner-defined) (we say degree of truth T of the operation), on another region the operation is indeterminate (we say degree of indeterminacy I of the operation), and on the third region the operation is outer-defined (we say degree of falsehood F of the operation), such that the union of the regions covers the whole set, while the regions may or may not be disjoint, where (T, I, F) is different from $(1, 0, 0)$ and from $(0, 0, 1)$.

Antisophication of an Operation on a given set X, means to have the operation outer-defined on the whole set X (we say total degree of falsehood F of the axiom), or $(0, 0, 1)$.

Definition 4. (Neutro-hyperoperations and Neutro-semihypergroup)

The neutro-hyperoperation of the hyperoperation (degree of well-defined, degree of indeterminacy, degree of outer-defined)

(NHO) $(\exists a, b \in H)(a \circ b \subseteq H)$ {degree of truth T} and $(\exists c, d \in H)(c \circ d$ is an indeterminate subset {degree of indeterminacy I} and $(\exists e, f \in H)$ $(e \circ f \subsetneq H)$ {degree of falsehood F}, where (T,I,F) $\notin$ {(1,0,0), (0,0,1)}.

The neutro-hyperaxiom is also characterized by degree of truth, degree of indeterminacy, and degree of falsehood.

Therefore, the neutro-hyperassociativity (NHA) is defined as below:

(NHA) $(\exists a, b, c \in H)(a \circ (b \circ c) = (a \circ b) \circ c)$ {degree of truth T}, $(\exists d, e, f \in H)(d \circ (e \circ f)$ *or* $(d \circ e) \circ f)$isindeterminate {degree of indeterminacy I}, and

$(\exists g, h, k \in H)(g \circ (h \circ k) \neq (g \circ h) \circ k)$ {degree of falsehood F}, where (T,I,F)$\notin$ {(1, 0, 0), (0, 0, 1)}.

Definition 5. (Neutro-H_v-semigroup)

The neutro-H_v-semigroup has an H_v-semigroup axiom characterized by degree of truth, degree of indeterminacy, and degree of falsehood, called neutro-weakassociativity (NWA), defined as below:

(NWA) $(\exists a, b, c \in H)(a \circ (b \circ c) \cap (a \circ b) \circ c \neq \emptyset)$ {degree of truth T},

$(\exists d, e, f \in H)(d \circ (e \circ f)$ or $(d \circ e) \circ f$ are indeterminate {degree on indeterminacy I}, and $(\exists g, h, k \in H)(g \circ (h \circ k) \cap (g \circ h) \circ k = \emptyset)$ {degree of falsehood F}, where (T,I,F) $\notin$ {(1,0,0), (0,0,1)}.

We define neutro-commutative law on $(H, \circ)$ as follows:

(NHC) $(\exists a, b \in H)(a \circ b = b \circ a)$ {degree of truth T}, and

$(\exists c, d \in H)$ $(c \circ d$ or $d \circ c$ *is indeterminate* {degree of indeterminacy I}, and $(\exists e, f \in H)(e \circ f \neq e \circ f)$ {degree of false hood F}), where (T,I,F) $\notin$ {(1,0,0), (0,0,1)}.

Also, we define neutro-weak commutative law on $(H, \circ)$ as follows:

(NHWC) $(\exists a, b \in H)\left(a \circ b \cap b \circ a \neq \emptyset\right)$ {degree of truth T},

$(\exists c, d \in H)$ $(c \circ d$ *or* $d \circ c$, is indeterminate {degree of indeterminacy I}, and $(\exists e, f \in H)$ $(e \circ f \cap f \circ e = \emptyset)$ {degree of falsehood F}), where (T,I,F)$\notin$ {(1, 0, 0), (0, 0, 1)}.

Now, we define a neutro-hyperalgebraic system (neutro-H_v-algebraic system) $S = H, O, A$, where H is a classical set or a neutrosophic set, O is a set of the hyperoperations or a set of neutro-hyperoperations of the hyperoperations, and A is a set of semihypergroup axioms (H_v-semigroup axioms) or the neutro-semihypergroup axioms (neutro-H_v-semigroup S) of the semihypergroup axioms (H_v-semigroup axioms).

Definition 6. (Anti-hyperoperations and Anti-semihypergroup)

The anti-hyperoperation of the hyperoperation (totally outer-defined) is defined as follows:

(AHO) $(\forall x, y \in H)$ $(x \circ y \subsetneq H)$.

The anti-semihypergroup of the semihypergroup axioms (totally false) is defined as follows:

(AA) $(\forall x, y, z \in H)(x \circ (y \circ z) \neq (x \circ y) \circ z)$.

Definition 7. (Anti-hyperoperations and Anti-H_v-semigroup)

The anti-hyperoperation of the hyperoperation (totally outer-defined) is defined as follows:

(AHO) $(\forall x, y \in H)(x \circ y \subsetneq H)$.

The anti-H_v-semigroup of the H_v-semigroup axioms (totally false) is defined as follows:

(AWA) $(\forall x, y, z \in H)((x \circ (y \circ z) \cap (x \circ y) \circ z) = \emptyset)$.

We define anti-commutative law on $(H, \circ)$ as follows:

(AC) $(\forall a, b \in H$ with $a \neq b)(a \circ b \neq b \circ a)$.

Also, we define anti-weak commutative law on $(H, \circ)$ as follows:

(AWC) $(\forall a, b \in H$ with $a \neq b)(a \circ b \cap b \circ a = \emptyset)$.

Definition 8. A neutro-semihypergroup is an alternative of semihypergroup that has at least an (NHO) or an (NHA) satisfied, with no anti-hyperoperation and no anti-semihypergroup axiom.

Remark 1. Every H_v-semigroup that is not a semihypergroup is a neutro-semihypergroup or an anti-semihypergroup.

Example 5. (i) Let $H = \{a, b, c\}$ and $\mathcal{U} = \{a, b, c, d\}$ be a universe of discourse that contains H. We define the neutro-hyperoperation $(\circ_5)$ on H in Table 3.

Table 3
Cayley table for the neutro-semihypergroup $(H, \circ_5)$

$\circ_5$	a	b	c
a	a	a	b
b	b	$\{a, b\}$	d
c	b	?	b

Then $(H, \circ_5)$ is a neutro-semihypergroup. (NHO) is valid, since $a \circ_5 b \subseteq H, b \circ_5 c = \{d\} \subsetneq H$ and $c \circ_5 b =$ indeterminate. Thus, (NHO) holds.

(ii) Let $H = \{a, b, c\}$. Define the hyperoperation $(\circ_6)$ on H in Table 4.

Table 4
Cayley table for the neutro-semihypergroup $(H, \circ_6)$

$\circ_6$	a	b	c
a	a	a	a
b	b	$\{a, b\}$	$\{a, b\}$
c	c	$\{b, c\}$	H

Then $(H, \circ_6)$ is a neutro-semihypergroup. (NHA) is valid, since $(b \circ_6 c) \circ_6 a = \{a, b\} \circ_6 a = (a \circ_6 a) \cup (b \circ_6 a) = \{a\} \cup \{b\} = \{a, b\}$ and $b \circ_6 (c \circ_6 a) = b \circ_6 \{c\} = b \circ_6 c = \{a, b\}$. Hence $(b \circ_6 c) \circ_6 a = b \circ_6 (c \circ_6 a)$. Also, $(b \circ_6 a) \circ_6 c = \{b\} \circ_6 c = b \circ_6 c = \{a, b\}$ and $b \circ_6 (a \circ_6 c) = b \circ_6 \{a\} = b \circ_6 a = \{b\}$, and so $(b \circ_6 a) \circ_6 c \neq b \circ_6 (a \circ_6 c)$. Thus, (NHA) holds.

Definition 9. An anti-semihypergroup is an alternative of semihypergroup that has at least an (AHO) or an (AA).

Example 6. (i) Let $H = \mathbb{N}$ be the set of natural numbers. Define hyperoperation $(\circ_7)$ on $\mathbb{N}$ by $x \circ_7 y = \left\{\frac{x^2}{x^2+1}, y\right\}$. Then $(H, \circ_7)$ is an anti-semihypergroup. (AHO) is valid, since for all $x, y \in \mathbb{N}$, $x \circ_7 y \subsetneq \mathbb{N}$. Thus, (AHO) holds.

(ii) Let $H = \{a, b, c\}$. Define the hyperoperation $(\circ_8)$ on H in Table 5.

Table 5
Cayley table for the anti-H_v-semigroup $(H, \circ_8)$

$\circ_8$	a	b	c
a	b	a	b
b	b	a	b
c	b	a	b

Then $(H, \circ_8)$ is an anti-semihypergroup. The (AA) law is valid, since for all $x, y, z \in H$, $x \circ_8 (y \circ_8 z) \neq (x \circ_8 y) \circ_8 z$.

Definition 10. A neutro-H_v-semigroup is an alternative of H_v-semigroup that has at least a (NHO) or satisfies (NWA), with no anti-hyperoperation and no anti-H_v-semigroupaxiom.

Example 7. (i) Neutro-semihypergroup $(H, \circ_5)$ in Example 5, is a neutro-H_v-semigroup.

(ii) Let $H = \{a, b, c, d\}$. Define the hyperoperation $(\circ_9)$ on H in Table 6.

Table 6
Cayley table for the anti-H_v-semigroup $(H, \circ_9)$

$\circ_9$	a	b	c	d
a	H	$\{a, c\}$	$\{a, b\}$	a
b	$\{b, d\}$	$\{a, c\}$	b	a
c	$\{c, d\}$	c	$\{a, b\}$	a
d	d	c	b	a

Then $(H, \circ_9)$ is a neutro-H_v-semigroup but it is not a neutro-semihypergroup. (NWA) is valid, since $(b \circ_9 c) \circ_9 a = b \circ_9 a = \{b, d\}$ and $b \circ_9 (c \circ_9 a) = b \circ_9 \{c, d\} = \{a, b\}$. Hence $(b \circ_9 c) \circ_9 a \cap b \circ_9 (c \circ_9 a) \neq \emptyset$. Also, $(a \circ_9 a) \circ_9 d = H \circ_9 d = \{a\}$ and $a \circ_9 (a \circ_9 d) = b \circ_9 a = \{a, c\}$, and so $(a \circ_9 a) \circ_9 d \cap b \circ_9 (a \circ_9 c) = \emptyset$. Thus, (NWA) holds. Also, we have for every $x, y, z \in H$, $(x \circ_9 y) \circ_9 z \neq x \circ_9 (y \circ_9 z)$ and therefore $(H, \circ_9)$ is not a neutro-semihypergroup.

Definition 11. An anti-H_v-semigroup is an alternative of H_v-semigroup that has at least a (AHO) or (AWA).

Example 8. (i) The anti-semihypergroup $(H, \circ_7)$ in Example 6, is an anti-H_v-semigroup.

(ii) Let $H = \{a, b, c\}$. Define the hyper operation $(\circ_{10})$ on H in Table 7.

Table 7
Cayley table for the anti-H_v-semigroup $(H, \circ_{10})$

$\circ_{10}$	a	b	c
a	b	a	b
b	b	a	b
c	b	a	b

Then $(H, \circ_{10})$ is an anti-semihypergroup. (AA) law is valid, since for all $x, y, z \in H$, $x \circ_{10} (y \circ_{10} z) \neq (x \circ_{10} y) \circ_{10} z$.

Lemma 1. *Every anti-H_v-semigroup is an anti-semihypergroup.*

The converse of Lemma 1 may not hold. This is because an anti-semihypergroup may be H_v-semigroup or a neutro-H_v-semigroup or an anti-H_v-semigroup.

Example 9. Let $|H| \geq 3$ and for every $x, y \in H$ we set $x \circ_{11} y = H - \{y\}$. Then

$$(x \circ_{11} y) \circ_{11} z = (H - \{y\}) \circ_{11} z = \bigcup_{u \in H - \{y\}} u \circ_{11} z = H - \{z\}$$

and

$$x \circ_{11} (y \circ_{11} z) = x \circ_{11} (H - \{z\}) = \bigcup_{v \in H-\{z\}} x \circ_{11} v = \bigcup_{v \in H-\{z\}} H - \{v\} = H.$$

Therefore, $(H, \circ_{11})$ is an anti-semihypergroup and an H_v-semigroup.

Example 10. Let $H = \{a, b\}$ and define the operation $(\circ_{12})$ on H in Table 8.

Table 8
Cayley table for the anti-H_v-semigroup $(H, \circ_{12})$

$\circ_{12}$	a	b
a	b	a
b	b	a

Then

Table 9
Test (A) and (WA)

x	y	z	$(x \circ_{12} y) \circ_{12} z$	$=$ or $\cap$ or $\emptyset$	$x \circ_{12} (y \circ_{12} z)$
a	a	a	b	$\emptyset$	a
a	a	b	a	$\emptyset$	b
a	b	a	b	$\emptyset$	a
a	b	b	a	$\emptyset$	b
b	a	a	b	$\emptyset$	a
b	a	b	a	$\emptyset$	b
b	b	a	b	$\emptyset$	a
b	b	b	a	$\emptyset$	b

Therefore, by Table 9 $(H, \circ_{12})$ is an anti-semihypergroup and an anti-Hv-semigroup.

Example 11. Let $H = \{a, b\}$ and define the hyperoperation $(\circ_{13})$ on H in Table 10.

Table 10
Cayley table for the anti-semihypergroup $(H, \circ_{13})$

$\circ_{13}$	a	b
a	H	a
b	b	a

Then

Table 11
Test (A) and (WA)

x	y	z	$(x \circ_{13} y) \circ_{13} z$	$=$ or $\cap$ or $\emptyset$	$x \circ_{13} (y \circ_{13} z)$
a	a	a	H	$\cap$	a
a	a	b	a	$\emptyset$	b
a	b	a	b	$\cap$	H
a	b	b	a	$\emptyset$	b
b	a	a	H	$\cap$	a
b	a	b	a	$\cap$	H
b	b	a	b	$\cap$	H
b	b	b	a	$\cap$	H

Therefore, by Table 11 $(H, \circ_{13})$ is an anti-semihypergroup and a neutro-H_v-semigroup.

Lemma 2. *Every neutro-semihypergroup is a neutro-H_v-semigroup.*

Proof. A neutro-semihypergroup is endowed with a neutro-associativity, which is also a neutro-weak associativity that characterizes the neutro-H_v-semigroup.

Example 12. Let $H = \{a, b, c\}$ and define the operation $(\circ_{14})$ on H in Table 12.

Table 12
Cayley table for neutro-H_v-semigroup and a neutro-semihypergroup $(H, \circ_{14})$

$\circ_{14}$	a	b	c
a	b	a	a
b	b	a	a
c	a	b	c

We have $(a \circ_{14} a) \circ_{14} a \cap a \circ_{14} (a \circ_{14} a) = \emptyset$ and $(c \circ_{14} c) \circ_{14} c = c \circ_{14} (c \circ_{14} c)$. Therefore, $(H, \circ_{14})$ is a neutro-H_v-semigroup and a neutro-semihypergroup.

Remark 2. From Example 11, we can see that there exists a neutro-H_v-semigroup such that it is an anti-semihypergroup.

Theorem 1. *Let $(H, \circ)$ be a weak commutative hypergroupoid. Then it cannot be an anti-H_v-semigroup.*

Proof. Let $x \in H$ and $b \in x \circ x$. Then there exists $c \in x \circ b \cap b \circ x$. Therefore

$$c \in x \circ b \cap b \circ x \subseteq x \circ (x \circ x) \cap (x \circ x) \circ x.$$

This implies that $x \circ (x \circ x) \cap (x \circ x) \circ x \neq \emptyset$, and so $(H, \circ)$ cannot be an anti-H_v-semigroup.

The next example shows that there exists a weak commutative anti-semihypergroup.

Example 13. Let $(H, \circ_{11})$ be the anti-semihypergroup in the Example 9. Since $|H| \geq 3$ then there exists

$z \in H$ such that $z \in x \circ_{11} y = H - \{y\}$ and $z \in y \circ_{11} x = H - \{x\}$. Therefore, $(H, \circ_{11})$ is a weak commutative anti-semihypergroup.

Theorem 2. ([30]) *Let $(H, \circ)$ be a commutative hypergroupoid. Then it cannot be an anti-semihypergroup.*

Definition 12. ([42]) Let $(H_1, \circ_1)$ and $(H_2, \circ_2)$ be two hypergroupoids. We say that $(\circ_1)$ is less than or equal to $(\circ_2)$, and note $\leq$, if and only if there exists $f \in Aut(H, \circ_2)$ such that $x \circ_1 y \subseteq f(x \circ_2 y)$ for any x, y of H.

From this definition we can deduce the following theorem:

Theorem 3. ([42]) *If a hyperoperation is (WA), then any hyperoperation superior to it and defined on the same set is (WA), too.*

Note that if a hyperoperation is (A), then any hyperoperation superior to it and defined on the same set may not be true in (A) law, but it is true (WA) law.

Example 14. Let $H = \{a, b\}$ and define the operation $(\circ_{15})$ on H in Table 13.

Table 13
Cayley table for the Null semigroup $(H, \circ_{15})$

$\circ_{15}$	a	b
a	a	a
b	a	a

$(H, \circ_{15})$ is a semigroup and called Null semigroup. Now, we define the hyperoperation $(\circ_{16})$ on H in Table 14.

Table 14
Cayley table for the H_v-semigroup $(H, \circ_{16})$

$\circ_{16}$	a	b
a	a	a
b	H	a

Then $(\circ_{15})$ is less than or equal to $(\circ_{16})$ and $(H, \circ_{15})$ is a semihypergroup. It easy to see that $(H, \circ_{16})$ is not a semihypergroup, but it is a neutro-semihypergroup and an H_v-semigroup.

Theorem 4. *If a hyperoperation $(\circ_2)$ is (AWA) and $(\circ_1)$ is less than or equal to $(\circ_2)$, then $(\circ_1)$ is (AWA), too.*

Proof. Suppose there exist $x, y, z \in H$ such that $x \circ_2 (y \circ_2 z) \cap (x \circ_2 y) \circ_2 z \neq \emptyset$. Since $(\circ_1)$ is less than or equal to $(\circ_2)$, then $x \circ_1 (y \circ_1 z) \subseteq x \circ_2 (y \circ_2 z)$, and $(x \circ_2 y) \circ_2 z \subseteq (x \circ_2 y) \circ_2 z$. Therefore, $x \circ_1 (y \circ_1 z) \cap (x \circ_1 y) \circ_1 z \neq \emptyset$ and this is a contradiction with (AWA) law of hyperoperation $(\circ_1)$.

Note that if a hyperoperation $(\circ_2)$ is (AA) and $(\circ_1)$ is less than or equal to $(\circ_2)$, then $(\circ_1)$ may not be true in (AA) law or (AWA) law.

Example 15. Let $H = \{a, b, c\}$ and define the hyperoperation $(\circ_{17})$ on H in Table 15.

Table 15
Cayley table for the anti-semihypergroup $(H, \circ_{17})$

$\circ_{17}$	a	b	c
a	$\{b, c\}$	$\{a, c\}$	$\{a, b\}$
b	$\{b, c\}$	$\{a, c\}$	$\{a, b\}$
c	$\{b, c\}$	$\{a, c\}$	$\{a, b\}$

$(H, \circ_{17})$ is an anti-semihypergroup. We define the hyperoperation $(\circ_{18})$ on H in Table 16.

Table 16
Cayley table for the neutro-H_v-semigroup $(H, \circ_{18})$

$\circ_{18}$	a	b	c
a	b	$\{c\}$	$\{b\}$
b	b	$\{a, c\}$	b
c	b	$\{a, c\}$	b

We obtain $(a \circ_{18} a) \circ_{18} a \cap a \circ_{18} (a \circ_{18} a) = \emptyset$ and $(c \circ_{18} c) \circ_{18} c = c \circ_{18} (c \circ_{18} c)$. Therefore, $(H, \circ_{18})$ is a neutro-H_v-semigroup and a neutro-semihypergroup.

By the above theorems we have the following:

Theorem 5. *Let $(H, \circ_1)$ and $(H, \circ_2)$ be two hypergroupoids and $\circ_1 \leq \circ_2$. Then*

1. *If $(H, \circ_1)$ is an H_v-semigroup, then $(H, \circ_2)$ is an H_v-semigroup.*
2. *If $(H, \circ_1)$ is a weak commutative hypergroupoid, then $(H, \circ_2)$ is a weak commutative hypergroupoid.*
3. *If $(H, \circ_1)$ is a neutro-H_v-semigroup, then $(H, \circ_2)$ cannot be an anti-H_v-semigroup.*
4. *If $(H, \circ_1)$ is a neutro-weak commutative hypergroupoid, then $(H, \circ_2)$ cannot be an anti-weak commutative hypergroupoid.*
5. *If $(H, \circ_2)$ is a neutro-H_v-semigroup, then $(H, \circ_1)$ cannot be an H_v-semigroup.*
6. *If $(H, \circ_2)$ is an anti-weak commutative hypergroupoid, then $(H, \circ_1)$ cannot be a weak commutative hypergroupoid.*
7. *If $(H, \circ_2)$ is an anti-H_v-semigroup, then $(H, \circ_1)$ is an anti-H_v-semigroup.*

8. *If $(H, \circ_2)$ is an anti-weak commutative hypergroupoid, then $(H, \circ_1)$ is an anti-weak commutative hypergroupoid.*

Proof.

1. For every $x, y, z \in H$, $x \circ_1 (y \circ_1 z) \subseteq x \circ_2 (y \circ_2 z)$ and $(x \circ_1 y) \circ_1 z \subseteq (x \circ_2 y) \circ_2 z$. So $x \circ_1 (y \circ_1 z) \cap (x \circ_1 y) \circ_1 z \neq \emptyset$ implies that $x \circ_2 (y \circ_2 z) \cap (x \circ_2 y) \circ_2 z \neq \emptyset$. Therefor $(H, \circ_2)$ is an H_v-semigroup.
2. It is straightforward.
3. Let $(H, \circ_1)$ is a neutro-H_v-semigroup, then there exists $x, y, z \in H$ such that $x \circ_1 (y \circ_1 z) \cap (x \circ_1 y) \circ_1 z \neq \emptyset$ and this implies that $x \circ_2 (y \circ_2 z) \cap (x \circ_2 y) \circ_2 z \neq \emptyset$. Therefor $(H, \circ_2)$ cannot be an anti-H_v-semigroup.
4. It proves that similar part 3.
5. It obtains from 1.
6. It is straightforward.
7. For every $x, y, z \in H$, $x \circ_1 (y \circ_1 z) \subseteq x \circ_2 (y \circ_2 z)$ and $(x \circ_1 y) \circ_1 z \subseteq (x \circ_2 y) \circ_2 z$. If for some $x, y, z \in H$, $x \circ_1 (y \circ_1 z) \cap (x \circ_1 y) \circ_1 z \neq \emptyset$ then $x \circ_2 (y \circ_2 z) \cap (x \circ_2 y) \circ_2 z \neq \emptyset$. Therefor $(H, \circ_2)$ is not an anti-H_v-semigroup and this is a contradiction.
8. It is straightforward.

Definition 13. Let $(H, \circ)$ be a hypergroupoid. We call the complement of $(H, \circ)$ is the partial hypergroupoid $(H, *)$ when for all $x, y \in H$, we define $x * y = H - x \circ y$.

Lemma 3. *If $(H, \circ)$ is a group and $|H| \geq 2$, then $(H, *)$ is a semihypergroup.*

Proof. For all $x, y \in H$,

$$x \star (y \star z) = x \star (H - y \circ z) = \bigcup_{u \in H - y \circ z} x \star u = \bigcup_{u \in H - y \circ z}$$

$$H - x \circ u = H - \bigcap_{u \in H - y \circ z} x \circ u = H - \emptyset = H$$

and

$$(x \star y) \star z = (H - x \circ y) \star z = \bigcup_{v \in H - x \circ y} v \star z = \bigcup_{v \in H - x \circ y}$$

$$H - v \circ z = H - \bigcap_{v \in H - x \circ y} v \circ z = H - \emptyset = H$$

Example 16. Let $(H = \{a, b\}, \circ_{19})$ be the semihypergroup in Table 17.

Table 17
Cayley table for the semihypergroup $(H, \circ_{19})$

$\circ_{19}$	a	b
a	a	b
b	a	b

Then by Example 10, $(\circ_{12}) = (\star)$ and $(H, \star)$ is the anti-semihypergroup and anti-H_v-semigroup.

Example 17. Let $(H = \{a, b\}, \circ_{20})$ be the semihypergroup in Table 18.

Table 18
Cayley table for the semihypergroup $(H, \circ_{20})$

$\circ_{20}$	a	b
a	a	a
b	a	b

Then $(H, \star)$ is a semihypergroup in Table 19.

Table 19
Cayley table for the semihypergroup $(H, \star)$

$\star$	a	b
a	b	b
b	b	a

Example 18. ([14]) The semihypergroup $H = \{a, b, c\}$ in Table 20 is a semihypergroup.

Table 20
Cayley table for the semihypergroup $(H, \circ_{21})$

$\circ_{21}$	a	b	c
a	b	$\{b, c\}$	$\{b, c\}$
b	$\{b, c\}$	$\{b, c\}$	$\{b, c\}$
c	$\{b, c\}$	$\{b, c\}$	$\{b, c\}$

Then $(H, \star)$ has the Table 21.

Table 21
Cayley table for the neutro-semihypergroup $(H, \star)$

$*$	a	b	c
a	$\{a, c\}$	a	a
b	a	a	a
c	a	a	a

We have

$$a \star (a \star a) = a \star \{a, c\} = \{a, c\} = \{a, c\} \star a = (a \star a) \star a$$

and

$$b \star (a \star a) = b \star \{a, c\} = a \neq \{a, c\} = b \star a = (b \star a) \star a.$$

Therefore, $(H, \star)$ is a neutro-semihypergroup.

Theorem 6. *Let $(H, \circ)$ be an H_v-semigroup then for every $x, y \in H$, define $x \star y = H - x \circ y$. Then $(H, \star)$ is a neutro-H_v-semigroup or an anti-H_v-semigroup or an H_v-semigroup.*

Proof. If there exist $x, y \in H$ such that $x \circ y = H$, then (NHO) or (NHO) is valid for partially hyperoperation $(\star)$. Then $(H, \star)$ is a neutro-H_v-semigroup or an anti-H_v-semigroup. So, let for every $x, y \in H$, $x \circ y \neq H$. Then $(\star)$ is a hyperoperation.

Let $(H, \circ)$ be an H_v-semigroup. For every $x, y \in H$, define $x \star y = H - x \circ y$. Then $(H, \star)$ can be a neutro-H_v-semigroup or an anti-H_v-semigroup or an H_v-semigroup.

Example 19. Let $(H, \circ)$ be a total semihypergroup, i.e., for every $x, y \in H$, $x \circ y = H$. Then $x \star y = H - x \circ y = H - H = \emptyset$, and so $(H, \star)$ is an anti-H_v-semigroup.

Lemma 4. *If for some $x \in H$, $x \in x \circ x$, then $(H, \circ)$ cannot be an anti-H_v-semigroup.*

Proof. Let $x \in x \circ x$. Then $x \in ((x \circ x) \circ x) \cap (x \circ (x \circ x))$, and so $(H, \circ)$ cannot be an anti-H_v-semigroup.

Theorem 7. *Let $(H, \circ)$ be an anti-H_v-semigroup then for every $x, y \in H$. Then $(H, \star)$ cannot be an anti-H_v-semigroup.*

Proof. Let $x \in H$. By Lemma 4, $x \notin x \circ x$, and so $x \in x \star x = H - x \circ x$. Therefore, by Lemma 4, $(H, \star)$ cannot be an anti-H_v-semigroup.

Corollary 1. *Let $(H, \circ)$ be an anti-H_v-semigroup. For every $x, y \in H$, define $x \star y = H - x \circ y$. Then $(H, \star)$ is a neutro-H_v-semigroup or an H_v-semigroup.*

Example 20. Let $(H = \{a, b\}, \circ_{12})$ be the anti-H_v-semigroup in Table 22 (see Examples 10 and 17).

Table 22
Cayley table for the anti-H_v-semigroup $(H, \circ_{12})$

$\circ_{12}$	a	b
a	b	a
b	b	a

Then by Example 17, $(\circ_{19}) = (\star)$ and $(H, \star)$ is an H_v-semigroup.

Lemma 5. *Every anti-weak commutative hypergroupoid is an anti-commutative hypergroupoid.*

Theorem 8. *Let $(H_i, \circ)$, where $i \in \Lambda$, be a family of neutro-H_v-semigroups. Then $\left(\bigcap_{i\in\Lambda} H_i, \circ\right)$ is a neutro-H_v-semigroup or anti-H_v-semigroup or H_v-semigroup.*

Theorem 9. *Let $(H_i, \circ)$, where $i \in \Lambda$, be a family of anti-H_v-semigroups. Then $\left(\bigcap_{i\in\Lambda} H_i, \circ\right)$ is an anti-H_v-semigroup.*

Theorem 10. *Let $(H, \circ)$ be a hypergroupoid and $(G, \circ)$ be an anti-H_v-semigroup. Then $(H \cap G, \circ)$ is an anti-H_v-semigroup.*

Theorem 11. *Let $(H, \circ_H)$ be a neutro-H_v-semigroup and $(G, \circ_G)$ be an anti-H_v-semigroup and $H \cap G = \emptyset$. Define a hyperoperation $\circ$ on $H \cup G$ by:*

$$x \circ y = \begin{cases} x \circ_H y & if\ x, y \in H; \\ x \circ_G y & if\ x, y \in G; \\ x \circ_H x & if\ x \in H, y \in G; \\ x \circ_G x & if\ x \in G, y \in H. \end{cases}$$

Then $(H \cup G, \circ)$ is an anti-H_v-semigroup.

Theorem 12. *Let $(H, \circ_H)$ be a neutro-H_v-semigroup and $(G, \circ_G)$ be an anti-H_v-semigroup and $H \cap G = \emptyset$. Define a hyperoperation $\circ$ on $H \cup G$ by:*

$$x \circ y = \begin{cases} x \circ_H y & if\ x, y \in H; \\ x \circ_G y & if\ x, y \in G; \\ H & x \in H, y \in G; \\ G & x \in G, y \in H. \end{cases}$$

Then $(H \cup G, \circ)$ is a neutro-H_v-semigroup.

Proof. If $x, y, z \in G$ we have $(x \circ_G y) \circ_G z \cap x \circ_G (y \circ_G z) = \emptyset$, and so $(x \circ y) \circ z \cap x \circ (y \circ z) = \emptyset$. If $x \in H$ and $y \in G$, then $(x \circ y) \circ x = H \circ x = H \circ_H x$ and $x \circ (y \circ x) = x \circ G = H$. Hence $(x \circ y) \circ x = H \circ_H x \subseteq H = x \circ (y \circ x)$ and $(x \circ y) \circ x \cap x \circ (y \circ x) \neq \emptyset$. Therefore, $(H \cup G, \circ)$ is a neutro-H_v-semigroup.

Let $(H_1, \circ_1)$ and $(H_2, \circ_2)$ be two hypergroupoids. Then $(H \times G, *)$ is a hypergroupoid, where $*$ is defined on $H \times G$ by: for any $(x_1, y_1), (x_2, y_2) \in H \times G$

$$(x_1, y_1) * (x_2, y_2) = (x_1 \circ_1 x_2, y_1 \circ_2 y_2).$$

Then we obtain the Table 23.

By the Table 23, we have the following:

Theorem 13. *Set $\mathcal{H} := \{SHG, HvSG, NSHG, NHvSG, ASHG, AHvSG\}$. Then $(\mathcal{H}, \times)$ is a commutative monoid (i.e. semigroup with identity). SHG*

Table 23
Semigroup $(\mathcal{H}, \times)$

$\times$	SHG	HvSG	NSHG	NHvSG	ASHG	AHvSG
SHG	SHG	HvSG	NSHG	NHvSG	ASHG	AHvSG
HvSG	HvSG	HvSG	NHvSG	NHvSG	ASHG	AHvSG
NSHG	NSHG	NHvSG	NSHG	NHvSG	ASHG	AHvSG
NHvSG	NHvSG	NHvSG	NHvSG	NHvSG	ASHG	AHvSG
ASHG	ASHG	ASHG	ASHG	ASHG	ASHG	AHvSG
AHvSG	AHvSG	AHvSG	AHvSG	AHvSG	AHvSG	AHvSG

Abbreviations: SHG, Semihypergroup; HvSG, H_v-semigroup; NSHG, Neutro-semihypergroup; NHvSG, Neutro-H_v-semigroup; ASHG, Anti-semihypergroup; AHvSG, Anti-H_v-semigroup.

is an identity element of $(\mathcal{H}, \times)$ *and* $AHvSG$ *is a zero element of* $(\mathcal{H}, \times)$.

Corollary 2. *Every subset of* $\mathcal{H}$ *is a semigroup such that it has an identity and a zero element.*

Theorem 14. *Let* $(H, \circ)$ *be a neutro-*H_v*-semigroup and let* $H_1 := \bigcup_{x\circ y\notin \mathcal{P}^*(H)}\{x, y\}$. *If* $H_1 \neq \emptyset$, *then* $(H_1, \circ)$ *is a neutro-*H_v*-semigroup or an anti-*H_v*-semigroup.*

Theorem 15. *Let* $(H, \circ)$ *be a neutro-*H_v*-semigroup and let* $H_2 := \bigcup_{x\circ(y\circ z)\,\neq\,(x\circ y)\circ z}\{x, y, z\}$. *If* $H_2 \neq \emptyset$, *then* $(H_2, \circ)$ *is a neutro-*H_v*-semigroup or an anti-*H_v*-semigroup.*

Suppose that $(H, \circ_1)$ *and* $(G, \circ_2)$ *are two hypergroupoids. A function* $f : H \to G$ *is called a homomorphism if, for all* $a, b \in H$, $f(a \circ_1 b) = f(a) \circ_2 f(b)$ *(see [10] and [14] for details).*

Theorem 16. *Let* $(H, \circ_1)$ *be an* H_v*-semigroup,* $(G, \circ_2)$ *be a neutro-*H_v*-semigroup and* $f : H \to G$ *be a homomorphism. Then* $(f(H), \circ_2)$ *is an* H_v*-semigroup, where* $f(H) = \{f(h) : h \in H\}$.

Proof. Assume that $(H, \circ_1)$ is an H_v-semigroup and $x, y, z \in f(H)$. Then there exist $h_1, h_2, h_3 \in f(H)$ such that $f(h_1) = x$, $f(h_2) = y$ and $f(h_3) = z$, and there exists $u \in h_1 \circ (h_2 \circ h_3) \cap (h_1 \circ h_2) \circ h_3$. So

$$x \circ (y \circ z) = f(h_1) \circ (f(h_2) \circ f(h_3)) = f(h_1)$$
$$\circ f(h_2 \circ h_3) = f(h_1 \circ (h_2 \circ h_3)) \ni \text{f(u)}$$
$$\in f((h_1 \circ h_2) \circ h_3) = f(h_1 \circ h_2) \circ f(h_3)$$
$$= (f(h_1) \circ f(h_2)) \circ f(h_3) = (x \circ y) \circ z.$$

Therefore, $(f(H), \circ_2)$ is an H_v-semigroup.

4. Conclusion

The classical algebraic structures were generalized to neutro-algebraic structurers [*neutro-algebras*] and anti-algebraic structures [*anti-algebras*] in 2019 and 2020 by Smarandache. This generalization was done thanks to the processes of neutrosophication and respectively antisophication of the operations and axioms.

In this paper we defined the neutro-semihypergroups, neutro-H_v-semigroups, anti-semihypergroups and anti-H_v-semigroups, and we proved several of their properties and illustrated the paper with many examples.

For future work, it will be interesting to introduce neutrosophication and antisophication on other hyperstructures. Also, it will be interesting to enumerate neutro-semihypergroups, neutro-H_v-semigroups, anti-semihypergroups and anti-H_v-semigroups of small order.

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