Journal of Intelligent & Fuzzy Systems 37 (2019) 2869–2885
DOI:10.3233/JIFS-190036
IOS Press

Single-valued neutrosophic directed (Hyper)graphs and applications in networks

Mohammad Hamidi[a,*] and Florentin Smarandache[b]
[a]*Department of Math., Faculty of Mathematics, Payame Noor University, Tehran, Iran*
[b]*Department of Math., University of New Mexico, Gallup, NM, USA*

Abstract. This paper considers networks as wireless sensor (hyper)networks and social (hyper)networks by single–valued neutrosophic (directed)(hyper)graphs.The notion of single–valued neutrosophic hypergraphs are extended to single–valued neutrosophic directed hypergraphs and conversely. We derived single–valued neutrosophic digraphs from single–valued neutrosophic directed hypergraphs via a positive equivalence relation. It tries to use single–valued neutrosophic directed hypergraphs and positive equivalence relation to create the sensor clusters and to access to cluster heads in wireless sensor (hyper)networks. Finally, the concept of α-derivable single–valued neutrosophic digraph is considered as the energy-efficient protocol of wireless sensor networks and is applied this concept as a tool in wireless sensor (hyper)networks.

Keywords: Single–valued neutrosophic directed (graphs)hypergraphs, positive equivalence relation, α-(semiself-self)derivable single–valued neutrosophic digraph, SN, WSN

1. Introduction

As a generalization of the classical set theory, fuzzy set theory was introduced by Zadeh [33] to deal with uncertainties. Fuzzy set theory is playing an important role in modeling and controlling unsure systems in nature, society and industry. Fuzzy set theory also plays a vital role in phenomena which is not easily characterized by classical set theory. Smarandache proposed the idea of neutrosophic sets and mingled thee component logic, non-standard analysis, and philosophy, in 1998 [26, 27]. Smarandache [26] and Wang et al. presented the notion of single-valued neutrosophic sets in real life problems more conveniently [32]. A single-valued neutrosophic set has three components: truth membership degree, indeterminacy membership degree and falsity membership degree. These three components of a single-valued neutrosophic set are not dependent and their values are contained in the standard unit interval [0, 1]. Single-valued neutrosophic sets have been a new hot research topic and many researchers have addressed this issue. Majumdar and Samanta studied similarity and entropy of single-valued neutrosophic sets [16]. Smarandache [28, 29] have defined four main categories of neutrosophic graphs, two based on literal indeterminacy (I), whose name were; I–edge neutrosophic graph and I–vertex neutrosophic graph, these concepts have been deeply studied and have gained popularity among the researchers due to their applications in real world problems [10, 30]. Akram et al. defined the concepts of single-valued neutrosophic hypergraph, line graph of single-valued neutrosophic hypergraph, dual single-valued neutrosophic hypergraph, transversal single-valued

*Corresponding author. Mohammad Hamidi, Department of Math., Faculty of Mathematics, Payame Noor University, Tehran, Iran. E-mail: m.hamidi@pnu.ac.ir.

neutrosophic hypergraph [1, 3]. A directed hypergraph is a powerful tool to solve the problems that arise in different fields, including computer networks, social networks and collaboration networks. Akram et al. applied the concept of single–valued neutrosophic sets to directed hypergraphs and introduced certain new concepts, including single-valued neutrosophic directed hypergraphs, single-valued neutrosophic line directed graphs and dual single-valued neutrosophic directed hypergraphs. They described applications of single-valued neutrosophic directed hypergraphs in manufacturing and production networks, collaboration networks and social networks [2, 4]. Further materials regarding graph and hypergraph are available in the literature too [3, 5–7, 12–15, 17–25]. Wireless sensor networks (WSNs) have gained world wide attention in recent years, particularly with the proliferation of micro–electro–mechanical systems technology, which has facilitated the development of smart sensors. WSNs are used in numerous applications, such as environmental monitoring, habitat monitoring, prediction and detection of natural calamities, medical monitoring, and structural health monitoring. WSNs consist of tiny sensing devices that are spread over a large geographic area and can be used to collect and process environmental data such as temperature, humidity, light conditions, seismic activities, images of the environment, and so on.

Regarding these points, this paper aims to generalize the notion of single-valued neutrosophic directed graphs by considering the notion of a positive equivalence relation and trying to define a concept of derivable single-valued neutrosophic directed graphs. The relationships between derivable single–valued neutrosophic directed graphs and single-valued neutrosophic directed hypergraphs are considered as a natural question. The quotient of single-valued neutrosophic directed hypergraphs via equivalence relations is one of our motivations of this research. Moreover, by using a positive equivalence relation, we define a well–defined operation on single-valued neutrosophic directed hypergraphs that the quotient of any single–valued neutrosophic directed hypergraphs via this relation is a single–valued neutrosophic directed graph. We use single–valued neutrosophic directed hypergraphs to represent wireless sensor hypernetworks and social hypernetworks. By considering the concept of the wireless sensor networks, the use of wireless sensor hypernetworks appears to be a necessity for exploring these systems and representation their relationships. We have introduced several valuable measures as truth–membership, indeterminacy and falsity–membership values for studying wireless sensor hypernetworks, such as node and hypergraph centralities as well as clustering coefficients for both hypernetworks and networks. Clustering is one of the basic approaches for designing energy–efficient, robust and highly scalable distributed sensor networks. A sensor network reduces the communication overhead by clustering, and decreases the energy consumption and the interference among the sensor nodes, so we via the concept of single-valued neutrosophic (hyper)graphs and equivalence relations considered the wireless sensor hypernetworks. A single–valued neutrosophic directed hypergraphs in a similar way, can also be used to study and understand the social networks, using people as nodes (or vertices) and relationships between two or more than two peoples as single valued neutrosophic directed hyperedges. The main our motivation in this study is a simulation and modeling of social network and sensor network to single-valued neutrosophic hypergraphs to solve a considered applied issue. Indeed single-valued neutrosophic directed hypergraphs connected some sets of nodes such that single-valued neutrosophic directed graphs could not connect them. For solving this problem, we modelify any (hyper)network to a single-valued neutrosophic directed hypergraph and by using a positive equivalence relation, convert the single-valued neutrosophic directed hypergraph to a single-valued neutrosophic directed graph. So we extract a single-valued neutrosophic directed graph from a (hyper)network by some algorithms in single-valued neutrosophic directed hypergraphs.

2. Preliminaries

In this section, we recall some definitions and results, which we need in what follows.

Let X be an arbitrary set. Then we denote $P^*(X) = P(X) \setminus \emptyset$, where $P(X)$ is the power set of X.

Definition 2.1. [9] A hypergraph on a finite set G is a pair $H = (G, \{E_i\}_{i=1}^{m})$ such that for all $1 \leq i \leq m$, we have, $E_i \in P^*(G)$ and $\bigcup_{i=1}^{m} E_i = G$.

The elements of G are called *vertices*, and the sets $E_1, E_2, \ldots, E_m$ are said the *hyperedges* of the hypergraph H. For any $1 \leq k \leq m$, if $|E_k| \geq 2$, then E_k is represented by a solid line surrounding its vertices, if $|E_k| = 1$ by a cycle on the element (loop). If for all $1 \leq k \leq m, |E_k| = 2$, the hypergraph becomes an ordinary (undirected) graph.

Theorem 2.2. [12] *Let* $H = (G, \{E_x\}_{x\in G})$ *be a hypergraph,* $\mathbb{N}^* = \mathbb{N} \cup \{0\}$ *and* $\eta = \eta^*$. *Then for every* $i \in \mathbb{N}^*$, *there exists a relation "*$*_i$*" on* G/η *such that* $H/\eta = (G/\eta, *_i)$ *is a graph.*

Definition 2.3. [31] Let X be a set. A single valued neutrosophic set A in X (SVN–S A) is a function $A : X \longrightarrow [0, 1] \times [0, 1] \times [0, 1]$ with the form $A = \{(x, \alpha_A(x), \beta_A(x), \gamma_A(x)) \mid x \in X\}$, where the functions $\alpha_A, \beta_A, \gamma_A$ define respectively a truth–membership function, an indeterminacy–membership function and a falsity–membership function of the element $x \in X$ to the set A such that $0 \leq \alpha_A(x) + \beta_A(x) + \gamma_A(x) \leq 3$.

Moreover, $Supp(A) = \{x \mid \alpha_A(x) \neq 0, \beta_A(x) \neq 0, \gamma_A(x) \neq 0\}$ is a crisp set.

Definition 2.4. [8]

(*i*) A single valued neutrosophic hypergraph (SVN–HG) is defined to be a pair $\mathcal{H}' = (H, \{E_i\}_{i=1}^m)$, that $H = \{v_1, \dots, v_n\}$ is a finite set of vertices and $\{E_i = \{(v_j, \alpha_{E_i}(v_j), \beta_{E_i}(v_j), \gamma_{E_i}(v_j))\}\}_{i=1}^m$ is a finite family of non–trivial neutrosophic subsets of the vertex H, such that $H = \bigcup_{i=1}^{m} supp(E_i)$.
Also $\{E_i\}_{i=1}^m$ is called the family of single valued neutrosophic hyperedges of $\mathcal{H}'$ and H is the crisp vertex set of $\mathcal{H}'$.

(*ii*) Let $1 \leq \epsilon_1, \epsilon_2, \epsilon_3 \leq 1$, then $A^{(\epsilon_1,\epsilon_2,\epsilon_3)} = \{x \in X \mid \alpha_A(x) \geq \epsilon_1, \beta_A(x) \geq \epsilon_2, \gamma_A(x) \leq \epsilon_3\}$ is called an $(\epsilon_1, \epsilon_2, \epsilon_3)$–level subset of A.

Definition 2.5. [11] Let G be a set and $F \subseteq P^*(G) \times P(G)$. Then $F = (T(F), H(F))$ is called a directed hyperedge or hyperarc, if $T(F) \cap H(F) = \emptyset$, where $T(F)$ is called the tail of F and $H(V)$ is called its head. A hypergraph $\mathcal{G}' = (G, \{F_i\}_{i=1}^n = \{(T(F_i), H(F_i))\}_{i=1}^n)$ is called a directed hypergraph (dihypergraph), if for every $1 \leq i \leq n$, F_i is a directed hyperedge.

Definition 2.6. [13] Let $\mathcal{G}' = (G, \{F_i\}_{i=1}^n)$ be a dihypergraph. Then define, $\alpha_1 = \{(x, x) \mid x \in G\}$ and for every integer $n \geq 2$, α_n is defined as follows:

$x\alpha_n y \Longleftrightarrow \exists 1 \leq k \leq n$ such that $\{x, y\} \subseteq T(F_k) \cup H(F_k)$, where for any $1 \leq i \neq k \leq n$, $x, y \notin T(F_i) \cup H(F_i)$ and $n = |T(F_k)| = |H(F_k)|$. Obviously the relation $\alpha = \bigcup_{n\geq 1} \alpha_n$ is an equivalence relation on $\mathcal{G}'$. We denote the set of all equivalence classes of α by $\mathcal{G}'/\alpha$. Hence $\mathcal{G}'/\alpha = \{\alpha(x) \mid x \in G\}$.

Theorem 2.7. [13] *Let* $\mathcal{G}' = (G, \{F_i\}_{i=1}^n)$ *be a dihypergraph. Then there exists a relation "*$*$*" on* $\mathcal{G}'/\alpha$ *such that* $(\mathcal{G}'/\alpha, *)$ *is a digraph.*

Definition 2.8. [1] A single-valued neutrosophic directed hypergraph (SVN–DHG) on a non-empty set X is defined as an ordered pair $\mathcal{G}' = \big(G = \{G_j\}_{j=1}^n, \{F_j\big(T(G_j), H(G_j)\big)\}_{j=1}^n\big)$, where for all $1 \leq j \leq n$, $G_j = \big\{T(G_j) = \{\big(v_j, \alpha_G(v_j), \beta_G(v_j), \gamma_G(v_j)\big)\}_{v_j\in X}, H(G_j) = \{\big(v'_j, \alpha_G(v'_j), \beta_G(v'_j), \gamma_G(v'_j)\}_{v'_j\in X}\big\}$ is a family of non-trivial single-valued neutrosophic subsets on X and $F_j\big(T(G_j), H(G_j)\big) = (\alpha_{F_j}, \beta_{F_j}, \gamma_{F_j})$ in such a way that

$$(i)\ \alpha_{F_j} \leq \bigwedge_{v_j\in T(G_j), v_{j'}\in H(G_j)} (\alpha_G(v_j) \wedge \alpha_G(v_{j'})),$$

$$(ii)\ \beta_{F_j} \leq \bigwedge_{v_j\in T(G_j), v_{j'}\in H(G_j)} (\beta_G(v_j) \wedge \beta_G(v_{j'})),$$

$$(iii)\ \gamma_{F_j} \leq \bigvee_{v_j\in T(G_j), v_{j'}\in T(G_j)} (\gamma_G(v_j) \wedge \gamma_G(v_{j'}))$$

$$\text{and } (iv)\ X = \bigcup_{j=1}^{n} supp(G_j).$$

3. Derivable single–valued neutrosophic directed hypergraph

In this section, we apply the concept of single–valued neutrosophic hypergraphs, construct the single–valued neutrosophic directed hypergraphs and present an associated algorithm. The quotient single–valued neutrosophic hypergraph, is constructed via the equivalence relations and the notation of single–valued neutrosophic graphs is reintroduced.

Theorem 3.1 *From every SVN–HG* $\mathcal{H}' = (H, \{E_i\}_{i=1}^m)$, *(where for all* $1 \leq i \leq m, |E_i| \geq 2$*), can construct at least an SVN–DHG* $\mathcal{G}' = (G = \{G_j\}_{j=1}^n, \{F_j\big(T(G_j), H(G_j)\big)\}_{j=1}^n\big)$ *such that*

(*i*) $G = H$,
(*ii*) $m = n$,

Table 1
Algoritm 1

1. Input the SVN–HG $\mathcal{H}' = (H, \{E_i\}_{i=1}^{m})$ and equivalence relation R on H.
2. If $\|H/R\| = k$, then for all $1 \leq i \leq k$ and $1 \leq s, t \leq n$ set $G_i = \Big\{\{(x_i, \alpha_{x_i}, \beta_{x_i}, \gamma_{x_i})\}_{i=1}^{s}, \{(y_i, \alpha_{y_i}, \beta_{y_i}, \gamma_{y_i})\}_{i=1}^{t}\Big\}$, where $s + t = k$ and $G_i \in H/R$.
3. For all $1 \leq i \leq k$, set $\alpha_{F_i} = \Big(\bigwedge_{i=1}^{t} \alpha_{x_i}\Big) \wedge \Big(\bigwedge_{i=1}^{s} \alpha_{y_i}\Big)\Big)$, $\beta_{F_i} = \Big(\bigwedge_{i=1}^{t} \beta_{x_i}\Big) \wedge \Big(\bigwedge_{i=1}^{s} \beta_{y_i}\Big)\Big)$ and $\gamma_{F_i} = \Big(\bigvee_{i=1}^{t} \gamma_{x_i}\Big) \wedge \Big(\bigvee_{i=1}^{s} \gamma_{y_i}\Big)\Big)$.
4. $\mathcal{G}' = (\{G_i\}_{i=1}^{k}, \{\alpha_{F_i}, \beta_{F_i}, \gamma_{F_i}\}_{i=1}^{k})$ is an SVN–DHG.

(iii) for any $1 \leq i \leq m$, there exists $1 \leq j \leq n$, such that $T(G_j) \cup H(G_j) = E_i$.

Proof. Let $\mathcal{H}' = (H, \{E_i\}_{i=1}^{m})$ be an SVN–HG. Then $H = \{v_1, v_2, \ldots, v_n\}$ is a finite set of vertices and $\{E_i = \{(v_j, \alpha_{E_i}(v_j), \beta_{E_i}(v_j), \gamma_{E_i}(v_j))\}\}_{i=1}^{m}$ is a finite family of non–trivial neutrosophic subsets of the vertex H such that $H = \bigcup_{i=1}^{m} supp(E_i)$. For every $1 \leq i \leq m$ and $1 \leq j \leq n$, define an equivalence relation R_i on E_i and consider $v_j^i, v_{j'}^i \in E_i$ in such a way that $E_i = R_i(v_j^i) \cup R_i(v_{j'}^i)$. Now, for every $1 \leq i \leq m$, we set
$T(G_j) = \{(R_i(v_j^i), \alpha_{R_i}(v_j), \beta_{R_i}(v_j), \gamma_{R_i}(v_j))\}$,
$H(G_j) = \{(R_i(v_{j'}^i, \alpha_{R_i}(v_{j'}^i), \beta_{R_i}(v_{j'}^i), \gamma_{R_i}(v_{j'}^i))\}$
and $F_j\big(T(G_j), H(G_j)\big) = (\alpha_{F_j}, \beta_{F_j}, \gamma_{F_j})$, where $\alpha_{F_j} = \Big(\bigwedge_{xR_iv_j^i} \alpha_{R_i}(x)\Big) \wedge \Big(\bigwedge_{yR_iv_{j'}^i} \alpha_{R_i}(y)\Big)\Big)$, $\beta_{F_j} = \Big(\bigwedge_{xR_iv_j^i} \beta_{R_i}(x)\Big) \wedge \Big(\bigwedge_{yR_iv_{j'}^i} \beta_{R_i}(y)\Big)$ and $\gamma_{F_j} = \Big(\bigvee_{xR_iv_j^i} \gamma_{R_i}(x)\Big) \wedge \Big(\bigvee_{yR_iv_{j'}^i} \gamma_{R_i}(y)\Big)$. Some modifications and computations show that $\mathcal{H}' = (H = \{E_j\}_{j=1}^{m}, \{F_i\big(T(E_i), H(E_i)\big)\}_{i=1}^{m})$, is a single-valued neutrosophic directed hypergraph (SVN–DHG), where for all $1 \leq j \leq m$, $G_j = E_j = \{(v_j, \alpha_{E_j}(v_j), \beta_{E_j}(v_j), \gamma_{E_j}(v_j)))\}$. Clearly for any $1 \leq i \leq m$, $T(E_i) \cup H(E_i) = E_i$ and $H = \bigcup_{i=1}^{m} supp(E_i)$ implies that $G = \bigcup_{j=1}^{m} supp(G_j)$. $\square$

Corollary 3.2. *From all SVN–HG, $\mathcal{H}' = (H, \{E_i\}_{i=1}^{m})$, can construct at least an SVN–DHG, $\mathcal{G}' = (G = \{G_j\}_{j=1}^{n}, \{F_j\big(T(G_j), H(G_j)\big)\}_{j=1}^{n})$ such that*

(i) $G = H$,

(ii) $n = m$,

(iii) for any $1 \leq i \leq m$, there exists $1 \leq j \leq n$, such that $T(G_j) \cup H(G_j) = E_i$.

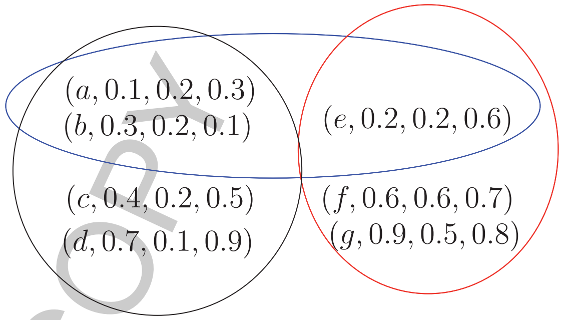

Fig. 1. SVN–HG

Let $\mathcal{H}' = (H, \{E_i\}_{i=1}^{m'})$ be an SVN–HG. We will call the SVN–DHG $\mathcal{G}' = (G = \{G_j\}_{j=1}^{n}, \{F_j\big(T(G_j), H(G_j)\big)\}_{j=1}^{n})$ which satisfied in Corollary 3, by a derived single-valued neutrosophic directed hypergraph (derived SVN–DHG) from SVN–HG, $\mathcal{H}' = (H, \{E_i\}_{i=1}^{m'})$ and will show by $\mathcal{G}' = \mathcal{H}'^{\uparrow}$.

The method for the construction of an SVN–DHG $\mathcal{G}'$ from an SVN–HG $\mathcal{H}'$ is explained in Algorithm 1 in Table 1.

Example 3.3. Let $\mathcal{H}' = (\{a, b, c, d, e, f, g\}, \{E_1, E_2, E_3\})$ be an SVN–HG in Figure 1, where $E_1 = \{(a, 0.1, 0.2, 0.3), (b, 0.3, 0.2, 0.1), (e, 0.2, 0.2, 0.6)\}$, $E_2 = \{(a, 0.1, 0.2, 0.3), (b, 0.3, 0.2, 0.1), (c, 0.4, 0.2, 0.5), (d, 0.7, 0.1, 0.9)\}$ and $E_3 = \{(e, 0.2, 0.2, 0.6), (f, 0.6, 0.6, 0.7), (g, 0.9, 0.5, 0.8)\}$. Then, by Theorem 3, we obtain an SVN–DHG in Figure 2, where $G_1 = \Big\{\{(a, 0.1, 0.2, 0.3), (b, 0.3, 0.2, 0.1)\}, \{(e, 0.2, 0.2, 0.6)\}\Big\}$, $G_2 = \Big\{\{(a, 0.1, 0.2, 0.3), (b, 0.3, 0.2, 0.1)\}, \{(c, 0.4, 0.2, 0.5), (d, 0.7, 0.1, 0.9)\}\Big\}$, $G_3 = \Big\{\{(e, 0.2, 0.2, 0.6)\}, \{(f, 0.6, 0.6, 0.7), (g, 0.9, 0.5, 0.8)\}\Big\}$, $(\alpha_{F_1}, \beta_{F_1}, \gamma_{F_1}) = (0.1, 0.2, 0.3)$, $(\alpha_{F_2}, \beta_{F_2}, \gamma_{F_2}) = (0.1, 0.1, 0.3)$ and $(\alpha_{F_3}, \beta_{F_3}, \gamma_{F_3}) = (0.2, 0.2, 0.6)$.

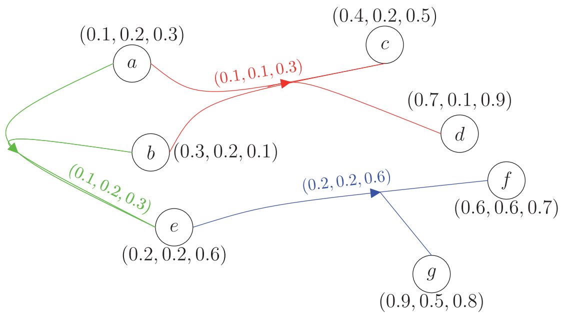

Fig. 2. Derived SVN–DHG $\mathcal{H}'^{\uparrow}$ from Figure 1

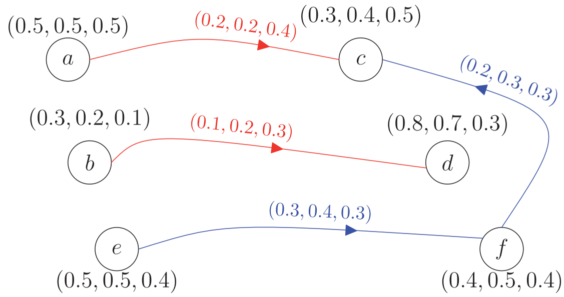

Fig. 3. SVN–DHG $\mathcal{G}'$

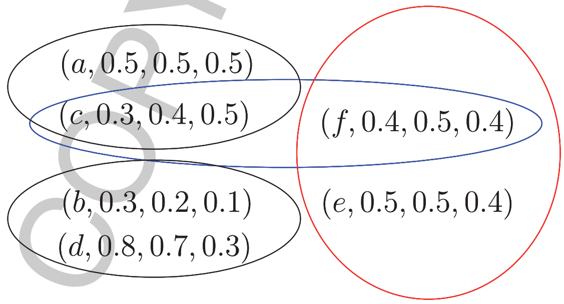

Fig. 4. SVN–HG $\mathcal{G}'^{\downarrow}$ from Figure 3

Definition 3.4. Let $\mathcal{G}' = (G = \{G_j\}_{j=1}^n, \{F_j\big(T(G_j), H(G_j)\big)\}_{j=1}^n)$ be an SVN–DHG. We call $\mathcal{G}'$ is a derivable SVN–DHG, if there exists an SVN–HG as $\mathcal{H}' = (H, \{E_i\}_{i=1}^m)$ such that $\mathcal{G}'$ is derived from $\mathcal{H}'$.

Theorem 3.5. *Every SVN–DHG is a derivable SVN–DHG.*

Proof. Let $\mathcal{G}' = (G = \{G_j\}_{j=1}^n, \{F_j\big(T(G_j), H(G_j)\big)\}_{j=1}^n)$ be an SVN–DHG. Then consider $H = G$ and for every $1 \leq i \leq n$, $E_i = \bigcup_{i=1}^{n}(T(G_i) \cup H(G_i))$. Since $G = \bigcup_{j=1}^{n} supp(G_j)$, we get $H = \bigcup_{i=1}^{n} supp(E_i)$ and so $\mathcal{H}' = (H, \{E_i\}_{i=1}^n)$ is an SVN–HG. Now, if consider $\alpha_{F_i} = \bigwedge_{x\in E_i} \alpha_{E_i}(x)$, $\beta_{F_i} = \bigwedge_{x\in E_i} \beta_{E_i}(x)$ and $\gamma_{F_i} = \bigwedge_{x\in E_i} \gamma_{E_i}(x)$, then $\mathcal{G}' = (G = \{G_j\}_{j=1}^n, \{F_j\big(T(G_j), H(G_j)\big)\}_{j=1}^n)$ is derived from $\mathcal{H}' = (H, \{E_i\}_{i=1}^n)$ and so it is a derivable SVN–DHG. □

We will show the SVN–HG $\mathcal{H}' = (H, \{E_i\}_{i=1}^n)$ in Theorem 3.5, by $\mathcal{H}' = \mathcal{G}'^{\downarrow}$.

Corollary 3.6. *Let $\mathcal{G}'$ be an SVN–DHG. Then $(\mathcal{G}'^{\downarrow})^{\uparrow} \cong \mathcal{G}'$.*

Example 3.7. Consider the SVN–DHG, $\mathcal{G}'$ in Figure 3, where $G_1 = \big\{\{(a, 0.5, 0.5, 0.5)\}, \{(c, 0.3, 0.4, 0.5)\}\big\}$, $G_2 = \big\{\{(b, 0.3, 0.2, 0.1)\}, \{(d, 0.8, 0.7, 0.3)\}\big\}$, $G_3 = \big\{\{(e, 0.5, 0.5, 0.4)\}, \{(f, 0.4, 0.5, 0.4)\}\big\}$, $G_4 = \big\{\{(f, 0.4, 0.5, 0.4)\}, \{(c, 0.3, 0.4, 0.5)\}\big\}$, $(\alpha_{F_1}, \beta_{F_1}, \gamma_{F_1}) = (0.2, 0.2, 0.4)$, $(\alpha_{F_2}, \beta_{F_2}, \gamma_{F_2}) = (0.1, 0.2, 0.3)$, $(\alpha_{F_3}, \beta_{F_3}, \gamma_{F_3}) = (0.3, 0.4, 0.3)$ and $(\alpha_{F_4}, \beta_{F_4}, \gamma_{F_4}) = (0.2, 0.3, 0.3)$.

Then, by Theorem 3, we obtain an SVN–HG in Figure 4, where $E_1 = \{(a, 0.5, 0.5, 0.5), (c, 0.3, 0.4, 0.5)\}$, $E_2 = \{(c, 0.3, 0.4, 0.5), (f, 0.4, 0.5, 0.4)\}$, $E_3 = \{(b, 0.3, 0.2, 0.1), (d, 0.8, 0.7, 0.3)\}$ and $E_4 = \{(f, 0.4, 0.5, 0.4), (e, 0.5, 0.5, 0.4)\}$.

Definition 3.8 (i) A single valued neutrosophic digraph(SVN–DG) is a pair $D = (V, A)$, where $V = \{(v_j, \alpha_V(v_j)), \beta_V(v_j)), \gamma_V(v_j))\}_{j=1}^n$, is a family of non-trivial single-valued neutrosophic subsets on V, $A = \{(v_i, v_j), (v_j, v_i) \mid v_i, v_j \in V\}$, such that (1), $\alpha_A(v_i, v_j) \leq \min\{\alpha_V(v_i), \alpha_V(v_j)\}$, (2), $\beta_A(v_i, v_j) \geq \max\{\beta_V(v_i), \beta_V(v_j)\}$, (3), $\gamma_A(v_i, v_j) \geq \max\{\gamma_V(v_i), \gamma_V(v_j)\}$ and for every $(v_i, v_j) \in A$, (4), $S_{\alpha}^{\beta}(A, v_i) = \alpha_A(v_i) + \beta_A(v_i) \geq S_{\alpha}^{\beta}(A, v_j) = \alpha_A(v_j) + \beta_A(v_j)$,

(ii) an SVN–DG is called a weak single–valued neutrosophic graph(WSVN–DG), if $supp(A) = V$;

(iii) an SVN–DG is called a regular single–valued neutrosophic graph(RSVN–DG), if it is a WSVN–DG and for any $v_i, v_j \in V$ have $\alpha_B(v_i, v_j) = \min\{\alpha_A(v_i), \alpha_A(v_j)\}$, $\beta_B(v_i, v_j) = \max\{\beta_A(v_i), \beta_A(v_j)\}$ and $\gamma_B(v_i, v_j) = \max\{\gamma_A(v_i), \gamma_A(v_j)\}$.

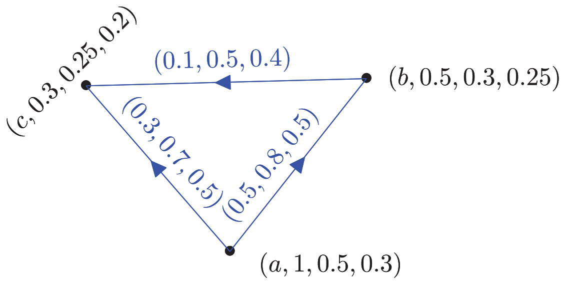

Fig. 5. SVN–DG K_3

Proposition 3.9. *Let* $V = \{a_1, a_2, \ldots, a_n\}$. *Consider the complete graph* K_n *and define* $V = \{(a_i, \frac{1}{i}, \frac{1}{i+1}, \frac{1}{i+2})\}_{i=1}^{n}$.

(*i*) *If* $A = \{\big((a_i, a_j), \frac{1}{ij}, \frac{i+j+2}{ij+j+i+1}, \frac{i+j+4}{ij+2j+2i+4}\big)\}$, *then* $D = (V, A)$ *is a WSVN–DG.*

(*ii*) *If* $A = \{\big((a_i, a_j), \frac{1}{j}, \frac{1}{i}, \frac{1}{i}\big) \mid j < i\}$, *then* $D = (V, A)$ *is an RSVN–DG.*

Example 3.10. Let $V = \{a, b, c\}$. Then $D = (V, A)$ is an SVN–DG in Figure 5.

Corollary 3.11. *Any finite set can be an RSVN–DG and a WSVN–DG.*

Proof. Let G be a finite set and R be an equivalence relation on G. Then consider, $H = \big(G, \{R(x) \times R(y)\}_{x,y \in G}\big)$, whence it is a complete graph. Applying Proposition 3, the proof is obtained. $\square$

Lemma 3.12. *Let* X *be a finite set and* $A = \{(x, \alpha_A(x), \beta_A(x), \gamma_A(x)) \mid x \in X\}$ *be an SVN–S in* X. *If* R *is an equivalence relation on* X, *then* $A/R = \{(R(x), T_{R(A)}(R(x)), I_{R(A)}(R(x)), F_{R(A)}(R(x)) \mid x \in X\}$ *is an SVN–S, where* $\alpha_{R(A)}(R(x)) = \bigvee_{t\ R\ x} \alpha_A(t)$, $\beta_{R(A)}(R(x)) = \bigvee_{t\ R\ x} \beta_A(t)$ *and* $\gamma_{R(A)}(R(x)) = \bigvee_{t\ R\ x} \gamma_A(t)$.

Proof. Let $X = \{x_1, x_2, \ldots, x_n\}$ and $\mathcal{P} = \{R(x_1), R(x_2), \ldots, R(x_k)\}$ be a partition of X, where $k \leq n$. Since for any $x_i \in X, \alpha_A(x_i) \leq 1, \beta_A(x_i) \leq 1$ and $\gamma_A(x_i) \leq 1$, we get that $\bigvee_{t\ R\ x_i} \alpha_A(t) \leq 1$, $\bigvee_{t\ R\ x_i} \beta_A(t) \leq 1$ and $\bigvee_{t\ R\ x_i} \gamma_A(t) \leq 1$. Hence for any $1 \leq i \leq k$, $0 \leq \bigvee_{t\ R\ x_i} \alpha_A(t) + \bigvee_{t\ R\ x_i} \beta_A(t) + \bigvee_{t\ R\ x_i} \gamma_A(t) \leq 3$ and so $R(A) = \{(R(x_i), \bigvee_{t\ R\ x_i} \alpha_A(t), \bigvee_{t\ R\ x_i} \beta_A(t), \bigvee_{t\ R\ x_i} \gamma_A(t))\}_{i=1}^{k}$ is a single–valued neutrosophic set in X/R. $\square$

Theorem 3.13 *Let* $V = \{v_1, v_2, \ldots, v_n\}$ *and* $\mathcal{G}' = \big(G = \{G_j\}_{j=1}^{n}, \{F_j\big(T(G_j), H(G_j)\big)\}_{j=1}^{n}\big)$ $\}_{i=1}^{m})$ *be an SVN–DHG. If* R *is an equivalence relation on* H, *then* $\mathcal{G}'/R = (R(G) = \{G_j/R\}_{j=1}^{n'}$, $\{F_j/R\big(T(G_j/R), H(G_j/R)\big)\}_{j=1}^{n'})$ *is an SVN–DHG, where* $n' \leq n$.

Proof. By Lemma 3, $\{R(v_j), \alpha_{R(F_i)}(R(v_j)), \beta_{R(F_i)}(R(v_j)), \gamma_{R(F_i)}(R(v_j))\}_{i=1}^{n}$ is a finite family of single–valued neutrosophic subsets of V/R. Since $V = \bigcup_{j=1}^{n} supp(G_j)$, we get that $\bigcup_{i=1}^{n'} supp(R(G_j)) = R(\bigcup_{j=1}^{n} supp(G_j)) = R(V)$. Now, for all $1 \leq i \leq n'$, define $F_i/R\big(T(F_i/R), H(F_i/R)\big) = (\alpha_{F_i/R}, \beta_{F_i/R}, \gamma_{F_i/R})$ as follows;
if $R(x) \in T(G_i/R)$ and $R(y) \in H(G_i/R)$, then for all $a \in R(x), b \in R(y)$ there exist $1 \leq j \leq n, a' \in T(G_j), b' \in H(G_j)$ such that $(a, a') \in R$, $(b, b') \in R$, $\alpha_{F_i/R} = \bigwedge \alpha_{F_j}, \beta_{F_i/R} = \bigwedge \beta_{F_j}$ and $\gamma_{F_i/R} = \bigwedge \gamma_{F_j}$. It follows that $\mathcal{G}'/R$ is an SVN–DHG. $\square$

Example 3.14. Consider the SVN–DHG $\mathcal{G}'$, in Figure 2. If R is an equivalence relation on G such that $R(a) = \{a\}, R(b) = \{b\}$, $R(c) = \{e, c\}$ and $R(d) = \{d, g, f\}$. Since $G_1 = \big\{\{(a, 0.1, 0.2, 0.3), (b, 0.3, 0.2, 0.1)\}, \{(e, 0.2, 0.2, 0.6)\}\big\}$ and $R(c) = R(e)$, we get that $R(G_2) = \big\{\{(R(a), 0.1, 0.2, 0.3), (R(b), 0.3, 0.2, 0.1)\}, \{(R(c), 0.4, 0.2, 0.6)\}\big\}$. In a similar a way, $G_2 = \big\{\{(a, 0.1, 0.2, 0.3), (b, 0.3, 0.2, 0.1)\}, \{(c, 0.4, 0.2, 0.5), (d, 0.7, 0.1, 0.9)\}\big\}$ and $R(d) = R(g) = R(f)$ imply that $R(G_1) = \big\{\{(R(a), 0.1, 0.2, 0.3), (R(b), 0.3, 0.2, 0.1)\}, \{(R(d), 0.9, 0.6, 0.9)\}\big\}$. Because $G_3 = \big\{\{(e, 0.2, 0.2, 0.6)\}, \{(f, 0.6, 0.6, 0.7), (g, 0.9, 0.5, 0.8)\}\big\}$, $R(c) = R(e)$ and $R(d) = R(g) = R(f)$, we have $R(G_3) = \big\{\{(R(c), 0.4, 0.2, 0.6)\}, \{(R(d), 0.9, 0.6, 0.9)\}\big\}$. Thus by Theorem 3, we obtain the SVN–DHG $\mathcal{G}'/R$, in Figure 6, where $(\alpha_{F_1}, \beta_{F_1}, \gamma_{F_1}) = (0.1, 0.1, 0.3), (\alpha_{F_1/R}, \beta_{F_1/R}, \gamma_{F_1/R}) =$

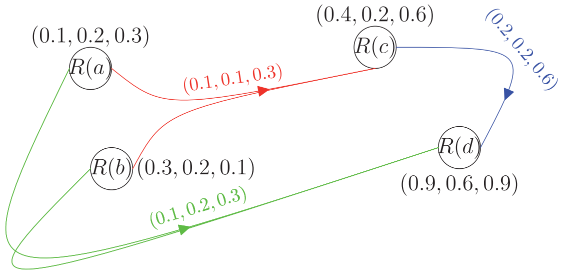

Fig. 6. SVN–DHG $\mathcal{G}'/R$ from Figure 2

$(0.1, 0.2, 0.3)$ and $(\alpha_{F_1/R}, \beta_{F_1/R}, \gamma_{F_1/R}) = (0.2, 0.2, 0.6)$.

4. α–Derivable SVN–DG

In this section, we introduce the concept of α–derivable single–valued neutrosophic digraphs via the equivalence relation α on single–valued neutrosophic directed hypergraphs. It is shown that any single–valued neutrosophic digraph is not necessarily an α–derivable single–valued neutrosophic digraph and it is proved under some conditions. Furthermore, it can show that directed path graphs, directed cyclic graphs, directed star graphs, directed complete graphs can be single–valued neutrosophic directed graphs and can be α–derivable single–valued neutrosophic directed graphs. Also we define the concept of α–(semi)self derivable single–valued neutrosophic digraphs and prove that some class of directed graphs are not α–self derivable single–valued neutrosophic digraphs, while are α–semiself derivable single–valued neutrosophic digraphs.

Theorem 4.1. *Let $\mathcal{G}' = (G = \{G_j\}_{j=1}^n, \{F_j(T(G_j), H(G_j))\}_{j=1}^n)$ be an SVN–DHG. Then there exists a relation "$*$" on $\mathcal{G}'/\alpha$ such that $(\mathcal{G}'/\alpha, *)$ is an SVN–DG.*

Proof. By Theorem 3, $\mathcal{G}'/\alpha = (\alpha(G) = \{G_j/\alpha\}_{j=1}^{n'}, \{F_i/\alpha(T(F_i/\alpha), H(F_i/\alpha))\}_{i=1}^{n'})$ is an SVN–DHG, where $\alpha_{\alpha(G_j)}(\alpha(x)) = \bigvee_{x\ \alpha\ t \in G} \alpha_{G_j}(t)$, $\beta_{\alpha(G_j)}(\alpha(x)) = \bigvee_{x\ \alpha\ t \in G} \beta_{G_j}(t)$ and $\gamma_{\alpha(G_j)}(\alpha(x)) = \bigvee_{x\ \alpha\ t \in G} \gamma_{G_j}(t)$. Let $\alpha(x) = \alpha((x, \alpha_{G_j}(x), \beta_{G_j}(x), \gamma_{G_j}(x)))$ and $\alpha(y) = \alpha((y, \alpha_{G_j}(y), \beta_{G_j}(y), \gamma_{G_j}(y)) \in \mathcal{G}'/\alpha$. Then define an operation "$*$" on $\mathcal{G}'/\alpha$ by

$$\alpha(x) * \alpha(y) = \begin{cases} (\alpha(x), \alpha(y)) & \text{if satisfies in T} \\ \emptyset & \text{otherwise,} \end{cases}$$

where $T: \exists 1 \leq k \leq n, \alpha(x) \cap T(G_k) \neq \emptyset$ and $\alpha(y) \cap H(G_k) \neq \emptyset$ and for any $x, y \in H$, $(\alpha(x), \alpha(y))$ is represented as an ordinary directed edge from vertex $\alpha(x)$ to vertex $\alpha(y)$ and $\emptyset = (\alpha(x), \alpha(x))$ means that there is no edge. We show that $*$ is a well-defined relation. Let $\alpha(x) = \alpha(x')$ and $\alpha(y) = \alpha(y')$. Then there exists uniquely $1 \leq k, s \leq n$ such that $\{x, x'\} \subseteq T(G_k) \cup H(G_k)$, and $\{y, y'\} \subseteq T(G_s) \cup H(G_s)$. If $\alpha(x) * \alpha(y) = (\alpha(x), \alpha(y))$, then there exists $1 \leq m \leq n$ such that $\alpha(x) \cap T(G_m) \neq \emptyset$ and $\alpha(y) \cap H(G_m) \neq \emptyset$. It follows that $T(G_k) \cap T(G_m) \neq \emptyset$ and so $\alpha(x') \cap T(G_m) \neq \emptyset$. In a similar way $\alpha(y') \cap H(G_m) \neq \emptyset$ and so $\alpha(x') * \alpha(y') = (\alpha(x'), \alpha(y')) = (\alpha(x), \alpha(y))$. If $\alpha(x) * \alpha(y) = \emptyset$, then for any $1 \leq m \leq n, \alpha(x) \cap T(G_m) = \emptyset$ or $\alpha(y) \cap H(G_m) = \emptyset$. It follows that $T(G_k) \cap T(G_m) = \emptyset$ and so $\alpha(x') \cap T(G_m) = \emptyset$. In a similar way, $\alpha(y') \cap H(G_m) = \emptyset$ and so $\alpha(x') * \alpha(y') = (\alpha(x'), \alpha(y')) = (\alpha(x), \alpha(y))$. It is easy to see that $(\mathcal{G}'/\alpha, *)$ is a simple graph. Consider $\overrightarrow{(\alpha(x), \alpha(y))}$ as a directed edge from vertex $\alpha(x)$ to vertex $\alpha(y)$ and define an operation "$*'$" on $\mathcal{G}'/\alpha$ by

$$\alpha(x) *' \alpha(y) = \begin{cases} \overrightarrow{(\alpha(x), \alpha(y))} & \text{if satisfies in A,} \\ \overrightarrow{(\alpha(y), \alpha(x))} & \text{if satisfies in B,} \end{cases}$$

where $A: S_\alpha^\beta(\alpha(G_j), \alpha(x)) \geq S_\alpha^\beta(\alpha(G_j), \alpha(y))$ and $B: S_\alpha^\beta(\alpha(G_j), \alpha(x)) \leq S_\alpha^\beta(\alpha(G_j), \alpha(y))$ Now, define $\alpha_{\alpha(G_i)}, \beta_{\alpha(G_i)}, \gamma_{\alpha(G_i)} : \alpha(G) \times \alpha(G) \longrightarrow [0, 1]$ by $\alpha_{\alpha(G_i)}(\alpha(x), \alpha(y)) = \bigwedge_{a\alpha x,\ b\alpha y} (\alpha_{\alpha(G_i)}(a) \wedge \alpha_{\alpha(G_i)}(b))$,

$\beta_{\alpha(G_i)}(\alpha(x), \alpha(y)) = \bigvee_{a\alpha x,\ b\alpha y} (\beta_{\alpha(G_i)}(a) \vee \beta_{\alpha(G_i)}(b))$

and $\gamma_{\alpha(G_i)}(\alpha(x), \alpha(y)) = \bigvee_{a\alpha x,\ b\alpha y} (\gamma_{\alpha(G_i)}(a) \vee \gamma_{\alpha(G_i)}(b))$. It is easy to see that

$$\alpha_{\alpha(G_i)}(\alpha(x), \alpha(y)) \leq (\alpha_{\alpha(G_i)}(\alpha(x)) \wedge \alpha_{\alpha(G_i)}(\alpha(y))),$$

$$\beta_{\alpha(G_i)}(\alpha(x), \alpha(y)) \geq (\beta_{\alpha(G_i)}(\alpha(x)) \vee \beta_{\alpha(G_i)}(\alpha(y)))$$

and

$$\gamma_{\alpha(G_i)}(\alpha(x), \alpha(y)) \geq (\gamma_{\alpha(G_i)}(\alpha(x)) \vee \gamma_{\alpha(G_i)}(\alpha(y))).$$

Table 2
Algorithm 2

1. Input the SVN–HG $\mathcal{G}' = (\{G_i\}_{i=1}^{k}, \{\alpha_{F_i}, \beta_{F_i}, \gamma_{F_i}\}_{i=1}^{k})$, where $G_i = \Big\{\{(x_i, \alpha_{x_i}, \beta_{x_i}, \gamma_{x_i})\}_{i=1}^{s}, \{(x_i, \alpha_{x_i}, \beta_{x_i}, \gamma_{x_i})\}_{i=1}^{t}\Big\}$.
2. Input the $x, y \in \mathcal{G}'$. If $\exists!\ 1 \leq i \leq k$ such that $x, y \in G_i$, then $y \in \alpha(x)$, $\|\alpha(x)\| \geq 2$ and if $\exists 1 \leq i \neq i' \leq k$ such that $x, y \in G_i \cap G_{i'}$, then $\alpha(x) = \{x\}$.
3. Input the $x, y \in \mathcal{G}'$. If for a fixed $1 \leq i \leq k$, we have $\alpha(x) \cap T(G_i) \neq \emptyset$ and $\alpha(y) \cap H(G_i) \neq \emptyset$, then $\alpha(x) * \alpha(y) = (\alpha(x), \alpha(y))$ and in else case $\alpha(x) * \alpha(y) = \emptyset$(no edge).
4. Input the $x, y \in \mathcal{G}'$. If $S_\alpha^\beta(\alpha(G_j), \alpha(x)) \geq S_\alpha^\beta(\alpha(G_j), \alpha(y))$, then $\alpha(x) *' \alpha(y) = \overrightarrow{(\alpha(x), \alpha(y))}$ as a directed edge from vertex $\alpha(x)$ to vertex $\alpha(y)$, and in else case $\alpha(x) *' \alpha(y) = \overrightarrow{(\alpha(y), \alpha(x))}$, where $S_\alpha^\beta(G, x) = \alpha_G(x) + \beta_G(x)$.

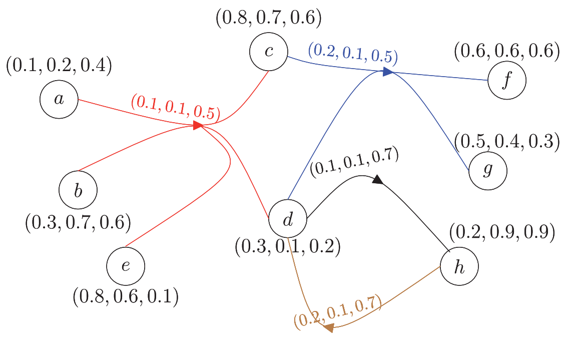

Fig. 7. SVN–DHG

Hence $(\mathcal{G}'/\alpha, *') = \big(\alpha(G), \{\alpha(v_j), \alpha_{\alpha(G_i)}(\alpha(v_j)), \beta_{\alpha(G_i)}(\alpha(v_j)), \gamma_{\alpha(G_i)}(\alpha(v_j))\}_{i=1}^{n'}, *'\big)$ is an SVN–DG. □

The method for the construction of an SVN–DG $\mathcal{G}'/\alpha$ from an SVN–DHG $\mathcal{G}'$ is explained in Algorithm 2 in Table 2.

Example 4.2. Let $G = \{a, b, c, d, e, f, g, h\}$. Consider the SVN–DHG, $\mathcal{G}'$ in Figure 7, where $G_1 = \Big\{\{(a, 0.1, 0.2, 0.4), (b, 0.3, 0.7, 0.6)\}, \{(c, 0.8, 0.7, 0.6), (d, 0.3, 0.1, 0.2), (e, 0.8, 0.6, 0.1)\}\Big\}$, $G_2 = \Big\{\{(c, 0.8, 0.7, 0.6), (d, 0.3, 0.1, 0.2)\}, \{(f, 0.6, 0.6, 0.6), (g, 0.5, 0.4, 0.3)\}\Big\}, 0.2)\}\Big\}$, $(\alpha_{F_1}, \beta_{F_1}, \gamma_{F_1}) = (0.1, 0.1, 0.5)$, $(\alpha_{F_2}, \beta_{F_2}, \gamma_{F_2}) = (0.2, 0.1, 0.5)$, $(\alpha_{F_3}, \beta_{F_3}, \gamma_{F_3}) = (0.1, 0.1, 0.7)$ and $(\alpha_{F_4}, \beta_{F_4}, \gamma_{F_4}) = (0.2, 0.1, 0.7)$.

Since

$$G_1 = (\{a, b\}, \{c, d, e\}), G_2 = (\{c, d\}, \{f, g\}),$$

$$G_3 = (\{d\}, \{h\}) \text{ and } F_4 = (\{h\}, \{e\}),$$

we get that

$$\alpha(a) = \{a, b\}, \alpha(c) = \{c\}, \alpha(d) = \{d\},$$

$$\alpha(e) = \{e\}, \alpha(f) = \{f, g\} \text{ and } \alpha(h) = \{h\}.$$

Hence we obtain $\mathcal{G}'/\alpha = \{\alpha(a), \alpha(c), \alpha(d), \alpha(e), \alpha(f), \alpha(h)\}$. Since

$$\alpha(a) \cap T(G_1) \neq \emptyset \text{ and } \alpha(c) \cap H(G_1) \neq \emptyset,$$

$$\alpha(a) \cap T(G_1) \neq \emptyset \text{ and } \alpha(d) \cap H(G_1) \neq \emptyset,$$

$$\alpha(a) \cap T(G_1) \neq \emptyset \text{ and } \alpha(e) \cap H(G_1) \neq \emptyset,$$

$$\alpha(c) \cap T(G_2) \neq \emptyset \text{ and } \alpha(f) \cap H(G_2) \neq \emptyset,$$

$$\alpha(d) \cap T(G_2) \neq \emptyset \text{ and } \alpha(f) \cap H(G_2) \neq \emptyset,$$

$$\alpha(d) \cap T(G_3) \neq \emptyset \text{ and } \alpha(h) \cap H(G_3) \neq \emptyset,$$

$$\alpha(h) \cap T(G_4) \neq \emptyset \text{ and } \alpha(e) \cap H(G_4) \neq \emptyset,$$

we get that

$$\alpha(a) * \alpha(c) = (\alpha(a), \alpha(c)), \alpha(a) * \alpha(d) = (\alpha(a), \alpha(d)),$$

$$\alpha(a) * \alpha(e) = (\alpha(a), \alpha(e)), \alpha(c) * \alpha(f) = (\alpha(c), \alpha(f)),$$

$$\alpha(d) * \alpha(f) = (\alpha(d), \alpha(f)), \alpha(d) * \alpha(h) = (\alpha(d), \alpha(h)),$$

$$\text{and } \alpha(h) * \alpha(e) = (\alpha(h), \alpha(e)).$$

Since $S_\alpha^\beta(\alpha(G), \alpha(c)) \geq S_\alpha^\beta(\alpha(G), \alpha(a))$, $S_\alpha^\beta(\alpha(G), \alpha(c)) \geq S_\alpha^\beta(\alpha(G), \alpha(f))$ $S_\alpha^\beta(\alpha(G), \alpha(f)) \geq S_\alpha^\beta(\alpha(G), \alpha(d))$, $S_\alpha^\beta(\alpha(G), \alpha(d)) \leq S_\alpha^\beta(\alpha(G), \alpha(b))$ $S_\alpha^\beta(\alpha(G), \alpha(e)) \geq S_\alpha^\beta(\alpha(G), \alpha(h))$, $S_\alpha^\beta(\alpha(G), \alpha(e)) \geq S_\alpha^\beta(\alpha(G), \alpha(a))$ $S_\alpha^\beta(\alpha(G), \alpha(d)) \geq S_\alpha^\beta(\alpha(G), \alpha(a))$. So we obtain the SVN–DG, $(\mathcal{G}'/\alpha, *')$ in Figure 8.

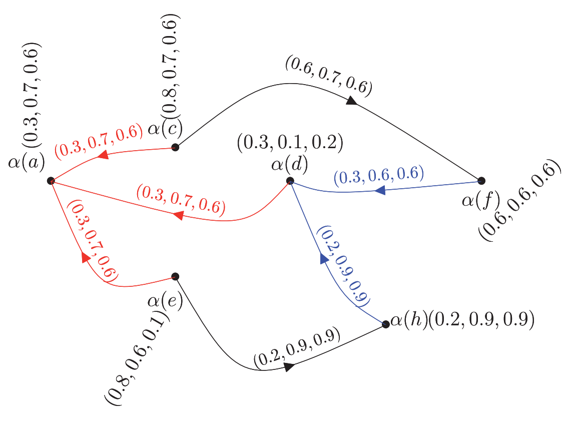

Fig. 8. SVN–DG $(\mathcal{G}'/\alpha, *')$

Definition 4.3. An SVN–DG $G = (V, A)$ is said to be:

(i) an *α-subderivable SVN–DG* if there exists a nontrivial SVN–DHG $\mathcal{G}' = (G = \{G_j\}_{j=1}^{n}, \{F_j\big(T(G_j), H(G_j)\big)\}_{j=1}^{n})$ such that $G = (V, A)$ is isomorphic to a subgraph of $\mathcal{G}'/\alpha$ and $\sum\limits_{i=1}^{m}(\alpha_{F_i} + \beta_{F_i} + \gamma_{F_i}) \geq \sum\limits_{i=1}^{m}(\alpha_{\alpha(F_i)} + \beta_{\alpha(F_i)} + \gamma_{\alpha(F_i)})$. An α-subderivable SVN–DG $G = (V, A)$ is called an α-derivable SVN–DG, if $G = (V, A) \cong \mathcal{G}'/\alpha$, also $\mathcal{G}'$ is called an associated SVN–DHG with SVN–DG G;

(ii) an α-semiself derivable SVN–DG, if it is an α-subderivable SVN–DG by itself;

(iii) an α-self derivable SVN–DG, if it is an α-derivable SVN–DG by itself.

Example 4.4. Consider the SVN–DG, $G = (V, A)$ in Figure 9, where $V = \{a, b, c\}$ and

$$A = \{\big((a, b), (0.2, 0.9, 0.4), \big((b, c), (0.1, 0.8, 0.6)\big)\}.$$

Now we construct an SVN–DHG $\mathcal{G}'$ in Figure 10.

Figure 8: SVN–DG $(\mathcal{G}'/\alpha, *')$

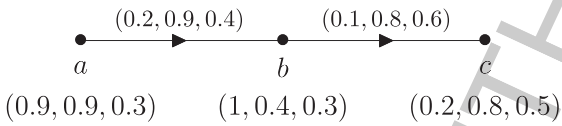

Fig. 9. SVN–DG G

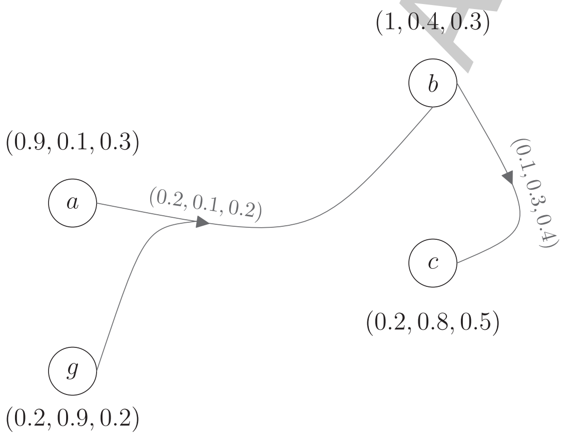

Fig. 10. SVN–DHG $\mathcal{G}'$

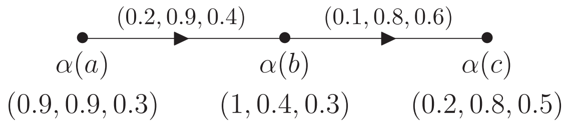

Fig. 11. SVN–DG G

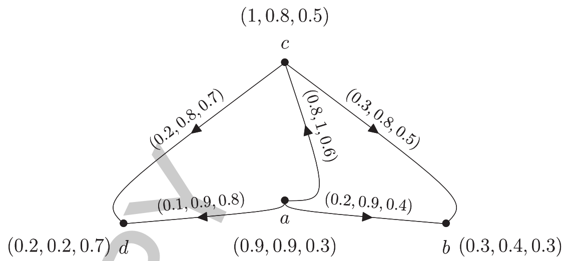

Fig. 12. SVN–DG G

Clearly $\mathcal{G}' = (G = \{G_j\}_{j=1}^{2}, \{F_j\big(T(G_j), H(G_j)\big)\}_{j=1}^{2})$ is a nontrivial SVN–DHG, where $G = \{a, b, c, g\}$, $G_1 = \{\{(a, 0.9, 0.1, 0.3), (g, 0.2, 0.9, 0.2)\}, \{(b, 1, 0.4, 0.3)\}\}$, $G_2 = \{\{(b, 1, 0.4, 0.3)\}, \{(c, 0.2, 0.8, 0.5)\}\}$, and $F_1\big(T(G_1), H(G_1) = (0.2, 0.1, 0.2)$, $F_2\big(T(G_2), H(G_2) = (0.1, 0.3, 0.4)\big)$. Computations show that $\alpha(a) = \{a, g\}, \alpha(b) = \{b\}$ and$\alpha(c) = \{c\}$. By Theorem 4, it is easy to see that digraph $\mathcal{G}'/\alpha$ is obtained in Figure 11. Clearly $\mathcal{G}'/\alpha \cong G$. Since $|G| \neq |V|$, we have digraph $G = (V, A)$ is an α-derivable SVN–DG and it is not an α-self derivable SVN–DG.

Example 4.5. Consider the SVN–DG, $G = (V, A)$ in Figure 12, where $V = \{a, b, c, d\}$ and $A = \{\big((a, b), (0.2, 0.9, 0.4), \big((a, c), (0.8, 1, 0.6), \big((a, d), (0.1, 0.9, 0.8), \big((c, d), (0.2, 0.8, 0.7), \big((c, b), (0.3, 0.8, 0.5)\big)\}$. Now we construct an SVN–DHG $\mathcal{G}'$ in Figure 13. Clearly $\mathcal{G}' = (G =$

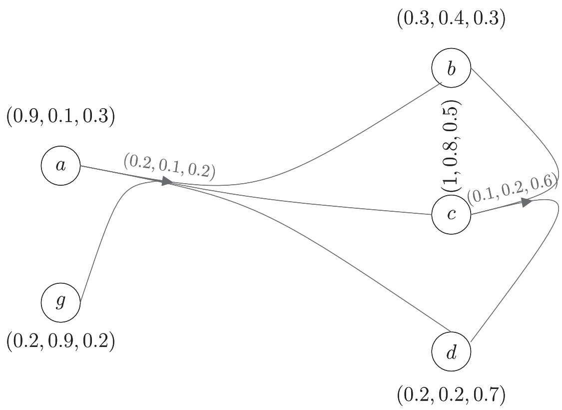

Fig. 13. SVN–DHG $\mathcal{G}'$

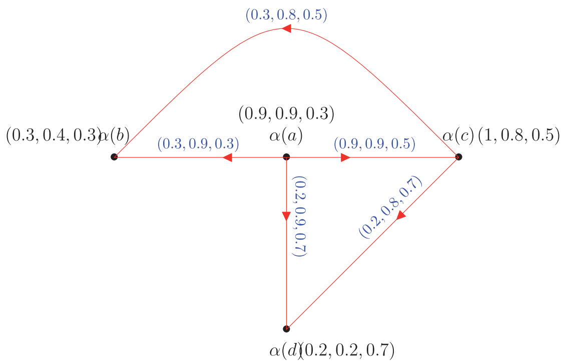

Fig. 14. SVN–DG, $\mathcal{G}'/\alpha$

$\{G_j\}_{j=1}^{2}, \{F_j\big(T(G_j), H(G_j)\big)\}_{j=1}^{2})$ is a nontrivial SVN–DHG, where $G = \{a, b, c, d, g\}$, $G_1 = \{\{(a, 0.9, 0.1, 0.3), (g, 0.2, 0.9, 0.2)\}, \{(c, 1, 0.8, 0.5), (b, 0.3, 0.4, 0.3), (d, 0.2, 0.2, 0.7)\}\}$, $G_2 = \{\{(c, 1, 0.8, 0.5)\}, \{(b, 0.3, 0.4, 0.3), (d, 0.2, 0.2, 0.7)\}\}$, and $F_1\big(T(G_1), H(G_1) = (0.2, 0.1, 0.2), F_2\big(T(G_2), H(G_2) = (0.1, 0.2, 0.6)\big)$. Computations show that $\alpha(a) = \{a, g\}, \alpha(b) = \{b\}, \alpha(c) = \{c\}$ and $\alpha(d) = \{d\}$. By Theorem 4.1, it is easy to see that digraph $\mathcal{G}'/\alpha$ is obtained in Figure 14.

Clearly $\mathcal{G}'/\alpha \cong G$. Since $|G| = |V|$, we have digraph $G = (V, A)$ is an α-self derivable SVN–DG.

Let $V = \{a_1, a_2, \ldots, a_n\}$. Then we denote the directed path graph in Figure 15 by DP_n.

Theorem 4.6. *If $DP_n = (V, E)$ is an SVN–DG, then for all $1 \leq i \leq n-1$, $\alpha_V(a_i) \geq \alpha_V(a_{i+1})$ or $\beta_V(a_i) \geq \beta_V(a_{i+1})$;*

Proof. Since $A = \{(a_i, a_{i+1}) \mid 1 \leq i \leq n-1\}$, we get that

$$S_\alpha^\beta(A, a_1) \geq S_\alpha^\beta(A, a_2) \geq S_\alpha^\beta(A, a_2) \geq \ldots \geq S_\alpha^\beta(A, a_n).$$

Thus for all $1 \leq i \leq n-1$, $\alpha_V(a_i) \geq \alpha_V(a_{i+1})$ or $\beta_V(a_i) \geq \beta_V(a_{i+1})$. □

Theorem 4.7. *Let $2 \leq n \in \mathbb{N}$. Then*

(i) SVN–DG, DP_n is an α-derivable SVN–DG.
(ii) SVN–DG, DP_2 is not an α-self derivable SVN–DG.

Proof. (i) Let $DP_n = (V, A)$ be a path SVN–DG, where $V = \{(a_j, \alpha_V(a_j)), \beta_V(a_j)), \gamma_V(a_j))\}_{j=1}^{n}$. Then for any $a, b \notin V$ consider $G_1 = \big(\{(a_1, \alpha_V(a_1)), \beta_V(a_1)), \gamma_V(a_1)), (a, t_1, t_2, t_3)\}, \{(a_2, \alpha_V(a_2)), \beta_V(a_2)), \gamma_V(a_2))\}\big)$, where $0 < t_1 \leq \alpha_V(a_1))$, $0 < t_2 \leq \beta_V(a_1))$ and $0 < t_3 \leq \gamma_V(a_1))$. Also for any $2 \leq i \leq n-2$, $G_i = (\{(a_i, \alpha_V(a_i)), \beta_V(a_i)), \gamma_V(a_i))\}$ and $G_{n-1} = \big(\{(a_{n-1}, \alpha_V(a_{n-1})), \beta_V(a_{n-1})), \gamma_V(a_{n-1}), \{(a_n, \alpha_V(a_n)), \beta_V(a_n)), \gamma_V(a_n)), (b, s_1, s_2, s_3)\}\big)$, where $0 < s_1 \leq \alpha_V(a_n))$, $0 < s_2 \leq \beta_V(a_n))$ and $0 < s_3 \leq \gamma_V(a_n))$. It can see that $\alpha(a_1) = \alpha(a) = \{a_1, a\}$, $\alpha(a_n) = \alpha(b) = \{a_n, b\}$ and for any $2 \leq i \leq n-1$, $\alpha(a_i) = \{a_i\}$. If $G = V \cup \{(a, t_1, t_2, t_3), (b, s_1, s_2, s_3)\}$, then $\mathcal{G}' = (G = \{G_i\}_{i=1}^{n-1}, \{F_i\big(T(G_i), H(G_i)\big)\}_{i=1}^{n-1}))$ is a nontrivial SVN–DHG, where for any $1 \leq i \leq n-1$ we have $F_i\big(T(G_i), H(G_i)\big)\}_{i=1}^{n-1}) = \big(\bigwedge_{a\alpha x,\, b\alpha y} (\alpha_{\alpha(G_i)}(a) \wedge \alpha_{\alpha(G_i)}(b)), \bigvee_{a\alpha x,\, b\alpha y} (\beta_{\alpha(G_i)}(a) \vee \beta_{\alpha(G_i)}(b)), \bigvee_{a\alpha x,\, b\alpha y} (\gamma_{\alpha(G_i)}(a) \vee \gamma_{\alpha(G_i)}(b))\big)$. Since for any $1 \leq i \leq n$ which is an odd, we have $\alpha(a_i) \cap T(G_i) \neq \emptyset$ and for any $1 \leq i \leq n$ which is an even, we have $\alpha(a_i) \cap H(G_i) \neq \emptyset$, we get that $\alpha(a_i) * \alpha(a_{i+1}) = (\alpha(a_i), \alpha(a_{i+1})) = e_{ii+1}$, where for all $1 \leq i \leq n$, $\alpha(a_i) = (\alpha(a_i), \alpha_V(a_i)), \beta_V(a_i)), \gamma_V(a_i)))$ and for all $1 \leq i \leq n-1$, $e_i = F_i\big(T(G_i), H(G_i)\big)\}_{i=1}^{n-1}))$. Hence we obtain an SVN–DG in Figure 16. Clearly $\mathcal{G}'/\alpha \cong DP_n$ and so for any $n \geq 2$, DP_n is an α-derivable SVN–DG.

(ii) Let DP_2 be an α-self derivable SVN–DG. Then there exists an associated SVN–DHG, $\mathcal{G}' = (G, \{F_j\big(T(G_j), H(G_j)\big)\}_{j=1}^{n})$ with SVN–DG, DP_2 such that $\mathcal{G}'/\alpha \cong DP_2$ and $|G| = 2$. Suppose that $G = \{x, y\}$, since $\mathcal{G}'$ is a nontrivial SVN–DHG, must be $2 \leq m$. But $|G| = 2$ implies that $m = 1$ which is a contradiction. □

Corollary 4.8. *Let $2 \leq n \in \mathbb{N}$. Then DP_n is an α-derivable SVN–DG but is not an α-self derivable SVN–DG.*

We introduce SVN–DG G' in Figure 17. From now on, we apply the SVN–DG, G' in Figure 17, in the following Theorem.

Theorem 4.9. *Let $G' = (\{a, b\}, A')$ be an SVN–DG. Then the following properties hold.*

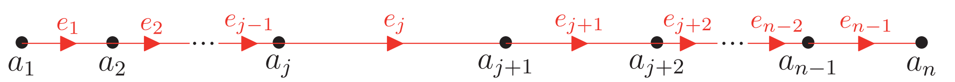

Fig. 15. Path graph DP_n.

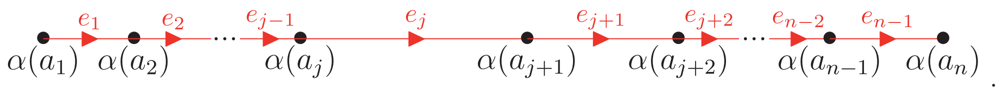

Fig. 16. SVN–DG $\mathcal{G}'/\alpha$.

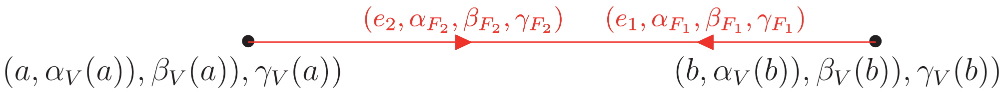

Fig. 17. SVN–DG G'

(*i*) $S^{\beta}_{\alpha}(A', a) = S^{\beta}_{\alpha}(A', b)$;
(*ii*) *G' is not an α-derivable SVN–DG.*

Proof. (*i*) Since $G' = (\{a, b\}, A')$ is an SVN–DG and $e_1, e_2 \in A'$, we get that $\alpha_V(a) + \beta_V(a) \geq \beta_V(b) + \alpha_V(b)$ and $\alpha_V(a) + \beta_V(a) \leq \alpha_V(b) + \beta_V(b)$. It follows that $S^{\beta}_{\alpha}(A', a) = S^{\beta}_{\alpha}(A', b)$.

(*ii*) Let $G' = (\{a, b\}, A')$ be an α-derivable SVN–DG. Then there exist a nontrivial SVN–DHG, $\mathcal{G}' = (G = \{G_j\}_{j=1}^{n}, \{F_i(T(G_i), H(G_i))\}_{i=1}^{m}))$ and $1 \leq k, l \leq n$ such that $\{\alpha(x_k), \alpha(x_l)\} = \mathcal{G}'/\alpha \cong G'$, where $G = \{x_1, x_2, \ldots, x_n\}$. Since $|V| = 2$, we get $m = 2$. In addition, $\alpha(x_k) *' \alpha(x_l) = \overrightarrow{(\alpha(x_k), \alpha(x_l))}, \alpha(x_l) *' \alpha(x_k) = \overrightarrow{(\alpha(x_l), \alpha(x_k))}$ implies that there exists $1 \leq j \leq n$ such that $\alpha(x_k) \cap T(G_j) \neq \emptyset$ and $\alpha(x_l) \cap H(G_j) \neq \emptyset$ or $\alpha(x_k) \cap H(G_j) \neq \emptyset$ and $\alpha(x_l) \cap T(G_j) \neq \emptyset$. It follows that $H(G_1) \cap T(G_2) \neq \emptyset$ and $H(G_2) \cap T(G_1) \neq \emptyset$ and so $|\mathcal{G}'/\alpha| \geq 3$ which is a contradiction. □

Corollary 4.10. *Let $G = (V, A)$ be an SVN–DG. If G is homeomorphic to SVN–DG, G', then G is not an α-derivable SVN–DG.*

Let $2 \leq n \in \mathbb{N}$, $V = \{v_1, v_2, v_3, \ldots, v_n\}$ and $E = \{e_1, e_2, e_3, \ldots, e_{n-1}\}$, where $e_i = v_1 v_{i+1}$, for every $1 \leq i \leq n-1$, and $DS_n = (V, E)$ be a star directed graph as Figure 18.

Theorem 4.11. *If $DS_n = (V, E)$ is an SVN–DG, then for all $2 \leq i \leq n$, $\alpha_V(v_1) \geq \alpha_V(v_i)$ or $\beta_V(v_1) \geq \beta_V(v_i)$.*

Proof. Since $A = \{(v_1, v_i) \mid 1 \leq i \leq n\}$, for all $1 \leq i \leq n$, we have $S^{\beta}_{\alpha}(A, v_1) \geq S^{\beta}_{\alpha}(A, v_i)$. It follows that for all $2 \leq i \leq n$, $\alpha_V(v_1) \geq \alpha_V(v_i)$ or $\beta_V(v_1) \geq \beta_V(v_i)$. □

Theorem 4.12. *Let $2 \leq n \in \mathbb{N}$. Then*

(*i*) *SVN–DG, DS_n is an α-derivable SVN–DG.*
(*ii*) *SVN–DG, DS_2 is not an α-self derivable SVN–DG.*

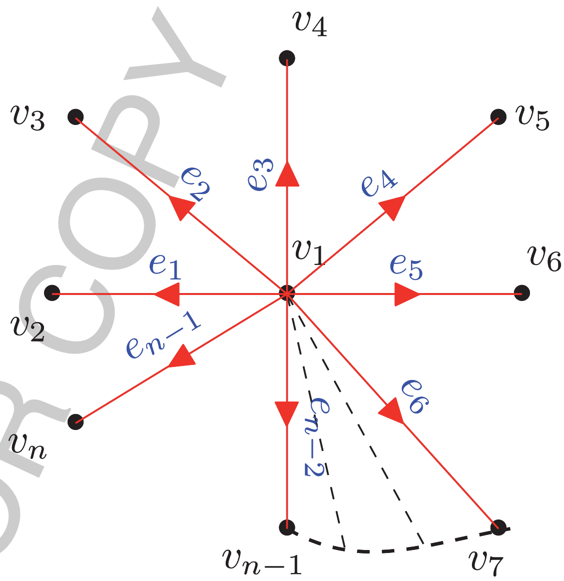

Fig. 18. Star digraph DS_n

(*iii*) *For all $n \geq 3$, SVN–DG, DS_n is an α-self derivable SVN–DG.*

Proof. (*i, ii*) The proof is similar to Theorem 4 and Corollary 4.

(*iii*) Let $DS_n = (V, A)$ be a path SVN–DG, where $V = \{(v_j, \alpha_V(v_j)), \beta_V(v_j)), \gamma_V(v_j))\}_{j=1}^{n}$. For simplifying we denote $(v_j, \alpha_V(v_j)), \beta_V(v_j), \gamma_V(v_j))$ by v_j and consider

$$G_1 = \big(\{v_1\}, \{v_2, v_3, v_4\}\big), G_2 = \big(\{v_1\}, \{v_3, v_4, v_5\}\big),$$

and for all $3 \leq i \leq n-3$, $G_i = \big(\{v_1\}, \{v_{i+3}\}$. One can see that for any $1 \leq i \leq n, \alpha(a_i) = \{a_i\}$ and $\mathcal{G}' = (G = \{G_i\}_{i=1}^{n-3}, \{F_i\big(T(G_i), H(G_i)\big)\}_{i=1}^{n-3}))$ is a nontrivial SVN–DHG, where for any $1 \leq i \leq n-3$, we have $F_i\big(T(G_i), H(G_i)\big)\}_{i=1}^{n-3}) = (\bigwedge_{a\alpha x,\, b\alpha y} (\alpha_{\alpha(G_i)}(a) \wedge$

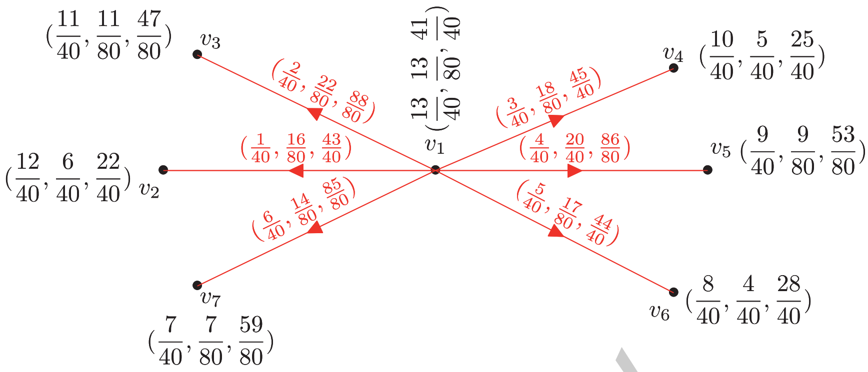

Fig. 19. SVN–DG, DS_7

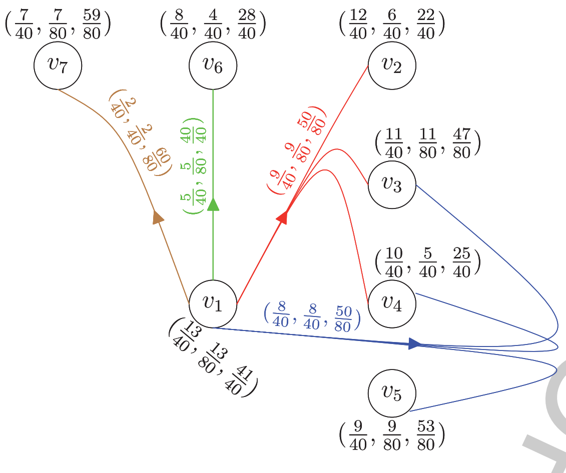

Fig. 20. SVN–DHG

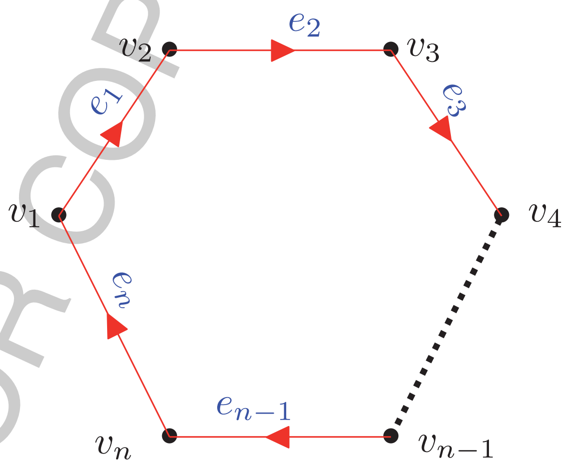

Fig. 21. Cycle digraph DC_n^*

$\alpha_{\alpha(G_i)}(b)), \bigvee_{a\alpha x,\ b\alpha y} (4\beta_{\alpha(G_i)}(a) \vee \beta_{\alpha(G_i)}(b)),$
$\bigvee_{a\alpha x,\ b\alpha y} (\gamma_{\alpha(G_i)}(a) \vee \gamma_{\alpha(G_i)}(b))\big)$. In a similar way of Theorem 4.7, $\mathcal{G}'/\alpha \cong DS_n$ and so for any $n \geq 3$, DS_n is an α-self derivable SVN–DG. □

Corollary 4.13. *Let $2 \leq n \in \mathbb{N}$. Then DS_n is an α-derivable SVN–DG and for $3 \leq n$, it is an α-self derivable SVN–DG.*

Example 4.14. Consider the SVN–DG, DS_7 in Figure 19. Now, construct the SVN–DHG, $\mathcal{G}'$ in Figure 20. Clearly $G_1 = (\{(v_1, \frac{13}{40}, \frac{13}{80}, \frac{41}{40})\}, \{(v_2, \frac{12}{40}, \frac{6}{40}, \frac{22}{40}), (v_3, \frac{11}{40}, \frac{11}{80}, \frac{47}{80}), (v_4, \frac{10}{40}, \frac{5}{40}, \frac{25}{40})\}), G_2 = (\{(v_1, \frac{13}{40}, \frac{13}{80}, \frac{41}{40})\}, \{(v_3, \frac{11}{40}, \frac{11}{80}, \frac{47}{80}), (v_4, \frac{10}{40}, \frac{5}{40}, \frac{25}{40}), (v_5, \frac{9}{40}, \frac{9}{80}, \frac{53}{80}\})$ $G_3 = (\{(v_1, \frac{13}{40}, \frac{13}{80}, \frac{41}{40})\}, \{(v_6, \frac{8}{40}, \frac{4}{40}, \frac{28}{40})\}$ and $G_4 = (\{(v_1, \frac{13}{40}, \frac{13}{80}, \frac{41}{40})\}, \{(v_7, \frac{7}{40}, \frac{7}{80}, \frac{59}{80})\}$. Obviously, for all $1 \leq i \leq 6$, we have $\alpha(v_i) = \{v_i\}$ and so$\mathcal{G}'/\alpha \cong DS_7$.

Let $3 \leq n \in \mathbb{N}$ and $V = \{v_1, v_2, \ldots, v_n\}$. Then we denote the directed cyclic graph DC_n^* in Figure 21.

Theorem 4.15. *If $DC_n^* = (V, A)$ is an SVN–DG, then for all $v, v' \in V$ $S_\alpha^\beta(A, v) = S_\alpha^\beta(A, v')$.*

Proof. Since $DS_n = (V, E)$ is an SVN–DG, for every $1 \leq i \leq n$, we get that $\alpha_V(v_i) + \beta_V(v_i) \geq \alpha_V(v_{i+1}) + \beta_V(v_{i+1})$. Consider $e_n = (v_n, v_1)$, so $\alpha_V(v_n) + \beta_V(v_n) \geq \alpha_V(v_1) + \beta_V(v_1) \geq \alpha_V(v_2) + \beta_V(v_2) \geq \alpha_V(v_3) + \beta_V(v_3) \geq \alpha_V(v_4) + \beta_V(v_4) \geq \ldots \geq \alpha_V(v_{n-1}) + \beta_V(v_{n-1}) \geq \alpha_V(v_n) + \beta_V(v_n) \geq \alpha_V(v_1) + \beta_V(v_1)$. Hence for all $v, v' \in V$ we get that $S_\alpha^\beta(A, v) = S_\alpha^\beta(A, v')$. □

Theorem 4.16. *Let $3 \leq n \in \mathbb{N}$. Then*

(*i*) *SVN–DG, DC_3^* is not an α-derivable SVN–DG.*
(*i*2) *SVN–DG, DC_n^* is not an α-derivable SVN–DG.*

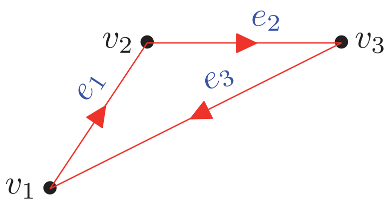

Fig. 22. Cycle digraph DC_3^*

$(ii3)$ *SVN–DG, DC_n^* is not an α-self derivable SVN–DG.*

(iv) *SVN–DG, DC_n^* is an α-semiself derivable SVN–DG.*

Proof. (i) Consider the SVN–DG, DC_3^* in Figure 22. If DC_3^* is an α-derivable SVN–DG, then we can consider the smallest associated SVN-DHG $\mathcal{G}' = (G = \{G_j\}_{j=1}^n, \{\{F_j\big(T(G_j), H(G_j)\big)\}_{j=1}^n),$ where there exists $1 \leq t \leq n$, in such a way that $2 \in \{|T(G_t)|, |H(G_t)|\}$ and for any $1 \leq i \neq t \leq n, |T(E_i)| = |H(E_i)| = 1$. Since for any $1 \leq i \leq n, ud(v_i) = id(v_i) = 1$(output degree and input degree of v_i), for all $1 \leq i \neq j \leq n$ we get that $\{v_i, v_j\} \nsubseteq T(E_i), \{v_i, v_j\} \nsubseteq H(E_i), \{v_i, v_j\} \nsubseteq T(E_j)$ and $\{v_i, v_j\} \nsubseteq H(E_j)$. Hence there exists $x' \notin H$ such that $x' \in T(E_t) \cup H(E_t)$ and so $m = n = 3$. In addition, for some $1 \leq k \leq n, v_t \in (T(E_t) \cup H(E_t)) \cap (T(E_k) \cup H(E_k))$ implies that $\alpha(v_t) \neq \alpha(x')$ and for $1 \leq i \leq n, \alpha(v_i) = \{v_i\}$. It follows that $\mathcal{G}'/\alpha = \{\alpha(x'), \alpha(v_1), \ldots, \alpha(v_n)\}$ and so $\mathcal{G}'/\alpha \ncong DC_3^*$, which is a contradiction.

(ii) Since every SVN–DG, DC_n^* is homeomorphic to SVN–DG, DC_3^*, by item (i) we get that for every $4 \leq n \in \mathbb{N}$, SVN–DG, DC_n^* is not an α-derivable SVN–DG.

(iii) Since SVN–DG, DC_n^* is not an α-derivable SVN–DG, we get that it is not an α-self derivable SVN–DG.

(iv) Let $DC_n^* = (V, A)$ be a cyclic SVN–DG, where $V = \{(a_j, \alpha_V(a_j)), \beta_V(a_j)),$ $\gamma_V(a_j))\}_{j=1}^n$. For simplifying we denote $(a_j, \alpha_V(a_j)),$ $\beta_V(a_j), \gamma_V(a_j))$ by a_j and consider $G_1 = \big(\{a_1 a_2\}, \{$ $a_3\}\big)$, for any $2 \leq i \leq n-2, G_i = \big(\{a_i\}, \{a_{i+1}\}\big)$, $G_{n-1} = \big(\{a_n\}, \{a_1\}\big)$ and $G_n = \big(\{a_1\}, \{a_2\}\big)$. It can see that for any $1 \leq i \leq n, \alpha(a_i) = \{a_i\}$ and by Theorem 4, for all $v, v' \in V$ $S_\alpha^\beta(A, v) = S_\alpha^\beta(A, v')$. Also, $\mathcal{G}' = (V = \{G_i\}_{i=1}^{n-1}, \{F_i\big(T(G_i), H(G_i)\big)\}_{i=1}^{n-1}$ $))$ is a nontrivial SVN–DHG, where for any $1 \leq i \leq n-1$ we have $F_i\big(T(G_i), H(G_i)\big)\}_{i=1}^{n-1}) =$ $\Big(\bigwedge\limits_{a\alpha x,\ b\alpha y}(\alpha_{\alpha(G_i)}(a) \wedge \alpha_{\alpha(G_i)}(b)), \bigvee\limits_{a\alpha x,\ b\alpha y}(\beta_{\alpha(G_i)}(a) \vee$ $\beta_{\alpha(G_i)}(b)), \bigvee\limits_{a\alpha x,\ b\alpha y}(\gamma_{\alpha(G_i)}(a) \vee \gamma_{\alpha(G_i)}(b))\Big)$. In a similar way to Theorem 4, one can see that DC_n^* is isomorphic to a subgraph of $\mathcal{G}'/\alpha$ and $\sum\limits_{i=1}^{m}(\alpha_{F_i} +$ $\beta_{F_i} + \gamma_{F_i}) \geq \sum\limits_{i=1}^{m}(\alpha_{\alpha(F_i)} + \beta_{\alpha(F_i)} + \gamma_{\alpha(F_i)})$. □

Let $3 \leq n \in \mathbb{N}$ and $V = \{v_1, v_2, \ldots, v_n\}$. Then we denote the directed complete graph by DK_n.

Corollary 4.17. *Let $3 \leq n \in \mathbb{N}$. Then*

(i) *for all $v, v' \in V$, we have $S_\alpha^\beta(A, v) = S_\alpha^\beta(A, v')$;*

(ii) *SVN–DG, DK_n is not both an α-derivable SVN–and α-self derivable SVN–DG.*

(iii) *SVN–DG, DK_n is an α-semiself derivable SVN–DG.*

4.1. Applications of α-driveable SVN–DG

In this subsection, we describe some applications of the concept of α-derivable single–valued neutrosophic digraphs and single–valued neutrosophic dihypergraphs.

Graphs and hypergraphs can be used to describe the network systems. The network systems, including social networks, world wide web, neural networks are investigated by means of simple graphs and digraphs. The graphs take the nodes as a set of objects or people and the edges define the relations between them. In many cases, it is not possible to give full description of real world systems using the simple graphs or digraphs. For example, if a collaboration network is represented through a simple graph. We only know that whether the two researchers are working together or not. We can not know if three or more researchers, which are connected in the network, are coauthors of the same article or not. Further, in various situations, the given data contains the information of existence, indeterminacy and non-existence. We represented these systems by SVN–DG(SVN–DHG) that consist of sets of nodes representing the objects or group under investigation, joined together in pairs by links if the corresponding nodes or sets are related by some kind of relationship. Consequently, we will formally apply the SVN–DHG concept as a generalization for representing weighted networks and will call them weighted hypernetworks. A cluster in

WNS consists of three main different elements: sensor nodes (SNs), base station (BS), and cluster–heads (CH). The SNs are the set of sensors present in the network, arranged to sense the environment and collect the data. The main task of an SN in a sensor field is to detect events, perform quick local data processing, and then transmit the data. The BS is the data processing point for the data received from the sensor nodes, and where the data are accessed by the end-user. It is generally considered fixed and at a far distance from the sensor nodes.The CH acts as a gateway between the SNs and the BS. The function of the cluster–head is to perform common functions for all the nodes in the cluster, like aggregating the data before sending it to the BS. In some way, the CH is the sink for the cluster nodes, and the BS is the sink for the cluster–heads. This structure formed between the sensor nodes, the sink, and the base station can be replicated as many times as it is needed, creating the different layers of the hierarchical WSN. The SNs and the communication links between them can be represented by an undirected graph $G = (V, E)$, where each vertex $v \in V$ (the set of vertices in the graph) represents a sensor node with a unique ID. An edge $(u, v) \in E$ (the set of edges in the graph) represents a communication link if the corresponding nodes u and v are within the transmission range of each other. We apply the concept of SVN–DG for clustering WSNs via the notation of positive relation and obtain directed clustering graphs.

Example 4.18. (Lifetime in wireless sensor network) The proposed protocol weight-based clustering routing (WCR) is a clustering-based, energy-efficient protocol for wireless sensor networks. The objective of the protocol is to reduce the energy dissipation of nodes for routing data to the base station and prolong the network lifetime. In WCR, a cluster–head selection algorithm is designed for periodically selecting cluster–heads based on the node position information and residual energy of node. This cluster–head selection scheme is a central controlled algorithm performed by the base station which is assumed to have no energy constraint. Distributed weight-based energy-efficient hierarchical clustering (DWEHC) as an algorithm, aims at high energy efficiency by generating balanced cluster sizes and optimizing the intra cluster topology. DWEHC algorithm has been shown to generate more well-balanced clusters as well as to achieve significantly lower energy consumption in intra cluster and intra cluster communication. Let

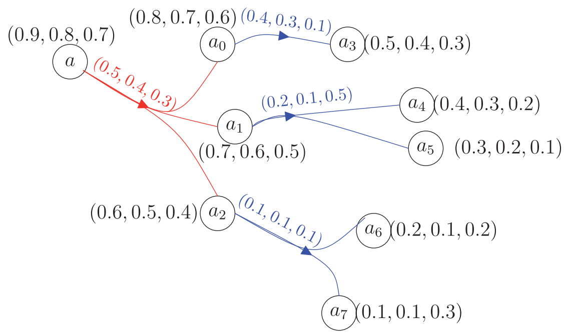

Fig. 23. DWEHC multi-hop intracluster topology

$H = \{a, a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9\}$ be a set of nodes in a wireless sensor networks as a hyper network. Figure 23, shows a multi-level cluster generated by DWEHC, where a is the cluster–head, first level children are a_0, a_1, a_2, second level children are a_3, a_4, a_5, a_6 and a_7. Let the degree of contribution in the energy-efficient protocol relationships of a is $90/100$, degree of indeterminacy of energy-efficient protocol is $80/100$ and degree of false–energy-efficient protocol is $70/100$, i.e. the truth–energy-efficient protocol, indeterminacy–energy-efficient protocol and falsity–energy-efficient protocol values of the vertex of wireless sensor network is $(0.9, 0.8, 0.7)$.

Since $\alpha(a) = \{a\}, \alpha(a_0) = \{a_0\}, \alpha(a_1) = \{a_1\}, \alpha(a_2) = \{a_2\}, \alpha(a_3) = \{a_3\}, \alpha(a_4) = \{a_4, a_5\}$ and $\alpha(a_7) = \{a_6, a_7\}$, we get the α-derivable digraph $\mathcal{G}'/\alpha$ in Figure 24.

The directed graph model of SVN–DG $\mathcal{G}'/\alpha$ associated to a lifetime in wireless sensor network or DWEHC multi-hop intracluster topology $\mathcal{G}'$ is explained in Algorithm 3 in Table 3 and in Figure 24.

Example 4.19. (Social networking) In social networks nodes represent people or groups of people, normally called actors, that are connected by pairs according to some pattern of contact or interactions between them. Such patterns can be of friendship, collaboration, business relationships, etc. There are some cases in which hypergraph representations of the social network are indispensable. Let $X = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$ be a society and $a_1, a_2, a_3, a_4, a_5, a_6, a_7$ be names of its people. These people create some groups as $E_1 = \{a_1, a_2, a_3\}, E_2 = \{a_4, a_3\}$ and $E_3 = \{a_4, a_5, a_6, a_7\}$. Let the degree of contribution in the business relationships of a_1 is $10/100$, degree of indeterminacy of contribution is $15/100$ and

Table 3
Algorithm 3

1. Consider the wireless sensor network and design a SVN–DHG model $\mathcal{G}'$.
3. By Algorithm 2 in Table 2, construct the SVN–DG $\mathcal{G}'/\alpha$.

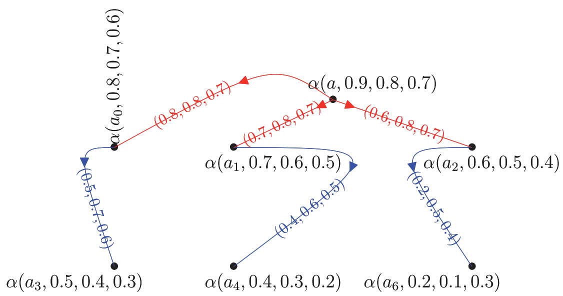

Fig. 24. Digraph $\mathcal{G}'/\alpha$

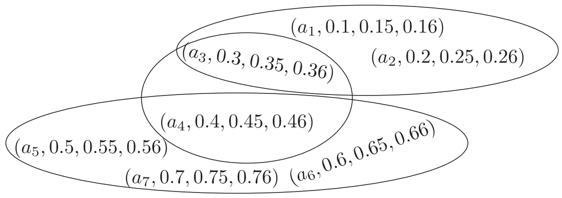

Fig. 25. Social network $\mathcal{H}'$

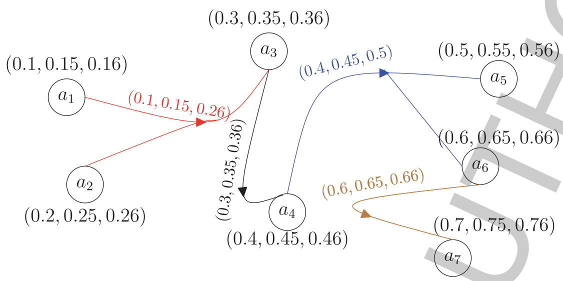

Fig. 26. SVN–DHG $\mathcal{G}'$

degree of false–contribution is $16/100$, i.e. the truth–membership, indeterminacy–membership and falsity–membership values of the vertex human is $(0.1, 0.15, 0.16)$. The likeness, indeterminacy and dislikeness of contribution in the business relationships this society is shown in the Figure 25.

By Theorem 3, the SVN–DHG $\mathcal{G}'$ is obtained in Figure 26.

By Theorem 4, and some computations, we obtain the SVN–DG, $(\mathcal{G}'/\alpha, *)$ in Figure 27.

The mathematical model of SVN–DG $\mathcal{G}'/\alpha$ associated to a social network $\mathcal{H}'$ is explained in Algorithm 4 in Table 4 and in Figure 27.

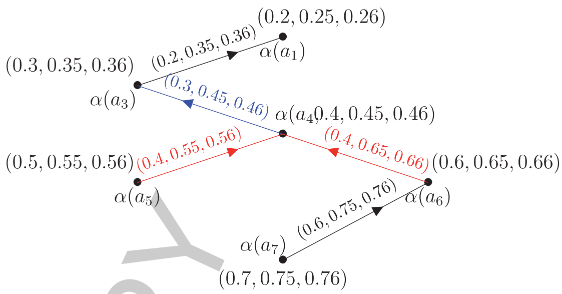

Fig. 27. SVN–DG $(\mathcal{G}'/\alpha, *)$

5. Conclusion

The current paper considered the concepts of single–valued neutrosophic hypergraphs(SVN-HG), single–valued neutrosophic directed hypergraphs(SVN-DHG) and constructed the single–valued neutrosophic directed hypergraphs from single-valued neutrosophic hypergraphs. Moreover

(i) It is introduced the notation of derived SVN–DHG and is shown that every SVN-DHG is a derivable-SVN-DHG.

(ii) We defined a concept of weak single valued neutrosophic digraph(WSVN-DG) and proved that any finite set can be a WSVN-DG.

(iii) We defined an equivalence relation (titled α) on single–valued neutrosophic directed hypergraphs and investigated the relation between of SVN–DHG and SVN-DG via α.

(iv) It is corresponded the single–valued neutrosophic directed (hyper)graphs with wireless sensor (hyper)networks such that the set of vertices (V) represent the sensors and the set of links (E) represents the connections between vertices.

(v) Using the relation α, the sensor clusters of wireless sensor (hyper)networks are considered as a class of wireless sensor (hyper)networks under relation α.

(vi) This study introduced the concept of α-(self-semi)derivable directed graph and investigated some conditions such that a single–valued neutrosophic directed graph is an α-(self-semi)derivable directed graph.

Table 4
Algorithm 4

1. Consider the social network and design a SVN–HG model $\mathcal{H}' = (H, \{E_i\}_{i=1}^{m})$.
2. By Algorithm 1 in Table 1, construct the SVN–DHG $\mathcal{G}'$.
3. By Algorithm 2 in Table 2, construct the SVN–DG $\mathcal{G}'/\alpha$.

(vii) Some algorithms are presented in such a way that analyze the application of single–valued neutrosophic directed (hyper)graph in (hyper)networks.

We hope that these results are helpful for further studies in single–valued neutrosophic directed (graphs)hypergraphs theory. In our future studies, we hope to obtain more results in coding theory and single–valued neutrosophic directed (hyper)graphs and their applications in (hyper)networks.

Acknowledgments

The authors would like to express their gratitude to anonymous referees for their comments and suggestions which improved the paper.

References

[1] M. Akram, S. Shahzadi and A. Borumand Saeid, Neutrosophic hypergraphs, *TWMS J App Eng Math* **8**(1) (2018), 122–135.
[2] M. Akram and A. Luqman, Certain concepts of bipolar fuzzy directed hypergraphs, *Mathematics* **5**(1) (2017).
[3] M. Akram and A. Luqman, Certain network models using single-valued neutrosophic directed hypergraphs, *Journal of Intelligent and Fuzzy Systems* **33**(1) (2017), 575–588.
[4] M. Akram and A. Luqman, Directed hypergraphs under mpolar fuzzy environment, *Journal of Intelligent and Fuzzy Systems* **34**(6) (2018), 4127–4137.
[5] M. Akrama, M. Sarwara and R.A. Borzooei, A novel decision-making approach based on hypergraphs in intuitionistic fuzzy environment, *Journal of Intelligent & Fuzzy Systems* **35**(2) (2018), 1905–1922.
[6] M. Akram and W.A. Dudek, Intuitionistic fuzzy hypergraphs with applications, *Information Sciences* **218** (2013), 182–193.
[7] M. Akram and M. Sarwar, Novel applications of m-polar fuzzy hypergraphs, *Journal of Intelligent and Fuzzy Systems* **32**(3) (2016), 2747–2762.
[8] M. Akram, S. Shahzadi and A. Borumand Saeid, single–valued neutrosophic hypergraph, *TWMS J App Eng Math* **8**(1) (2018), 122–135.
[9] C. Berge, *Graphs and Hypergraphs*, North Holland, 1979.
[10] A.V. Devadoss, A. Rajkumar and N.J.P. Praveena, A study on miracles through holy bible using neutrosophic cognitive maps (NCMS), *International Journal of Computer Applications* **69**(3) (2013).
[11] G. Gallo, G. Longo and S. Pallottino, Directed hypergraphs and applications, *Discrete Appl. Math* **42** (1993), 177–201.
[12] M. Hamidi and A. Borumand Saeid, Creating and computing graphs from hypergraphs, *Kragujevac Journal of Mathematics* **43**(1) (2019), 139–164.
[13] M. Hamidi and A. Borumand saeid, On derivable trees, *Transactions on Combinatorics* (2019) (accepted).
[14] M. Hamidi and A. Borumand Saeid, Accessible single-valued neutrosophic graphs, *J Appl Math Comput* **57** (2018), 121–146.
[15] M. Hamidi and A. Borumand Saeid, Achievable single-valued neutrosophic graphs in wireless sensor networks, *New Mathematics and Natural Computation* **14**(2) (2018), 157–185.
[16] P. Majumdar and S.K. Samanta, On similarity and entropy of neutrosophic sets, *Journal of Intelligent & Fuzzy Systems* **26**(3) (2014), 1245–1252.
[17] H. Rashmanlou, S. Sahoo, R.A. Borzooei, M. Pal and A. Lakdashti, Computation of shortest path in a vague network by euclidean distance, *Journal of Mult-Valued Logic and Soft Computing* **30** (2017), 115–123.
[18] S. Sahoo and M. Pal, Certain types of edge irregular intuitionistic fuzzy graphs, *Journal of Intelligent and Fuzzy Systems* **34**(1) (2018), 295–305.
[19] S. Sahoo and M. Pal, Different types of products on intuitionistic fuzzy graphs, *Pacific Science Review A: Natural Science and Engineering* **17**(3) (2015), 87–96.
[20] S. Sahoo and M. Pal, Intuitionistic fuzzy competition graph, *Journal of Applied Mathematics and Computing* **52** (2015), 37–57.
[21] S. Sahoo and M. Pal, Intuitionistic fuzzy tolerance graph with application, *Journal of Applied Mathematics and Computing* **55** (2016), 495–511.
[22] S. Sahoo and M. Pal, Modular and homomorphic product of intuitionistic fuzzy graphs and their degree, *International Journal of Computing Science and Mathematics* **8**(5) (2017), 395–404.
[23] S. Sahoo and M. Pal, Product of intuitionistic fuzzy graphs and degree, *Journal of Intelligent and Fuzzy Systems* **32**(1) (2016), 1059–1067.
[24] S. Sahoo, M. Pal, H. Rashmanlou and R.A. Borzooei, Covering and paired domination in intuitionistic fuzzy graphs, *Journal of Intelligent and Fuzzy Systems* **33**(6) (2017), 4007–4015.
[25] S. Sahoo, G. Ghorai and M. Pal, Embedding and genus of intuitionistic fuzzy graphs on spheres, *Journal of Mult.-Valued Logic and Soft Computing* **31** (2018), 139–154.
[26] F. Smarandache, *Neutrosophy neutrosophic probability*, Set and Logic, Amer Res Press, Rehoboth, USA, 1998.
[27] F. Smarandache, *A unifying field in logics*, Neutrosophy: Neutrosophic probability, set and logic, American Research Press, Rehoboth, 1999.
[28] F. Smarandache, Refined literal indeterminacy and the multiplication law of sub-indeterminacies, *Neutrosophic Sets and Systems* **9** (2015), 58.
[29] F. Smarandache, Symbolic Neutrosophic Theory, Europanova asbl, Brussels, p. 195.

[30] W.B. Vasantha Kandasamy and K. Ilanthenral, Florentin Smarandache Neutrosophic Graphs, A New Dimension to Graph Theory, Kindle Edition, 2015.

[31] H. Wang, F. Smarandache, Y. Zhang and R. Sunderraman, Single valued neutrosophic sets, *Multisspace and Multistructure* **4** (2010), 410–413.

[32] H. Wang, F. Smarandache, Y.Q. Zhang and R. Sunderraman, Single-valued neutrosophic sets, *Multisspace and Multistruct* **4** (2010), 410–413.

[33] L.A. Zadeh, Fuzzy sets, *Information and Control* **8**(3) (1965), 338–353.