Journal of Innovation and Development
ISSN: 2958-4752 | Vol. 2, No. 3, 2023

A TODIM Method Based on 2- Tuple Linguistic Neutrosophic Numbers and Its Application

Shujue Tang

School of Management, Shanghai University, Shanghai, China

Abstract: This paper proposes a TODIM method based on 2-Tuple Linguistic Neutrosophic Numbers for the decision-making problem of bounded rational online shopping consumers. Firstly, the definition of 2- Tuple Linguistic Neutrosophic sets, normalized Hamming distance and the operation method of traditional TODIM method are introduced. Then the paper puts forward the operation steps of TODIM method based on 2-Tuple Linguistic Neutrosophic Numbers. Finally, an actual example analysis application is carried out to illustrate the operability and rationality of the method. Suggestions on recommendation function of shopping website and online business operators are put forward.

Keywords: Bounded rationality; Multiple attribute group decision making problems; 2-Tuple linguistic neutrosophic numbers; TODIM.

1. Introduction

With the reduction of online payment risks and the booming development of e-commerce, online shopping has become a common choice for most people, and major shopping festivals have emerged, with people's online shopping consumption gradually increasing. In the process of online shopping, many consumers are always hesitant to make a decision among many similar products, or even have bad feelings. Due to the limitations of human cognitive ability, they will eventually grasp a few important information after a mental struggle to make their own satisfaction rather than the best shopping decision.

Classical economics is based on the theory of economic man, which means that man is rational and that he is a subject who carries out economic activities with the sole purpose of pursuing material interests [1], hoping to obtain the maximum economic benefit with the least possible economic effort. However, there are often psychological accounts, endowment effects, risk aversion and other incomplete rational behaviors in the actual decision-making process. The limited rationality theory proposed by Simon is therefore more suitable to explain consumer decision-making behaviors.

According to finite rationality, an individual's mental resources are scarce and there is a tendency for individuals to use fewer mental resources, so individuals can be regarded as 'cognitive misers', that is, people always balance between improving the quality of decision-making and reducing cognitive efforts. This trade-off is not equal. Consumers generally prefer to reduce cognitive efforts rather than improve the quality of decision-making. The satisfaction principle is the core of bounded rationality theory [2].

Lu Yanfeng et al. believe that in the online shopping environment, individuals still follow the minimization of cognitive efforts, and a variety of information in the online shopping information environment has an interactive impact on consumers' information processing methods [2]. In actual decision-making, individuals cannot obtain all alternatives and cannot predict the benefits of all alternatives. Decision preferences are often self-constructed temporarily according to the decision-making situation [3]. Decision seeking is not optimal, but satisfaction. They believe that brand familiarity (consumers' familiarity with the brand), presence (consumers' immersive shopping experience), and online evaluation (online evaluation given by people who have purchased the product) in the online shopping environment are independent variables. The importance of the task (the importance of the decision-making task), consistency (the consistency of the role of brand familiarity and presence) as moderators will affect consumers' final evaluation of the product to varying degrees. Information load (the amount of information faced by consumers in decision-making) will increase consumers' decision-making time, so it is not the greater the better.

Therefore, online shopping decision-making is a typical multi-attribute group decision-making problem, and the information that affects consumers' final evaluation of products and purchase decisions is usually suitable for evaluation in language, such as online evaluation of goods, usually in 'good' or 'poor'. TODIM (an acronym in Portuguese of interactive and multi-attribute decision making) method can well reflect the psychological activity process of consumers, and can effectively deal with the problem of multi-attribute group decision making with uncertain decision information [4]. Therefore, this paper uses the TODIM method to construct the hesitant fuzzy language evaluation model of commodity evaluation, in order to provide reference for the operation of online merchants and the recommendation function of shopping websites.

2. Preliminary knowledge

2.1. Two-tuple linguistic neutrosophic set

Based on the basic theory of two-tuple linguistic fuzzy sets (2TLS) and single-valued neutrosophic sets (SVNS), the definition of two-tuple linguistic neutrosophic sets (2TLNS) proposed by Wang et al.is described as follows.

Definition 1 [5] Let $\eta 1$, $\eta 2$, ..., ηk be a set of language phrases. The label ηi represents a possible semantic variable, and η= {$\eta 0$: quite low, $\eta 1$: very low, $\eta 2$: low, $\eta 3$: general， $\eta 4$: high, $\eta 5$: very high, $\eta 6$: quite high}. The 2TLNSs are described as follows:

$$\eta = \{(s\alpha, \varphi), (s\beta, \phi), (s\chi, \gamma)\} \tag{1}$$

There are $s\alpha, s\beta, s\chi \in \eta$， $\varphi, \phi, \gamma \in [0, 5, 0.5)$. ($s\alpha$, φ),($s\beta$, ϕ) and ($s\chi$, γ) represent real membership, uncertain membership and false membership, respectively. Membership is expressed by binary semantic neutrosophic set. $(s\alpha, \varphi)$, $(s\beta, \phi)$ and $(s\chi, \gamma)$ satisfy the following conditions: $\Delta^{-1}(s\alpha, \varphi)$, $\Delta^{-1}(s\beta, \phi)$, $\Delta^{-1}(s\chi, \gamma) \in [0, k]$, and $0 \le \Delta^{-1}(s\alpha, \varphi) + \Delta^{-1}(s\beta, \phi) + \Delta^{-1}(s\chi, \gamma) \le 3k$.

Definition 2[5] Suppose there are three two-tuple linguistic neutrosophic numbers: $\eta1=\{(s\alpha1,\varphi1),(s\beta1,\phi1),(s\chi1,\gamma1)\}$, $\eta2=\{(s\alpha2,\varphi2),(s\beta2,\phi2),(s\chi2,\gamma2)\}$ and $\eta=\{(s\alpha,\varphi),(s\beta,\phi),(s\chi,\gamma)\}$. The operation is defined as follows:

$$\eta_1 \oplus \eta_2 = \left\{\begin{array}{c}\Delta\left(k\left(\frac{\Delta^{-1}(s_{\alpha1},\varphi_1)}{k}+\frac{\Delta^{-1}(s_{\alpha2},\varphi_2)}{k}-\frac{\Delta^{-1}(s_{\alpha1},\varphi_1)}{k}\cdot\frac{\Delta^{-1}(s_{\alpha2},\varphi_2)}{k}\right)\right),\\ \Delta\left(k\left(\frac{\Delta^{-1}(s_{\beta1},\phi_1)}{k}\cdot\frac{\Delta^{-1}(s_{\beta2},\phi_2)}{k}\right)\right),\Delta\left(k\left(\frac{\Delta^{-1}(s_{\chi1},\gamma_1)}{k}\cdot\frac{\Delta^{-1}(s_{\chi2},\gamma_2)}{k}\right)\right)\end{array}\right\};$$

$$\eta_1 \otimes \eta_2 = \left\{\begin{array}{c}\Delta\left(k\left(\frac{\Delta^{-1}(s_{\alpha1},\varphi_1)}{k}\cdot\frac{\Delta^{-1}(s_{\alpha2},\varphi_2)}{k}\right)\right),\\ \Delta\left(k\left(\frac{\Delta^{-1}(s_{\beta1},\phi_1)}{k}+\frac{\Delta^{-1}(s_{\beta2},\phi_2)}{k}-\frac{\Delta^{-1}(s_{\beta1},\phi_1)}{k}\cdot\frac{\Delta^{-1}(s_{\beta2},\phi_2)}{k}\right)\right),\\ \Delta\left(k\left(\frac{\Delta^{-1}(s_{\chi1},\gamma_1)}{k}+\frac{\Delta^{-1}(s_{\chi2},\gamma_2)}{k}-\frac{\Delta^{-1}(s_{\chi1},\gamma_1)}{k}\cdot\frac{\Delta^{-1}(s_{\chi2},\gamma_2)}{k}\right)\right)\end{array}\right\};$$

$$\lambda\eta = \left\{\Delta\left(k\left(1-\left(1-\frac{\Delta^{-1}(s_\alpha,\varphi)}{k}\right)^\lambda\right)\right),\Delta\left(k\left(\frac{\Delta^{-1}(s_\beta,\phi)}{k}\right)^\lambda\right),\Delta\left(k\left(\frac{\Delta^{-1}(s_\chi,\gamma)}{k}\right)^\lambda\right)\right\},\lambda>0;$$
$$\eta^\lambda = \left\{\Delta\left(k\left(\frac{\Delta^{-1}(s_\alpha,\varphi)}{k}\right)^\lambda\right),\Delta\left(k\left(1-\left(1-\frac{\Delta^{-1}(s_\beta,\phi)}{k}\right)^\lambda\right)\right),\Delta\left(k\left(1-\left(1-\frac{\Delta^{-1}(s_\chi,\gamma)}{k}\right)^\lambda\right)\right)\right\},\lambda>0.$$

According to Definition 2, it is obvious that the operation has the following properties:

$$\eta_1 \oplus \eta_2 = \eta_2 \oplus \eta_1, \eta_1 \otimes \eta_2 = \eta_2 \otimes \eta_1, ((\eta_1)^{\lambda_1})^{\lambda_2} = (\eta_1)^{\lambda_1\lambda_2}; \quad (2)$$

$$\lambda(\eta_1 \oplus \eta_2) = \lambda\eta_1 \oplus \lambda\eta_2, (\eta_1 \otimes \eta_2)^\lambda = (\eta_1)^\lambda \otimes (\eta_2)^\lambda; \quad (3)$$

$$\lambda_1\eta_1 \oplus \lambda_2\eta_1 = (\lambda_1+\lambda_2)\eta_1, (\eta_1)^{\lambda_1} \otimes (\eta_1)^{\lambda_2} = (\eta_1)^{(\lambda_1+\lambda_2)} \quad (4)$$

Definition 3 [5] Let $\eta=\{(s\alpha, \varphi),(s\beta,\phi),(s\chi,\gamma)\}$ be a 2TLNN, the score function and accuracy function of η can be expressed as:

$$s(\eta) = \frac{\left(2k+\Delta^{-1}(s_\alpha,\varphi)-\Delta^{-1}(s_\beta,\phi)-\Delta^{-1}(s_\chi,\gamma)\right)}{3k}, s(\eta) \in [0,1] \quad (5)$$

$$h(\eta) = \Delta^{-1}(s_\alpha,\varphi) - \Delta^{-1}(s_\chi,\gamma), h(\eta) \in [-k,k] \quad (6)$$

For the two two-tuple linguistic neutrosophic numbers $\eta1$ and $\eta2$, based on Definition 3, the comparison rules are as follows:

If $s(\eta_1) \prec s(\eta_2)$, then $\eta_1 \prec \eta_2$;
If $s(\eta_1) \succ s(\eta_2)$, then $\eta_1 \succ \eta_2$;
If $s(\eta_1) = s(\eta_2)$ and $h(\eta_1) \prec h(\eta_2)$, then $\eta_1 \prec \eta_2$;
If $s(\eta_1) = s(\eta_2)$ and $h(\eta_1) \succ h(\eta_2)$, then $\eta_1 \succ \eta_2$;
If $s(\eta_1) = s(\eta_2)$ and $h(\eta_1) = h(\eta_2)$, then $\eta_1 = \eta_2$.

2.2. Normalized Hamming distance

Definition 4 [6] Let $\eta1=\{(s\alpha1, \varphi1),(s\beta1,\phi1),(s\chi1,\gamma1)\}$ and $\eta2=\{(s\alpha2,\varphi2),(s\beta2,\phi2),(s\chi2,\gamma2)\}$ be two 2-tuple linguistic neutrosophic numbers (2TLNNs), then the normalized Hamming distance:

$$d(\eta_1,\eta_2) = \frac{1}{3k}\left(\begin{array}{c}|\Delta^{-1}(s_{\alpha1},\varphi_1)-\Delta^{-1}(s_{\alpha2},\varphi_2)|+|\Delta^{-1}(s_{\beta1},\phi_1)-\Delta^{-1}(s_{\beta2},\phi_2)|\\ |\Delta^{-1}(s_{\chi1},\gamma_1)-\Delta^{-1}(s_{\chi2},\gamma_2)|\end{array}\right) \quad (7)$$

2.3. The original TODIM method

The TODIM method is based on the prospect theory (PT). Considering the subjectivity of the decision maker's behavior, the advantages of each scheme over other schemes are obtained through specific operation formulas, which is more reasonable and scientific in multi-attribute group decision making.

Suppose $\{\eta_1,\eta_2,\ldots,\eta_m\}$ is a set of alternatives and $\{c_1,c_2,\ldots c_n\}$ is a standard list with a weighted vector $\{w_1,w_2,\ldots,w_n\}$ and satisfies $w_i \in [0,1]$ and $\sum_{i=1}^n w_i = 1$. Construct a decision matrix $\eta = [d_{ij}]_{m\times n}$, where d_{ij} denotes the estimated result based on each alternative $\eta_i(i=1,2,\ldots,n)$ of criterion $c_j(j=1,2,\ldots,n)$. Let $w_{jk} = \frac{w_j}{w_k}(0 \le w_{jk} \le 1)$ is the relative weight of c_j to c_t, where $w_k = max(w_j)$ $(k,j=1,2,\ldots,n)$. The traditional TODIM method decision operation steps are as follows:

Steps 1: Standardized decision matrix $\eta = [d_{ij}]_{m\times n}$, which is standardized to $\eta' = [d'_{ij}]_{m\times n}$.

Steps 2: Based on c_j the dominance of η_i relative to each alternative η_t is calculated. Let ρ be the loss attenuation coefficient, then:

$$\delta(\eta_i,\eta_t) = \sum_{j=1}^n \vartheta_j(\eta_i,\eta_t) \ (i,t=1,2,\ldots,m) \quad (8)$$

$$\vartheta_j^{\lambda}(\eta_i,\eta_t) = \begin{cases}\sqrt{\frac{\omega_{jk}(d_{ij}-d_{tj})}{\sum_{j=1}^n w_{jk}}}, d_{ij}-d_{tj}>0\\ 0, d_{ij}-d_{tj}=0\\ -\frac{1}{\rho}\sqrt{\frac{(\sum_{j=1}^n w_{jk})(d_{ij}-d_{tj})}{w_{jk}}}, d_{ij}-d_{tj}<0\end{cases} \quad (9)$$

where $\vartheta_j(\eta_i,\eta_j)(d_{ij}-d_{tj}>0)$ denotes gain and $\vartheta_j(d_{ij}-d_{tj})$ $(d_{ij}-d_{tj}<0)$ denotes loss.

Steps 3: Calculate the overall value of $\delta(\eta_i)$ with formula (10):

$$\delta(\eta_i) = \frac{\sum_{t=1}^m \delta(\eta_i,\eta_t) - \min_i\{\sum_{t=1}^m \delta(\eta_i,\eta_t)\}}{\max_i\{\sum_{t=1}^m \delta(\eta_i,\eta_t)\} - \min_i\{\sum_{t=1}^m \delta(\eta_i,\eta_t)\}} \quad (10)$$

Steps 4: By sorting the value of $\delta(\eta_i)$, the best alternative is selected, and the alternative of the maximum value is the best choice.

3. TODIM method based on 2-tuple linguistic neutrosophic numbers (2TLNNs)

Assume that $\{\eta_1,\eta_2,\ldots,\eta_m\}$ is a set of alternatives, $\{d_1,d_2,\ldots,d_\lambda\}$ is a list of experts with a weighted vector $\{v_1,v_2,\ldots,v_t\}$, $\{c_1,c_2,\ldots,c_t\}$ is a standard list with a weighted vector $\{w_1,w_2,\ldots,w_n\}$, and satisfies $w_i \in [0,1]$, $v_i \in [0,1]$, $\sum_{i=1}^n w_i = 1$ and $\sum_{i=1}^t v_i = 1$. Construct a decision matrix $\eta^\lambda = [r_{ij}{}^\lambda]_{m\times n}$, where $r_{ij}{}^\lambda = \left\{(s_{\alpha_{ij}},\phi_{ij})^\lambda,(s_{\beta_{ij}},\varphi_{ij})^\lambda,(s_{\chi_{ij}},\gamma_{ij})^\lambda\right\}$ represents an estimate of option $\eta_i(i=1,2,\ldots,m)$ by d^λ based on criterion $c_j(j=1,2,\ldots,n)$. $(s_{\alpha_{ij}},\phi_{ij})^\lambda$ indicates True Membership (TMD), $(s_{\beta_{ij}},\varphi_{ij})^\lambda$ indicates Indeterminate Membership (IMD), $(s_{\chi_{ij}},\gamma_{ij})^\lambda$ indicates False Membership (FMD), and

$0 \le \Delta^{-1}\left(s_{\alpha_{ij}}, \varphi_{ij}\right)^{\lambda} + \Delta^{-1}\left(s_{\beta_{ij}}, \phi_{ij}\right)^{\lambda} + \Delta^{-1}\left(s_{\chi_{ij}}, \gamma_{ij}\right)^{\lambda} \le 3k$ $(i = 1,2,\ldots,m;\ j = 1,2,\ldots,n)$.

Let $w_{jk} = \frac{w_j}{w_k}(0 \le w_{jk} \le 1)$ be the relative weight of$c_j$toc_t, where $w_k = max(w_j)$ $(k, j = 1,2,\ldots,n\quad)$.

Considering that the 2TLNNs theory and the traditional TODIM method are based on the prospect theory (PT), we try to propose a TODIM method based on 2TLNNs to effectively solve the multi-attribute group decision making problem. The model is described as follows:

Steps 1: Calculate the value of $w_{jk} = \frac{w_j}{w_k}(0 \le w_{jk} \le 1)$, where $w_k = max(w_j)$ $(k, j = 1,2,\ldots,n\quad)$.

Steps 2: According to the calculation results of the relative weight w_{jk}, we can calculate the dominance of the c_j based on ${\eta_i}^{\lambda}$ of expert d_λ to each alternative ${\eta_t}^{\lambda}$. Let ρ be the loss decay coefficient, then

$$\vartheta_j{}^{\lambda}(\eta_i, \eta_t) = \begin{cases} \sqrt{\dfrac{w_{jk} d({r_{ij}}^{\lambda} - {r_{tj}}^{\lambda})}{\sum_{j=1}^{n} w_{jk}}}, {r_{ij}}^{\lambda} - {r_{tj}}^{\lambda} > 0 \\ 0, {r_{ij}}^{\lambda} - {r_{tj}}^{\lambda} = 0 \\ -\dfrac{1}{\rho}\sqrt{\dfrac{\left(\sum_{j=1}^{n} w_{jk}\right) d({r_{ij}}^{\lambda} - {r_{tj}}^{\lambda})}{w_{jk}}}, {r_{ij}}^{\lambda} - {r_{tj}}^{\lambda} < 0 \end{cases} \tag{11}$$

$$d({r_{ij}}^{\lambda} - {r_{tj}}^{\lambda}) = \frac{1}{3k}\left(\begin{array}{c} \left|\Delta^{-1}\left(s_{\alpha_{ij}}, \phi_{ij}\right)^{\lambda} - \Delta^{-1}\left(s_{\alpha_{tj}}, \phi_{tj}\right)^{\lambda}\right| + \left|\Delta^{-1}\left(s_{\beta_{ij}}, \varphi_{ij}\right)^{\lambda} - \Delta^{-1}\left(s_{\beta_{tj}}, \varphi_{tj}\right)^{\lambda}\right| + \\ \left|\Delta^{-1}\left(s_{\chi_{ij}}, \gamma_{ij}\right)^{\lambda} - \Delta^{-1}\left(s_{\chi_{tj}}, \gamma_{tj}\right)^{\lambda}\right| \end{array} \right), \tag{12}$$

where $\vartheta_j{}^{\lambda}(\eta_i, \eta_j)({r_{ij}}^{\lambda} - {r_{tj}}^{\lambda} > 0)$ denotes gain and $\vartheta_j^{\lambda}(\eta_i, \eta_t)(r_{ij}^{\lambda} - r_{tj}^{\lambda} < 0)$ denotes loss. Based on Definition 4, $d({r_{ij}}^{\lambda} - {r_{tj}}^{\lambda})$ represents the normalized Hamming distance of ${r_{ij}}^{\lambda}$ and ${r_{tj}}^{\lambda}$.

Then we can construct the dominance matrix model $\vartheta_j{}^{\lambda} = \left[\vartheta_j{}^{\lambda}(\eta_i, \eta_t)\right]_{m \times m}$ of expert d_λ under standard c_j to express the calculation results of Formula (11) more clearly.

$$\vartheta_j^{\lambda}(\eta_i, \eta_t) = \begin{array}{c} \\ \eta_1 \\ \eta_2 \\ \vdots \\ \eta_m \end{array} \begin{array}{c} \begin{array}{cccc} \eta_1 & \eta_2 & \cdots & \eta_m \end{array} \\ \begin{bmatrix} 0 & \vartheta_j^{\lambda}(\eta_1, \eta_2) & \cdots & \vartheta_j^{\lambda}(\eta_1, \eta_m) \\ \vartheta_j^{\lambda}(\eta_2, \eta_1) & 0 & \cdots & \vartheta_j^{\lambda}(\eta_2, \eta_m) \\ \vdots & \vdots & 0 & \vdots \\ \vartheta_j^{\lambda}(\eta_m, \eta_1) & \vartheta_j^{\lambda}(\eta_m, \eta_2) & \cdots & 0 \end{bmatrix} \end{array}, \quad j = 1,2,\ldots,n \tag{13}$$

Steps 3: Calculate the overall dominance $\vartheta_j{}^{\lambda} = \left[\vartheta_j{}^{\lambda}(\eta_i, \eta_t)\right]_{m \times m}$ to get the matrix model $\vartheta^{\lambda} = \left[\vartheta^{\lambda}(\eta_i, \eta_t)\right]_{m \times m}$.

$$\vartheta^{\lambda}(\eta_i, \eta_t) = \sum_{j=1}^{n} \vartheta_j{}^{\lambda}(\eta_i, \eta_t)\ (i, t = 1,2,\ldots,m) \tag{14}$$

$$\vartheta^{\lambda}(\eta_i, \eta_t) = \begin{array}{c} \\ \eta_1 \\ \eta_2 \\ \vdots \\ \eta_m \end{array} \begin{array}{c} \begin{array}{cccc} \eta_1 & \eta_2 & \cdots & \eta_m \end{array} \\ \begin{bmatrix} 0 & \vartheta^{\lambda}(\eta_1, \eta_2) & \cdots & \vartheta^{\lambda}(\eta_1, \eta_m) \\ \vartheta^{\lambda}(\eta_2, \eta_1) & 0 & \cdots & \vartheta^{\lambda}(\eta_2, \eta_m) \\ \vdots & \vdots & 0 & \vdots \\ \vartheta^{\lambda}(\eta_m, \eta_1) & \vartheta^{\lambda}(\eta_m, \eta_2) & \cdots & 0 \end{bmatrix} \end{array} \tag{15}$$

Steps 4: The overall advantage $\delta(\eta_i, \eta_t)$ is calculated based on the results of expert weighted vector $\{v_1, v_2, \ldots, v_t\quad\}$ and formula (15).

$$\delta(\eta_i, \eta_t) = \sum_{j=1}^{\lambda} v_\lambda \vartheta^{\lambda}(\eta_i, \eta_t)\ (i, t = 1,2,\ldots,m) \tag{16}$$

The overall advantage $\delta(\eta_i, \eta_t)$ can be constructed using equation (17) as follows:

$$\delta_j(\eta_i, \eta_t) = \begin{array}{c} \\ \eta_1 \\ \eta_2 \\ \vdots \\ \eta_m \end{array} \begin{array}{c} \begin{array}{cccc} \eta_1 & \eta_2 & \cdots & \eta_m \end{array} \\ \begin{bmatrix} 0 & \delta_j(\eta_1, \eta_2) & \cdots & \delta_j(\eta_1, \eta_m) \\ \delta_j(\eta_2, \eta_1) & 0 & \cdots & \delta_j(\eta_2, \eta_m) \\ \vdots & \vdots & 0 & \vdots \\ \delta_j(\eta_m, \eta_1) & \delta_j(\eta_m, \eta_2) & \cdots & 0 \end{bmatrix} \end{array}, \quad j = 1,2,\ldots,n \tag{17}$$

Steps 5: Calculate the overall value of $\delta(\eta_i)$ with formula (18):

$$\delta(\eta_i) = \frac{\sum_{t=1}^{m} \delta(\eta_i, \eta_t) - min\{\sum_{t=1}^{n} \delta(\eta_i, \eta_t)\}}{\underset{i}{max}\{\sum_{t=1}^{m} \delta(\eta_i, \eta_t)\} - \underset{i}{min}\{\sum_{t=1}^{m} \delta(\eta_i, \eta_t)\}} \tag{18}$$

Steps 6: By sorting the values of $\delta(\eta_i)$, the best alternative is selected, and the maximum alternative is the best choice.

4. Simulation example

4.1. The calculation steps based on multi-attribute group decision making problem

Online shopping consumers how to choose the right and satisfactory commodities (solutions). The TODIM scheme evaluation method based on the binary semantic neutrosophic number proposed in this paper is used here for the evaluation of the characteristics of different goods. Through market research and initial screening, there are five potential commodities (solutions) $\eta_i (i = 1,2,3,4,5)$ that need to be evaluated. The decision-making group is composed of three experts $d^k (k = 1,2,3)$. After discussion, the evaluation indicators of the scheme include: G1 (online evaluation), G2 (brand familiarity), G3 (retailer reputation), and G4 (information load). And the weight of the four indicators is $w = (0.4, 0.15, 0.3, 0.15)^T$, the weight of the three experts is $v = (0.45, 0.15, 0.40)^T$. Experts evaluate the candidate solutions based on a pre-defined set of semantic terms, as shown in Table 2-4.

Table 1. Indicator meaning

index	meaning
online evaluation	People who have purchased this product give online evaluation of the product.
brand familiarity	Consumers ' familiarity with product brands
retailer reputation	Consumers ' trust in online merchants
information load	The amount of information faced by consumers in the decision-making process

Steps 1: Calculate the value of w_{jk}: $w_{jk} = w_j / w_k$ $(0 \le w_{jk} \le 1)$, $w_k = max(w_j)$ $(k, j = 1,2,\ldots,n)$.

$$w_k = max(0.4, 0.15, 0.3, 0.15) = 0.4$$

$$w_{jk} = w_j / w_k = (1.000, 0.3750, 0.7500, 0.3750)^T$$

Steps 2: According to the calculation results of the relative weight w_{jk}, we can calculate the dominance η_i^{λ} of scheme η_i relative to each alternative η_t based on the index c_j by the λ expert. The results are as follows: $(\rho = 2.4)$

For expert d_1, calculate the dominance of η_i^1:

$$\vartheta_1^1 = \begin{array}{c} \\ \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \end{array} \begin{array}{c} \begin{array}{ccccc} \eta_1 & \eta_2 & \eta_3 & \eta_4 & \eta_5 \end{array} \\ \begin{bmatrix} 0.0000 & 0.4472 & 0.2981 & 0.3651 & 0.3944 \\ -0.4658 & 0.0000 & -0.3472 & -0.2690 & -0.3106 \\ -0.3106 & 0.3333 & 0.0000 & 0.2108 & 0.2582 \\ -0.3804 & 0.2582 & -0.2196 & 0.0000 & 0.2582 \\ -0.4108 & 0.2981 & -0.2690 & -0.2690 & 0.0000 \end{bmatrix} \end{array}$$

$$\vartheta_2^1 = \begin{array}{cccccc} & \eta_1 & \eta_2 & \eta_3 & \eta_4 & \eta_5 \\ \eta_1 & 0.0000 & 0.2415 & 0.2236 & 0.2415 & 0.2041 \\ \eta_2 & -0.6709 & 0.0000 & -0.2536 & 0.1826 & 0.1826 \\ \eta_3 & -0.6211 & 0.0913 & 0.0000 & 0.2041 & 0.1581 \\ \eta_4 & -0.6709 & -0.5072 & -0.5670 & 0.0000 & -0.3586 \\ \eta_5 & -0.5670 & -0.5072 & -0.4392 & 0.1291 & 0.0000 \end{array}$$

$$\vartheta_3^1 = \begin{array}{cccccc} & \eta_1 & \eta_2 & \eta_3 & \eta_4 & \eta_5 \\ \eta_1 & 0.0000 & 0.3162 & 0.2887 & -0.3106 & 0.2887 \\ \eta_2 & -0.4392 & 0.0000 & -0.3106 & -0.4744 & -0.1793 \\ \eta_3 & -0.4009 & 0.2236 & 0.0000 & -0.3586 & 0.2582 \\ \eta_4 & 0.2236 & 0.3416 & 0.2582 & 0.0000 & 0.3162 \\ \eta_5 & -0.4009 & 0.1291 & -0.3586 & -0.4392 & 0.0000 \end{array}$$

$$\vartheta_4^1 = \begin{array}{cccccc} & \eta_1 & \eta_2 & \eta_3 & \eta_4 & \eta_5 \\ \eta_1 & 0.0000 & 0.1826 & 0.1826 & 0.1581 & 0.2415 \\ \eta_2 & -0.5072 & 0.0000 & -0.7172 & -0.2536 & 0.2415 \\ \eta_3 & -0.5072 & 0.2582 & 0.0000 & 0.2415 & 0.2041 \\ \eta_4 & -0.4392 & 0.0913 & -0.6709 & 0.0000 & 0.2582 \\ \eta_5 & -0.6709 & -0.6709 & -0.5670 & -0.7172 & 0.0000 \end{array}$$

For expert d_2, calculate the dominance of η_i^2:

$$\vartheta_1^2 = \begin{array}{cccccc} & \eta_1 & \eta_2 & \eta_3 & \eta_4 & \eta_5 \\ \eta_1 & 0.0000 & 0.4944 & 0.3944 & 0.3651 & 0.3333 \\ \eta_2 & -0.5150 & 0.0000 & -0.3106 & -0.3472 & -0.3804 \\ \eta_3 & -0.4108 & 0.2981 & 0.0000 & -0.4108 & -0.4392 \\ \eta_4 & -0.3804 & 0.3333 & 0.3944 & 0.0000 & -0.1553 \\ \eta_5 & -0.3472 & 0.3651 & 0.4216 & 0.1491 & 0.0000 \end{array}$$

$$\vartheta_2^2 = \begin{array}{cccccc} & \eta_1 & \eta_2 & \eta_3 & \eta_4 & \eta_5 \\ \eta_1 & 0.0000 & 0.2582 & 0.1826 & 0.1826 & 0.0913 \\ \eta_2 & -0.7172 & 0.0000 & -0.6211 & -0.5072 & -0.6709 \\ \eta_3 & -0.5072 & 0.2236 & 0.0000 & 0.2236 & -0.4392 \\ \eta_4 & -0.5072 & 0.1826 & -0.6211 & 0.0000 & -0.5670 \\ \eta_5 & -0.2536 & 0.2415 & 0.1581 & 0.2041 & 0.0000 \end{array}$$

$$\vartheta_3^2 = \begin{array}{cccccc} & \eta_1 & \eta_2 & \eta_3 & \eta_4 & \eta_5 \\ \eta_1 & 0.0000 & 0.2236 & 0.3162 & 0.2236 & -0.3106 \\ \eta_2 & -0.3106 & 0.0000 & -0.4009 & -0.2536 & -0.3586 \\ \eta_3 & -0.4392 & 0.2887 & 0.0000 & -0.4009 & -0.4744 \\ \eta_4 & -0.3106 & 0.1826 & 0.2887 & 0.0000 & -0.2536 \\ \eta_5 & 0.2236 & 0.2582 & 0.3416 & 0.1826 & 0.0000 \end{array}$$

$$\vartheta_4^2 = \begin{array}{cccccc} & \eta_1 & \eta_2 & \eta_3 & \eta_4 & \eta_5 \\ \eta_1 & 0.0000 & 0.2415 & 0.2041 & 0.1581 & 0.2739 \\ \eta_2 & -0.6709 & 0.0000 & -0.5072 & -0.6211 & 0.1291 \\ \eta_3 & -0.5670 & 0.1826 & 0.0000 & -0.5072 & 0.2236 \\ \eta_4 & -0.4392 & 0.2236 & 0.1826 & 0.0000 & 0.2582 \\ \eta_5 & -0.7607 & -0.3586 & -0.6211 & -0.7172 & 0.0000 \end{array}$$

For expert d_3, calculate the dominance of η_i^3:

$$\vartheta_1^3 = \begin{array}{cccccc} & \eta_1 & \eta_2 & \eta_3 & \eta_4 & \eta_5 \\ \eta_1 & 0.0000 & 0.4472 & 0.3651 & 0.2981 & 0.2582 \\ \eta_2 & -0.4658 & 0.0000 & -0.3472 & -0.4108 & -0.3804 \\ \eta_3 & -0.3804 & 0.3333 & 0.0000 & -0.2196 & -0.3472 \\ \eta_4 & -0.3106 & 0.3944 & 0.2108 & 0.0000 & 0.3333 \\ \eta_5 & -0.2690 & 0.3651 & 0.3333 & -0.3472 & 0.0000 \end{array}$$

$$\vartheta_2^3 = \begin{array}{cccccc} & \eta_1 & \eta_2 & \eta_3 & \eta_4 & \eta_5 \\ \eta_1 & 0.0000 & 0.1826 & 0.1581 & 0.2415 & 0.0913 \\ \eta_2 & -0.5072 & 0.0000 & 0.1581 & 0.2041 & -0.4392 \\ \eta_3 & -0.4392 & -0.4392 & 0.0000 & 0.1826 & -0.3586 \\ \eta_4 & -0.6709 & -0.5670 & -0.5072 & 0.0000 & -0.6211 \\ \eta_5 & -0.2536 & 0.1581 & 0.1291 & 0.2236 & 0.0000 \end{array}$$

$$\vartheta_3^3 = \begin{array}{cccccc} & \eta_1 & \eta_2 & \eta_3 & \eta_4 & \eta_5 \\ \eta_1 & 0.0000 & 0.3651 & 0.2582 & 0.2887 & 0.3162 \\ \eta_2 & -0.5072 & 0.0000 & -0.4392 & -0.3106 & -0.2536 \\ \eta_3 & -0.3586 & 0.3162 & 0.0000 & 0.2236 & 0.2582 \\ \eta_4 & -0.4009 & 0.2236 & -0.3106 & 0.0000 & 0.1291 \\ \eta_5 & -0.4392 & 0.1826 & -0.3586 & -0.1793 & 0.0000 \end{array}$$

$$\vartheta_4^3 = \begin{array}{cccccc} & \eta_1 & \eta_2 & \eta_3 & \eta_4 & \eta_5 \\ \eta_1 & 0.0000 & 0.1826 & -0.3586 & 0.1826 & -0.4392 \\ \eta_2 & -0.5072 & 0.0000 & -0.6211 & -0.5072 & -0.5670 \\ \eta_3 & 0.1291 & 0.2236 & 0.0000 & 0.1826 & 0.1581 \\ \eta_4 & -0.5072 & 0.1826 & -0.5072 & 0.0000 & -0.4392 \\ \eta_5 & 0.1581 & 0.2041 & -0.4392 & 0.1581 & 0.0000 \end{array}$$

Steps 3: The total dominance of $\vartheta_j^\lambda = \left[\vartheta_j^\lambda(\eta_i,\eta_t)\right]_{m\times m}$ is calculated to obtain matrix $\varphi^\lambda = \left[\varphi^\lambda(\phi_i,\phi_t)\right]_{m\times m}$.

$$\vartheta^1 = \begin{array}{cccccc} & \eta_1 & \eta_2 & \eta_3 & \eta_4 & \eta_5 \\ \eta_1 & 0.0000 & 1.1875 & 0.9930 & 0.4542 & 1.1287 \\ \eta_2 & -2.0831 & 0.0000 & -1.6286 & -0.8144 & -0.0658 \\ \eta_3 & -1.8398 & 0.9064 & 0.0000 & 0.2979 & 0.8786 \\ \eta_4 & -1.2669 & 0.1839 & -1.1993 & 0.0000 & 0.4740 \\ \eta_5 & -2.0497 & -0.7508 & -1.6338 & -1.2963 & 0.0000 \end{array}$$

$$\vartheta^2 = \begin{array}{cccccc} & \eta_1 & \eta_2 & \eta_3 & \eta_4 & \eta_5 \\ \eta_1 & 0.0000 & 1.2177 & 1.0973 & 0.9294 & 0.3879 \\ \eta_2 & -2.2137 & 0.0000 & -1.8398 & -1.7291 & -1.2808 \\ \eta_3 & -1.9242 & 0.9930 & 0.0000 & -1.0953 & -1.1292 \\ \eta_4 & -1.6373 & 0.9221 & 0.2445 & 0.0000 & -0.7177 \\ \eta_5 & -1.1379 & 0.5063 & 0.3002 & -0.1814 & 0.0000 \end{array}$$

$$\vartheta^3 = \begin{array}{cccccc} & \eta_1 & \eta_2 & \eta_3 & \eta_4 & \eta_5 \\ \eta_1 & 0.0000 & 1.1775 & 0.4229 & 1.0109 & 0.2265 \\ \eta_2 & -1.9873 & 0.0000 & -1.2494 & -1.0244 & -1.6402 \\ \eta_3 & -1.0491 & 0.4340 & 0.0000 & 0.3692 & -0.2895 \\ \eta_4 & -1.8896 & 0.2336 & -1.1140 & 0.0000 & -0.5979 \\ \eta_5 & -0.8036 & 0.9100 & -0.3354 & -0.1448 & 0.0000 \end{array}$$

Steps 4: Based on expert weight vectors (0.45,0.15,0.40) and the results of $\vartheta^\lambda = \left[\vartheta^\lambda(\eta_i,\eta_t)\right]_{m\times m}$, the total dominance $\delta(\eta_i,\eta_t)$ is calculated.

$$\delta(\eta_i,\eta_t) = \begin{array}{cccccc} & \eta_1 & \eta_2 & \eta_3 & \eta_4 & \eta_5 \\ \eta_1 & 0.0000 & 1.1881 & 0.7806 & 0.7482 & 0.6567 \\ \eta_2 & -2.0644 & 0.0000 & -1.5086 & -1.0356 & -0.8778 \\ \eta_3 & -1.5362 & 0.7304 & 0.0000 & 0.1174 & 0.1102 \\ \eta_4 & -1.5715 & 0.3145 & -0.9486 & 0.0000 & -0.1335 \\ \eta_5 & -1.4145 & 0.1021 & -0.8243 & -0.6685 & 0.0000 \end{array}$$

Steps 5: Calculate the overall value of $\delta(\eta_i)$ with formula (18):

$\delta(\eta_1) = 1.0000$,$\delta(\eta_2) = 0.0000$,$\delta(\eta_3) = 0.5540$,$\delta(\eta_4) = 0.3552$,$\delta(\eta_5) = 0.3026$

Steps 6: The values of $\delta(\eta_i)$ are sorted to select the best alternative, and the alternative with the maximum value is the best choice. According to step 5, η_i is sorted as $\eta_1 > \eta_3 > \eta_4 > \eta_5 > \eta_2$, and η_1 is obviously the best choice.

Table 2. The 2TLNNs evaluation matrix given by expert d^1

	G_1	G_2	G_3	G_4
η_1	$\{(s_5,0),(s_1,0),(s_2,0)\}$	$\{(s_4,0),(s_3,0),(s_1,0)\}$	$\{(s_4,0),(s_2,0),(s_1,0)\}$	$\{(s_5,0),(s_1,0),(s_2,0)\}$
η_2	$\{(s_2,0),(s_4,0),(s_5,0)\}$	$\{(s_3,0),(s_1,0),(s_5,0)\}$	$\{(s_2,0),(s_3,0),(s_4,0)\}$	$\{(s_2,0),(s_1,0),(s_3,0)\}$
η_3	$\{(s_4,0),(s_3,0),(s_3,0)\}$	$\{(s_3,0),(s_1,0),(s_4,0)\}$	$\{(s_2,0),(s_1,0),(s_3,0)\}$	$\{(s_5,0),(s_4,0),(s_1,0)\}$
η_4	$\{(s_3,0),(s_4,0),(s_3,0)\}$	$\{(s_2,0),(s_4,0),(s_5,0)\}$	$\{(s_5,0),(s_1,0),(s_2,0)\}$	$\{(s_2,0),(s_1,0),(s_2,0)\}$
η_5	$\{(s_4,0),(s_5,0),(s_4,0)\}$	$\{(s_2,0),(s_3,0),(s_4,0)\}$	$\{(s_3,0),(s_3,0),(s_4,0)\}$	$\{(s_4,0),(s_4,0),(s_5,0)\}$

Table 3. The 2TLNNs evaluation matrix given by expert d^2

	G_1	G_2	G_3	G_4
η_1	$\{(s_5,0),(s_1,0),(s_1,0)\}$	$\{(s_5,0),(s_1,0),(s_2,0)\}$	$\{(s_3,0),(s_3,0),(s_1,0)\}$	$\{(s_4,0),(s_2,0),(s_1,0)\}$
η_2	$\{(s_1,0),(s_4,0),(s_5,0)\}$	$\{(s_2,0),(s_4,0),(s_4,0)\}$	$\{(s_3,0),(s_4,0),(s_3,0)\}$	$\{(s_2,0),(s_4,0),(s_4,0)\}$
η_3	$\{(s_5,0),(s_4,0),(s_5,0)\}$	$\{(s_3,0),(s_2,0),(s_1,0)\}$	$\{(s_2,0),(s_1,0),(s_4,0)\}$	$\{(s_4,0),(s_5,0),(s_3,0)\}$
η_4	$\{(s_2,0),(s_1,0),(s_4,0)\}$	$\{(s_5,0),(s_4,0),(s_3,0)\}$	$\{(s_4,0),(s_3,0),(s_3,0)\}$	$\{(s_5,0),(s_2,0),(s_3,0)\}$
η_5	$\{(s_2,0),(s_1,0),(s_3,0)\}$	$\{(s_4,0),(s_1,0),(s_2,0)\}$	$\{(s_5,0),(s_3,0),(s_2,0)\}$	$\{(s_1,0),(s_4,0),(s_5,0)\}$

Table 4. The 2TLNNs evaluation matrix given by expert d^3

	G_1	G_2	G_3	G_4
η_1	$\{(s_4,0),(s_2,0),(s_1,0)\}$	$\{(s_5,0),(s_3,0),(s_2,0)\}$	$\{(s_4,0),(s_1,0),(s_1,0)\}$	$\{(s_3,0),(s_2,0),(s_2,0)\}$
η_2	$\{(s_1,0),(s_4,0),(s_5,0)\}$	$\{(s_2,0),(s_3,0),(s_1,0)\}$	$\{(s_3,0),(s_4,0),(s_5,0)\}$	$\{(s_2,0),(s_4,0),(s_3,0)\}$
η_3	$\{(s_5,0),(s_4,0),(s_4,0)\}$	$\{(s_3,0),(s_4,0),(s_2,0)\}$	$\{(s_2,0),(s_1,0),(s_3,0)\}$	$\{(s_4,0),(s_1,0),(s_2,0)\}$
η_4	$\{(s_5,0),(s_4,0),(s_2,0)\}$	$\{(s_2,0),(s_4,0),(s_5,0)\}$	$\{(s_3,0),(s_2,0),(s_4,0)\}$	$\{(s_2,0),(s_1,0),(s_4,0)\}$
η_5	$\{(s_3,0),(s_2,0),(s_3,0)\}$	$\{(s_4,0),(s_3,0),(s_2,0)\}$	$\{(s_3,0),(s_3,0),(s_4,0)\}$	$\{(s_2,0),(s_1,0),(s_1,0)\}$

4.2. The influence of loss attenuation coefficient ρ

In the process of using the TODIM method based on the binary semantic neutrosophic number, by changing the loss attenuation coefficient ρ, the overall dominance and ranking results of each scheme are calculated as follows.

According to the calculation results in Table 5, by changing the value of ρ, we can easily determine that the best alternative is η_1.

Table 5. Overall dominance and ranking of schemes for different loss attenuation coefficients ρ

ρ	$\delta(\eta_1)$	$\delta(\eta_2)$	$\delta(\eta_3)$	$\delta(\eta_4)$	$\delta(\eta_5)$	Ordering
1.0	1.0000	0.0000	0.5732	0.3434	0.3096	$\eta_1 > \eta_3 > \eta_4 > \eta_5 > \eta_2$
1.1	1.0000	0.0000	0.5715	0.3445	0.3089	$\eta_1 > \eta_3 > \eta_4 > \eta_5 > \eta_2$
1.2	1.0000	0.0000	0.5698	0.3455	0.3083	$\eta_1 > \eta_3 > \eta_4 > \eta_5 > \eta_2$
1.5	1.0000	0.0000	0.5652	0.3483	0.3067	$\eta_1 > \eta_3 > \eta_4 > \eta_5 > \eta_2$
1.7	1.0000	0.0000	0.5624	0.3501	0.3056	$\eta_1 > \eta_3 > \eta_4 > \eta_5 > \eta_2$
2.0	1.0000	0.0000	0.5585	0.3524	0.3043	$\eta_1 > \eta_3 > \eta_4 > \eta_5 > \eta_2$
2.3	1.0000	0.0000	0.5551	0.3546	0.3030	$\eta_1 > \eta_3 > \eta_4 > \eta_5 > \eta_2$
2.5	1.0000	0.0000	0.5529	0.3559	0.3022	$\eta_1 > \eta_3 > \eta_4 > \eta_5 > \eta_2$
3.0	1.0000	0.0000	0.5482	0.3588	0.3005	$\eta_1 > \eta_3 > \eta_4 > \eta_5 > \eta_2$
4.0	1.0000	0.0000	0.5405	0.3635	0.2977	$\eta_1 > \eta_3 > \eta_4 > \eta_5 > \eta_2$

According to Table 5, with the change of attenuation loss coefficient ρ, the ranking of alternatives does not change, and the best choice is still η_1, indicating that the value of attenuation loss coefficient ρ is not sensitive to the calculation results of TODIM method based on binary semantic neutrosophic number. Compared with the original TODIM method, the TODIM method based on binary semantic neutrosophic numbers can capture the psychological behavior of decision makers and reflect the bounded rationality of decision makers, which is more accurate and reasonable in the application of multi-attribute group decision making problems.

5. Conclusion

In this paper, we propose a TODIM method based on binary semantic neutrosophic number theory and traditional TODIM method. Firstly, we introduce the definition of binary semantic neutrosophic set, normalized Hamming distance and the operation method of traditional TODIM method. Then we propose the operation steps of TODIM method based on binary semantic neutrosophic number. Finally, we carry out a practical example analysis and application, which shows that this method has strong feasibility and effectiveness, and has certain methodological significance. For online business operators, in order to allow consumers to quickly choose their own suitable and satisfactory goods, businesses in the provision of product information is not the more the better, to ensure that goods cannot cheat consumers, establish a good reputation in the minds of consumers, and gradually establish their own brand, store image. For the recommendation function of the website, after obtaining the category of goods that users may be interested in according to the user 's browsing history and operation behavior, considering that consumers are limited rational, the products in the same category can be inferred through the above method. More satisfied goods, then these goods should be recommended to users first to improve the accuracy of the shopping website recommendation function.

References

[1] ZONG Ji-chuan, ZHU Ting-ting. Bounded rationality triumphs over perfect rationality again -- on the contribution of Richard Thaler, winner of the 2017 Nobel Prize in Economics [J]. Journal of Dongbei University of Finance and Economics, 2018, (1): 13-22.

[2] FAN Xiao-ping,LU Yan-feng,HAN Hong-ye. The Effect of Online Shopping Information Context on Consumers' Decision-Making: A Perspective of Bounded Rationality [J]. Journal of Industrial Engineering/Engineering Management, 2016, 30(2): 38-47.

[3] Lu Yanfeng, FAN Xiaoping, SUN Jiaqi. The Impact of Online Shopping Multiple Cues Context on Consumer's Products Evaluation: A Perspective of Bounded Rationality [J]. Chinese Journal of Management, 2016,13(10): 1546-1556.

[4] KONG Ling-yan, TAN Qian-yun. Hesitant fuzzy language TODIM method and its application [J]. Statistics & Decision, 2017(5): 98-100.

[5] WANG Jie, WEI Gui-wu, WEI Yu. Models for Green Supplier Selection with Some 2-Tuple Linguistic Neutrosophic Number Bonferroni Mean Operators [J]. Symmetry, 2018,10(5): 131-166.

[6] WANG Jie, WEI Gui-wu, LU Mao. TODIM Method for Multiple Attribute Group Decision Making Under 2-Tuple Linguistic Neutrosophic Environment [J]. Symmetry, 2018, 10(10): 486-501.