Neutrosophical fuzzy modeling and optimization approach for multiobjective four-index transportation problem

Firoz Ahmad[1,2*], Florentin Smarandache[3] and Arup Kumar Das[1]
[1]SQC & OR Unit, Indian Statistical Institute, Kolkata, India.
[2]Department of Management Studies, Indian Institute of Science, Bangalore, India.
[3]Department of Mathematics and Science, University of New Mexico, Gallup, NM, USA
*Corresponding e-mail: firoz.ahmad02@gmail.com

Abstract

This study investigates a four-index multiobjective transportation problem (F-IMOTPs) with uncertain supply and demand coverage. Different echelons having uncertain parameters' values are considered. An inter-connected multi-product F-IMOTPs is assumed for the smooth flow of items, enhancing supply chain reliability under uncertainty. A mixed-integer multiobjective programming problem that minimizes total transportation costs, time, safety costs, and carbon emissions abatement is depicted under an intuitionistic fuzzy environment. Further, three different interactive approaches, namely extended fuzzy programming approach (EFPA), extended intuitionistic fuzzy programming approach (EIFPA), and extended neutrosophic programming approach (ENPA), are developed to solve the proposed F-IMOTPs model. Different membership functions elicit each objective's marginal evaluation. The proposed F-IMOTPs model is implemented in a logistic company and solved using three interactive approaches that reveal the proposed methods' validity and applicability. An ample opportunity to generate the compromise solution is suggested by tuning various weight parameters. The outcomes are evaluated with practical managerial implications based on the significant findings. Finally, conclusions and future research scope are addressed based on the proposed work. The discussed F-IMOTPs model can be merged with and extended by considering inventory and supply chain facilities, which are not included in this study. Uncertainty among parameters due to randomness can be incorporated and tackled with historical data. Besides the proposed conventional solution methods, various metaheuristic approaches may be applied to solve the proposed F-IMOTPs model as a future research scope. The strategy advised is to provide an opportunity to create valuable decision-making policies within India by helping existing transportation networks, safety features, and imports only if necessary to meet timelines. The reduction in carbon emissions abatement also ensures less burden on environmental impacts. Thus, any logistics/transportation company or organization can adopt the distribution management initiatives amongst the supply and demand points to strengthen and enable the company to handle the uncertainties. Finally, managers or policy-makers can take advantage of the current study and extract fruitful information and knowledge regarding the optimal distribution strategies while making decisions. This research work manifests the supply-demand oriented extension of the integrated F-IMOTPs model design with minimum total transportation costs, time, safety costs, and carbon emissions abatement under flexible uncertainty. The practical managerial implications are explored that immensely support the managers or practitioners to adopt the distribution policies for the PIs to ensure sustainability in the designed F-IMOTPs model.

Keywords: Intuitionistic fuzzy parameters; Conventional optimization methods; Neutrosophic set theory; Multiobjective transportation problem.

1 Introduction

Transportation problem is a special case of linear programming problem. The objective is to determine the amount that should be transported from each source to each destination, so that the total transportation cost is minimized. It consists with a linear objective function and linear constraints. In this article we consider to model multiobjective transportation problem using fuzzy set theory. We face many situations where more than one conflicting/non-conflicting objectives are to be optimized under a set of well-defined constraints. In optimization theory, this class of problems is known as multiobjective programming problems (MOPP) and identified as an important class of optimization problem. Because of the presence of multiple objective functions, the problems become harder to obtain a single solution that satisfies each objective function efficiently. Instead, attempts are taking to obtain the compromised solution sets which satisfy each objective function marginally. A multiobjective optimization problem (MOOP) refers to obtain a solution $x \in G \subset R^E$ which minimizes an objective function vector $f : G \rightarrow R^H$ such that G denotes the $E-$dimensional solution space, and R^H represents the $H-$dimensional objective space. Most commonly, the sole target of MOOPs is to determine a set of non-dominated solution which attains the approximates of Pareto front in the same objective spaces. Mathematically, MOOPs can be expressed as follows:

$$\begin{aligned} &\underset{\zeta \in R^n}{\text{Min}} F(\zeta) = [f_1(\zeta), f_2(\zeta), \cdots, f_m(\zeta)] \in R^m \\ &\text{Subject to} \\ &\qquad p(\zeta) \leq 0 \\ &\qquad q(\zeta) = 0 \\ &\qquad \zeta_{ri} \leq \zeta_i \leq \zeta_{si}, \;\; i = [1, 2, \cdots, n] \end{aligned} \tag{1}$$

where $\zeta = [\zeta_1, \zeta_1, \cdots, \zeta_n]$ is defined as the decision variables, $F(\zeta)$ is the objective vector, $p(\zeta)$ represents the inequality and $q(\zeta)$ denotes equality constraint vectors, respectively, ζ_{ri} and ζ_{si} are the lower and upper bounds in the decision space of the ζ_i variable. The solutions methods can classified into three broad categories namely classical technique, fuzzy-based solution approach, and nature-inspired algorithm. In this context we mention that vector optimization is a subclass of optimization problems with a vector-valued objective function for a given partial ordering. A multi-objective optimization problem is a special case of a vector optimization problem. The classical techniques contemplate the use of priority information while optimizing the MOOPs. Various methods such as Weighted sum method, $\epsilon-$Constraint method, Weighted metric method, Benson's method, Value function method and Goal programming method. The Weighted sum method is based on the working principle that the objectives are transformed into a single objective by multiplying the pre-determined weight. The $\epsilon-$Constraint method resolves the problems that are encountered while the weighted sum method is applied. It alleviates obtaining the solution having non-convex objective spaces by solving the single objectives and keeping the objectives within a well-specified value. The weighted metric method considers the metrics such as l_p and l_∞ distance metrics are commonly used in place of the weighted sum of the objectives. Hence the weighted metric methods convert the multiple objectives into a single objective. The weighted metric method and the Bensons method are similar to each other except that the reference solution is obtained as the feasible non-Pareto optimal solution. The value function methods determine the mathematical value function $U : R^M \rightarrow R$, concerning all objectives. The validity of the value function should be over the whole feasible solution search space. The goal programming technique tries to search the pre-targeted values of one or more than one objective function at a time. When no solution attains the pre-specified target values, the task is to determine such a solution that minimizes deviations from the targets. If a solution with desired target values exists, then the task is to determine that specific solution. For more details, visit Das and Jana [17], Mohan et al. [22] etc.

The fuzzy programming approach (FPA) is basically concerned with maximizing satisfaction degree for the decision-maker(s) while dealing with multiple objectives simultaneously. In last several decades, a tremendous amount of research was presented based on the fuzzy decision set. The limitation of the fuzzy set has been examined because it cannot define the non-membership function of the element into the fuzzy set. The intuitionistic fuzzy programming approach (IFPA) is a more flexible and realistic optimization technique compared to the fuzzy technique because it deals with the membership function and the non-membership function of the element into a feasible decision set.

Therefore, an efficient algorithm is needed to solve the MOPP. The fuzzy set (FS) was initially proposed by Zadeh et al. [32], and later on, it was extensively used in multiple criteria, multiple attributes, and multiobjective decision-making problems. Afterward, Zimmermann [37] investigated the fuzzy programming technique for the multiobjective optimization problem, which was based on the membership function for the marginal evaluation of each objective function. Therefore, the fuzzy set's extension was first presented by Atanassov [15] which is based on more intuition compared to the fuzzy set and termed as the intuitionistic fuzzy set (IFS). Later on, the potential applications of IFS have been presented in many decision-making processes and emerged as useful tools while dealing with uncertainty.

Based on IFS, Angelov [14] first addressed the intuitionistic fuzzy programming approach (IFPA) for real-life decision-making problems. Peng and Yang [24] also obtained some useful results based on the Pythagorean fuzzy set for multi-attribute decision-making problems. Peng and Selvachandran [23] addressed some well-known results and also discussed some future direction of research-based Pythagorean fuzzy set. Wan et al. [30] also presented the Pythagorean fuzzy mathematical optimization technique for multi-attribute group decision-making problem under the Pythagorean fuzzy scenario. Zhang and Xu [33] developed a new model for multiple criteria decision-making problem under Pythagorean fuzzy environment and also proposed a technique for order preference by similarity to ideal solution (TOPSIS) method to determine the degree of closeness to the ideal solution. Unlike IFS, the flexible nature of PFS would be immensely adopted for further research scope. Ye [31] presented a study on multi-attribute decision-making method with the single-valued neutrosophic hesitant fuzzy information. Zhang et al. [34] addressed a multiple criteria decision-making problem with the hesitant fuzzy information regarding the values of different parameters. Zhou and Xu [36] also presented a portfolio optimization technique under a hesitant fuzzy environment.

Alcantud and Torra [13] have proposed some decomposition theorem and extensions principle for the hesitant fuzzy set. Bharati [16] suggested a hesitant fuzzy technique to solve the multiobjective optimization problem. Faizi et al. [18] also discussed multiple criteria decision-making problems under hesitant fuzzy set theory. Lan et al. [20] extended the application of hesitant fuzzy set to hesitant preference degree in multiple attribute decision-making problem. Liu and Luo [21] also proposed a new aggregation operator based on a neutrosophic hesitant fuzzy set and applied it to multiple attribute decision making. Zhang [35] also discussed hesitant fuzzy multi-criteria decision-making problem under the unknown weight information. Akram et al. [10] suggested a novel model based on the combination of hesitant fuzzy set and N-soft set. The application of hesitant N-soft set in group decision-making problem is also presented. Alcantud and Santos-García [12] also performed a study on expanded hesitant fuzzy sets along with the application in group decision-making problems. Alcantud and Giarlotta [11] also investigated a new model for group decision-making problems with the aid of necessary and possible hesitant fuzzy sets.

Nature-inspired algorithms are categorized into three different approaches namely, aggregating functions, population-based approaches and Pareto-based methods. The aggregation functions convert all the objective functions into a single objective employing some arithmetical operations. These methods contain the linear aggregation functions, which make it trivial and not that much impressive. Often, the population-based approaches are based on the EA's population to initiates the search. A Vector Evaluated Genetic Algorithm considered the conventional example of population-based approaches. At each generation, sub-populations are generated by proportional selection. For example, if the population size is N and n is the total number of objectives, the sub-population size will be N/n. The population-based optimization method is straightforward to apply, but the main difficulty is to find the appropriate selection scheme, which is not based on Pareto-optimality. Pareto-based methods are the most popular and extensively used techniques, which are divided into two different generations. The first generation comprises the fitness sharing, niching combined with Pareto ranking, second generation with elitism.

Motivation and research contribution

The TPs are well-known in the field of continuous optimization. Various advance mathematical programming models are presented in the literature based on the different scenarios and assumptions. The best estimation of uncertain parameters is always a challenging task in TPs because it decides the optimal allocation policies of the products and determines the optimal transportation cost and other objectives defined in the model. One more essential issue arising these days is the existence of multiple objectives in the TPs. More than one commensurable and conflicting objectives are to be optimized, such as transportation time, carbon emissions, and the transportation cost that addresses TPs sustainability. Thus, we have presented the F-IMOTPs under an intuitionistic fuzzy environment. Due to incomplete, inconsistent, and inappropriate information or knowledge, the different parameters are represented by intuitionistic fuzzy numbers comprising the membership grades, covering a wide vagueness. The extended version of various conventional approaches such as EFPA, EIFPA, and ENPA is developed to solve the F-IMOTPs. The robustness of the solution approaches has been established with the help of solution results. The ample opportunity to obtain different solution sets has been introduced for decision-makers or managers by tuning the feasibility degree (λ). The selection of the best compromise solution set among multiple outcomes has been determined by the fuzzy TOPSIS ranking method. A numerical study of Indian-based transportation companies has been done along with the significant findings.
The remaining portion of the article is structured as follows. In Section 2, some definitions, results and the concept regarding the intuitionistic fuzzy parameters are presented which will be used in the subsequent sections. Section 3 contains several novel modeling approaches of F-IMOTP along with the related results. The extended version of solution approaches are proposed in Section 4. In Section 5, a numerical example is presented to illustrate the proposed methods. The conclusions and future research scope are given in Section 6.

2 Preliminaries

In this section we consider some definitions, results and concepts which are used in the following sections.
Definition 1: [15] (Intuitionistic fuzzy set) Assume that there be a univeral set X. Then, an intuitionistic fuzzy set (IFS) $\widetilde{Y}$ in X is defined by the ordered triplets as follows:

$$\widetilde{Y} = \{x,\ \mu_{\widetilde{Y}}(x),\ \ \nu_{\widetilde{Y}}(x) \mid x \in\ X\}$$

where $\mu_{\widetilde{Y}}(x) : X \rightarrow [0,\ 1]$ denotes the membership function and $\nu_{\widetilde{Y}}(x) : X \rightarrow [0,\ 1]$ denotes the non-membership function of the element $x \in X$ into the set $\widetilde{Y}$, respectively, with the conditions $0 \leq \mu_{\widetilde{Y}}(x) + \nu_{\widetilde{Y}}(x) \leq 1$. The value of $\phi_{\widetilde{Y}}(x) = 1 - \mu_{\widetilde{Y}}(x) - \nu_{\widetilde{Y}}(x)$, is called the degree of uncertainty of the element $x \in X$ to the IFS $\widetilde{Y}$. If $\phi_{\widetilde{Y}}(x) = 0$, an IFS chnages into fuzzy set and becomes $\widetilde{Y} = \{x,\ \mu_{\widetilde{Y}}(x),\ 1 - \mu_{\widetilde{Y}}(x) \mid x \in\ X\}$.

Definition 2: [5] (Intuitionistic fuzzy number) An IFS $\widetilde{Y} = \{x,\ \mu_{\widetilde{Y}}(x),\ \ \nu_{\widetilde{Y}}(x) \mid x \in\ X\}$ is said to be an intuitionistic fuzzy number iff

1. There exists a real number $x_0 \in \mathbb{R}$ for which $\mu_{\widetilde{Y}}(x) = 1$ and $\nu_{\widetilde{Y}}(x) = 0$.
2. The membership function $\mu_{\widetilde{Y}}(x)$ of $\widetilde{Y}$ is fuzzy convex and non-membership function $\nu_{\widetilde{Y}}(x)$ of $\widetilde{Y}$ is fuzzy concave.
3. Also, $\mu_{\widetilde{Y}}(x)$ is upper semi-continuous and $\nu_{\widetilde{Y}}(x)$ is lower semi-continuous.
4. The support of $\widetilde{Y}$ is depicted as $\big(x \in \mathbb{R} : \nu_{\widetilde{Y}}(\mathrm{x}) \leq 1\big)$.

Definition 3: [5] (Triangular intuitionistic fuzzy number) A triangular intuitionistic fuzzy number (TrIFN) is represented by $\widetilde{Y} = ((y_1, y_2, y_3); (z_1, y_2, z_3))$ where $z_1, y_1, y_2, y_3, z_3 \in \mathbb{R}$ such that $z_1 \leq y_1 \leq y_2 \leq y_3 \leq z_3$; and its membership function $\mu_{\widetilde{Y}}(x)$ and non-membership function $\nu_{\widetilde{Y}}(x)$ is of the form

$$\mu_{\widetilde{Y}}(x) = \begin{cases} \dfrac{x - y_1}{y_2 - y_1}, & \text{if } y_1 < x < y_2, \\ 1, & \text{if } x = y_2, \\ \dfrac{y_3 - x}{y_3 - y_2}, & \text{if } y_2 < x < y_3, \\ 0, & \text{if otherwise.} \end{cases} \quad \text{and } \nu_{\widetilde{Y}}(x) = \begin{cases} \dfrac{y_2 - x}{y_2 - z_1}, & \text{if } z_1 < x < y_2,, \\ 0, & \text{if } x = y_2, \\ \dfrac{x - y_2}{z_3 - y_2}, & \text{if } y_2 < x < z_3, \\ 1, & \text{if otherwise.} \end{cases}$$

Definition 4: [5] Consider that a TrIFN is given by $\widetilde{Y} = ((y_1, y_2, y_3); (z_1, y_2, z_3))$ where $z_1, y_1, y_2, y_3, z_3 \in \mathbb{R}$ such that $z_1 \leq y_1 \leq y_2 \leq y_3 \leq z_3$. Then the parametric form of $\widetilde{Y}$ are $u(\tau) = \left(\overline{u(\tau)}, \underline{u(\tau)}\right)$ and $v(\tau) = \left(\overline{v(\tau)}, \underline{v(\tau)}\right)$. Further, $u(\tau)$ and $v(\tau)$ are the parametric form of TrIFN corresponding to membership and non-membership functions such that $\overline{u(\tau)} = y_3 - \tau(y_3 - y_1)$, $\underline{u(\tau)} = y_1 - \tau(y_2 - y_1)$ and $\overline{v(\tau)} = y_2 - (1 - \tau)(y_2 - z_1)$, $\underline{v(\tau)} = y_2 + (1 - \tau)(z_3 - y_2)$ respectively. A TrIFN $\widetilde{Y} = ((y_1, y_2, y_3); (z_1, y_2, z_3))$ is said to be positive TrIFN if $z_1 > 0$ and hence y_1, y_2, y_3, z_3 are all positive numbers.

Remark 1: Assume that $\widetilde{Y} = ((y_1, y_2, y_3); (z_1, y_2, z_3))$ and $\widetilde{W} = ((w_1, w_2, w_3); (v_1, w_2, v_3))$ are two TrIFNs. Then addition of $\widetilde{Y}$ and $\widetilde{W}$ is again a TrIFN.

$$\widetilde{Y} + \widetilde{W} = [(y_1 + w_1, y_2 + w_2, y_3 + w_3)\,; (z_1 + v_1, y_2 + w_2, z_3 + v_3)]$$

Remark 2: Consider that $\widetilde{Y} = ((y_1, y_2, y_3); (z_1, y_2, z_3))$ be a TrIFN and $k \in \mathbb{R}$. Then scaler multiplication of $\widetilde{Y}$ is again a TrIFN.

$$k(\widetilde{Y}) = \begin{cases} (ky_1, ky_2, ky_3; kz_1, ky_2, kz_3) & k > 0 \\ (ky_3, ky_2, ky_1; kz_3, ky_2, kz_1) & k < 0 \\ (0, 0, 0; 0, 0, 0), & k = 0 \end{cases}$$

Remark 3: The two TrIFNs $\widetilde{Y} = ((y_1, y_2, y_3); (z_1, y_2, z_3))$ and $\widetilde{W} = ((w_1, w_2, w_3); (v_1, w_2, v_3))$ are said to be equal iff $y_1 = w_1, y_2 = w_2, y_3 = w_3; z_1 = v_1, y_{2=w_2}, z_3 = v_3$.

Definition 5: [9] (Expected interval and expected value of TrIFNs) Suppose that $\widetilde{Y} = ((y_1, y_2, y_3); (z_1, y_2, z_3))$ be a TrIFN and EI^μ and EI^ν depict the expected intervals for membership and non-membership functions respectively. Thus, these can be defined as follows:

$$\begin{aligned} EI^\mu(\widetilde{Y}) &= \left[\int_0^1 \underline{u(\tau)} d_k\tau, \int_0^1 \overline{u(\tau)} d_k\tau\right] \\ &= \left[\int_0^1 y_3 - \tau(y_3 - y_1) d_k\tau, \int_0^1 y_1 - \tau(y_2 - y_1) d_k\tau\right] \end{aligned}$$

$$\begin{aligned} EI^\nu(\widetilde{Y}) &= \left[\int_0^1 \underline{v(\tau)} d_k\tau, \int_0^1 \overline{v(\tau)} d_k\tau\right] \\ &= \left[\int_0^1 y_2 - (1-\tau)(y_2 - z_1) d_k\tau, \int_0^1 y_2 + (1-\tau)(z_3 - y_2) d_k\tau\right] \end{aligned}$$

Moreover, consider that $EV^\mu(\widetilde{Y})$ and $EV^\nu(\widetilde{Y})$ represent the expected values corresponding to membership and non-membership functions respectively. These can be depicted as follows:

$$EV^\mu(\widetilde{Y}) = \frac{\int_0^1 \underline{u(\tau)} d_k\tau + \int_0^1 \overline{u(\tau)} d_k\tau}{2} = \frac{y_1 + 2y_2 + y_3}{4} \tag{2}$$

$$EV^\nu(\widetilde{Y}) = \frac{\int_0^1 \underline{v(\tau)} d_k\tau + \int_0^1 \overline{v(\tau)} d_k\tau}{2} = \frac{z_1 + 2y_2 + z_3}{4} \tag{3}$$

The expected value EV of a TrIFN $\widetilde{Y} = ((y_1, y_2, y_3); (z_1, y_2, z_3))$ is given as follows:

$$EV(\widetilde{Y}) = \psi EV^\mu(\widetilde{Y}) + (1 - \psi) EV^\nu(\widetilde{Y}), \text{ where } \psi \in [0,\ 1]$$

Definition 6: [6] (Accuracy function) The expected value (EV) for TrIFN

$$\widetilde{Y} = ((y_1, y_2, y_3); (z_1, y_2, z_3))$$

with the help of Eqs. (2) and (3) and for $\psi = 0.5$ can be represented as follows:

$$EV(\widetilde{Y}) = \frac{y_1 + y_3 + 4y_2 + z_1 + z_3}{8}$$

Thus $EV(\widetilde{Y})$ is also known as accuracy function of $\widetilde{Y}$.

Theorem 1: [26] Suppose that $\widetilde{Y}$ be a TrIFN. Then for any $EV : IF(\mathbb{R}) \rightarrow \mathbb{R}$; the expected value $EV(k\widetilde{A}) = kEV(\widetilde{A})$ for all $k \in \mathbb{R}$.

Theorem 2: [26] Suppose that $\widetilde{Y}$ and $\widetilde{W}$ be two TrIFNs. Then the accuracy function $EV : IF(\mathbb{R}) \rightarrow \mathbb{R}$ is a linear function i.e., $EV(\widetilde{Y} + k\widetilde{W}) = EV(\widetilde{Y}) + kEV(\widetilde{W}) \forall k \in \mathbb{R}$.

Theorem 3: [26] Suppose that $\widetilde{Y} = ((y_1, y_2, y_3); (z_1, y_2, z_3))$ be a TrIFN. If $z_1 = y_1, z_3 = y_3$, then $EV(\widetilde{Y}) = \dfrac{y_1 + 2y_2 + y_3}{4}$, represents a defuzzified value of triangular fuzzy number.

Theorem 4: [26] The expected value $EV(k) = k$, where $k \in \mathbb{R}$.

Definition 7: [3] (Neutrosophic set) Suppose $x \in X$ denotes the universal discourse. A neutrosophic set (NS) A in X can be depicted by the truth $\mu_A(x)$, indeterminacy $\lambda_A(x)$ and a falsity $\nu_A(x)$ membership functions and is expressed as follows:

$$A = \{< x, \mu_A(x), \lambda_A(x), \nu_A(x) > | x \in X\}$$

where $\mu_A(x), \lambda_A(x)$ and $\nu_A(x)$ are real standard or non-standard subsets belong to $]0^-, 1^+[$, also given as, $\mu_A(x) : X \rightarrow]0^-, 1^+[$, $\lambda_A(x) : X \rightarrow]0^-, 1^+[$, and $\nu_A(x) : X \rightarrow]0^-, 1^+[$. Also, the sum of $\mu_A(x), \lambda_A(x)$ and $\nu_A(x)$ is free from all restrictions. Thus, we have

$$0^- \leq \ sup\ \mu_A(x) + \lambda_A(x) + \ sup\ \nu_A(x) \leq 3^+$$

Definition 8: [3] A NS is said to be single valued neutrosophic set A if the following condition will holds:

$$A = \{< x, \mu_A(x), \lambda_A(x), \nu_A(x) > | x \in X\}$$

where $\mu_A(x), \lambda_A(x)$ and $\nu_A(x) \in [0, 1]$ and $0 \leq \mu_A(x) + \lambda_A(x) + \nu_A(x) \leq 3$ for each $x \in X$.

Definition 9: [5] The union of two single valued neutrosophic sets A and B is also a single valued neutrosophic set C, i.e., $C = (A \cup B)$ with the truth $\mu_C(x)$, indeterminacy $\lambda_C(x)$ and falsity $\nu_C(x)$ membership functions as follows:

$$\mu_C(x) = \max\ (\mu_A(x),\ \mu_B(x))$$
$$\lambda_C(x) = \max\ (\lambda_A(x),\ \lambda_B(x))$$

$$\nu_C(x) = \min\left(\nu_A(x),\ \nu_B(x)\right)$$

for each $x \in X$.

Definition 10: [5] The intersection of two single valued neutrosophic sets A and B is also a single valued neutrosophic set C, i.e., $C = (A \cap B)$ with the truth $\mu_C(x)$, indeterminacy $\lambda_C(x)$ and falsity $\nu_C(x)$ membership functions as follows:

$$\mu_C(x) = \min\left(\mu_A(x),\ \mu_B(x)\right)$$
$$\lambda_C(x) = \min\left(\lambda_A(x),\ \lambda_B(x)\right)$$
$$\nu_C(x) = \max\left(\nu_A(x),\ \nu_B(x)\right)$$

for each $x \in X$.

3 Multiobjective transportation problem

Transportation problems (TPs) are concerned with the transporting of different kinds of products from one place to another to achieve the optimal prescribed objective(s). The classical transportation model can be defined as follows:

$$Min\ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \qquad i = 1, 2, 3, \ldots, m$$
$$\sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1, 2, 3, \ldots, n$$
$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j, \quad x_{ij} \geq 0, \ \forall i \ \&\ j$$

Ahmad and Adhami [3] presented the multiobjective transportation model under fuzziness. The performances of the fuzzy approach are analyzed using the family of the distance function. Adhami and Ahmad [1] also solved the multiobjective transportation problem using the interactive fuzzy programming approaches. Singh and Yadav [28] represented the cost parameter with the triangular intuitionistic fuzzy number, and the ordering of fuzzy number have used to develop intuitionistic fuzzy modified distribution method with the help of accuracy function for finding the optimal solution of TPs. Singh and Yadav [27] used the ranking function to deal with all uncertain parameters and consequently proposed an intuitionistic fuzzy method to find the initial basic feasible solution of TPs. Jana [19] solved a type-2 intuitionistic fuzzy transportation problem by the ranking function for the mean interval method by taking all the parameters type-2 intuitionistic fuzzy number.Here, we propose a new F-IMOTP with k $(= 1, 2, 3, ..., K)$ objectives which are to be optimized under each m origins having $a_i(i = 1, 2, 3, ..., m)$ units of availability and to be transported among n destinations having $b_j(j = 1, 2, 3, ..., n)$ units of demand level. The different cost associated with the k objectives is represented as c_{ijk}. A decision variable x_{ijk} is defined, which is an unknown quantity and are to be transported from i^{th} origin to j^{th} destination in such a way that the total transportation cost, labor cost, and safety cost is minimum. The useful notations are summarized in Table 1.

Table 1: Notions and descriptions

Indices	Descriptions
i	Represents the sources
j	Represents the destinations
k	Represents the conveyance
g	Represents the types of products
Decision variable	
x^g_{ijk}	Unit quantity of products
y^g_{ijk}	Binary variable such that $y^g_{ijk} = \begin{cases} 1, & x^g_{ijk} > 0 \\ 0, & x^g_{ijk} = 0 \end{cases}$
Parameters	
$\widetilde{c}^g_{ijk}$	Unit transportation cost
$\widetilde{p}^g_k$	Unit penalty cost
$\widetilde{t}^g_{ijk}$	Unit transportation time
$\widetilde{s}^g_{ijk}$	The safety factor
$\widetilde{sc}^g_{ijk}$	Unit safety cost
$\widetilde{ce}^g_{ijk}$	Unit carbon emissions cost
$\widetilde{a}^g_i$	Total availabiltiy
$\widetilde{b}^g_j$	Total demand
$\widetilde{e}^g_i$	Total coveyance capacity
B_j	Total budget
B	Desired safety value

So, the mathematical model for F-IMOTPs can be given as follows (4):

$$Min\ \widetilde{Z}_1^{IF} = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{g=1}^{G}\widetilde{c}_{ijk}^{g}x_{ijk}^{g} \quad (Transportation\ cost)$$

$$Min\ \widetilde{Z}_2^{IF} = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{g=1}^{G}\widetilde{t}_{ijk}^{g}y_{ijk}^{g} \quad (Transportation\ time)$$

$$Min\ \widetilde{Z}_3^{IF} = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{g=1}^{G}\widetilde{sc}_{ijk}^{g}y_{ijk}^{g} \quad (Safety\ cost)$$

$$Min\ \widetilde{Z}_4^{IF} = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{g=1}^{G}\widetilde{p}_{k}^{g}\left(\widetilde{ce}_{ijk}^{g}x_{ijk}^{g}\right) \quad (Carbon\ emissions)$$

Subject to

$$\begin{aligned}
&\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{g=1}^{G}x_{ijk}^{g} \le \widetilde{a}_i^{g}, \quad i = 1,2,3,\dots,m\\
&\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{g=1}^{G}x_{ijk}^{g} \ge \widetilde{b}_j^{g}, \quad j = 1,2,3,\dots,n\\
&\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{g=1}^{G}x_{ijk}^{g} \le \widetilde{e}_i^{g}, \quad i = 1,2,3,\dots,m\\
&\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{g=1}^{G}\widetilde{c}_{ijk}^{g}x_{ijk}^{g} \le B_j, \quad j = 1,2,3,\dots,n\\
&\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{g=1}^{G}\widetilde{s}_{ijk}^{g}y_{ijk}^{g} > B, \quad i = 1,2,3,\dots,m\\
&\sum_{i=1}^{m}\widetilde{a}_i \ge \sum_{j=1}^{n}\widetilde{b}_j, \quad x_{ij} \ge 0, \ \forall i\ \&\ j
\end{aligned} \tag{4}$$

where notations $(\widetilde{.})$ over different parameters represents the triangular intuitionistic fuzzy number for all indices' set.
The equivalent intuitionistic fuzzy multiobjective transprotation problem (4) can be sum-

marized as follows (5):

$$
\begin{array}{ll}
\text{Min} & \widetilde{Z}^{IF}(x) = \left[\widetilde{Z}_1^{IF}(x_{ijk}^g), \widetilde{Z}_2^{IF}(x_{ijk}^g), \widetilde{Z}_3^{IF}(x_{ijk}^g), \widetilde{Z}_4^{IF}(x_{ijk}^g)\right] \\
\text{Subject to} & \\
& \sum_{j=1}^{J} \widetilde{A}_{ij} x_{ijk}^g \geq \widetilde{B}_i, \;\; i = 1, 2, \cdots, I_1, \\
& \sum_{j=1}^{J} \widetilde{A}_{ij} x_{ijk}^g \leq \widetilde{B}_i, \;\; i = I_1 + 1, I_1 + 2, \cdots, I_2, \\
& \sum_{j=1}^{J} \widetilde{A}_{ij} x_{ijk}^g = \widetilde{B}_i, \;\; i = I_2 + 1, I_2 + 2, \cdots, I. \\
& x_{ijk}^g \geq 0, \;\; j = 1, 2, \cdots, J.
\end{array}
\tag{5}
$$

where $\widetilde{Z}_k^{IF}(x) = \sum_{k=1}^{4} (\cdot)^{IF} x_{ijk}^g, \;\; \forall \;\; k = 1, 2, \cdots, 4$ is the k-th objective function with trapeziodal intuitionistic fuzzy parameters.
With the aid of accuracy function (Theorem 1) which is linear, the intuitionistic fuzzy programming problem (IFMOPP) (5) can be converted into the following deterministic MOPP (6):

$$
\begin{array}{ll}
\text{Min} & Z^{'}(x_{ijk}^g) = \left[Z_1^{'}(x_{ijk}^g), Z_2^{'}(x_{ijk}^g), Z_3^{'}(x_{ijk}^g), Z_4^{'}(x_{ijk}^g)\right] \\
\text{Subject to} & \\
& \sum_{j=1}^{J} A_{ij}^{'} x_{ijk}^g \geq B_i^{'}, \;\; i = 1, 2, \cdots, I_1, \\
& \sum_{j=1}^{J} A_{ij}^{'} x_{ijk}^g \leq B_i^{'}, \;\; i = I_1 + 1, I_1 + 2, \cdots, I_2, \\
& \sum_{j=1}^{J} A_{ij}^{'} x_{ijk}^g = B_i^{'}, \;\; i = I_2 + 1, I_2 + 2, \cdots, I. \\
& x_{ijk}^g \geq 0, \;\; j = 1, 2, \cdots, J.
\end{array}
\tag{6}
$$

where $Z_k^{'}(x_{ijk}^g) = EV\left(\widetilde{Z}_k^{IF}(x_{ijk}^g)\right) = \sum_{k=1}^{K} EV\left(\widetilde{c}_{ijk}\right) x_{ijk}^g, \; \forall \; k = 1, 2, \cdots, K; \; B_i^{'} = EV\left(\widetilde{B}_i\right)$ and $A_{ij}^{'} = EV\left(\widetilde{A}_{ij}\right)$, for all $i = 1, 2, \cdots, I, \;\; j = 1, 2, \cdots, J$ are the crisp version of all the objective functions and parameters.

Of particular interest, we have proven the existence of an efficient solution of the problem (5) and the convexity property of crisp MOPP (6) in Theorems 5 and 6, respectively.

Definition 13: Assume that X be the set of feasible solution for the crisp MOPP (6). Then a point x^* is said to be an efficient or Pareto optimal solution of the crisp MOPP (6) if and only iff there does not exist any $x \in X$ such that, $O_k(x^*) \geq O_k(x), \;\; \forall \;\; k = 1, 2, \cdots, 4$ and $O_k(x^*) > O_k(x)$ for all at least one $\forall \;\; k = 1, 2, \cdots, 4$. Here, k is the number of objective function present in the crisp MOPP (6).

Definition 14: A point $x^* \in X$ is said to be weak Pareto optimal solution for the crisp MOPP (6) iff there does not exist any $x \in X$ such that, $O_k(x^*) \geq O_k(x), \;\; \forall \;\; k = 1, 2, \cdots, 4$.

We prove the following theorem to establish the existence of efficient solution which has one-one correspondence between MOPP and IFMOPP.

Theorem 5: An efficient solution of the crisp MOPP (6) is also an efficient solution for the IFMOPP (5).

Proof: Consider that $x \in X$ be an efficient solution of the crisp MOPP (6). Then X is also feasible for the crisp MOPP (6). It means that the following condition will hold:

$$\begin{aligned}
&\textstyle\sum_{j=1}^{J} A_{ij}^{'} x_{ijk}^{g} \geq B_{i}^{'}, \;\; i = 1, 2, \cdots, I_1, \\
&\textstyle\sum_{j=1}^{J} A_{ij}^{'} x_{ijk}^{g} \leq B_{i}^{'}, \;\; i = I_1 + 1, I_1 + 2, \cdots, I_2, \\
&\textstyle\sum_{j=1}^{J} A_{ij}^{'} x_{ijk}^{g} = B_{i}^{'}, \;\; i = I_2 + 1, I_2 + 2, \cdots, I. \\
&x_{ijk}^{g} \geq 0, \;\; j = 1, 2, \cdots, J.
\end{aligned}$$

Since it is proven that EV is a linear function (Theorem 2), we have

$$\begin{aligned}
&\textstyle\sum_{j=1}^{J} EV\left(\widetilde{A}_{ij}\right) x_{ijk}^{g} \geq EV\left(\widetilde{B}_{i}\right), \;\; i = 1, 2, \cdots, I_1, \\
&\textstyle\sum_{j=1}^{J} EV\left(\widetilde{A}_{ij}\right) x_{ijk}^{g} \leq EV\left(\widetilde{B}_{i}\right), \;\; i = I_1 + 1, I_1 + 2, \cdots, I_2, \\
&\textstyle\sum_{j=1}^{J} EV\left(\widetilde{A}_{ij}\right) x_{ijk}^{g} = EV\left(\widetilde{B}_{i}\right), \;\; i = I_2 + 1, I_2 + 2, \cdots, I. \\
&x_{j} \geq 0, \;\; j = 1, 2, \cdots, J.
\end{aligned}$$

Consequently, we have

$$\begin{aligned}
&\textstyle\sum_{j=1}^{J} \widetilde{A}_{ij} x_{ijk}^{g} \leq \widetilde{B}_{i}, \;\; i = I_1 + 1, I_1 + 2, \cdots, I_2, \\
&\textstyle\sum_{j=1}^{J} \widetilde{A}_{ij} x_{ijk}^{g} = \widetilde{B}_{i}, \;\; i = I_2 + 1, I_2 + 2, \cdots, I. \\
&x_{ijk}^{g} \geq 0, \;\; j = 1, 2, \cdots, J.
\end{aligned}$$

Hence, X is a feasible solution for the IFMOPP (5).
Moreover, since X is an efficient solution for the crisp MOPP (6), there does not exist any $X^* = \left(x_1^*, x_2^*, \cdots, x_n^*\right)$ such that $Z_k(X^*) \leq Z_k(X) \; \forall \;\; k = 1, 2, \cdots, 4$ and $Z_k(X^*) < Z_k(X)$ for at least one $k = 1, 2, \cdots, 4$. Thus we have no X^* such that Min $\sum_{k=1}^{K} EV\left(\widetilde{Z}_k(X)\right) \leq$ Min $\sum_{k=1}^{K} EV\left(\widetilde{Z}_k(X^*)\right) \; \forall \; k = 1, 2, \cdots, 4$ and Min $\sum_{k=1}^{K} EV\left(\widetilde{Z}_k(X)\right) <$ Min $\sum_{k=1}^{K} EV\left(\widetilde{Z}_k(X^*)\right) \; \forall \; k = 1, 2, \cdots, 4$ for at least one $k = 1, 2, \cdots, 4$.
Since EV is a linear function (Theorem 2), we have no X^* such that Min $\sum_{k=1}^{K} EV\left(\widetilde{Z}_k(X)\right) \leq$ Min $\sum_{k=1}^{K} EV\left(\widetilde{Z}_k(X^*)\right) \; \forall \; k = 1, 2, \cdots, 4$ and Min $\sum_{k=1}^{K} EV\left(\widetilde{Z}_k(X)\right) <$ Min $\sum_{k=1}^{K} EV\left(\widetilde{Z}_k(X^*)\right) \; \forall \; k = 1, 2, \cdots, 4$ for at least one $k = 1, 2, \cdots, 4$. Thus X is an efficient solution for the IFMOPP (5). ■

We propose the following model which is equivalent the crisp MOPP.

Let Z_1 and Z_2 be comonotonic functions, then for any intuitionistic fuzzy parameter $\widetilde{Y}$, we have

$$EV\left[Z_1(\widetilde{Y}) + Z_2(\widetilde{Y})\right] = EV\left[Z_1(\widetilde{Y})\right] + EV\left[Z_2(\widetilde{Y})\right]$$

For the sake of simplicity, let us consider an auxiliary model (7) which is an equivalent to the crisp MOPP (6) and can be given as follows:

$$\begin{array}{ll} \text{Min} & EV\left[Z(X,\widetilde{Y})\right] \ = \left(EV\left[Z_1(X,\widetilde{Y})\right], \ldots, EV\left[Z_k(X,\widetilde{Y})\right]\right) \ \forall\ k = 1,2,3,4. \\ \text{Subject to} & \\ & \sum_{j=1}^{J} A'_{ij} x^{g}_{ijk} \geq B'_{i}, \ \ i = 1,2,\cdots,I_1, \\ & \sum_{j=1}^{J} A'_{ij} x^{g}_{ijk} \leq B'_{i}, \ \ i = I_1+1, I_1+2,\cdots,I_2, \\ & \sum_{j=1}^{J} A'_{ij} x^{g}_{ijk} = B'_{i}, \ \ i = I_2+1, I_2+2,\cdots,I. \\ & x^{g}_{ijk} \geq 0, \ \ j = 1,2,\cdots,J. \end{array} \tag{7}$$

Where $EV[\cdot]$ in auxiliary model (7) represents the expected values (accuracy function) of the intuitionistic fuzzy parameters.

In Theorem 5, we have already proven the expected value EV efficient solution for the IFMOPP (5). This concept is obtained by presenting the crisp MOPP (6), which comprise the expected value of intuitionistic fuzzy uncertain objectives of the IFMOPP (5).
Intuitionally, if the intuitionistic fuzzy uncertain vectors in the auxiliary model (7) degenerate into intuitionistic fuzzy parameters, then the following convexity Theorem 6 of the auxiliary model (7) can be proved.

Theorem 6: Suppose that the function $Z(X,\widetilde{Y})$ is differentiable and a convex vector function with respect to X and $\widetilde{Y}$. Thus, for any given $X_1,\ X_2 \in X$, if $Z_k(X_1,\widetilde{Y})$ and $Z_k(X_2,\widetilde{Y})$ are comonotonic on intuitionistic fuzzy parameters $\widetilde{Y}$, then the auxiliary model (7) is a convex programming problem.

Proof: Since, the feasible solution set X is a convex set, intuitionally, it is sufficient to obtain that the auxiliary model (7) is a convex vector function.
Note that the $Z(X,\widetilde{Y})$ is a convex vector function on X for any given $\widetilde{Y}$, the inequality

$$Z\left(\delta X_1 + (1-\delta)X_2, \widetilde{Y}\right) \leqq \delta Z(X_1,\widetilde{Y}) + (1-\delta)Z(X_2,\widetilde{Y})$$

holds for any $\delta \in [0.1]$ and $X_1,\ X_2 \in X$, i.e;

$$Z_k\left(\delta X_1 + (1-\delta)X_2, \widetilde{Y}\right) \leqq \delta Z_k(X_1,\widetilde{Y}) + (1-\delta)Z_k(X_2,\widetilde{Y})$$

holds for each $k,\ 1 \leq k \leq 4$.
By using the assumed condition that $Z_k(X_1, \widetilde{Y})$ and $Z_k(X_2, \widetilde{Y})$ are comonotonic on $\widetilde{Y}$, it follows from Definition 13 that

$$EV\left[Z_k\left(\delta X_1 + (1-\delta)X_2, \widetilde{Y}\right)\right] \leqq \delta EV\left[Z_k(X_1, \widetilde{Y})\right] + (1-\delta)EV\left[Z_k(X_2, \widetilde{Y})\right],\ \forall\ k;$$

which implies that

$$EV\left[Z\left(\delta X_1 + (1-\delta)X_2, \widetilde{Y}\right)\right] \leqq \delta EV\left[Z(X_1, \widetilde{Y})\right] + (1-\delta)EV\left[Z(X_2, \widetilde{Y})\right]$$

The above inequality shows that $EV\left[Z(X, \widetilde{Y})\right]$ is a convex vector function. Hence the auxiliary model (7) is a convex programming problem. Consequently, the crisp MOPP (6) is also a convex programming problem. Thus Theorem 6 is proved. ■

4 Solution approach

4.1 Extended Fuzzy Programming Approach

Based on fuzzy set theory [37], fuzzy programming is developed to solve the multiobjective optimization problem. The fuzzy programming approach (FPA) deals with the degree of belongingness (membership function) lying between 0 to 1. It shows the marginal evaluation of each objective function into the feasible solution sets. The membership functions can be defined by a mapping function (say $\mu(Z_k) \rightarrow [0,1]|\lambda \in [0,1]$) that assigned the values between 0 to 1 to each objective function which shows the decision makers' preferences have been fulfilled up to λ level of satisfaction. Mathematically, it can be expressed as follows:

$$\mu(Z_k(x)) = \begin{cases} 1 & if\ \ Z_k(x) < L_k \\ \frac{U_k - Z_k(x)}{U_k - L_k} & if\ \ L_k \leq Z_k(x) \leq U_k \\ 0 & if\ \ Z_k(x) > U_k \end{cases}$$

where U_k and L_k are the lower and upper bound for each objective function and obtained by the minimization and maximization of each objective function individually.

Hence, the mathematical formulation of EFPA to solve the transportation problem can be represented as below:

$$\begin{aligned} Max\ \psi(x)\ & = \lambda\,(\alpha) + (1-\lambda)\left(\textstyle\sum_{k=1}^{4} \mu_k(Z_k(x))\right) \\ \text{Subject to}\ & \\ & \mu_k(Z_k(x)) \geq \alpha, \\ & \alpha \geq 0,\ \ 0 \leq \alpha \leq 1, \\ & \lambda \in [0,1] \\ & \text{constraints (4)} \end{aligned} \qquad (8)$$

where $\mu_k(Z_k(x))$ represents the membership degree of k-th objective function and α depicts the satisfaction level for each objective function and provides a compromise solution under the given set of constraints under fuzzy environment.

We prove that the unique optimal solution of LTMFA is efficient.
Theorem 7: A unique optimal solution of problem (8) (LTMFA) is also an efficient solution for the problem (5).

Proof: Suppose that $(\bar{x}, \bar{\alpha})$ be a unique optimal solution of problem (8) (LTMFA). Then, $(\bar{\alpha}) > (\alpha)$ for any (x, α) feasible to the problem (8) (LTMFA). On the contrary, assume that $(\bar{x}, \bar{\alpha},)$ is not an efficient solution of the crisp IP-TPP (8). For that, there exists x^* $(x^* \neq \bar{x})$ feasible to the crisp IP-TPP (8), such that $O_m(x^*) \leq O_m(\bar{x})$ for all $m = 1, 2, \cdots, M$ and $O_m(x^*) < O_m(\bar{x})$ for at least one m.
Therefore, we have $\frac{O_m(x^*)-L_m}{U_m-L_m} \leq \frac{O_m(\bar{x})-L_m}{U_m-L_m}$ for all $m = 1, 2, \cdots, M$ and $\frac{O_m(x^*)-L_m}{U_m-L_m} < \frac{O_m(\bar{x})-L_m}{U_m-L_m}$ for at least one m.
Hence, $\max_m \left(\frac{O_m(x^*)-L_m}{U_m-L_m}\right) \leq (<) \max_m \left(\frac{O_m(\bar{x})-L_m}{U_m-L_m}\right) = \bar{\alpha}$.
Assume that $\alpha^* = \min_m \left(\frac{U_m-O_m(x^*)}{U_m-L_m}\right)$, this gives $(\bar{\alpha}) < (\alpha^*)$ which means that the solution is not unique optimal. Thus, we have arrived at a contradiction with the fact that $(\bar{x}, \bar{\alpha})$ is the unique optimal solution of (LTMFA). Therefore, it is also an efficient solution for the problem (5). This completes the proof of Theorem 7. ■

4.2 Extended Intuitionistic Fuzzy Programming Approach

Based on intuitionistic fuzzy set theory [15], intuitionistic fuzzy programming is developed to solve the multiobjective optimization problem. The intuitionistic fuzzy programming approach (IFPA) deals with the degree of belongingness (membership function) and degree of non-belongingness (non-membership function) simultaneously, lying between 0 to 1. It shows the marginal evaluation of each objective function into the feasible solution sets. The membership functions can be defined by a mapping function (say $\mu(Z_k), \nu(Z_k) \rightarrow [0, 1] \mid \alpha, \beta \in [0, 1]$) that assigned the values between 0 to 1 to each objective function which shows the decision makers' preferences have been fulfilled up to $(\alpha - \beta)$ level of satisfaction. Mathematically, it can be expressed as follows:

$$\mu(Z_k(x)) = \begin{cases} 1 & if \;\; Z_k(x) < L_k \\ \frac{U_k - Z_k(x)}{U_k - L_k} & if \;\; L_k \leq Z_k(x) \leq U_k \\ 0 & if \;\; Z_k(x) > U_k \end{cases}$$

and

$$\nu(Z_k(x)) = \begin{cases} 0 & if \;\; Z_k(x) < L_k \\ \frac{Z_k(x) - L_k}{U_k - L_k} & if \;\; L_k \leq Z_k(x) \leq U_k \\ 1 & if \;\; Z_k(x) > U_k \end{cases}$$

Therefore the mathematical formulation of EIFPA to solve the transportation problem can be represented as below:

$$\begin{array}{ll} Max\ \psi(x) & = \lambda\,(\alpha-\beta) + (1-\lambda)\sum_{k=1}^{K}\left(\mu_k(Z_k(x)) - \nu_k(Z_k(x))\right) \\ \text{Subject to} & \\ & \mu_k(Z_k(x)) \geq \alpha, \\ & \nu_k(Z_k(x)) \leq \beta, \\ & \alpha \geq \beta,\ \ 0 \leq \alpha+\beta \leq 1, \\ & \lambda \in [0,1] \\ & \text{constraints (4)} \end{array} \tag{9}$$

where $\mu_k(Z_k(x))$ and $\nu_k(Z_k(x))$ represent the satisfaction and dissatisfaction degrees of k-th objective function under intuitionistic fuzzy environment. Also, $\alpha = \min\,[\mu_k(Z_k(x))]$ and $\beta = \max\,[\nu_k(Z_k(x))]$ denote the minimum satisfaction and maximum dissatisfaction degrees of each objectives, respectively. Moreover, $(\alpha-\beta)$ represents the overall degree of satisfaction for each objective function and provides a compromise solution under the given set of constraints.

Theorem 8: A unique optimal solution of problem (9) (LTMFA) is also an efficient solution for the problem (5).

Proof: Suppose that $\left(\bar{x},\bar{\alpha},\bar{\beta}\right)$ be a unique optimal solution of problem (9) (LTMFA). Then, $\left(\bar{\alpha}-\bar{\beta}\right) > (\alpha-\beta)$ for any (x,α,β) feasible to the problem (9) (LTMFA). On the contrary, assume that $\left(\bar{x},\bar{\alpha},\bar{\beta}\right)$ is not an efficient solution of the crisp IP-TPP (9). For that, there exists x^* $(x^* \neq \bar{x})$ feasible to the crisp IP-TPP (9), such that $O_m(x^*) \leq O_m(\bar{x})$ for all $m = 1,2,\cdots,M$ and $O_m(x^*) < O_m(\bar{x})$ for at least one m.
Therefore, we have $\frac{O_m(x^*)-L_m}{U_m-L_m} \leq \frac{O_m(\bar{x})-L_m}{U_m-L_m}$ for all $m = 1,2,\cdots,M$ and $\frac{O_m(x^*)-L_m}{U_m-L_m} < \frac{O_m(\bar{x})-L_m}{U_m-L_m}$ for at least one m.
Hence, $\max\limits_m\left(\frac{O_m(x^*)-L_m}{U_m-L_m}\right) \leq (<) \max\limits_m\left(\frac{O_m(\bar{x})-L_m}{U_m-L_m}\right)$.
Suppose that that $\beta^* = \max\limits_m\left(\frac{U_m-O_m(x^*)}{U_m-L_m}\right)$, then $\beta^* \leq (<)\,\bar{\beta}$.
In the same manner, we have $\frac{U_m-O_m(x^*)}{U_m-L_m} \geq \frac{U_m-O_m(\bar{x})}{U_m-L_m}$ for all $m = 1,2,\cdots,M$ and $\frac{U_m-O_m(x^*)}{U_m-L_m} > \frac{U_m-O_m(\bar{x})}{U_m-L_m}$ for at least one m.
Thus, $\min\limits_m\left(\frac{U_m-O_m(x^*)}{U_m-L_m}\right) \geq (>) \min\limits_m\left(\frac{U_m-O_m(\bar{x})}{U_m-L_m}\right)$.
Assume that $\alpha^* = \min\limits_m\left(\frac{U_m-O_m(x^*)}{U_m-L_m}\right)$, this gives $\left(\bar{\alpha}-\bar{\beta}\right) < (\alpha^*-\beta^*)$ which means that the solution is not unique optimal. Thus, we have arrived at a contradiction with the fact that $\left(\bar{x},\bar{\alpha},\bar{\beta}\right)$ is the unique optimal solution of (LTMFA). Therefore, it is also an efficient solution for the problem (5). This completes the proof of Theorem 8. ■

4.3 Extended Neutrosophic Programming Approach

In reality, the characteristic of indeterminacy is the most trivial concern in the decision-making process. It seldom happens that DM(s) has(have) neutral thoughts about any specific value about membership degree of the element into feasible decision set. In this situation, indeterminacy/neutral values are possible to assign it. Inspired with such cases, Smarandache [29] proposed a set named neutrosophic set (NS), which is the extension of FS and IFS. The NS deals with an indeterminacy degree of the element into a feasible decision set. The neutrosophic programming approach (NPA) has been widely used by researchers. Ahmad et al. [4] have proposed a new computational algorithm based on a single-valued neutrosophic hesitant fuzzy decision set and applied it to the multiobjective nonlinear optimization problem of the manufacturing system. Ahmad et al. [6] and Ahmad et al. [5] have also addressed modified neutrosophic fuzzy optimization technique for multiobjective programming problem under uncertainty. For more details, please visit [2**?**] Thus, the upper and lower bound for each objective as given below:

$$U_k = max[Z_k(X^k)] \;\; and \;\; L_k = min[Z_k(X^k)] \quad \forall\; k = 1, 2, 3, ..., K.$$

The bounds for k objective function is given as follows:

$$U_k^\mu = U_k, \quad L_k^\mu = L_k \quad for\ truth\ membership$$

$$U_k^\sigma = L_k^\mu + s_k, \quad L_k^\sigma = L_k^\mu \quad for\ indeterminacy\ membership$$

$$U_k^\nu = U_k^\mu, \quad L_k^\nu = L_k^\mu + t_k \quad for\ falsity\ membership$$

where s_k and $t_k \in (0, 1)$ are predetermined real numbers assigned by DM(s).
Hence, the different membership functions can be defined as follows:

$$\mu_k(Z_k(x)) = \begin{cases} 1 & if \;\; Z_k(x) < L_k^\mu \\ \frac{U_k^T - Z_k(x)}{U_k^\mu - L_k^\mu} & if \;\; L_k^\mu \leq Z_k(x) \leq U_k^\mu \\ 0 & if \;\; Z_k(x) > U_k^\mu \end{cases}$$

$$\sigma_k(Z_k(x)) = \begin{cases} 1 & if \;\; Z_k(x) < L_k^\sigma \\ \frac{U_k^\sigma - Z_k(x)}{U_k^\sigma - L_k^\sigma} & if \;\; L_k^\sigma \leq Z_k(x) \leq U_k^\sigma \\ 0 & if \;\; Z_k(x) > U_k^\sigma \end{cases}$$

$$\nu_k(Z_k(x)) = \begin{cases} 1 & if \;\; Z_k(x) > U_k^\nu \\ \frac{Z_k(x) - L_k^\nu}{U_k^\nu - L_k^\nu} & if \;\; L_k^\nu \leq Z_k(x) \leq U_k^\nu \\ 0 & if \;\; Z_k(x) < L_k^\nu \end{cases}$$

Therefore the mathematical formulation of ENPA to solve the transportation problem can be represented as below:

$$\begin{aligned}
Max\ \psi(x) \quad &= \lambda\,(\alpha-\beta-\gamma)+(1-\lambda)\textstyle\sum_{k=1}^{K}\left(\mu_k(Z_k(x))-\lambda_k(Z_k(x))-\nu_k(Z_k(x))\right)\\
\text{Subject to:} \quad &\\
&\mu_k(Z_k(x))\ge\alpha,\ \ \sigma_k(Z_k(x))\le\beta,\ \ \nu_k(Z_k(x))\le\gamma,\\
&\alpha\ge\beta,\ \ 0\le\alpha+\beta+\gamma\le 3,\\
&\lambda\in[0,1]\\
&\text{constraints (4)}
\end{aligned} \tag{10}$$

where $(\alpha+\beta-\gamma)$ represents the overall degree of satisfaction for each objective function and provides a compromise solution under the given set of constraints.

Theorem 9: A unique optimal solution of problem (10) (LTMFA) is also an efficient solution for the problem (5).

Proof: Suppose that $\left(\bar{x},\bar{\alpha},\bar{\beta},\bar{\gamma}\right)$ be a unique optimal solution of problem (10) (LTMFA). Then, $\left(\bar{\alpha}-\bar{\beta}-\bar{\gamma}\right) > (\alpha-\beta-\gamma)$ for any (x,α,β,γ) feasible to the problem (10) (LTMFA). On the contrary, assume that $\left(\bar{x},\bar{\alpha},\bar{\beta},\bar{\gamma}\right)$ is not an efficient solution of the problem (10). For that, there exists x^* $(x^*\neq\bar{x})$ feasible to problem (10), such that $O_k(x^*)\le O_k(\bar{x})$ for all $k=1,2,\cdots,K$ and $O_k(x^*)<O_k(\bar{x})$ for at least one k.
Therefore, we have $\frac{O_k(x^*)-L_k}{U_k-L_k}\le\frac{O_k(\bar{x})-L_k}{U_k-L_k}$ for all $k=1,2,\cdots,K$ and $\frac{O_k(x^*)-L_k}{U_k-L_k}<\frac{O_k(\bar{x})-L_k}{U_k-L_k}$ for at least one k.
Hence, $\max_k\left(\frac{O_k(x^*)-L_k}{U_k-L_k}\right)\le(<)\max_k\left(\frac{O_k(\bar{x})-L_k}{U_k-L_k}\right)$.
Suppose that $\gamma^*=\max_k\left(\frac{U_k-O_k(x^*)}{U_k-L_k}\right)$, then $\gamma^*\le(<)\,\bar{\gamma}$.
Also, consider that $\beta^*=\max_k\left(\frac{U_k-O_k(x^*)}{U_k-L_k}\right)$, then $\beta^*\le(<)\,\bar{\beta}$.
In the same manner, we have $\frac{U_k-O_k(x^*)}{U_k-L_k}\ge\frac{U_k-O_k(\bar{x})}{U_k-L_k}$ for all $k=1,2,\cdots,K$ and $\frac{U_k-O_k(x^*)}{U_k-L_k}>\frac{U_k-O_k(\bar{x})}{U_k-L_k}$ for at least one k.
Thus, $\min_k\left(\frac{U_k-O_k(x^*)}{U_k-L_k}\right)\ge(>)\min_k\left(\frac{U_k-O_k(\bar{x})}{U_k-L_k}\right)$.
Assume that $\alpha^*=\min_k\left(\frac{U_k-O_k(x^*)}{U_k-L_k}\right)$, this gives $\left(\bar{\alpha}-\bar{\beta}-\bar{\gamma}\right)<(\alpha^*-\beta^*-\gamma^*)$ which means that the solution is not unique optimal. Thus, we have arrived at a contradiction with the fact that $\left(\bar{x},\bar{\alpha},\bar{\beta},\bar{\gamma}\right)$ is the unique optimal solution of (LTMFA). Therefore, it is also an efficient solution of the problem (5). This completes the proof of Theorem 9. ■

4.4 Sensitivity analyses

Three different robust solution approaches have been suggested to solve the proposed F-IMOTPs. A variety of obtained solution results may not reflect the most appropriate technique to solve the F-IMOTPs in a generalized way. To select the most promising solution techniques and solution sets, it is further presented with the different sensitivity measures. The following are various criteria to analyze the performances of different approaches.

Savings compared to baseline solution: The most reasonable compromise solution is assumed to be a baseline solution for each objective function. The comparison is made with a different optimal solution which is then selected in terms of more savings (See, [2]).

Co-efficient of variation: It is a relative measure and most suitable method to compare two series. The size of the measure of dispersion also depends on the size of the measurement. Thus, it is an appropriate measure of dispersion to compare two series that differ largely in respect of their means. Moreover, a series or a set of values having a lesser co-efficient of variation than others is more consistent. It also indicates how much fluctuation is happening in the existing mean response. The lower value of co-efficient of variation indicates the more homogeneous and robustness of the data (See, [2, 8]).

Degrees of desirability: The concept of degrees of desirability has been first proposed by [7, 8]. Linear physical programming [8] is a method that is used to depict the degrees of desirability (priority) for each objective function of the MOOP. The degree of desirability is a beneficial and handy tool for assigning the target values (T_l) for the objective function and categorizing the solutions. By obtaining the individual best and worst solution of each objective function, the upper and lower bound for target values (T_l) can be determined directly. By using the pay-off matrix (individual best and worst solutions of each objective function), bound ($T_{l\ max}$) and ($T_{l\ min}$) can be obtained. These bound provides the reduction in solvability set which can be denoted as $S^{'}$ and mathematically it can be shown expressed as $S^{'} = \{S|T_{l\ min} \leq T_l \leq T_{l\ max};\ \ \forall l = 1, 2, ..., L\}$ where S is a set of parameter values for which the problem is solvable. Thus, the reduced solvability set can be used for defining the degree of desirability in the form of linguistic preferences. For more information and a stepwise procedure, one can visit the research paper by [8].

Stepwise solution algorithm

The stepwise solution procedures for the proposed F-IMOTP can be summarized as follows:
Step-1. Model the proposed F-IMOTP under uncertainty.
Step-2. Convert each intuitionistic fuzzy parameters involved in F-IMOTP into its crisp form using the accuracy function (Definition 6).
Step-3. Formulate the model (6) and solve each objective function individually in order to obtain the best and worst solution.
Step-4. Apply the different solution approach such as EFPA, EIFPA and ENPA discussed

in Sub-sections (4.1), (4.2) and (4.3) respectively.
Step-5. Solve the final model for F-IMOTP to get the compromise solution at different feasibility degree λ using suitable techniques or some optimizing software packages.
Step-6. Use the snesitivity analysis to analyze the better performance of different solution techniques at various feasibility degree λ and choose the desired compromise solution.

5 Numerical illustration

A logistic company transports two kinds of products from three sources to three different destinations using two types of conveyances. The relevant data are summarized in Table 2. The logistic company's decision-maker intends to find optimal units of different products that should be transported from various sources to different destinations using the suitable mode of conveyance, which minimizes the cost and time according to the input parameters, respectively. We have considered the three different source and destinations using two conveyances for the shipment of two types products. The decision-maker(s) wants to determine the optimal shipment policies for which the total transportation cost, time, safety cost and carbon emissions are minimized by maintaining the resource restrictions. The transportation problem is coded in AMPL language and solved using solver Knitro available on NEOS server online facility provided by Wisconsin Institutes for Discovery at the University of Wisconsin in Madison for solving optimization problems, see [25]. The solution results are summarized in Table 3. From the Table 3, it can be observed that by tuning the weight parameters between 0 to 1, various solution results are obtained. Due to space limitations, the optimal allocation of products among different echelon has not presented in this paper. The compromise solution for all four objectives has obtained at a different weight parameter (λ).
From Table 3, it can be observed that by using EFPA; the minimum value of all the objectives are found to be \$ 4360.10, 887.644 hrs., \$ 88.7644 and 5.97024 mt (metric ton), at weight parameter $\lambda = 0.1$ respectively. As for weight parameter λ increases, the values of each objective also reach towards its worst solution, and at $\lambda = 0.9$ the worst values of each objective are \$ 4357.69, 886.239 hrs., \$ 88.6283 and 5.96459 mt, which shows the more consciousness of decision makers towards the uncertainty. Similarly, EIFPA techniques also yield in different compromise solutions. At $\lambda = 0.1$, the values of each objectives by using EIFPA have been found to be \$ 4324.70, 868.882 hrs., \$ 86.8882 and 5.83967 mt, respectively. As for weight parameter λ increases, all the objectives reach towards their worst solution and at $\lambda = 0.9$, it approaches to \$ 4325.80, 870.357 hrs., \$ 87.0357 and 5.85055 mt, due to supreme importance has been given to risk violation by decision makers. Furthermore, ENPA technique results in different objective values at various weight parameter λ. At $\lambda = 0.1$, the magnitude of each objectives have been obtained as \$ 4306.83, 853.027 hrs., \$ 85.3027 and 5.7299 mt, respectively. With the increase in weight parameter λ, it has observed that each objective reaches towards their

worst outcomes which reveals that the decision makers have given more importance to the risk violation under uncertainty. Moreover, if we perform the comparison among all four approaches with respect to objective functions then it can be observed that EFPA results in better outcomes for the second and fourth objective over the other two approaches whereas EIFPA and ENPA methods yield in better results for first and third objectives for each weight parameter λ respectively. Hence all three approaches are well capable of providing the best solution for different objectives.

The corresponding achievement degree of each compromise solution has presented in Table 3. As the weight parameter λ is increasing, the values of the membership function μ is decreasing which shows the inverse trade-off between the feasibility degree and acceptance level of each compromise solution set. Interestingly, each methods EFPA, EIFPA and ENPA have been assigned with top three ranks at minimum weight parameter $\lambda = 0.1$ respectively. All the techniques have outperformed for this presented study over others. Initial few ranks have assigned to the solution set yielded by ENPA approach whereas systematic and deserving ranks have allocated to the solution sets obtained by different methods at each weight parameter λ.

The two critical aspects have highlighted that inherently involved in decision-making processes: (1) violation of risk under uncertainty and (2) balancing the global optimality of each objective. By applying EIFPA, the budget are found to be decreasing as the weight parameter λ increases. Likewise, EFPA and ENPA techniques also result in the same declining pattern of the objectives which ensures the potential management of different products. Hence, from the decision-making point of view, the computational results cope with all the prime target of the company to survive in the competitive market. An extensive opportunity to select the most promising compromise solution set is a significant advantage for the decision makers. The ENPA yield comparatively better compromise results at different weight parameters. There are ample opportunity to choose the most desired solution sets based on the decision-makers satisfaction level. Thus the decision-makers can be reached towards the optimal policies and strategies by adopting the most desired solution methods and the corresponding results. The Table 4 represents the the most desirble, desirable and most undesirable values for each objectives based on the degrees of desirability.

Table 2: Intuitionistic fuzzy input parameters

Parameters	Product 1	Product 2
$\widetilde{s}_{ijk}$	(83,85,87;81,85,89),(80,83,86;79,83,87),(86,87,88;85,87,89)	(82,84,86;80,84,88),(81,82,83;80,82,84),(74,75,76;73,75,77)
	(83,85,87;81,85,89),(81,83,85;80,83,86),(75,78,81;74,78,82)	(84,85,86;83,85,87),(80,83,85;78,83,87),(76,78,80;75,78,81)
	(83,84,85;82,84,86),(80,82,84;78,82,86),(78,79,80;77,79,81)	(84,85,86;83,85,87),(82,83,84;81,83,85),(83,84,85;82,84,86)
	(83,84,85;82,84,86),(80,82,84;78,82,86),(76,78,80;75,78,81)	(84,85,86;83,85,87),(82,83,84;81,83,85),(86,87,88;85,87,89)
	(83,84,85;82,84,86),(80,82,84;78,82,86),(72,75,78;71,75,79)	(82,84,86;80,84,88),(81,82,83;80,82,84),(78,79,80;77,79,81)
	(84,85,86;83,85,87),(80,83,86;79,83,87),(76,78,80;75,78,81)	(83,84,85;82,84,86),(80,82,84;78,82,86),(77,79,81;76,79,82)
$\widetilde{c}_{ijk}$	(14,15,16;13,15,17),(20,23,25;18,23,28),(16,17,18;15,17,19)	(42,44,46;40,44,48),(40,42,44;38,42,46),(44,45,46;43,45,47)
	(53,55,57;51,55,59),(70,73,76;69,73,77),(37,38,39;36,38,40)	(24,25,26;23,25,27),(61,63,65;60,63,66),(36,38,40;35,38,41)
	(53,54,55;52,54,56),(50,52,54;48,52,56),(28,29,30;27,29,31)	(24,25,26;23,25,27),(21,23,25;20,23,26),(12,14,16;11,14,17)
	(72,74,76;70,74,78),(42,43,44;41,43,45),(26,28,30;25,28,31)	(64,65,66;63,65,67),(42,43,44;41,43,45),(26,27,28;25,27,29)
	(42,44,46;40,44,48),(20,22,24;18,22,26),(44,45,46;43,45,47)	(12,14,16;11,14,17),(30,32,34;28,32,36),(48,49,50;47,49,51)
	(44,45,46;43,45,47)(52,53,54;50,53,56),(16,18,20;15,18,21)	(22,24,26;20,24,28),(70,72,74;68,72,76),(38,39,40;37,39,41)
$\widetilde{t}_{ijk}$	(3,5,7;2,5,8),(1,2,3;0,2,4),(6,7,8;5,7,9)	(3,4,5;2,4,6),(2,4,6;0,4,8),(4,5,6;3,5,7)
	(3,5,7;2,5,8),(5,7,9;4,7,10),(5,8,11;4,8,12)	(4,5,6;3,5,7),(3,6,9;2,6,10),(7,8,9;6,8,10)
	(3,4,5;2,4,6),(3,5,7;2,5,8),(7,9,11;6,9,12)	(2,5,8;1,5,9),(1,2,3;0,2,4),(2,4,6;0,4,8)
	(3,4,5;2,4,6),(2,4,6;1,4,7),(6,8,10;5,8,11)	(4,5,6;3,5,7) ,(2,4,6;1,4,7), (5,7,9;4,7,10)
	(3,4,5;2,4,6), (1,2,3;0,2,4),(4,5,6;3,5,7)	(3,4,5;2,4,6),(2,3,4;1,3,5),(7,9,11;6,9,12)
	(4,5,6;3,5,7),(3,5,7;2,5,8),(6,8,10;5,8,11)	(2,4,6;1,3,7),(5,7,9;4,7,10),(8,9,10;7,9,11)
$\widetilde{sc}_{ijk}$	(0.3,0.5,0.8;0.2,0.5,0.9),(0.1,0.2,0.3;0,0.2,0.4),(0.5,0.7,0.9;0.4,0.7,1.0)	(0.3,0.4,0.5;0.2,0.4,0.6),(0.2,0.4,0.6;0.1,0.4,0.7),(0.4,0.5,0.6;0.3,0.5,0.8)
	(0.3,0.5,0.7;0.2,0.5,0.8),(0.6,0.7,0.8;0.5,0.7,0.9),(0.6,0.8,1.0;0.5,0.8,1.1)	(0.4,0.5,0.6;0.3,0.5,0.7),(0.5,0.6,0.7;0.4,0.6,0.8),(0.6,0.8,1;0.5,0.8,1.1)
	(0.3,0.4,0.5;0.2,0.4,0.6),(0.4,0.5,0.6;0.3,0.5,0.7),(0.8,0.9,1;0.7,0.9,1.1)	(0.3,0.5,0.7;0.2,0.5,0.8),(0.1,0.2,0.3;0,0.2,0.4),(0.3,0.4,0.5;0.2,0.4,0.6)
	(0.3,0.4,0.5;0.2,0.4,0.6),(0.3,0.4,0.5;0.2,0.4,0.6),(0.6,0.8,1;0.5,0.8,1.1)	(0.3,0.5,0.7;0.2,0.5,0.8),(0.2,0.4,0.6;0.1,0.4,0.7),(0.5,0.7,0.9;0.4,0.7,1.0)
	(0.3,0.4,0.5;0.2,0.4,0.6),(0.1,0.2,0.3;0,0.2,0.4),(0.3,0.5,0.7;0.2,0.5,0.8)	(0.3,0.4,0.5;0.2,0.4,0.6),(0.2,0.3,0.4;0.1,0.3,0.5),(0.7,0.9,1.1;0.6,0.9,1.2)
	(0.3,0.5,0.7;0.2,0.5,0.8),(0.3,0.5,0.7;0.2,0.5,0.8),(0.7,0.8,0.9;0.6,0.8,1)	(0.3,0.4,0.5;0.2,0.4,0.6),(0.5,0.7,0.9;0.4,0.7,1),(0.8,0.9,1;0.7,0.9,1.1)
$\widetilde{ce}_{ijk}$	(0.03,0.05,0.07;0.02,0.05,0.08),(0.01,0.02,0.03;0.0,0.02,0.04),(0.06,0.07,0.08;0.05,0.07,0.09)	(0.03,0.04,0.05;0.02,0.04,0.06),(0.03,0.04,0.05;0.02,0.04,0.06),(0.04,0.05,0.06;0.03,0.05,0.0
	(0.04,0.05,0.06;0.03,0.05,0.07),(0.06,0.07,0.08;0.05,0.07,0.09),(0.07,0.08,0.09;0.06,0.08,0.1)	(0.03,0.05,0.07;0.02,0.05,0.08),(0.05,0.06,0.07;0.04,0.06,0.08),(0.07,0.08,0.09;0.06,0.08,0.
	(0.03,0.04,0.05;0.02,0.04,0.06),(0.04,0.05,0.06;0.03,0.05,0.07),(0.07,0.09,0.11;0.06,0.06,0.12)	(0.03,0.05,0.07;0.02,0.05,0.08),(0.01,0.02,0.03;0.0,0.02,0.04),(0.03,0.04,0.05;0.02,0.04,0.0
	(0.03,0.04,0.05;0.02,0.04,0.06),(0.03,0.04,0.05;0.02,0.04,0.06),(0.07,0.08,0.09;0.06,0.08,0.1)	(0.03,0.05,0.07;0.02,0.05,0.08),(0.03,0.04,0.05;0.02,0.04,0.06),(0.06,0.07,0.08;0.05,0.07,0.0
	(0.03,0.04,0.05;0.02,0.04,0.06),(0.01,0.02,0.03;0.0,0.02,0.04),(0.04,0.05,0.06;0.03,0.05,0.07)	(0.03,0.04,0.05;0.02,0.04,0.06),(0.03,0. 05,0.07;0.02,0.05,0.08),(0.07,0.09,0.11;0.06,0.06,0.
	(0.04,0.05,0.06;0.03,0.05,0.07),(0.04,0.05,0.06;0.03,0.05,0.07),(0.07,0.08,0.09;0.06,0.08,0.1)	(0.03,0.04,0.05;0.02,0.04,0.06),(0.06,0.07,0.08;0.05,0.07,0.09),(0.07,0.09,0.11;0.06,0.06,0.
$\widetilde{a}_i$	(206,208,210;205,208,211),(250,252,254;248,252,256),(224,226,228;222,226,230)	(252,254,256;250,254,258),(262,264,266;260,264,268),(244,245,246;243,245,247)
	(152,154,156;150,154,158),(562,564,566;560,564,568),(540,545,550;535,545,555)	(158,159,160;157,159,161),(146,148,150;144,148,152),(123,125,127;122,125,128)
	(272,274,276;270,274,278),(255,256,257;254,256,258),(254,255,256;253,255,257)	(272,274,276;270,274,278),(765,767,769;764,767,780),(453,455,457;452,455,458)
$\widetilde{b}_j$	(82,83,84;81,83,85),(85,87,89;84,87,91),(93,94,95;92,94,96)	(73,74,75;72,74,76),(50,52,54;48,52,56),(83,85,87;81,85,89)
	(82,84,86;80,84,88),(84,85,86;83,85,87),(92,95,98;91,95,99)	(41,45,49;40,45,50),(56,57,58;55,57,59),(74,75,76;73,75,77)
	(84,85,86;83,85,87),(65,67,69;64,67,70),(92,94,96;90,94,98)	(71,75,79;70,75,80),(85,87,89;84,87,90),(62,63,64;60,63,66)
$\widetilde{e}_i$	(22,24,26;20,24,28),(20,24,28;18,24,30) (34,35,36;33,35,37)	(65,67,69;64,67,70) (23,24,25;22,24,26),(42,44,46;40,44,48)
	(34,35,36;33,35,37),(33,35,37;32,35,38) (24,25,26;23,25,27)	(30,35,40;25,35,45) (32,35,38;31,35,39),(44,45,46;43,45,47)
$\widetilde{p}_k$	(0.2,0.4,0.6;0.1,0.4,0.7),(0.3,0.5,0.7;0.2,0.5,0.8)	(0.7,0.9,1.1;0.6,0.9,1.2),(0.6,0.8,1.0;0.5,0.8,1.1)
B_j	(2450,2445,2505) (1425,1665,1545) (1565,1685,1425)	(2450,2445,2505) (1425,1665,1545) (1565,1685,1425)
B	5	5

Table 3: Optimal solutions obtained by different methods

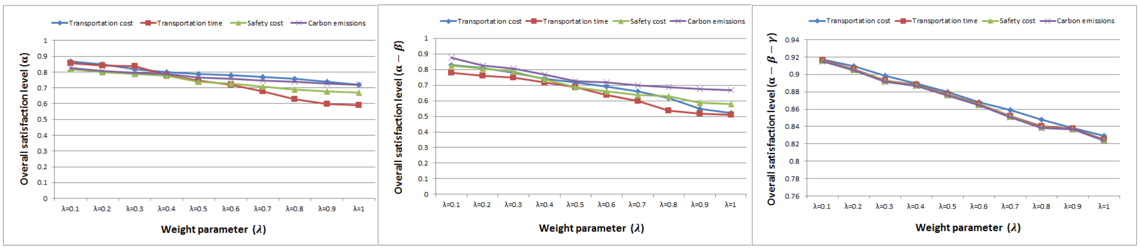
Weight (λ)	Objective values	Extended Fuzzy Programming Approach (EFPA)	Extended Intuitionistic Fuzzy Programming Approach (EIFPA)	Extended Neutrosophic Programming Approach (ENPA)
λ=0.1	$Min\ Z_1$	4360.1	4324.7	4306.83
	$Min\ Z_2$	887.644	868.882	853.027
	$Min\ Z_3$	88.7644	86.8882	85.3027
	$Min\ Z_4$	5.97024	5.83967	5.7299
λ=0.2	$Min\ Z_1$	4360.09	4358.87	3889.59
	$Min\ Z_2$	887.646	887.751	835.3
	$Min\ Z_3$	88.7646	88.7751	83.53
	$Min\ Z_4$	5.97025	5.97094	5.58824
λ=0.3	$Min\ Z_1$	4357.6	4322.77	4228.76
	$Min\ Z_2$	882.777	870.266	848.714
	$Min\ Z_3$	88.2777	87.0266	84.8714
	$Min\ Z_4$	5.93873	5.84968	5.69559
λ=0.4	$Min\ Z_1$	4356.27	4313.18	4057.54
	$Min\ Z_2$	883.088	857.292	846.627
	$Min\ Z_3$	88.3088	85.7292	84.6627
	$Min\ Z_4$	5.93522	5.72106	5.67529
λ=0.5	$Min\ Z_1$	4360.19	4337.69	4311.32
	$Min\ Z_2$	887.609	872.949	853.118
	$Min\ Z_3$	88.7609	87.2949	85.3118
	$Min\ Z_4$	5.97005	5.86669	5.731
λ=0.6	$Min\ Z_1$	4359.6	4324.34	3926.1
	$Min\ Z_2$	887.456	868.335	833.667
	$Min\ Z_3$	88.7456	86.8335	83.3667
	$Min\ Z_4$	5.96928	5.83305	5.57907
λ=0.7	$Min\ Z_1$	4349.47	4325.95	4206.61
	$Min\ Z_2$	871.233	870.301	848.91
	$Min\ Z_3$	87.1233	87.0301	84.891
	$Min\ Z_4$	5.87853	5.85014	5.69869
λ=0.8	$Min\ Z_1$	4355.73	4314.35	4146.7
	$Min\ Z_2$	883.183	851.972	834.551
	$Min\ Z_3$	88.3183	85.1972	83.4551
	$Min\ Z_4$	5.9364	5.70287	5.5971
λ=0.9	$Min\ Z_1$	4357.69	4325.8	4304.75
	$Min\ Z_2$	886.239	870.357	852.007
	$Min\ Z_3$	88.6283	87.0357	85.2007
	$Min\ Z_4$	5.96459	5.85055	5.72234

5.1 Sensitivity analyses

The three different solution sets based on the degree of desirability scenario have been generated, and the corresponding performances of each solution method under the dif-

ferent solution sets are also recorded. From Table 5 (solution 1), the EFPA reveals that transportation cost (T.C) can be reduced by 53.73%, transportation time (T.T) can be lowered by 20.69%, safety cost (S.C) can be mitigated by 79.36% and carbon emissions (C.E) can be mitigated by 79.36% as compared to the baseline solution. Furthermore, EIFPA technique yield in the reduction of T.C by 55.21%, a significant increment in the T.T by 17.23%, notably decrement in the S.C by 83.81% and C.E by 83.81% as compared to the baseline solution. Similarly, on applying ENPA technique, it is observed that the T.C can be reduced by 55.99 %, T.T can be lowered by 23.12 %, S.C can be mitigated by 79.36% and C.E can be mitigated by 87.45% as compared to the baseline solution. Likewise, from Table 6 (solution 2), the EFPA technique shows that T.C can be diminished by 55.81 %, T.T can be increased by 22.77 %, S.C can be mitigated by 79.36% and C.E can be reduced by 87.15 % as compared to the baseline solution. Furthermore, EIFPA technique results in the reduction of T.C by 53.63 %, a significant increment in the T.T by 19.56 %, notably decrement in the S.C by 83.55 % and C.E can be mitigated by 79.36% as compared to the baseline solution. Similarly, on applying ENPA technique, it is observed that the T.C can be mitigated by 55.01 %, T.T can be enhanced by 16.63 %, S.C can be reduced by 79.26% and C.E can be mitigated by 79.36% as compared to the baseline solution. From Table 7 (solution 3), EFPA ensures that T.C can be reduced by 53.77 %; T.T can be enhanced by 20.96 %, and S.C can be mitigated by 79.43% and C.E can be reduced by 79.36% as compared to the baseline solution. Furthermore, the EIFPA technique results in the reduction of T.C by 56.13 %, a significant increment in the T.T by 23.24 %, remarkable decrement in the S.C by 87.66 % and C.E can be mitigated by 79.36% as compared to the baseline solution. Similarly, on applying ENPA technique, it is observed that the T.C can be reduced by 55.01 %, T.T can be enhanced by 16.63 %, S.C can be mitigated by 79.26% and C.E can be mitigated by 79.36% as compared to the baseline solution.
For solution 1, a comparative study with the co-efficient of variation shows that all the objective functions are more homogeneous under variation while using ENPA techniques over others. Similarly, more robust (homogeneous) results of each objective function have been achieved for solution 2 while using EFPA technique. Furthermore, it is also observed that all the objective functions are more homogeneous under variation while using the EIFPA technique for solution 3. The trending behavior of the different techniques has been depicted in Figure 2 for each solution set. The graphical representation of solution 1 (sub-figure 2a), solution 2 (sub-figure 2b) and solution 3 (sub-figure 2c) by using different techniques reveals the performances of each solution approaches. In addition to the different solutions set, the behavior of the overall satisfaction level with the co-efficient of variations has also been shown in Figure 3. The representation of fluctuating behavior for solution 1 (sub-figure 3a), solution 2 (sub-figure 3b) and solution 3 (sub-figure 3c) by using different techniques reflects homogeneity or robustness under the variation. Finally, the optimal solution results for three different solution sets have been summarized in Table 8. From Table 8, all the solution sets are under the most desirable zone, which provides an opportunity to select a better one amongst the best solution sets. Thus these criteria

(savings compared to baseline solution, CV, and degrees of desirability) for selection of optimal solution results are proven to be quite helpful tools while dealing with multiple objective optimization problems. Moreover, the different solutions set, the behavior of the overall satisfaction level with the weight parameter (λ) has also been shown in Figure 1. The representation of fluctuating behavior for solution 1 (sub-figure 1a), solution 2 (sub-figure 1b) and solution 3 (sub-figure 1c) by using different techniques reflects homogeneity or robustness under the variation.



(a) Proposed EFPA (b) Proposed EIFPA (c) Proposed ENPA

Figure 1: Overall satisfaction level v/s Weight parameter (λ)

Table 4: Degrees of desirability for each objective function

Objective functions	Most Desirable (MD)	Desirable (D)	Most Undesirable (MU)
Minimum $Z_1(X)$ (Transportation cost)	4360.19	5230.80	5794.80
Minimum $Z_2(X)$ (Transportation time)	887.609	901.456	917.739
Minimum $Z_3(X)$ (Safety cost)	88.7609	93.8349	98.4924
Minimum $Z_4(X)$ (Carbon emissions)	5.97005	9.45924	13.8984

Table 5: Solution 1: ($Z_1 \leq 4360.19$, $Z_2 \leq 887.609$, $Z_3 \leq 88.7609$ and $Z_4 \leq 5.97005$)

Objective functions		EFPA		EIFPA		ENPA	
	Baseline solution	Solution	CV	Solution	CV	Solution	CV
$Z_1(X)$ (Transportation cost)	4276.34	4360.10	1.23	4324.70	1.34	4306.83	1.05
$Z_2(X)$ (Transportation time)	810.4543	887.644	0.93	868.882	1.02	853.027	0.91
$Z_3(X)$ (Safety cost)	72.8690	88.7644	1.17	86.8882	1.14	85.3027	1.09
$Z_4(X)$ (Carbon emissions)	2.03943	5.97024	1.03	5.83967	1.09	5.7299	1.14

6 Conclusions

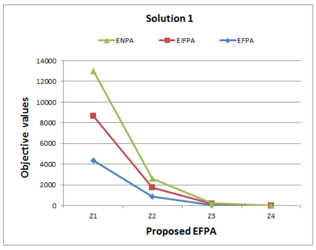
In this article we consider various modeling approaches for multiobjective transportation problem under intuitionistic fuzzy parameters. Minimization of transportation cost,

Table 6: Solution 2: ($Z_1 \leq 5230.80$, $Z_2 \leq 901.456$, $Z_3 \leq 93.8349$ and $Z_4 \leq 9.45924$)

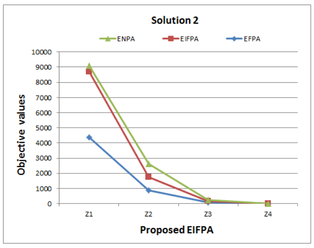
Objective functions		EFPA		EIFPA		ENPA	
	Baseline solution	Solution	CV	Solution	CV	Solution	CV
$Z_1(X)$ (Transportation cost)	4276.349	4360.19	1.39	4337.69	1.84	411.32	1.71
$Z_2(X)$ (Transportation time)	810.4543	887.609	0.78	872.949	0.89	853.118	0.98
$Z_3(X)$ (Safety cost)	72.8690	88.7609	1.29	87.2949	1.43	85.3118	1.67
$Z_4(X)$ (Carbon emissions)	2.03943	5.97005	1.11	5.86669	1.17	5.7310	1.23

Table 7: Solution 3: ($Z_1 \leq 5794.80$, $Z_2 \leq 917.739$, $Z_3 \leq 98.4924$ and $Z_4 \leq 13.8984$)

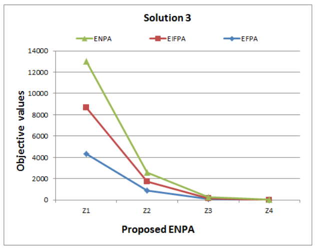
Objective functions		EFPA		EIFPA		ENPA	
	Baseline solution	Solution	CV	Solution	CV	Solution	CV
$Z_1(X)$ (Transportation cost)	4276.34	4357.69	1.37	4325.80	1.17	4304.75	1.49
$Z_2(X)$ (Transportation time)	810.4543	886.239	0.98	870.357	0.87	852.007	1.13
$Z_3(X)$ (Safety cost)	72.8690	88.6283	1.26	87.0357	1.01	85.2007	1.14
$Z_4(X)$ (Carbon emissions)	2.03943	5.96459	1.21	5.85055	1.11	5.72234	1.25



(a) (Solution 1)

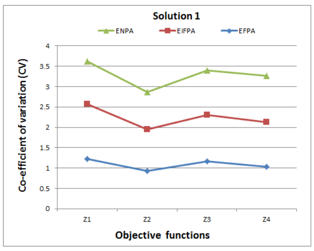


(b) (Solution 2)

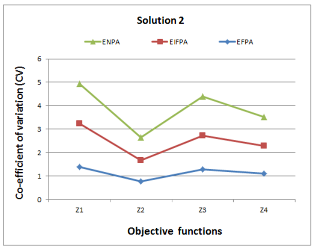


(c) (Solution 3)

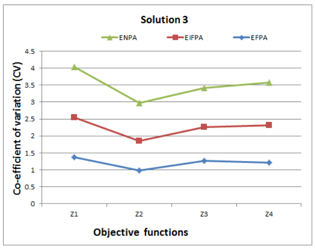
Figure 2: Objective functions v/s solution methods



(a) (Solution 1)



(b) (Solution 2)



(c) (Solution 3)

Figure 3: Co-efficient of variation v/s solution methods

Table 8: Comparision of optimal solutions with multiple criteria

Different approaches	Baseline solution	CV	Degree of desirability
Solution 1 (EFPA)	Z_1: 55.01 % ↓	1.05	4360.19 (MD)
	Z_2: 23.12 % ↓	0.91	887.609 (MD)
	Z_3: 87.45 % ↓	1.09	88.7609 (MD)
	Z_4: 72.34 % ↓	1.19	5.97005 (MD)
Solution 2 (EIFPA)	Z_1: 55.81 % ↓	1.39	4360.19 (MD)
	Z_2: 16.63 % ↓	0.78	887.609 (MD)
	Z_3: 87.15 % ↓	1.29	88.7609 (MD)
	Z_4: 72.03 % ↓	1.12	5.97005 (MD)
Solution 3 (ENPA)	Z_1: 56.13 % ↓	1.17	4360.19 (MD)
	Z_2: 23.24 % ↓	0.87	887.609 (MD)
	Z_3: 87.66 % ↓	1.01	88.7609 (MD)
	Z_4: 71.89 % ↓	1.03	5.97005 (MD)

time, safety cost and carbon-emissions are considered as objective functions under the supply,demand, capacity, safety and budget constraints. The extended version of various conventional approaches such as EFPA, EIFPA and ENPA are developed to solve the MOTPs. The robustness of the solution approaches have been established with the help of results. At different weight parameter, a set of compromised solution are obtained. The sensitivity analysis is also performed based on the different criteria such as baseline solution, CV and degrees of desirability which generates the variaty of solution setsbased on the satisfaction level of the decision-maker. It is observed that ENPA outperforms others. For all the solution approaches, when decision-maker(s) are more concerned about the vagueness then objective values reaches to its worst and vice-versa. We propose that the present work can be extended further for multi-level MOPP and applied to different real-life problems such as supply chain, inventory control and assignment problem as well.

References

[1] Adhami, A. Y. and Ahmad, F. [2020], 'Interactive pythagorean-hesitant fuzzy computational algorithm for multiobjective transportation problem under uncertainty', *International Journal of Management Science and Engineering Management* **15**(4), 288–297.

[2] Ahmad, F. [2021], 'Robust neutrosophic programming approach for solving intuitionistic fuzzy multiobjective optimization problems', *Complex & Intelligent Systems* **7**, 1935–1954.

[3] Ahmad, F. and Adhami, A. Y. [2019], 'Neutrosophic programming approach to mul-

tiobjective nonlinear transportation problem with fuzzy parameters', *International Journal of Management Science and Engineering Management* **14**(3), 218–229.

[4] Ahmad, F., Adhami, A. Y. and Smarandache, F. [2018], 'Single Valued Neutrosophic Hesitant Fuzzy Computational Algorithm for Multiobjective Nonlinear Optimization Problem', *Neutrosophic Sets and Systems* **22**, 76–86.

[5] Ahmad, F., Adhami, A. Y. and Smarandache, F. [2019], 'Neutrosophic optimization model and computational algorithm for optimal shale gas water management under uncertainty', *Symmetry* **11**(4).
URL: *http://www.mdpi.com/2073-8994/11/4/544*

[6] Ahmad, F., Adhami, A. Y. and Smarandache, F. [2020], Modified neutrosophic fuzzy optimization model for optimal closed-loop supply chain management under uncertainty, *in* 'Optimization theory based on neutrosophic and plithogenic sets', Elsevier, pp. 343–403.

[7] Ahmad, F., Ahmad, S. and Zaindin, M. [2021], 'A sustainable production and waste management policies for covid-19 medical equipment under uncertainty: A case study analysis', *Computers & Industrial Engineering* **157**(3), 107381.

[8] Ahmad, F., Alnowibet, K. A., Alrasheedi, A. F. and Adhami, A. Y. [2021], 'A multi-objective model for optimizing the socio-economic performance of a pharmaceutical supply chain', *Socio-Economic Planning Sciences* p. 101126.

[9] Ahmadini, A. A. H. and Ahmad, F. [2021], 'A novel intuitionistic fuzzy preference relations for multiobjective goal programming problems', *Journal of Intelligent & Fuzzy Systems* **40**(3), 4761–4777.

[10] Akram, M., Adeel, A. and Alcantud, J. C. R. [2019], 'Group decision-making methods based on hesitant n-soft sets', *Expert Systems with Applications* **115**, 95–105.

[11] Alcantud, J. C. R. and Giarlotta, A. [2019], 'Necessary and possible hesitant fuzzy sets: A novel model for group decision making', *Information Fusion* **46**, 63–76.

[12] Alcantud, J. C. R. and Santos-García, G. [2017], Expanded hesitant fuzzy sets and group decision making, *in* '2017 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)', IEEE, pp. 1–6.

[13] Alcantud, J. C. R. and Torra, V. [2018], 'Decomposition theorems and extension principles for hesitant fuzzy sets', *Information Fusion* **41**, 48–56.

[14] Angelov, P. P. [1997], 'Optimization in an intuitionistic fuzzy environment', *Fuzzy Sets and Systems* **86**(3), 299–306.

[15] Atanassov, K. T. [1986], 'Intuitionistic fuzzy sets', *Fuzzy Sets and Systems* **20**(1), 87–96.

[16] Bharati, S. K. [2018], 'Hesitant fuzzy computational algorithm for multiobjective optimization problems', *International Journal of Dynamics and Control* pp. 1–8.

[17] Das, A. K., D. and Jana, R. [2020], 'Some aspects on solving transportation problem', *Yugoslav Journal of Operations Research* **30**(1), 45–57.

[18] Faizi, S., Rashid, T., Saabun, W., Zafar, S. and Wtrbski, Jaros, j. v. n. p. y. p. [n.d.], 'Decision making with uncertainty using hesitant fuzzy sets'.

[19] Jana, D. K. [2016], 'Paci fi c Science Review A : Natural Science and Engineering Novel arithmetic operations on type-2 intuitionistic fuzzy and its applications to transportation problem', *Pacific Science Review A: Natural Science and Engineering* **18**(3), 178–189.
URL: *http://dx.doi.org/10.1016/j.psra.2016.09.008*

[20] Lan, J., Jin, R., Zheng, Z. and Hu, M. [2017], 'Priority degrees for hesitant fuzzy sets: Application to multiple attribute decision making', *Operations Research Perspectives* **4**, 67–73.

[21] Liu, C.-f. and Luo, Y.-S. [2017], 'New aggregation operators of single-valued neutrosophic hesitant fuzzy set and their application in multi-attribute decision making', *Pattern Analysis and Applications* pp. 1–11.

[22] Mohan, S., Neogy, S. and Das, A. [2004], 'A note on linear complementarity problems and multiple objective programming', *Mathematical programming* **100**(2), 339–344.

[23] Peng, X. and Selvachandran, G. [2017], 'Pythagorean fuzzy set: state of the art and future directions', *Artificial Intelligence Review* pp. 1–55.

[24] Peng, X. and Yang, Y. [2015], 'Some results for pythagorean fuzzy sets', *International Journal of Intelligent Systems* **30**(11), 1133–1160.

[25] Server, N. [2016], 'State-of-the-art solvers for numerical optimization'.

[26] Singh, S. K. and Yadav, S. P. [2015], 'Modeling and optimization of multi objective non-linear programming problem in intuitionistic fuzzy environment', *Applied Mathematical Modelling* **39**(16), 4617–4629.
URL: *http://dx.doi.org/10.1016/j.apm.2015.03.064*

[27] Singh, S. K. and Yadav, S. P. [2016*a*], 'Intuitionistic fuzzy transportation problem with various kinds of uncertainties in parameters and variables', *International Journal of Systems Assurance Engineering and Management* **7**(3), 262–272.

[28] Singh, S. and Yadav, S. [2016*b*], 'A new approach for solving intuitionistic fuzzy transportation problem of type-2', *Annals of Operations Research* **243**(1-2), 131–140.

[29] Smarandache, F. [1999], *A Unifying Field in Logics: Neutrosophic Logic.*
URL: *http://cogprints.org/1919/*

[30] Wan, S.-P., Jin, Z. and Dong, J.-Y. [2018], 'Pythagorean fuzzy mathematical programming method for multi-attribute group decision making with pythagorean fuzzy truth degrees', *Knowledge and Information Systems* **55**(2), 437–466.

[31] Ye, J. [2015], 'Multiple-attribute decision-making method under a single-valued neutrosophic hesitant fuzzy environment', *Journal of Intelligent Systems* **24**(1), 23–36.

[32] Zadeh, L. A. et al. [1965], 'Fuzzy sets', *Information and control* **8**(3), 338–353.

[33] Zhang, X. and Xu, Z. [2014], 'Extension of TOPSIS to multiple criteria decision making with pythagorean fuzzy sets', *International Journal of Intelligent Systems* **29**(12), 1061–1078.

[34] Zhang, X., Xu, Z. and Xing, X. [2016], 'Hesitant fuzzy programming technique for multidimensional analysis of hesitant fuzzy preferences', *OR Spectrum* **38**(3), 789–817.

[35] Zhang, Z. [2017], 'Hesitant fuzzy multi-criteria group decision making with unknown weight information', *International Journal of Fuzzy Systems* **19**(3), 615–636.

[36] Zhou, W. and Xu, Z. [2018], 'Portfolio selection and risk investment under the hesitant fuzzy environment', *Knowledge-Based Systems* **144**, 21–31.
URL: *https://doi.org/10.1016/j.knosys.2017.12.020*

[37] Zimmermann, H. J. [1978], 'Fuzzy programming and linear programming with several objective functions', *Fuzzy Sets and Systems* **1**(1), 45–55.