



A short history of fuzzy, intuitionistic fuzzy, neutrosophic and plithogenic sets

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Abstract

Recently, research on uncertainty modeling is progressing rapidly and many essential and breakthrough studies have already been done. There are various ways such as fuzzy, intuitionistic and neutrosophic sets to handle these uncertainties. Although these concepts can handle incomplete information in various real-world issues, they cannot address all types of uncertainty such as indeterminate and inconsistent information. Also, plithogenic sets as a generalization of crisp, fuzzy, intuitionistic fuzzy, and neutrosophic sets, which is a set whose elements are characterized by many attributes' values. In this paper, our aim is to demonstrate and review the history of fuzzy, intuitionistic and neutrosophic sets. For this purpose, we divided the paper as: section 1. History of Fuzzy Sets, section 2. History of Intuitionistic Fuzzy Sets and section 3. History of Neutrosophic Theories and Applications, section 4. History of Plithogenic Sets.

1 History of Fuzzy Sets

In the twentieth century, the problem of representing vagueness in logic, in physics, in linguistics and the questioning of the notion of set caused many first suggestions for the fuzzy set theory. The fuzzy set theory is not a unusual, arbitrary object that abruptly come out of nowhere, yet it clarified the intuitions of some spearheading scientists in the century. The philosophers Charles Peirce¹⁰⁹ and Bertrand Russell¹¹³ thought that vagueness is neglected in mathematical information. There were discussions on the relationships between logic and vagueness in the philosophical literature (⁴⁹ ⁷⁶). It is stated that notions in natural language do not have a apparent class of their properties but they have enlargeable boundaries.¹⁴⁵ To determine vague symbols, the American philosopher Max Black propounded consistency profiles which are the ancestors of fuzzy membership functions.³⁹ In 1940, H. Weyl gave a generalization of the usual characteristic function by substituting it with a continuous characteristic function.¹⁴⁴ Then Kaplan and Schott proposed a similar generalization of characteristic functions of vague predicates and the fundamental fuzzy set connectives in these works.⁸² In 1951, Karl Menger used the term "ensemble flou" which is the French counterpart of fuzzy set for the first-time.⁹⁹

Although the term "fuzzy set" is often worst defined and misunderstood, it is fashionable in scientific areas and daily life. The doctrine of fuzziness has been properly defined by M. M. Gupta as "a body of concepts and techniques aimed at providing a systematic framework for dealing with the vagueness and imprecision inherent in human thought processes".⁷³ Thus, the three basic keywords, the three pillars of this doctrine which form its philosophical basis, are following: thinking, vagueness, and imprecision.⁵⁸ Since the activity of thinking is not basically correlated with linguistic expressions,⁶³ it designates to operations or constructions in the mind, which are in charge of analysis and logical tests. On the other hand, the whole doctrine of fuzziness is founded on vagueness and imprecision, and so, it is handled as properties of language and knowledge. For centuries, scientist have started different enterprises in order to explain these phenomena. Various logical calculus were constructed to the aim of dealing with different aspects of language and information. The fuzzy theory is also one of them. Two different kinds of theories of perception and thought exist.¹¹⁵ Some suppose

that the content of perception is never identical with the perceived entity. However, others assert that this content is identical with the perceived object, and provided that perception is veridical. Identity theories were strongly criticized by B. Russell,¹¹³ who clearly distinguished the properties of words from the properties of things. In this famous paper, he writes that vagueness and precision are characteristics which can only belong to a language and that “apart from representation there can be no such thing as vagueness or precision”. He means that vagueness is a characteristic of words, not of things.⁵⁸ The aim of fuzzy set theory is to build a formal framework for unfinished and gradual knowledge in natural language. The concepts of ambiguity and imprecision of information can be variously understood by researchers as vagueness, uncertainty, etc. When these concepts are regarded as related for representational systems, everyone agree that language is a typical example.

In the twentieth century, Zadeh introduced the degree of membership/truth (T) in 1965 and defined the fuzzy set.¹⁴⁸ In contrast to classical binary logic, this aims to express thoughts and notions which do not mathematically explain in natural language such as uncertainty and randomness. Thus, Zadeh preferred classes of objects consisting of relative notions in natural language terms such as age, size, height, temperature. However, classical binary logic is not appropriate to analyze such classes where it appears that membership is a gradual notion rather than an all-or-nothing matter. The fuzzy theory summarizes significantly perceptive phenomena that expound the complexity of the world. Analytical representations of physical phenomena can model natural language but they are sometimes difficult to understand since they have nebulous explanations and is not be obvious for the non-specialist people. Mental representations has uncertainty and vagueness, the lack of specificity of linguistic terms and the lack of well-defined boundaries of the class of preferred objects. In natural language, objects are represented by proper names, properties are expressed by adjectives and nouns, whereas prepositions and verbs tend to express relations between two or more things.¹¹⁴ In artificial languages, for example, in a language based on first order logic, objects are represented by singular names, whereas properties and relations are represented by predicates.⁵⁸

For years, producing of the devices simulated human thoughts and behaviors is many scientists' dream. Since we have entered the era of information management as soon as the emergence of computers, this dream have started to realize. The developments of science and technology needing detailed knowledge to produce of the devices human thoughts and behaviors and easy attainableness to computers helps to realize the dream. An important problem is to stock and make use of the information in various areas. Because the fuzzy theory attend to this tendency,⁵⁶ it has close connection with artificial intelligence.

When Zadeh propounded fuzzy sets, his concerns concentrated on their potential contribution in the fields of model classification,³⁸ processing and transportation of knowledge, abstraction and summarization.¹⁴⁹ Despite the claims that fuzzy sets were related in these fields emerged in the early sixties, the developments of sciences, technologies and engineering showed that these intuitions were true.

The terms “depressed” and “old” are fuzzy concepts since they cannot be sharply defined. However, people use it to decide. The fuzzy concepts is absolutely contrast to the terms as married, under 25 years old or over 2.5 feet tall. Pure mathematics is interested in classes of objects such as the subset of rational numbers in the set of real numbers. However, when the subset of young people in a given set of people, it may be impossible to decide whether a person is in that subset or not. It is possible to answer it but there can be losing of information when it does. Even though this case has existed from time immemorial, the dominant context is the situation in which statements are exact.

In this day and age, quickly developing technology and the dream of producing devices that mimic human reasoning human thoughts and behaviors which is based on uncertain and imprecise knowledge has drawn many scientists' attention. The theory and applications of fuzzy concepts is largely in the fieds of engineering and applied sciences. The widely usage of fuzzy concepts in technology and science are witnessed by means of the developments of automatic control and expert systems. Although the mathematical elements that form the basis of fuzzy notions have existed for a long time, the rising of its applications has satisfied a motivation for mathematics. Until the rising of fuzzy set theory as a crucial tool in practical applications, there did not exist a force majeure to analyse its mathematics. By virtue of the practical significance of such improvements, studying the mathematical basis of this theory has become important.

In the literature of fuzzy sets, the word “fuzzy” means the word “vague”. According to the Oxford English Dictionary, the word “fuzzy” has some various meanings. Unlike any other meaning, “fuzzy” can be used to form a predicate of the form “something is fuzzy”. For instance, it can sound normal to say that “a fox is fuzzy” but sound weirdo to say that “old is fuzzy”. In this case, the adjective “old” is vague (but not fuzzy in the material sense) since its sense is not set by certain boundaries. However, the word “fuzzy” is applied to predicates and needs to state the gradual nature of some of these words that appear as vague. Additionally, “vagueness” specifies mostly a much larger kind of tabulation for words including ambiguity. Vagueness is interested the meaning of signs in a language and is treated as a certain type of ambiguity. If any notion

is not sharply defined, then it is called vague. Since there exists a graceful degradation between situations to which a vague concept fully applies and situations to which it does not never apply, there exist a whole range of situations to which a vague concept fractionally apply.⁵⁸ This is called “membership gradience” by Lakoff.⁹⁰ It is keynoted that the fuzziness is an main property of the notion of vagueness which is different from uncertainty. Many researchers think that full applicability of a vague notion to a particular situation may be dubious.¹¹² If a vague concept applies or not to a particular situation, then such doubt can only emerge in the case which is forced to decide. If a proposition involves gradual predicates, then it is called vague. A characteristic property of these propositions is that they can be neither true nor false when they applied to describe a given situation.⁴⁹ In a word, they can be stated as a degree. Symbolic forms of these degrees of truth exist in natural languages, for example, “very”, “rather”, “almost no”, “less”, etc. If a person of the class of “depressive people” characterizes “less depressive”, then it is said that the truth level of the statement “this person is depressive” is “less” and this person has a low degree of membership to the class of “depressive people”. These linguistic restrictions apply only to gradual predicates. For example, “married” is not a gradual predicate since the term “less married” does not exist.

There exist two reasons that there are fuzziness and ambiguity in all languages. A first reason is that any language is discrete and the external world seems to be continuous. According to Aristotle, this represents the extensity of ambiguity in languages. Specially, a certain number of words import ostensible continuous numerical scales. For example, let the word “old” apply to humans. In order to fix an age threshold below which old fully applies and it does not at all is difficult. There exists a conflict between the linguistic representation of the age scale (for example, {old, teen, young, mature}) and its numerical representation (for instance, the real interval [0, 120] years for humans). The range of functions which Zadeh¹⁴⁸ calls membership functions, can involve as many elements as in the age scale and continuous itself. Terms such as old, young, teen, etc. is called fuzzy predicates. A second reason is that natural language overlooks the existence of exceptions. For instance, some of snakes are venomous but other is not since some don’t have venom. Thus, the collection of snakes is not just a set, and also it is a set partially ordered by a relation of compared typicality. Then the membership function is only a partially arbitrary encoding of the partial ordering relation.

Imprecision is also characteristic of language and relates to measurable concepts. These concepts are represented by numbers. This imprecision originates from the fact that any measurement results bounded truth. Then it is possible to mention many correspondences between the results of measurement and real numbers. Chwistek suggested the usage of intervals instead of single numbers⁴⁷ and Mellor improved in various direction by Mellor (^{97 98}). Imprecision seems in the form of disjunctions in logic. For instance, a proposition “p or q is true” contains imprecision since it is not known whether p is true, q is true, or both. Hence, imprecision is represented by sets as a disjunction (not conjunction) of elements. R. Young attempts to improve a formal theory allowing manipulations with indefinite quantities.¹⁴⁷ This leading work is called interval mathematics. R. Young substituted a variable which supposes severally considered values by a notion of many-valuedness. This notion was stated as a set of values but it considered collectively. S. Lesniewski studied widely on these sets (^{91 92}).

Despite imprecision and vagueness mention the contents of a part of knowledge in a language, uncertainty states the ability of an agent to claim whether a proposition holds or not. Uncertainty of Boolean propositions can be gradual as in probability theory where partial doctrine ranges on the unit interval while uncertainty is three-valued in propositional logic, i.e., a proposition is known, its negation is known or both is not known. Even though stating and reasoning these three cases explicitly requires the features of modal logic,⁷⁸ the modal possibility has the lack of certainty and certainty reflects logical consistency. However, uncertainty modeled by probability is different from uncertainty modeled by propositional logic. Probability theory often models uncertainty irritating from conflicting, exactly observed, part of knowledge. This situation seems in statistics where a random experiment runs several times and it does not produce the same outcomes. Probability logic is a tool for making inferences from data. Also, Probability logic is used in cases where propositions are either true or false, but the knowledge is imprecise. If the knowledge is fuzzy, then a model of gradual uncertainty is produced since this uncertainty stems from a lack of knowledge. Hence, fuzzy sets causes gradual uncertainty which is different from the one of frequentest probability. This is possibility theory (^{58 59}).

In complex real world cases, many types of uncertainty exist. For example, “morality” or “beauty” of each population of humans chosen at random can be researched and so on. These examples are fuzzy concepts that is accurately formulated by fuzzy sets. Each type of uncertainty has its mathematical model. Different mathematical theories are like tools which are advantageous to use than another in a given situation. Some of these theories may be used individually or in conjunction. However, the right combination of mathematical theories should be chosen.

This discussion points out that uncertainty differs from imprecision and vagueness and only result from them. In order to see better the differences between the three notions discussed here, let us consider the

following assertions about some car:

- This car is between 10 and 15 years old (pure imprecision).
- This car is very big (imprecision and vagueness).
- This car was probably made in Germany (uncertainty).

In the first case there is a lack of knowledge, due to a lack of ability to measure or to evaluate numerical features. In the second case there is a lack of precise definition of the notion “big” and the modifier “very” indicates a rough degree of “bigness”, and the third case expresses uncertainty about a well-defined proposition, perhaps based on statistics.⁵⁸

The originality of fuzzy sets is to catch the idea of partial membership. The membership function of a fuzzy set is a function whose range is an ordered membership set containing more than two values. Thus, a fuzzy set can be thought as a function. Also, the fuzzy set theory are to handle functions as if they were subsets of their domains because such functions are used to state gradual classes. Therefore, the concepts such as intersection, union, complement, inclusion are enlarged in order to combine functions ranging on an ordered membership set. In fuzzy set theory, the union, intersection, inclusion and complementation of functions are committed by taking their point wise maximum, minimum, inequality between functions, and by means of order-reversing automorphism of the membership scale, respectively. Because degrees of membership may be thought as degrees of truth, intersection as conjunction, union as disjunction, complementation as negation and set-inclusion as implication, it seems easily that the fuzzy set theory is closely related to many-valued logics. Moreover, many mathematical notions are extended to fuzzy sets. It is defined special types of fuzzy sets such as convex fuzzy sets, fuzzy numbers, fuzzy intervals and fuzzy relations, and some types of non-classical sets that are different from fuzzy sets.

Gottwald⁷¹ and Ostasiewicz^(106 107) defined the early developments of fuzzy sets in detailed. Also, Ostasiewicz gave the basic philosophical parts of fuzzy set theory.¹⁰⁵ The first published book on fuzzy sets wrote by Kaufmann in French (1973) and translated into English in 1975.⁸³ Up to now. Then a mathematical dissertation is followed by Negoita and Ralescu.¹⁰⁰ Dubois and Prade^(60 61), Kandel,⁸⁰ Novak,¹⁰² Yager et al.¹⁴⁶ and Klir and Bo Yuan^(84 85) are fundamental books on fuzzy sets. Kruse et al. (1994) focused on probabilistic foundations.⁸⁹ Gottwald,⁷² Lowen⁹⁶ and Nguyen and Walker¹⁰¹ studied on a mathematical structure of fuzzy sets. Zimmerman,¹⁵⁰ Terano, Asai and Sugeno,¹⁴¹ Klir and Folger,⁸⁶ Klir et al⁸⁷ and Pedrycz and Gomide¹⁰⁸ investigate methodological problems and applications. Also, the anthology edited by Dubois, Prade and Yager is seen for some basic fuzzy papers of the first twenty five years and other books.⁵⁷

2 History of Intuitionistic Fuzzy Sets

The set theory based on the concept of element membership to sets has proved itself to be one of the most powerful tools of Modern Mathematics and it has allowed to model and to improve other sciences. However, the concept of element membership to a set is a bivalent concept, used by the values 0 (there is no membership) and 1 (there is membership) and it does not approve other set possibilities in logic.

In today's word, science, technology, biology, social sciences, linguistic, psychology and economics consists of complicated operations and situations for which complete information does not always exist. For these situations, mathematical models are improved to treat various types of systems containing elements of uncertainty. Some of these models are based fuzzy sets. The fuzzy set (FS, briefly) theory introduced by Zadeh regards membership degree and the non-membership degree is the complement of the membership degree. However, this linguistic complement does not provide the logical negation. Since a membership function can be Gaussian, triangular, exponential or any other membership function, and so, there exist a hesitation when a membership function is defined, non-membership degree is less than or equal to the complement of the membership degree. Because different results are obtained with different membership functions.

In 1985, Atanassov introduced the degree of non-membership (T) and falsehood (F) (or non-membership) and defined the intuitionistic fuzzy set (IFS, for short). An intuitionistic fuzzy set may be thought as an alternative approach to describe a fuzzy sets on is not sufficient for the definition of an incomplete notion by means of an ordinary fuzzy set. In the intuitionistic fuzzy set theory, the non-membership degree is not equal to the complement of the membership degree owing to the fact that a lack of information exists when the membership function is defined. In contrast to fuzzy sets, a intuitionistic fuzzy set has two uncertainties called membership and non-membership degrees. Actually, the idea of intuitionistic fuzziness was a coincidence as a mathematical game. Atanassov obtained extension of the ordinary fuzzy set by adding a second degree (degree of non-membership). Since he did not immediately notice properties of intuitionistic fuzzy sets that are different from properties of fuzzy sets, his first studies in this area followed step by step the existing results in fuzzy sets. Some notions of fuzzy sets are not very difficult to extend formally notions of intuitionistic fuzzy

sets. However, it is interesting to show that the relevant extension has certain properties which do not exist in the basic concept. Since the method of fuzzification has the idea of intuitionism, the name “intuitionistic fuzzy set” (IFS) is suggested by George Gargov who is Atanassov’s former lecturer in the Mathematical Faculty of Sofia University (^{6 77}). The intuitionistic fuzzy set is a successful generalization of the fuzzy set. The fuzzy set has already achieved a great success in theoretical researches and practical applications.⁹⁴ Therefore, it is expected that the intuitionistic fuzzy set could be used to simulate human decision-making processes and any activities requiring human expertise, experience, and knowledge, which are inevitably imprecise or not totally reliable. Thus, it is completely believed that the intuitionistic fuzzy set has a wide prospect of applications to the fields such as management, economics, business and environment as well as military.⁹⁵

The first example for intuitionistic fuzzy sets is given as follow:⁶

This is a story with Johnny and Mary, characters in many Bulgarian anecdotes. They bought a box of chocolates with 10 pieces inside. Being more nimble, Johnny ate seven of them, while Mary – only two.

One of the candies fell into the floor.

In this moment, a girl friend of Mary came and Mary said, “*We can’t treat you with chocolates, because Johnny ate them all*”.

Let us estimate the truth value of this statement at the moment of speaking, i.e., before we have any knowledge of subsequent events.

From classical logic point of view, which uses for estimations the members of the set $\{0, 1\}$, the statement has truth value of 0, since Mary has also taken part in eating the candies and Johnny was not the only one who has eaten them. On the other hand, we are intuitively convinced that the statement is more true than false. Mary greets her guest – statement’s truth value is obviously 0.7. However, at the next moment Johnny can take the fallen candy and place it back into the box of treat the guest, preserving the truth value of Mary’s statement at 0.7, and falsity 0.3. But he can always take advantage of the distraction and eat the last candy which would result in truth value of 0.8 and falsity of 0.2. In this sense the statement depends to a great extent on Johnny’s actions. Therefore, the apparatus of IFSs gives us the most accurate answer to the question: $[0.7, 0.2]$. The degree of uncertainty now is 0.1 and it corresponds to our ignorance of the boundaries of Johnny’s gluttony.

In the beginning of the last century, L. Brouwer introduced the notion of the intuitionism.⁴¹ He proposed to the mathematicians to remove Aristoteles’ law of excluded middle. He said

An immediate consequence was that for a mathematical assertion the two cases of truth and falsehood, formerly exclusively admitted, were replaced by the following three:

(1) has been proved to be true;

(2) has been proved to be absurd;

(3) has neither been proved to be true nor to be absurd, nor do we know a finite algorithm leading to the statement either that is true or that is absurd”.¹⁴³

Thus, if a proposition A exists, then it can be said that either A is true, or A is false, or that it is not known whether A is true or false. Also, the proposition $A \vee \neg A$ is always valid in the first order logic. It has truth value “true” (or 1) in Boolean algebras but this proposition can take value smaller than 1 in the ordinary fuzzy logic of Zadeh and in many-valued logics starting with Lukasiewicz. The same is true for intuitionistic fuzzy sets but the situation happens on semantical and prediction’ level. the prediction in Brouwer’s sense is fuzzified by clarifying the three possibilities. Thus, Gargov proposed the name “intuitionistic fuzzy set”.

By now, the relations between the intuitionistic fuzzy set theory and Brouwer’s intuitionism don’t have been researched in detail.

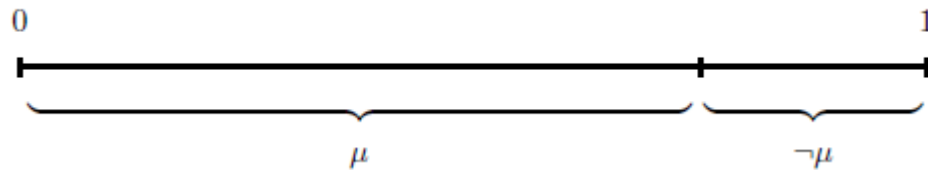
The intuitionistic fuzzy sets allow the definition of operators which are, in a way, similar to the modal operators. Since these operators reduce to identity, they are meaningless in ordinary fuzzy sets.⁷ G. Takeuti and S. Titani just attribute a very different meaning in the notion of an “intuitionistic fuzzy sets”.¹⁴⁰ Atanassov’s first two communications in Bulgarian⁷ and English.⁸

Atanassov started a discussion on the accuracy of the name of the intuitionistic fuzzy sets.⁶² Besides, T. Trifonov and Atanassov built a similar of Takeuti and Titani’s research (⁹¹⁰). H. Bustince and P. Burillo showed to coincide vague sets with intuitionistic fuzzy sets.⁴²

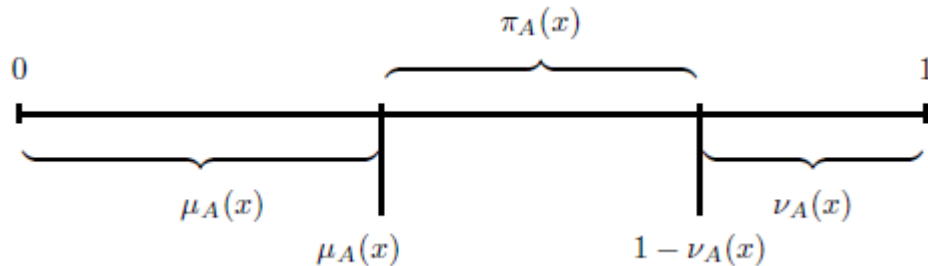
The notion of “interval-valued fuzzy set” (IVFS, shortly) are defined in.⁷⁰ Atanassov and Gargov discussed that this notion equals⁸⁸ to the intuitionistic fuzzy sets (^{11 33}). They showed that each intuitionistic fuzzy set can be stated by an interval-valued fuzzy set and vice versa. Since the intuitionistic fuzzy sets are extensions of the ordinary fuzzy sets, every studies on fuzzy sets may be described in terms of intuitionistic fuzzy sets.

The relations between ordinary fuzzy sets and intuitionistic fuzzy sets are examined from geometrical and probabilistical aspects. Also, some scientists discuss the statement $\mu \vee \neg\mu \leq 1$ in ordinary fuzzy sets as a declaration of the idea of intuitionism. This inequality does not provide the Law of Excluded Middle. However,

this is not the situation in a geometrical interpretation. Having in mind that in fuzzy set theory $\neg\mu = 1 - \mu$, it is obtained that the geometrical interpretation as follows:



The situation in the intuitionistic fuzzy sets case is different and as below:



Now, the geometrical sums of both degrees can really be smaller than 1, i.e., the Law of Excluded Middle is not valid. From probabilistic point of view, for case of the ordinary fuzzy sets, if $\mu \& \neg\mu = 0$, then the probability $p(\mu \vee \neg\mu) = p(\mu) + p(\neg\mu) = 1$ as in the geometrical case, while in the case of intuitionistic fuzzy sets, the inequality $p(\mu \vee \neg\mu) \leq 1$ follows that for proper elements of intuitionistic fuzzy sets will be strong. All of these constructions are only on the level of the definition of the set (fuzzy set or intuitionistic fuzzy sets), i.e., not related to the possible operations that can be defined over these sets.⁶

The notion of vague sets is defined.⁶⁴ It is proved that the notion of vague sets is equivalent to the notion of intuitionistic fuzzy sets.⁴²

A non-probabilistic type of entropy measure is offered for intuitionistic fuzzy sets.¹³⁴ Li and Cheng examined similarity measures of intuitionistic fuzzy sets and their application to pattern recognitions.⁵⁴

De et al. studied on Sanchez's approach for medical diagnosis, generalized this concept with the notion of intuitionistic fuzzy set theory and described some operations of intuitionistic fuzzy sets (^{50 51}).

Some types of fuzzy connectedness in intuitionistic fuzzy topological spaces are defined.¹⁴² Szmidi and Kacprzyk contemplated the usage of intuitionistic fuzzy sets for constructing soft decision-making models with incomplete information and they offered two solutions about the intuitionistic fuzzy core and the concurrence winner for group decision-making by using intuitionistic fuzzy sets (^{135 136 137 138}). Besides, they analyzed distances between intuitionistic fuzzy sets are analyzed.¹³⁹ A novel and effective approach to deal with decision-making in medical diagnosis using the composition of intuitionistic fuzzy relations was suggested.⁵⁵

Different theorems for building intuitionistic fuzzy relations on a set with preestablished features are given.⁴⁶

Different forms of fuzzy propositional representations and their relationships are investigated.⁴⁸

However, there has been barely existed researches on multicriteria or multiattribute in discrete decision situations and/or group decision-making using intuitionistic fuzzy sets, which is in fact one of the most crucial fields in decision analysis as in real world decision problems contain multiple criteria and a group of decision makers (⁵²⁹⁴). Multiattribute decision-making using intuitionistic fuzzy sets is researched, a lot of suitable linear programming models are built to produce optimal weights of attributes and the suitable decision-making methods are suggested.⁹³

Moreover, various types of intuitionistic fuzzy sets are introduced (^{30 31 32}). Two versions of intuitionistic fuzzy propositional calculus, intuitionistic fuzzy predicate calculus, two versions of intuitionistic fuzzy modal logic and temporal intuitionistic fuzzy logic (TIFL, briefly) were defined and investigated in a series of communications (^{33 34 35 65}).

Versions of FORTRAN, C, and PASCAL software packages implementing operations, relations and operators over the intuitionistic fuzzy sets are introduced.²⁹ V-fuzzy Petri nets, reduced V-fuzzy generalized nets,¹²⁸ intuitionistic fuzzy generalized nets of type I¹⁹ and II (^{20 21}) and intuitionistic fuzzy programs²² have

been investigated. A gravity field of many bodies is examined [428]. Intuitionistic fuzzy models of neural networks is improved on the basis of intuitionistic fuzzy sets. Intuitionistic fuzzy expert systems (^{23 24 25 110 111}), intuitionistic fuzzy systems,²⁶ intuitionistic fuzzy PROLOG (^{27 28 67}) and intuitionistic fuzzy constraint logic programming (^{16 17}) are studied. Some new results on the intuitionistic fuzzy sets theory and its applications were reported to the “Mathematical Foundation of Artificial Intelligence” Seminars held in Sofia in October, 1989 (^{18 66 67 74}), March, 1990 (^{14 15 30 68 129 133}), June, 1990 (^{130 131}), November, 1990 (^{13 132}) and October, 1994 (^{36 37 43 44 45}). A collection of open questions and problems in the theory of intuitionistic fuzzy sets is presented¹³ and discussed.²⁹

3 History of Neutrosophic Theories and Applications

The neutrosophic theory that was founded by F. Smarandache in 1998 constitutes a further generalization of fuzzy set, intuitionistic fuzzy set, picture fuzzy set, Pythagorean fuzzy set, spherical fuzzy set, etc. Since then, this logic has been applied in various domains of science and engineering. F. Smarandache introduced the degree of indeterminacy/neutrality (I) as independent component in 1995 (published in 1998) and he defined the neutrosophic set on three components: $(T, I, F) = (\text{Truth}, \text{Indeterminacy}, \text{Falsehood})$, where in general T, I, F are subsets of the interval $[0, 1]$; in particular T, I, F may be intervals, hesitant sets, or single-values; see F. Smarandache, *Neutrosophy / Neutrosophic probability, set, and logic*, Proquest, Michigan, USA, 1998, <https://arxiv.org/ftp/math/papers/0101/0101228.pdf> <http://fs.unm.edu/eBook-Neutrosophics6.pdf>; reviewed in Zentralblatt fuer Mathematik (Berlin, Germany): <https://zbmath.org/?q=an:01273000> and cited by Denis Howe in The Free Online Dictionary of Computing, England, 1999.

Neutrosophic Set and Logic are generalizations of classical, fuzzy, and intuitionistic fuzzy set and logic: <https://arxiv.org/ftp/math/papers/0404/0404520.pdf> <https://arxiv.org/ftp/math/papers/0303/0303009.pdf>

Nonstandard Neutrosophic Logic, Set, Probability (1998, 2019) <https://arxiv.org/ftp/arxiv/papers/1903/1903.04558.pdf>

While Neutrosophic Probability and Statistics are generalizations of classical and imprecise probability and statistics.

Etymology.

The words “neutrosophy” and “neutrosophic” were coined/invented by F. Smarandache in his 1998 book.

Neutrosophy: A branch of philosophy, introduced by F. Smarandache in 1980, which studies the origin, nature, and scope of neutralities, as well as their interactions with different idealization spectra. Neutrosophy considers a proposition, theory, event, concept, or entity $\langle A \rangle$ in relation to its opposite $\langle \text{anti}A \rangle$, and with their neutral $\langle \text{neut}A \rangle$. Neutrosophy (as dynamic of opposites and their neutrals) is an extension of the Dialectics (which is the dynamic of opposites only).

Neutrosophy is the basis of neutrosophic logic, neutrosophic probability, neutrosophic set, and neutrosophic statistics.

<https://arxiv.org/ftp/math/papers/0010/0010099.pdf>

Neutrosophic Logic is a general framework for unification of many existing logics, such as fuzzy logic (especially intuitionistic fuzzy logic), paraconsistent logic, intuitionistic logic, etc. The main idea of NL is to characterize each logical statement in a 3D-Neutrosophic Space, where each dimension of the space represents respectively the truth (T), the falsehood (F), and the indeterminacy (I) of the statement under consideration, where T, I, F are standard or non-standard real subsets of $]^{-0}, 1^{+}[$ with not necessarily any connection between them. For software engineering proposals the classical unit interval $[0, 1]$ may be used.

Degrees of Dependence and Independence between Neutrosophic Components T, I, F are independent components, leaving room for incomplete information (when their superior sum < 1), paraconsistent and contradictory information (when the superior sum > 1), or complete information (sum of components $= 1$).

For software engineering proposals the classical unit interval $[0, 1]$ is used.

For single valued neutrosophic logic, the sum of the components is:

$0 \leq t + i + f \leq 3$ when all three components are independent;

$0 \leq t + i + f \leq 2$ when two components are dependent, while the third one is independent from them;

$0 \leq t + i + f \leq 1$ when all three components are dependent.

When three or two of the components T, I, F are independent, one leaves room for incomplete information ($\text{sum} < 1$), paraconsistent and contradictory information ($\text{sum} > 1$), or complete information ($\text{sum} = 1$).

If all three components T, I, F are dependent, then similarly one leaves room for incomplete information ($\text{sum} < 1$), or complete information ($\text{sum} = 1$).

In general, the sum of two components x and y that vary in the unitary interval $[0, 1]$ is: $0 \leq x + y \leq 2 - d^0(x, y)$, where $d^0(x, y)$ is the degree of dependence between x and y , while $d^0(x, y)$ is the degree of independence between x and y .

<https://doi.org/10.5281/zenodo.571359>

<http://fs.unm.edu/NSS/DegreeOfDependenceAndIndependence.pdf>

In 2013 Smarandache refined the neutrosophic set to n components:

$(T_1, T_2, \dots; I_1, I_2, \dots; F_1, F_2, \dots)$;

<https://arxiv.org/ftp/arxiv/papers/1407/1407.1041.pdf>

<http://fs.unm.edu/n-ValuedNeutrosophicLogic-PiP.pdf>

The Most Important Books and Papers in the Advancement of Neutrosophics 1995-1998 – Smarandache generalized the dialectics to neutrosophy; introduced the neutrosophic set/logic/probability/statistics; introduces the single-valued neutrosophic set (pp. 7-8);

<https://arxiv.org/ftp/math/papers/0101/0101228.pdf> (fourth edition)

<http://fs.unm.edu/eBook-Neutrosophics6.pdf> (online edition)

2002 – Introduction of corner cases of sets / probabilities / statistics / logics, such as:

-Neutrosophic intuitionistic set (different from intuitionistic fuzzy set), neutrosophic paraconsistent set, neutrosophic faillibilist set, neutrosophic paradoxist set, neutrosophic pseudo-paradoxist set, neutrosophic tautological set, neutrosophic nihilist set, neutrosophic dialetheist set, neutrosophic trivialist set;

-Neutrosophic intuitionistic probability and statistics, neutrosophic paraconsistent probability and statistics, neutrosophic faillibilist probability and statistics, neutrosophic paradoxist probability and statistics, neutrosophic pseudo-paradoxist probability and statistics, neutrosophic tautological probability and statistics, neutrosophic nihilist probability and statistics, neutrosophic dialetheist probability and statistics, neutrosophic trivialist probability and statistics;

-Neutrosophic paradoxist logic (or paradoxism), neutrosophic pseudo-paradoxist logic (or neutrosophic pseudo-paradoxism), neutrosophic tautological logic (or neutrosophic tautologism):

<https://arxiv.org/ftp/math/papers/0301/0301340.pdf>

<http://fs.unm.edu/DefinitionsDerivedFromNeutrosophics.pdf>

2003 – Introduction by Kandasamy and Smarandache of Neutrosophic Numbers ($a + bI$, where $I = \text{indeterminacy}$, $I^2 = I$), I-Neutrosophic Algebraic Structures and Neutrosophic Cognitive Maps

<https://arxiv.org/ftp/math/papers/0311/0311063.pdf>

<http://fs.unm.edu/NCMs.pdf>

2005 - Introduction of Interval Neutrosophic Set/Logic

<https://arxiv.org/pdf/cs/0505014.pdf>

<http://fs.unm.edu/INSL.pdf>

2006 – Introduction of Degree of Dependence and Degree of Independence between the Neutrosophic Components T, I, F

<http://fs.unm.edu/eBook-Neutrosophics6.pdf> (p. 92)

<http://fs.unm.edu/NSS/DegreeOfDependenceAndIndependence.pdf>

2007 – The Neutrosophic Set was extended [Smarandache, 2007] to Neutrosophic Overset (when some neutrosophic component is > 1), since he observed that, for example, an employee working overtime deserves a degree of membership > 1 , with respect to an employee that only works regular full-time and whose degree of membership $= 1$; and to Neutrosophic Underset (when some neutrosophic component is < 0), since, for example, an employee making more damage than benefit to his company deserves a degree of membership < 0 , with respect to an employee that produces benefit to the company and has the degree of membership > 0 ; and to and to Neutrosophic Offset (when some neutrosophic components are off the interval $[0, 1]$, i.e. some neutrosophic component > 1 and some neutrosophic component < 0).

Then, similarly, the Neutrosophic Logic/Measure/Probability/Statistics etc. were extended to respectively Neutrosophic Over-/Under-/Off- Logic, Measure, Probability, Statistics etc.

<https://arxiv.org/ftp/arxiv/papers/1607/1607.00234.pdf>

<http://fs.unm.edu/NeutrosophicOversetUndersetOffset.pdf>

<http://fs.unm.edu/SVNeutrosophicOverset-JMI.pdf>

<http://fs.unm.edu/IV-Neutrosophic-Overset-Underset-Offset.pdf>

2007 – Smarandache introduced the Neutrosophic Tripolar Set and Neutrosophic Multipolar Set and consequently the Neutrosophic Tripolar Graph and Neutrosophic Multipolar Graph

<http://fs.unm.edu/eBook-Neutrosophics6.pdf> (p. 93)

<http://fs.unm.edu/IFS-generalized.pdf>

2009 – Introduction of N-norm and N-conorm

<https://arxiv.org/ftp/arxiv/papers/0901/0901.1289.pdf>

<http://fs.unm.edu/N-normN-conorm.pdf>

2013 - Development of Neutrosophic Measure and Neutrosophic Probability (chance that an event occurs, indeterminate chance of occurrence, chance that the event does not occur)

<https://arxiv.org/ftp/arxiv/papers/1311/1311.7139.pdf>

<http://fs.unm.edu/NeutrosophicMeasureIntegralProbability.pdf>

2013 – Smarandache Refined the Neutrosophic Components (T, I, F) as

$(T_1, T_2, \dots; I_1, I_2, \dots; F_1, F_2, \dots)$

<http://fs.unm.edu/n-ValuedNeutrosophicLogic-PiP.pdf>

2014 – Introduction of the Law of Included Multiple Middle

$(< A >; < neut1A >, < neut2A >, \dots, < antiA >)$

<http://fs.unm.edu/LawIncludedMultiple-Middle.pdf>

2014 - Development of Neutrosophic Statistics (indeterminacy is introduced into classical statistics with respect to the sample/population, or with respect to the individuals that only partially belong to a sample/population)

<https://arxiv.org/ftp/arxiv/papers/1406/1406.2000.pdf>

<http://fs.unm.edu/NeutrosophicStatistics.pdf>

2015 - Introduction of Neutrosophic Precalculus and Neutrosophic Calculus

<https://arxiv.org/ftp/arxiv/papers/1509/1509.07723.pdf>

<http://fs.unm.edu/NeutrosophicPrecalculusCalculus.pdf>

2015 – Refined Neutrosophic Numbers $(a + b_1 I_1 + b_2 I_2 + \dots + b_n I_n)$, where $I_1, I_2, \dots, I_n$ are subindeterminacies of indeterminacy I;

2015 – (t, i, f) -neutrosophic graphs;

2015 - Thesis-Antithesis-Neutrothesis, and Neutrosynthesis, Neutrosophic Axiomatic System, neutrosophic dynamic systems, symbolic neutrosophic logic, (t, i, f) -Neutrosophic Structures, I-Neutrosophic Structures, Refined Literal Indeterminacy, Quadruple Neutrosophic Algebraic Structures, Multiplication Law of Subindeterminacies:

<https://arxiv.org/ftp/arxiv/papers/1512/1512.00047.pdf>

<http://fs.unm.edu/SymbolicNeutrosophicTheory.pdf>

2015 – Introduction of the Subindeterminacies of the form $(I_0)^n = \frac{k}{0}$, for $k \in \{0, 1, 2, \dots, n-1\}$, into the ring of modulo integers $\mathbb{Z}_n$ - called natural neutrosophic indeterminacies (Vasanth-Smarandache)

<http://fs.unm.edu/MODNeutrosophicNumbers.pdf>

2015 – Introduction of Neutrosophic Crisp Set and Topology (Salama & Smarandache)

<http://fs.unm.edu/NeutrosophicCrispSetTheory.pdf>

2016 – Introduction of Neutrosophic Multisets (as generalization of classical multisets)

<http://fs.unm.edu/NeutrosophicMultisets.htm>

2016 – Introduction of Neutrosophic Triplet Structures and m-valued refined neutrosophic triplet structures [Smarandache - Ali]

<http://fs.unm.edu/NeutrosophicTriplets.htm>

2016 – Introduction of Neutrosophic Duplet Structures

<http://fs.unm.edu/NeutrosophicDuplets.htm>

2017 – 2020 - Neutrosophic Score, Accuracy, and Certainty Functions form a total order relationship on the set of (single-valued, interval-valued, and in general subset-valued) neutrosophic triplets (T, I, F); and these functions are used in MCDM (Multi-Criteria Decision Making):

<http://fs.unm.edu/NSS/TheScoreAccuracyAndCertainty1.pdf>

2017 - In biology Smarandache introduced the Theory of Neutrosophic Evolution: Degrees of Evolution, Indeterminacy or Neutrality, and Involution

<http://fs.unm.edu/neutrosophic-evolution-PP-49-13.pdf>

2017 - Introduction by F. Smarandache of Plithogeny (as generalization of Dialectics and Neutrosophy), and Plithogenic Set/Logic/Probability/Statistics (as generalization of fuzzy, intuitionistic fuzzy, neutrosophic set/logic/probability/statistics)

<https://arxiv.org/ftp/arxiv/papers/1808/1808.03948.pdf>

<http://fs.unm.edu/Plithogeny.pdf>

2018 – Introduction to Neutrosophic Psychology (Neutropsyche, Refined Neutrosophic Memory: conscious, aconscious, unconscious, Neutropsyche Personality, Eros/ Aoristos/ Thanatos, Neutropsyche Crisp

Personality)

<http://fs.unm.edu/NeutropsychicPersonality-ed3.pdf>

2019 - Introduction to Neutrosophic Sociology (Neutrosociology) [neutrosophic concept, or (T, I, F)-concept, is a concept that is T

<http://fs.unm.edu/Neutrosociology.pdf>

2019 & 2020 – Generalization of the classical Algebraic Structures to NeutroAlgebraic Structures (or NeutroAlgebras) [whose operations and axioms are partially true, partially indeterminate, and partially false] and AntiAlgebraic Structures (or AntiAlgebras) [with operations and axioms totally false]:

<http://fs.unm.edu/NeutroAlgebra.htm>

Applications in: Artificial Intelligence, Information Systems, Computer Science, Cybernetics, Theory Methods, Mathematical Algebraic Structures, Applied Mathematics, Automation, Control Systems, Big Data, Engineering, Electrical, Electronic, Philosophy, Social Science, Psychology, Biology, Biomedical, Engineering, Medical Informatics, Operational Research, Management Science, Imaging Science, Photographic Technology, Instruments, Instrumentation, Physics, Optics, Economics, Mechanics, Neurosciences, Radiology Nuclear, Medicine, Medical Imaging, Interdisciplinary Applications, Multidisciplinary Sciences etc.

[Xindong Peng and Jingguo Dai, A bibliometric analysis of neutrosophic set: two decades review from 1998 to 2017, Artificial Intelligence Review, Springer, 18 August 2018;

<http://fs.unm.edu/BibliometricNeutrosophy.pdf>]

Neutrosophic Sets and Systems (NSS) international journal started in 2013 and it is indexed by Scopus, Web of Science (ESCI), DOAJ, Index Copernicus, Redalyc - Universidad Autonoma del Estado de Mexico (IberoAmerica), Publons, CNKI, Google Scholar, Chinese Baidu Scholar etc.

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Submit papers on neutrosophic set/logic/probability/statistics and their applications to the editor-in-chief: smarand@unm.edu.

Encyclopedia of Neutrosophic Researchers

The authors who have published or presented papers on neutrosophics and are not included in the Encyclopedia of Neutrosophic Researchers (ENR), vols. 1, 2, 3, and 4,

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<http://fs.unm.edu/EncyclopediaNeutrosophicResearchers2.pdf>

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<http://fs.unm.edu/EncyclopediaNeutrosophicResearchers4.pdf>

are pleased to send their CV, photo, and List of Neutrosophic Publications to smarand@unm.edu in order to be included into the fifth volume of ENR.

4 History of Plithogenic Sets

In 2018, F. Smarandache introduced the plithogenic set (as generalization of crisp, fuzzy, intuitionistic fuzzy, and neutrosophic sets), which is a set whose elements are characterized by many attributes' values. An attribute value v has a corresponding (fuzzy, intuitionistic fuzzy, or neutrosophic) degree of appurtenance $d(x, v)$ of the element x , to the set P , with respect to some given criteria. In order to obtain a better accuracy for the plithogenic aggregation operators in the plithogenic set, and for a more exact inclusion (partial order), a (fuzzy, intuitionistic fuzzy, or neutrosophic) contradiction (dissimilarity) degree is defined between each attribute value and the dominant (most important) attribute value. The plithogenic intersection and union are linear combinations of the fuzzy operators t_{norm} and t_{conorm} , while the plithogenic complement, inclusion (inequality), equality are influenced by the attribute values contradiction (dissimilarity) degrees.¹²³

Plithogeny is the genesis or origination, creation, formation, development, and evolution of new entities from dynamics and organic fusions of contradictory and/or neutrals and/or non-contradictory multiple old entities. While plithogenic means what is pertaining to plithogeny.

A plithogenic set P is a set whose elements are characterized by one or more attributes, and each attribute may have many values. Each attribute's value v has a corresponding degree of appurtenance $d(x, v)$ of the element x , to the set P , with respect to some given criteria.

In order to obtain a better accuracy for the plithogenic aggregation operators, a contradiction (dissimilarity) degree is defined between each attribute value and the dominant (most important) attribute value.

{However, there are cases when such dominant attribute value may not be taking into consideration or may

not exist [therefore it is considered zero by default], or there may be many dominant attribute values. In such cases, either the contradiction degree function is suppressed, or another relationship function between attribute values should be established.}

The plithogenic aggregation operators (intersection, union, complement, inclusion, equality) are based on contradiction degrees between attributes' values, and the first two are linear combinations of the fuzzy operators' t_{norm} and t_{conorm} .

Plithogenic set is a generalization of the crisp set, fuzzy set, intuitionistic fuzzy set, and neutrosophic set, since these four types of sets are characterized by a single attribute value (appurtenance): which has one value (membership) – for the crisp set and fuzzy set, two values (membership, and nonmembership) – for intuitionistic fuzzy set, or three values (membership, nonmembership, and indeterminacy) – for neutrosophic set (see¹²³).

<http://fs.unm.edu/NSS/PlithogenicSetAnExtensionOfCrisp.pdf>

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