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On The Neutrosophic Formula of Some Matrix Equations Derived from Data Mining Theory and Control Systems

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Abstract

This paper is dedicated to studying the neutrosophic formula of some famous matrix equations used in theoretical data mining algorithms and control systems by using neutrosophic matrices and refined neutrosophic matrices over neutrosophic real fields. On the other hand, we concentrate on the neutrosophic formula of the Sylvester equation, and Lyapunov equation, where we study their formulas and properties in terms of theorems in the neutrosophic real number field and refined real number field. Also, we illustrate many different examples to clarify the validity of our work.

Keywords: Data mining; Control System; Matrix equation; Neutrosophic matrix; Refined neutrosophic matrix.

Introduction

In the theory of data mining algorithms and control systems, we find many essential matrix equations that describe the state of a system in real-life action [1,2]. For example, three famous matrix equations are used in these two research fields Ricatti, Sylvester, and Lyapunov equations [11,12,13].

In the literature, neutrosophic matrices and refined neutrosophic matrices were defined as generalizations of classical and fuzzy matrices [3]. Where these matrices were built by using the Smarandache idea about indeterminacy I. Many results about neutrosophic matrices and their related spaces, representations, and computations can be found in [4-10].

The motivation of this work is to find a novel application of neutrosophic algebraic structures in applied mathematics fields such as control systems theory and data mining algorithms. We will present a discussion of the neutrosophic formula and refined neutrosophic formula of some special and famous matrix equations.

Main Discussion

Definition (Sylvester Matrix Equation):

It is defined as follows:

$$AX + XB = W; A \in R^{n \times n}, B \in R^{m \times m}, W \in R^{n \times m}.$$

Definition (Lyapunov Matrix equation):

$$AX + XA^T = W; A \in R^{n \times n}, W = W^T \in R^{n \times n}.$$

Definition (Sylvester neutrosophic matrix equation):

We define it as follows:

$$(A_0 + A_1 I)(X_0 + X_1 I) + (X_0 + X_1 I)(B_0 + B_1 I) = W_0 + W_1 I; A_i \in R^{n \times n}, B_i \in R^{m \times m}, W_i \in R^{n \times m}, \text{ and } X = X_0 + X_1 I.$$

Example:

$$\text{Let } A_0 = \begin{pmatrix} 1 & 6 \\ 0 & -2 \end{pmatrix}, A_1 = \begin{pmatrix} 3 & 3 \\ 11 & 4 \end{pmatrix}, B_0 = \begin{pmatrix} 1 & 2 & -5 \\ 7 & 7 & 0 \\ 67 & 1 & 0 \end{pmatrix}, B_1 = \begin{pmatrix} 2 & 2 & 1 \\ 13 & 2 & 13 \\ 16 & -4 & -6 \end{pmatrix}, W_0 = \begin{pmatrix} 2 & 1 & 1 \\ 9 & 7 & 5 \end{pmatrix}, W_1 = \begin{pmatrix} -2 & 0 & 8 \\ 1 & 1 & 3 \end{pmatrix}. \text{ We get:}$$

$$\begin{pmatrix} 1+3I & 6+3I \\ 11I & -2+4I \end{pmatrix} X + X \begin{pmatrix} 1+2I & 2+2I & -5+I \\ 7+13I & 7+2I & 13I \\ 67+16I & 1-4I & -6I \end{pmatrix} = \begin{pmatrix} 2-2I & 1 & 1+8I \\ 9+I & 7+I & 5+3I \end{pmatrix}.$$

Definition (Sylvester refined neutrosophic matrix equation):

We define it as follows:

$$(A_0 + A_1 I_1 + A_2 I_2)(X_0 + X_1 I_1 + X_2 I_2) + (X_0 + X_1 I_1 + X_2 I_2)(B_0 + B_1 I_1 + B_2 I_2) = W_0 + W_1 I_1 + W_2 I_2; A_i \in R^{n \times n}, B_i \in R^{m \times m}, W_i \in R^{n \times m}.$$

Example:

$$\text{Let } A_0 = \begin{pmatrix} 1 & 6 \\ 0 & -2 \end{pmatrix}, A_1 = \begin{pmatrix} 3 & 3 \\ 11 & 4 \end{pmatrix}, A_2 = \begin{pmatrix} 0 & 1 \\ 22 & -3 \end{pmatrix}, B_0 = \begin{pmatrix} 1 & 2 & -5 \\ 7 & 7 & 0 \\ 67 & 1 & 0 \end{pmatrix}, B_1 = \begin{pmatrix} 2 & 2 & 1 \\ 13 & 2 & 13 \\ 16 & -4 & -6 \end{pmatrix}, B_2 = \begin{pmatrix} 1 & 0 & 2 \\ -4 & -1 & 1 \\ 5 & 0 & 0 \end{pmatrix}, W_0 = \begin{pmatrix} 2 & 1 & 1 \\ 9 & 7 & 5 \end{pmatrix}, W_1 = \begin{pmatrix} -2 & 0 & 8 \\ 1 & 1 & 3 \end{pmatrix}, W_2 = \begin{pmatrix} 1 & 0 & 1 \\ -5 & 3 & -8 \end{pmatrix}. \text{ We get:}$$

$$\begin{pmatrix} 1+3I_1 & 6+3I_1+I_2 \\ 11I_1+22I_2 & -2+4I_1-3I_2 \end{pmatrix} X + X \begin{pmatrix} 1+2I_1+I_2 & 2+2I_1 & -5+I_1+2I_2 \\ 7+13I_1-4I_2 & 7+2I_1-I_2 & 13I_1+I_2 \\ 67+16I_1+5I_2 & 1-4I_1 & -6I_1 \end{pmatrix} = \begin{pmatrix} 2-2I_1+I_2 & 1 & 1+8I_1+I_2 \\ 9+I_1-5I_2 & 7+I_1+3I_2 & 5+3I_1-8I_2 \end{pmatrix}.$$

Theorem:

Let $(A_0 + A_1 I)(X_0 + X_1 I) + (X_0 + X_1 I)(B_0 + B_1 I) = W_0 + W_1 I$ be the neutrosophic Sylvester matrix equation, then it is equivalent to the following two classical Sylvester equations:

$$(1) A_0 X_0 + X_0 B_0 = W_0.$$

$$(2) (A_0 + A_1)(X_0 + X_1) + (X_0 + X_1)(B_0 + B_1) = W_0 + W_1.$$

Proof:

By computing the value of the multiplication, we get:

$$A_0 X_0 + X_0 B_0 + I(A_0 X_1 + A_1 X_0 + A_1 X_1 + X_0 B_1 + X_1 B_0 + X_1 B_1) = W_0 + W_1 I, \text{ thus}$$

$A_0X_0 + X_0B_0 = W_0$, and $((A_0 + A_1)(X_0 + X_1))(X_0 + X_1)(B_0 + B_1) - (A_0X_0 + X_0B_0) = W_1$, so that $(A_0 + A_1)(X_0 + X_1) + (X_0 + X_1)(B_0 + B_1) = W_0 + W_1$.

Theorem:

Let $(A_0 + A_1I_1 + A_2I_2)(X_0 + X_1I_1 + X_2I_2) + (X_0 + X_1I_1 + X_2I_2)(B_0 + B_1I_1 + B_2I_2) = W_0 + W_1I_1 + W_2I_2$; $A_i \in R^{n \times n}$, $B_i \in R^{m \times m}$, $W_i \in R^{n \times m}$ be the refined neutrosophic Sylvester matrix equation, then it is equivalent to the following classical system of three Sylvester equations:

$$(1) A_0X_0 + X_0B_0 = W_0$$

$$(2) (A_0 + A_1 + A_2)(X_0 + X_1 + X_2) + (X_0 + X_1 + X_2)(B_0 + B_1 + B_2) = W_0 + W_1 + W_2.$$

$$(3) (A_0 + A_2)(X_0 + X_2) + (X_0 + X_2)(B_0 + B_2) = W_0 + W_2.$$

Proof:

By computing the value of the multiplication, we get:

$$A_0X_0 + X_0B_0 + I_1[A_0X_1 + A_1X_0 + A_1X_1 + A_1X_2 + X_0B_1 + X_1B_0 + X_1B_1 + X_1B_2 + A_2X_1 + X_2B_1] + I_2[A_0X_2 + A_2X_0 + A_2X_2 + X_0B_2 + X_2B_0 + X_2B_2] = W_0 + W_1I_1 + W_2I_2.$$

The previous formula can be written as follows:

$$A_0X_0 + X_0B_0 = W_0, (A_0 + A_1 + A_2)(X_0 + X_1 + X_2) + (X_0 + X_1 + X_2)(B_0 + B_1 + B_2) = W_0 + W_1 + W_2, (A_0 + A_2)(X_0 + X_2) + (X_0 + X_2)(B_0 + B_2) = W_0 + W_2.$$

Example:

Consider the following neutrosophic Sylvester equation:

$$\begin{pmatrix} 1 + 3I_1 & 6 + 3I_1 + I_2 \\ 11I_1 + 22I_2 & -2 + 4I_1 - 3I_2 \end{pmatrix} X + X \begin{pmatrix} 1 + 2I_1 + I_2 & 2 + 2I_1 & -5 + I_1 + 2I_2 \\ 7 + 13I_1 - 4I_2 & 7 + 2I_1 - I_2 & 13I_1 + I_2 \end{pmatrix} = \begin{pmatrix} 2 - 2I_1 + I_2 & 1 & 1 + 8I_1 + I_2 \\ 9 + I_1 - 5I_2 & 7 + I_1 + 3I_2 & 5 + 3I_1 - 8I_2 \end{pmatrix}. \text{ It can be written by classical description as follows:}$$

$$\left(\begin{pmatrix} 1 & 6 \\ 0 & -2 \end{pmatrix} \right) (X_0) + (X_0) \begin{pmatrix} 1 & 2 & -5 \\ 7 & 7 & 0 \\ 67 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 \\ 9 & 7 & 5 \end{pmatrix}.$$

$$\begin{pmatrix} 4 & 9 \\ 11 & 2 \end{pmatrix} (X_0 + X_1) + (X_0 + X_1) \begin{pmatrix} 3 & 4 & 4 \\ 20 & 9 & 13 \\ 87 & -3 & -6 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 9 \\ 10 & 8 & 8 \end{pmatrix}.$$

Definition (LyapunovNeutrosophic matrix equation):

We define the neutrosophic version of the Lyapunov matrix equation as follows:

$$(A_0 + A_1I)(X_0 + X_1I) + (X_0 + X_1I)(A_0^T + A_1^TI) = W_0 + W_1I; A_i \in R^{n \times n}, W_i = W_i^T \in R^{n \times n}.$$

Example :

$$A_0 = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 0 & 3 \\ 6 & -2 & 5 \end{pmatrix}, A_1 = \begin{pmatrix} 4 & -2 & -4 \\ 0 & 3 & 1 \\ 5 & 9 & 9 \end{pmatrix}, W_0 = \begin{pmatrix} 2 & 3 & 5 \\ 3 & 1 & 7 \\ 5 & 7 & 6 \end{pmatrix}, W_1 = \begin{pmatrix} 6 & -3 & 4 \\ -3 & 8 & 2 \\ 4 & 2 & 1 \end{pmatrix}.$$

Thus, the neutrosophic Lyapunov equation is:

$$\begin{pmatrix} 2 + 4I & 1 - 2I & 1 - 4I \\ 0 & 3I & 3 + I \\ 6 + 5I & -2 + 9I & 5 + 9I \end{pmatrix} (X_0 + X_1I) + (X_0 + X_1I) \begin{pmatrix} 2 + 4I & 0 & 6 + 5I \\ 1 - 2I & 3I & -2 + 9I \\ 1 - 4I & 3 + I & 5 + 9I \end{pmatrix} = \begin{pmatrix} 2 + 6I & 3 - 3I & 5 + 4I \\ 3 - 3I & 1 + 8I & 7 + 2I \\ 5 + 4I & 7 + 2I & 6 + I \end{pmatrix}.$$

Definition (Lyapunov refined neutrosophic equation):

We define the refined neutrosophic version of the Lyapunov matrix equation as follows:

$$(A_0 + A_1 I_1 + A_2 I_2)(X_0 + X_1 I_1 + X_2 I_2) + (X_0 + X_1 I_1 + X_2 I_2)(A_0^T + A_1^T I_1 + A_2^T I_2) = W_0 + W_1 I_1 + W_2 I_2; A_i \in R^{n \times n}, W_i = W_i^T \in R^{n \times n}.$$

Example:

$$A_0 = \begin{pmatrix} 2 & 1 & 4 \\ 3 & 3 & 2 \\ 0 & 6 & -1 \end{pmatrix}, A_1 = \begin{pmatrix} 1 & 0 & 7 \\ 7 & 2 & 5 \\ 8 & 8 & 2 \end{pmatrix}, A_2 = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 6 & 1 \\ 7 & 3 & -1 \end{pmatrix}, W_0 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & 3 & 2 \end{pmatrix}, W_1 = \begin{pmatrix} 2 & 5 & 2 \\ 5 & 1 & 4 \\ 2 & 4 & 1 \end{pmatrix}, W_2 = \begin{pmatrix} -4 & 3 & -4 \\ 3 & 1 & 1 \\ -4 & 1 & 0 \end{pmatrix}.$$

So, the corresponding refined neutrosophic Lyapunov equation is:

$$\begin{pmatrix} 2 + I_1 & 1 & 4 + 7I_1 \\ 3 + 7I_1 + I_2 & 3 + 2I_1 + 6I_2 & 2 + 5I_1 + I_2 \\ 8I_1 + 7I_2 & 6 + 8I_1 + 2I_2 & -1 + 2I_1 - I_2 \end{pmatrix} (X_0 + X_1 I_1 + X_2 I_2) + (X_0 + X_1 I_1 + X_2 I_2) \begin{pmatrix} 2 + I_1 & 3 + 7I_1 + I_2 & 8I_1 + 7I_2 \\ 1 & 3 + 2I_1 + 6I_2 & 6 + 8I_1 + 2I_2 \\ 4 + 7I_1 & 2 + 5I_1 + I_2 & -1 + 2I_1 - I_2 \end{pmatrix} = \begin{pmatrix} 1 + 2I_1 - 4I_2 & 1 + 5I_1 + 3I_2 & 1 + 2I_1 - 4I_2 \\ 1 + 5I_1 + 3I_2 & I_1 + I_2 & 3 + 4I_1 + I_2 \\ 1 + 2I_1 - 4I_2 & 3 + 4I_1 + I_2 & 2 + I_1 \end{pmatrix}.$$

Theorem:

Let $(A_0 + A_1 I)(X_0 + X_1 I) + (X_0 + X_1 I)(A_0^T + A_1^T I) = W_0 + W_1 I$; $A_i \in R^{n \times n}, W_i = W_i^T \in R^{n \times n}$ be the neutrosophic Lyapunov equation, hence it is equivalent to the following classical system of matrix equations:

$$(1) A_0 X_0 + X_0 A_0^T = W_0.$$

$$(2) (A_0 + A_1)(X_0 + X_1) + (X_0 + X_1)(A_0^T + A_1^T) = W_0 + W_1.$$

Theorem:

Let $(A_0 + A_1 I_1 + A_2 I_2)(X_0 + X_1 I_1 + X_2 I_2) + (X_0 + X_1 I_1 + X_2 I_2)(A_0^T + A_1^T I_1 + A_2^T I_2) = W_0 + W_1 I_1 + W_2 I_2$; $A_i \in R^{n \times n}, W_i = W_i^T \in R^{n \times n}$ be the refined neutrosophic Lyapunov equation, hence it is equivalent to:

$$(1) A_0 X_0 + X_0 A_0^T = W_0$$

$$(2) (A_0 + A_1 + A_2)(X_0 + X_1 + X_2) + (X_0 + X_1 + X_2)(A_0^T + A_1^T + A_2^T) = W_0 + W_1 + W_2.$$

$$(3) (A_0 + A_2)(X_0 + X_2) + (X_0 + X_2)(A_0^T + A_2^T) = W_0 + W_2$$

Example:

The neutrosophic Lyapunov equation

$$\begin{pmatrix} 2 + 4I & 1 - 2I & 1 - 4I \\ 0 & 3I & 3 + I \\ 6 + 5I & -2 + 9I & 5 + 9I \end{pmatrix} (X_0 + X_1 I) + (X_0 + X_1 I) \begin{pmatrix} 2 + 4I & 0 & 6 + 5I \\ 1 - 2I & 3I & -2 + 9I \\ 1 - 4I & 3 + I & 5 + 9I \end{pmatrix} = \begin{pmatrix} 2 + 6I & 3 - 3I & 5 + 4I \\ 3 - 3I & 1 + 8I & 7 + 2I \\ 5 + 4I & 7 + 2I & 6 + I \end{pmatrix}$$

is equivalent to:

$$(1) \begin{pmatrix} 2 & 1 & 1 \\ 0 & 0 & 3 \\ 6 & -2 & 5 \end{pmatrix} (X_0) + (X_0) \begin{pmatrix} 2 & 0 & 6 \\ 1 & 0 & -2 \\ 1 & 3 & 5 \end{pmatrix} = \begin{pmatrix} 2 & 3 & 5 \\ 3 & 1 & 7 \\ 5 & 7 & 6 \end{pmatrix}.$$

$$(2) \begin{pmatrix} 6 & -1 & -3 \\ 0 & 3 & 4 \\ 11 & 7 & 14 \end{pmatrix} (X_0 + X_1) + (X_0 + X_1) \begin{pmatrix} 6 & 0 & 11 \\ -1 & 3 & 7 \\ -3 & 4 & 14 \end{pmatrix} = \begin{pmatrix} 8 & 0 & 9 \\ 0 & 9 & 9 \\ 9 & 9 & 7 \end{pmatrix}.$$

Conclusion

In this paper, we have presented the neutrosophic formula of some matrix equations derived from the theory of control systems, where we have discussed the Lyapunov, and Sylvester matrix equation in the neutrosophic real field and refined neutrosophic real field. On the other hand, we have illustrated many examples to clarify the validity of this work.

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