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Pre-separation Axioms in Neutrosophic Topological Spaces

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Abstract

In this article, we first establish a few relationships among neutrosophic interior, neutrosophic closure, neutrosophic pre-open sets, and neutrosophic pre-closed sets in single-valued neutrosophic topological spaces. Thereafter, we defined neutrosophic pre-T_0 space, neutrosophic pre-T_1 space, and neutrosophic pre-T_2 space based on single-valued neutrosophic topological spaces and studied a few properties and relationships among them. We try to establish some relationships between existing neutrosophic separation axioms and newly defined neutrosophic pre-separation axioms. Finally, we study some hereditary properties of pre-separation axioms. Apart from these, we also explore some results implementing neutrosophic pre-open function, neutrosophic pre-continuous function, neutrosophic pre-irresolute function and neutrosophic pre*-function based on our defined definitions.

Keywords: Neutrosophic subspace ; Neutrosophic pre-T_0space ; Neutrosophic pre-T_1space ; Neutrosophic pre-T_2 space ; Neutrosophic pre-open set ; Neutrosophic pre-closed set ; Neutrosophic pre-open function ; Neutrosophic pre-continuous function ; Neutrosophic pre-irresolute function ; Neutrosophic pre*-continuous function.

1. Introduction:

The thought of a fuzzy set was revealed by L.A. Zadeh[22] in the year 1965 and a generalized model of a fuzzy set, known as the intuitionistic fuzzy set, was published by K. Atanassov[1] in 1986. Afterward, the notion of a neutrosophic set was developed and studied by Florentin Smarandache[13,14,15]. A neutrosophic set is entangled with three membership functions which are the truth-membership function, falsity-membership function, and indeterminacy-membership function. It is noteworthy that all these three neutrosophic components are impartial to one another. After Smarandache had brought the idea of neutrosophy to light, it was studied and taken in advance by many researchers[3,12,21] across the globe. There are some innate difficulties in the earlier methods (classical or fuzzy) because of the deficiency of parameterizing tools and so, those methods are insufficient to deal with several real-life problems. These problems can be handled in a more general and suitable way with the help of neutrosophic theory.

In 2012, Salama & Alblowi[16] revealed the idea of neutrosophic topological space as an extension of intuitionistic fuzzy topological space developed by D.Coker[6] in 1997. Later, a number of notions related to neutrosophic topological spaces had been developed by several researchers[7,9,10,17,18]. Rao & Rao[11] introduced the concepts of neutrosophic pre-open and pre-closed sets in 2017 and thereafter, Arokiarani *et al*[2]. developed the idea of neutrosophic pre-open, pre-closed, and pre-continuous functions. In recent years, separation properties were studied by some researchers[5,8,19,20]. In 2020, Ahu and Ferhat[4] introduced and studied the concept of

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pre-separation axioms in neutrosophic soft topological spaces. From the above discussion, it has been clear that the separation axioms in neutrosophic topological spaces via neutrosophic pre-open sets have not been studied so far.

In this article, our primary motive is to define and learn about the separation axioms using neutrosophic pre-open sets which we shall call neutrosophic pre-separation axioms. But, before proceeding to that, we set up a few definitions and propositions based on neutrosophic interior, neutrosophic closure, neutrosophic pre-open sets, and neutrosophic pre-closed sets in section 3, which will be utilized in studying neutrosophic pre-separation axioms. In section 4, we define neutrosophic pre-T_0 space, neutrosophic pre-T_1 space, neutrosophic pre-T_2 space and study their various properties. We try to establish some relationships between neutrosophic separation axioms[8] and neutrosophic pre-separation axioms. We also study some hereditary properties. Apart from these, we establish some significant results implementing some functions such as neutrosophic continuous function, neutrosophic pre-irresolute function, neutrosophic pre*-continuous function. In section 5, we bestow a conclusion.

The article is organized by conferring some basic notions in section 2. In section 3, we establish some results in connection with single-valued neutrosophic sets. We then define neutrosophic subspace with example and investigate some properties. In section 4, we define neutrosophic T_0, T_1, T_2-spaces and study various properties. In section 5, we confer a conclusion.

2. Preliminaries:

First Definition:[13] Let X be the universe of discourse. A neutrosophic set A over X is defined as $A = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in X\}$, where the functions $\mathcal{T}_A, \mathcal{I}_A, \mathcal{F}_A$ are real standard or non-standard subsets of $]^-0, 1^+[$, i.e., $\mathcal{T}_A : X \to\ ]^-0, 1^+[$, $\mathcal{I}_A : X \to\ ]^-0, 1^+[$, $\mathcal{F}_A : X \to\ ]^-0, 1^+[$ and $^-0 \leq \mathcal{T}_A(x) + \mathcal{I}_A(x) + \mathcal{F}_A(x) \leq 3^+$.

The neutrosophic set A is characterized by the truth-membership function $\mathcal{T}_A$, indeterminacy-membership function $\mathcal{I}_A$, falsehood-membership function $\mathcal{F}_A$.

Second Definition:[21] Let X be the universe of discourse. A single-valued neutrosophic set A over X is defined as $A = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in X\}$, where $\mathcal{T}_A, \mathcal{I}_A, \mathcal{F}_A$ are functions from X to $[0,1]$ and $0 \leq \mathcal{T}_A(x) + \mathcal{I}_A(x) + \mathcal{F}_A(x) \leq 3$.

The set of all single-valued neutrosophic sets over X is denoted by $\mathcal{N}(X)$.

Throughout this article, a neutrosophic set (NS, for short) will mean a single-valued neutrosophic set.

Third Definition:[10] Let $A, B \in \mathcal{N}(X)$. Then

(i) (Inclusion): If $\mathcal{T}_A(x) \leq \mathcal{T}_B(x), \mathcal{I}_A(x) \geq \mathcal{I}_B(x), \mathcal{F}_A(x) \geq \mathcal{F}_B(x)$ for all $x \in X$ then A is said to be a neutrosophic subset of B and which is denoted by $A \subseteq B$.

(ii) (Equality): If $A \subseteq B$ and $B \subseteq A$ then $A = B$.

(iii) (Intersection): The intersection of A and B, denoted by $A \cap B$, is defined as $A \cap B = \{\langle x, \mathcal{T}_A(x) \wedge \mathcal{T}_B(x), \mathcal{I}_A(x) \vee \mathcal{I}_B(x), \mathcal{F}_A(x) \vee \mathcal{F}_B(x)\rangle : x \in X\}$.

(iv) (Union): The union of A and B, denoted by $A \cup B$, is defined as $A \cup B = \{\langle x, \mathcal{T}_A(x) \vee \mathcal{T}_B(x), \mathcal{I}_A(x) \wedge \mathcal{I}_B(x), \mathcal{F}_A(x) \wedge \mathcal{F}_B(x)\rangle : x \in X\}$.

(v) (Complement): The complement of the NS A, denoted by A^c, is defined as $A^c = \{\langle x, \mathcal{F}_A(x), 1 - \mathcal{I}_A(x), \mathcal{T}_A(x)\rangle : x \in X\}$

(vi) (Universal Set): If $\mathcal{T}_A(x) = 1, \mathcal{I}_A(x) = 0, \mathcal{F}_A(x) = 0$ for all $x \in X$ then A is said to be neutrosophic universal set and which is denoted by $\tilde{X}$.

(vii) (Empty Set): If $\mathcal{T}_A(x) = 0, \mathcal{I}_A(x) = 1, \mathcal{F}_A(x) = 1$ for all $x \in X$ then A is said to be neutrosophic empty set and which is denoted by $\tilde{\emptyset}$.

Fourth Definition: [16] Let $\{A_i : i \in \triangle\} \subseteq \mathcal{N}(X)$, where $\triangle$ is an index set. Then

(i) $\cup_{i\in\triangle} A_i = \{\langle x, \vee_{i\in\triangle} \mathcal{T}_{A_i}(x), \wedge_{i\in\triangle} \mathcal{I}_{A_i}(x), \wedge_{i\in\triangle} \mathcal{F}_{A_i}(x)\rangle : x \in X\}$.

(ii) $\cap_{i\in\triangle} A_i = \{\langle x, \wedge_{i\in\triangle} \mathcal{T}_{A_i}(x), \vee_{i\in\triangle} \mathcal{I}_{A_i}(x), \vee_{i\in\triangle} \mathcal{F}_{A_i}(x)\rangle : x \in X\}$.

Fifth Definition:[12] Let $\mathcal{N}(X)$ be the set of all neutrosophic sets over X. An NS $P = \{\langle x, \mathcal{T}_P(x), \mathcal{I}_P(x), \mathcal{F}_P(x)\rangle : x \in X\}$ is called a neutrosophic point (NP, for short) iff for any element $y \in X$, $\mathcal{T}_P(y) = \alpha, \mathcal{I}_P(y) = \beta, \mathcal{F}_P(y) = \gamma$ for $y = x$ and $\mathcal{T}_P(y) = 0, \mathcal{I}_P(y) = 1, \mathcal{F}_P(y) = 1$ for $y \neq x$, where $0 < \alpha \leq 1, 0 \leq$

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$\beta < 1, 0 \le \gamma < 1$. A neutrosophic point $P = \{\langle x, \mathcal{T}_P(x), \mathcal{I}_P(x), \mathcal{F}_P(x)\rangle : x \in X\}$ will be denoted by $x_{\alpha,\beta,\gamma}$. For the NP $x_{\alpha,\beta,\gamma}$, x will be called its support. The complement of the NP $x_{\alpha,\beta,\gamma}$ will be denoted by $(x_{\alpha,\beta,\gamma})^c$. An NS $P = \{\langle x, \mathcal{T}_P(x), \mathcal{I}_P(x), \mathcal{F}_P(x)\rangle : x \in X\}$ is called a neutrosophic crisp point (NCP, for short) iff for any element $y \in X$, $\mathcal{T}_P(y) = 1, \mathcal{I}_P(y) = 0, \mathcal{F}_P(y) = 0$ for $y = x$ and $\mathcal{T}_P(y) = 0, \mathcal{I}_P(y) = 1, \mathcal{F}_P(y) = 1$ for $y \ne x$.

First Proposition:[8] Let X, Y, Z be three sets such that $\emptyset \ne Z \subseteq Y \subseteq X$. Let $A \in \mathcal{N}(X)$ and $\{A_\lambda : \lambda \in \triangle\} \subseteq \mathcal{N}(X)$, where $\triangle$ is an index set. Then

(i) $(\cup_{\lambda\in\triangle} A_\lambda)|_Y = \cup_{\lambda\in\triangle} (A_\lambda|_Y)$. (ii) $(\cap_{\lambda\in\triangle} A_\lambda)|_Y = \cap_{\lambda\in\triangle} (A_\lambda|_Y)$.

(iii) $A^c|_Y = (A|_Y)^c$. (iv) $(A|_Y)|_Z = A|_Z$.

Sixth Definition:[17] Let X and Y be two non-empty sets and $f: X \to Y$ be a function. Also let $A \in \mathcal{N}(X)$ and $B \in \mathcal{N}(Y)$. Then

1. Image of A under f is defined by

$f(A) = \{\langle y, f(\mathcal{T}_A)(y), f(\mathcal{I}_A)(y), (1 - f(1 - \mathcal{F}_A))(y)\rangle : y \in Y\}$, where

$$f(\mathcal{T}_A)(y) = \begin{cases} \sup\{\mathcal{T}_A(x) : x \in f^{-1}(y)\} & if \quad f^{-1}(y) \ne \emptyset \\ 0 & if \quad f^{-1}(y) = \emptyset \end{cases}$$

$$f(\mathcal{I}_A)(y) = \begin{cases} \inf\{\mathcal{I}_A(x) : x \in f^{-1}(y)\} & if \quad f^{-1}(y) \ne \emptyset \\ 1 & if \quad f^{-1}(y) = \emptyset \end{cases}$$

$$(1 - f(1 - \mathcal{F}_A))(y) = \begin{cases} \inf\{\mathcal{F}_A(x) : x \in f^{-1}(y)\} & if \quad f^{-1}(y) \ne \emptyset \\ 1 & if \quad f^{-1}(y) = \emptyset \end{cases}$$

2. Pre-image of B under f is defined by

$$f^{-1}(B) = \{\langle x, f^{-1}(\mathcal{T}_B)(x), f^{-1}(\mathcal{I}_B)(x), f^{-1}(\mathcal{F}_B)(x)\rangle : x \in X\}$$

Seventh Definition:[10] Let $\tau \subseteq \mathcal{N}(X)$. Then τ is called a neutrosophic topology on X if

(i) $\tilde{\emptyset}$ and $\tilde{X}$ belong to τ.
(ii) Arbitrary union of neutrosophic sets in τ is in τ.
(iii) Intersection of any two neutrosophic sets in τ is in τ.

If τ is a neutrosophic topology on X then the pair (X, τ) is called a neutrosophic topological space (NTS, for short) over X. The members of τ are called neutrosophic τ-open sets (neutrosophic open sets or open sets, for short) in X. If for an NS A, $A^c \in \tau$ then A is said to be a neutrosophic τ-closed set (neutrosophic closed set or closed set, for short) in X.

Eighth Definition:[10] Let (X, τ) be a NTS and $A \in \mathcal{N}(X)$. Then the neutrosophic

(i) interior of A, denoted by $int(A)$, is defined as $int(A) = \cup \{G : G \in \tau$ and $G \subseteq A\}$.

(ii) closure of A, denoted by $cl(A)$, is defined as $cl(A) = \cap \{G : G^c \in \tau$ and $G \supseteq A\}$.

First Theorem:[11] Let (X, τ) be a NTS and $A \in \mathcal{N}(X)$. Then.

(i) A is a neutrosophic pre-open set (NPO, for short) in X if and only if $A \subseteq int\big(cl(A)\big)$.
(ii) A is a neutrosophic pre-closed (NPC, for short) set in X if and only if $cl\big(int(A)\big) \subseteq A$.
(iii) A is an NPC set in X if and only if A^c is an NPO set in X.
(iv) Every neutrosophic open set in an NTS is an NPO set.
(vi) Every neutrosophic closed set in an NTS is an NPC set.
(vi) Arbitrary union of NPO sets in X is an NPO set in X.
(vi) Arbitrary intersection of NPC sets in X is an NPC set in X.

If G is an NPO (resp. NPC) set in XX then we may also say that G is a τ-NPO (resp. τ-NPC) set.

Ninth Definition:[2] Let $f: X \to Y$ be a function from an NTS (X, τ) to an NTS (Y, σ). If $f(G)$ is an NPO set in X for every neutrosophic open set G in X then f is called a neutrosophic pre-open function.

Tenth Definition:[2] Let $f: X \to Y$ be a function from an NTS (X, τ) to an NTS (Y, σ). If $f^{-1}(G)$ is an NPO set

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in X for every neutrosophic open set G in Y then f is called a neutrosophic pre-continuous function.

Eleventh Definition:[9] Let $f: X \to Y$ be a function from an NTS (X, τ) to an NTS (Y, σ). If $f^{-1}(G)$ is an NPO set in X for every NPO set G in Y then f is called a neutrosophic pre-irresolute function.

Twelfth Definition:[8] Let (X, τ) be an NTS. Let $\emptyset \neq Y \subseteq X$ and $\tau|_Y = \{G|_Y : G \in \tau\}$. Then $(Y, \tau|_Y)$ is an NTS. The topology $\tau|_Y$ is called the neutrosophic subspace topology of Y and the NTS $(Y, \tau|_Y)$ is called a neutrosophic subspace (or a subspace, for short) of the NTS (X, τ). Members of $\tau|_Y$ are called $\tau|_Y$-open sets in Y. An NS $A \in \mathcal{N}(Y)$ such that $A^c \in \tau|_Y$ is called a $\tau|_Y$-closed set in Y.

Thirteenth Definition:[8] Let $(Y, \tau |_Y)$ be a neutrosophic subspace of an NTS (X, τ) and $A \in \mathcal{N}(Y)$. Then the neutrosophic interior of A, denoted by $int_Y(A)$, is defined as $int_Y(A) = \cup \{G : G \in \tau |_Y$ and $G \subseteq A\}$ and the neutrosophic closure of A, denoted by $cl_Y(A)$, is defined as $cl_Y(A) = \cap \{G : G^c \in \tau |_Y$ and $G \supseteq A\}$.

Fourteenth Definition:[8] A property of an NTS (X, τ) is said to be hereditary if whenever the space X has that property, then so does every subspace of it.

Second Proposition:[8] Let $(Y, \tau |_Y)$ be a neutrosophic subspace of an NTS (X, τ) and $A \in \mathcal{N}(Y)$. Then A is $\tau |_Y$-closed NS iff $A = F |_Y$ for some τ-closed NS F in X.

Fifteenth Definition:[8] An NTS (X, τ) is called a neutrosophic

(i) T_0-space (NT_0-space, for short) iff for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists a $U \in \tau$ such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ or there exists a $V \in \tau$ such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$.

(ii) T_1-space (NT_1-space, for short) iff for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists a $U \in \tau$ such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ and there exists a $V \in \tau$ such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$.

(iii) T_2-space or neutrosophic Hausdorff space (NT_2-space or Hausdorff space, for short) iff for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists $U, V \in \tau$ such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \in V$ and $U \cap V = \widetilde{\emptyset}$.

3. Some Important Results:

First Proposition: Let $(Y, \tau |_Y)$ be a neutrosophic subspace of an NTS (X, τ). Then

(i) $cl_X(G) |_Y = cl_Y(G |_Y)$ for every $G \in \mathcal{N}(X)$, where $cl_Y(G |_Y)$ is the $\tau |_Y$-closure of $G |_Y$ and $cl_X(G)$ is the τ-closure of G.

(ii) $int_X(G) |_Y = int_Y(G |_Y)$ for every $G \in \mathcal{N}(X)$, where $int_Y(G |_Y)$ is the $\tau |_Y$-interior of $G |_Y$ and $int_X(G)$ is the τ-interior of G.

Proof: (i) $cl_X(G) |_Y = [\bigcap\{F : F$ is a τ-closed NS and $G \subseteq F\}] |_Y = \bigcap\{F |_Y : F$ is a τ-closed NS and $G \subseteq F\}$ [by 1st Prop., Sec 2] $= \bigcap\{F |_Y : F |_Y$ is a $\tau |_Y$-closed NS and $G |_Y \subseteq F |_Y\}$ [by 2nd Prop., Sec 2] $= cl_Y(G |_Y)$.

(ii) $int_X(G) |_Y = [\bigcup\{A : A$ is a τ-open NS and $A \subseteq G\}] |_Y = \bigcup\{A |_Y : A$ is a τ-open NS and $A \subseteq G\}$ [by 1st Prop., Sec 2] $= \bigcup\{A |_Y : A |_Y$ is a $\tau |_Y$-open NS and $A |_Y \subseteq G |_Y\}$ [by 12th Def., Sec 2] $= int_Y(G |_Y)$.

Second Proposition: Let $(\boldsymbol{Y}, \boldsymbol{\tau} |_{\boldsymbol{Y}})$ be a neutrosophic subspace of the NTS $(\boldsymbol{X}, \boldsymbol{\tau})$ and $\boldsymbol{A} \in \boldsymbol{\mathcal{N}}(\boldsymbol{Y})$. Then $\boldsymbol{A}$ is a $\boldsymbol{\tau} |_{\boldsymbol{Y}}$-NPC set in $\boldsymbol{Y}$ iff $\boldsymbol{A^c}$ is a $\boldsymbol{\tau} |_{\boldsymbol{Y}}$-NPO set in $\boldsymbol{Y}$.

Proof: A is a $\tau |_Y-$ NPC set $\Leftrightarrow cl_Y\big(int_Y(A)\big) \subseteq A \Leftrightarrow A^c \subseteq \big[cl_Y\big(int_Y(A)\big)\big]^c = int_Y\big(\big(int_Y(A)\big)^c\big) = int_Y\big(cl_Y(A^c)\big) \Leftrightarrow A^c \subseteq int_Y\big(cl_Y(A^c)\big) \Leftrightarrow A^c$ is a $\tau |_Y$-NPO set.

Third Proposition: Let $(\boldsymbol{Y}, \boldsymbol{\tau} |_{\boldsymbol{Y}})$ be a neutrosophic subspace of the NTS $(\boldsymbol{X}, \boldsymbol{\tau})$. Then

(i) $G |_Y$ is a $\tau |_Y$-NPO set in Y for every τ-NPO set G in X.

(ii) $G |_Y$ is a $\tau |_Y$-NPC set in Y for every τ-NPC set G in X.

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Proof: (i) G is a τ-NPO set in $X \Rightarrow G \subseteq int_X(cl_X(G)) \Rightarrow G \mid_Y \subseteq [int_X(cl_X(G))] \mid_Y \Rightarrow G \mid_Y \subseteq int_Y(cl_X(G) \mid_Y)$ [by 1st Prop.(ii), sec 3] $\Rightarrow G \mid_Y \subseteq int_Y(cl_Y(G \mid_Y))$ [by 1st Prop.(i), Sec 3] $\Rightarrow G \mid_Y$ is a $\tau \mid_Y$-NPO set in Y.

(ii) G is a τ-NPC set $\Rightarrow G^c$ is a τ-NPO set $\Rightarrow G^c \mid_Y$ is a $\tau \mid_Y$-NPO set [by 3rd Prop.(i), Sec 3] $\Rightarrow (G \mid_Y)^c$ is a $\tau \mid_Y$-NPO set [by 1st Prop., Sec 2] $\Rightarrow G \mid_Y$ is a $\tau \mid_Y$-NPC set in Y[by 2nd Prop., Sec 3].

First Definition: Let $(\boldsymbol{X}, \boldsymbol{\tau})$ be an NTS and $\boldsymbol{A} \in \boldsymbol{\mathcal{N}}(\boldsymbol{X})$. Then the neutrosophic

(i) pre-interior of A, denoted by $Pint(A)$, is defined as $Pint(A) = \cup\{G: G$ is an NPO set in X and $G \subseteq A\}$.

(ii) pre-closure of A, denoted by $Pcl(A)$, is defined as $Pcl(A) = \cap\{G: G$ is an NPC set in X and $G \supseteq A\}$.

Second Definition: Let $(\boldsymbol{X}, \boldsymbol{\tau})$ and $(\boldsymbol{X}, \boldsymbol{\tau}^*)$ be two NTSs. If every $\boldsymbol{\tau}$-NPO set in $\boldsymbol{X}$ is a $\boldsymbol{\tau}^*$-NPO set in $\boldsymbol{X}$ then $\boldsymbol{\tau}$ is said to be pre-coarser than $\boldsymbol{\tau}^*$ (denoted by $\boldsymbol{\tau} < \boldsymbol{\tau}^*$) or $\boldsymbol{\tau}^*$ is said to be pre-finer than $\boldsymbol{\tau}$ (denoted by $\boldsymbol{\tau}^* > \boldsymbol{\tau}$).

Example on Second definition: Let $\boldsymbol{X} = \{\boldsymbol{a}, \boldsymbol{b}\}$, $\boldsymbol{\tau}^* = \{\widetilde{\emptyset}, \widetilde{\boldsymbol{X}}\}$ and $\boldsymbol{\tau} = \{\widetilde{\emptyset}, \widetilde{\boldsymbol{X}}, \boldsymbol{A}\}$, where $\boldsymbol{A} = \{\langle \boldsymbol{a}, \boldsymbol{1}, \boldsymbol{0}, \boldsymbol{0}\rangle, \langle \boldsymbol{b}, \boldsymbol{0}, \boldsymbol{1}, \boldsymbol{1}\rangle\}$. Obviously both $(\boldsymbol{X}, \boldsymbol{\tau})$ and $(\boldsymbol{X}, \boldsymbol{\tau}^*)$ are NTSs. It is also clear that every $\boldsymbol{\tau}$-NPO set is a $\boldsymbol{\tau}^*$-NPO set. Therefore, $\boldsymbol{\tau}$ is pre-coarser than $\boldsymbol{\tau}^*$, i.e., $\boldsymbol{\tau}^*$ is pre-finer than $\boldsymbol{\tau}$.

Third Definition: Let $\boldsymbol{f}: \boldsymbol{X} \rightarrow \boldsymbol{Y}$ be a function from an NTS $(\boldsymbol{X}, \boldsymbol{\tau})$ to an NTS $(\boldsymbol{Y}, \boldsymbol{\sigma})$. If $\boldsymbol{f}^{-1}(\boldsymbol{G})$ is a neutrosophic open set set in $\boldsymbol{X}$ for every NPO set $\boldsymbol{G}$ in $\boldsymbol{Y}$ then $\boldsymbol{f}$ is called a neutrosophic pre*-continuous function.

Fourth Definition: Let (X, τ) be an NTS and $x_{\alpha,\beta,\gamma}$ be an NP in X. Then A is said to be a pre-neighbourhood of $x_{\alpha,\beta,\gamma}$ iff there exists a τ-NPO set B such that $x_{\alpha,\beta,\gamma} \in B \subseteq$ A.

Fourth Proposition: (X, τ) be an NTS and $A \subseteq X$. Then A is an NPO set in X iff for every NP $x_{\alpha,\beta,\gamma} \in A$, there exists a τ-NPO set B such that $x_{\alpha,\beta,\gamma} \in B \subseteq$ A.

Proof: Necessary part: Obvious.

Sufficient part: Since for every NP $x_{\alpha,\beta,\gamma} \in A$, there exists a τ-NPO set B such that $x_{\alpha,\beta,\gamma} \in B \subseteq$ A, so $A = \cup\{B: B$ is an τ-NPO set and $x_{\alpha,\beta,\gamma} \in B\}$. Since the arbitrary union of NPO sets is an NPO set, A is an NPO set.

Fifth Proposition: In an NTS, every NPO set is a pre-neighbourhood of each of its NP.

Proof: Obvious

4. Neutrosophic Pre-separation Axioms:

First Definition: An NTS (X, τ) is called a neutrosophic pre-T_0-space (NPT_0-space, for short) iff for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists an NPO set U in X such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ or there exists an NPO set V in X such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$.

Example on First Definition: Let $X = \{a, b\}$ and $\tau = \{\widetilde{\emptyset}, \tilde{X}, A, B\}$, where $A = \{\langle a, 1,0,0\rangle, \langle b, 0,1,1\rangle\}$ and $B = \{\langle a, 0,1,1\rangle, \langle b, 1,0,0\rangle\}$. Clearly the NTS (X, τ) is an NPT_0-space.

First Proposition: Let τ and τ^* be two neutrosophic topologies on a set X such that τ^* is pre-finer than τ. If (X, τ) is a NPT_0-space then (X, τ^*) is also an NPT_0-space.

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, be two NPs in X. Since (X, τ) is a NPT_0-space, so there exists a τ-NPO set G in X such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ or there exists a τ-NPO set H in X such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Since τ^* is pre-finer than τ, so every τ-NPO set in X is a τ^*-NPO set in X. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in X such that $x \neq y$, there exists a τ^*-NPO set G such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ or there exists a τ^*-NPO set H such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Hence (X, τ^*) is also an NPT_0-space.

Second Proposition: Let (X, τ) be an NTS. If X is NT_0-space then X is an NPT_0-space.

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, be two NPs in X. Since (X, τ) is a NT_0-space, so there exists a τ-open NS G in X such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ or there exists τ-open NS H in X such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$.

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Since every neutrosophic open set is an NPO set [by 1st Th., Sec 2], so for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists a τ-NPO set G such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ or there exists a τ-NPO set H in X such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Hence (X,τ) is an NPT_0-space.

Remark on Second Prop.: Converse of the 2nd proposition "Let(X,τ) be an NTS. If X is NT_0-space then X is an NPT_0-space" is not true in general.

We establish it by the following counter example.
Let $X = \{a,b\}$ and $\tau = \{\tilde{\emptyset}, \tilde{X}\}$.Clearly (X,τ) is not an NT_0-space.

Wshow that (X,τ) is an NPT_0-space. Let $a_{\alpha,\beta,\gamma}$ and $b_{\alpha',\beta',\gamma'}$ be two NPs in X $a \neq b$. Also let $A = \{\langle a,1,0,0\rangle, \langle b,0,1,1\rangle\}$. Obviously, $A \in \mathcal{N}(X)$, $a_{\alpha,\beta,\gamma} \in A$ but $b_{\alpha',\beta',\gamma'} \notin A$.

Now, $int(cl(A)) = int(\tilde{X}) = \tilde{X} \supseteq A$. Therefore A is an NPO set in$X$. Thus for any two NPs $a_{\alpha,\beta,\gamma}$ and $b_{\alpha',\beta',\gamma'}$, $a \neq b$, there exists an NPO set A in X such that $a_{\alpha,\beta,\gamma} \in A$ but $b_{\alpha',\beta',\gamma'} \notin A$. Therefore $(X,\tau)$is an NPT_0-space.

Thus the NTS (X,τ) is an NPT_0-space but not an NT_0-space.

Third Proposition: Let (X,τ) be a NPT_0-space. Then every neutrosophic subspace of X is an NPT_0-space and hence the property is hereditary.
Proof: Let $(Y,\tau|_Y)$ be a neutrosophic subspace of (X,τ), where $\tau|_Y = \{G|_Y : G \in \tau\}$. We want to show $(Y,\tau|_Y)$ is an NPT_0-space. Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be two NPs in Y such that $x \neq y$. Then $x_{\alpha,\beta,\gamma}, y_{\alpha',\beta',\gamma'} \in X$, $x \neq y$. Since (X,τ) is an NPT_0-space, so there exists a τ-NPO set U such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ or there exists a τ-NPO set V such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$. Then $(x_{\alpha,\beta,\gamma} \in U|_Y$, $y_{\alpha',\beta',\gamma'} \notin U|_Y)$ or $(x_{\alpha,\beta,\gamma} \notin V|_Y$, $y_{\alpha',\beta',\gamma'} \in V|_Y)$. Also by 3rd Prop. of Sec. 3, $U|_Y, V|_Y$ are $\tau|_Y$-NPO sets in Y as U and V are τ-NPO sets in X. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in Y such that $x \neq y$, there exists a $\tau|_Y$-NPO set $U|_Y$ such that $x_{\alpha,\beta,\gamma} \in U|_Y$, $y_{\alpha',\beta',\gamma'} \notin U|_Y$ or there exists a $\tau|_Y$-NPO set $V|_Y$ such that $x_{\alpha,\beta,\gamma} \notin V|_Y$, $y_{\alpha',\beta',\gamma'} \in V|_Y$. Therefore $(Y,\tau|_Y)$ is a NPT_0-space and hence the property is hereditary.

Fourth Proposition: Let (X,τ) be an NTS. Then X is a NPT_0-space iff for any two distinct neutrosophic crisp points $x_{1,0,0}$ and $y_{1,0,0}$ in X, $(x_{1,0,0})\hat{q}[Pcl(y_{1,0,0})]$ or $(y_{1,0,0})\hat{q}[Pcl(x_{1,0,0})]$.
Proof: Necessary part: Suppose that the statement $(x_{1,0,0})\hat{q}[Pcl(y_{1,0,0})]$ or $(y_{1,0,0})\hat{q}[Pcl(x_{1,0,0})]$ is false. Then $(x_{1,0,0})q[Pcl(y_{1,0,0})]$ and $(y_{1,0,0})q[Pcl(x_{1,0,0})]$ are true. Now $(x_{1,0,0})q[Pcl(y_{1,0,0})] \Rightarrow x_{1,0,0} \notin [Pcl(y_{1,0,0})]^c \Rightarrow x_{1,0,0} \notin [\cap\{G: G$ is a τ-NPC set and $y_{1,0,0} \in G\}]^c \Rightarrow x_{1,0,0} \notin \cup\{G^c: G^c$ is a τ-NPO set and $y_{1,0,0} \notin G^c\} \Rightarrow x_{1,0,0} \notin G^c$ for all τ-NPO sets G^c such that $y_{1,0,0} \notin G^c$. This ensures that if H is an τ-NPO set such that $y_{1,0,0} \in H$ then $x_{1,0,0} \in H$. Similarly $(y_{1,0,0})q[Pcl(x_{1,0,0})]$ implies that if K is a τ-NPO set such that $x_{1,0,0} \in K$ then $y_{1,0,0} \in K$. Thus every τ- NPO set containing one of $x_{1,0,0}$ and $y_{1,0,0}$ must contain the other. But this is a contradiction to our assumption that X is a NPT_0-space. Therefore $(x_{1,0,0})\hat{q}[Pcl(y_{1,0,0})]$ or $(y_{1,0,0})\hat{q}[Pcl(x_{1,0,0})]$.

Sufficient part: $x_{\alpha,\beta,\gamma}$ and $y_{p,q,r}$ be any two NPs in X such that $x \neq y$. Now, if $(x_{1,0,0})\hat{q}[Pcl(y_{1,0,0})]$ then $x_{1,0,0} \in [Pcl(y_{1,0,0})]^c$. Obviously, $x_{\alpha,\beta,\gamma} \in [Pcl(y_{1,0,0})]^c$. Obviously $y_{p,q,r} \notin [Pcl(y_{1,0,0})]^c$. Since $Pcl(y_{1,0,0})$ is a τ-NPC set, so $[Pcl(y_{1,0,0})]^c$ is a τ-NPO set. Thus there exists a τ-NPO set $[Pcl(y_{1,0,0})]^c$ in X such that $x_{\alpha,\beta,\gamma} \in [Pcl(y_{1,0,0})]^c$ but $y_{p,q,r} \notin [Pcl(y_{1,0,0})]^c$. Similarly if $(y_{1,0,0})\hat{q}[Pcl(x_{1,0,0})]$ then there exists a τ-NPO set $[Pcl(x_{1,0,0})]^c$ in X such that $x_{\alpha,\beta,\gamma} \notin [Pcl(x_{1,0,0})]^c$ and $y_{p,q,r} \in [Pcl(x_{1,0,0})]^c$. Therefore (X,τ) is an NPT_0-space.

Hence proved.

Fifth Proposition: Let f be a bijective neutrosophic pre-open function from an NTS (X,τ) to an NTS (Y,σ). If (X,τ) is NT_0 then (Y,σ) is an NPT_0-space.

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Proof: Let $y^1_{p,q,r}$ and $y^2_{p',q',r'}$ be any two NPs in Y such that $y^1 \neq y^2$. Since f is bijective, so there exist two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$, $x^1 \neq x^2$, in X such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$. Since X is NT_0, so there exists a τ-open NS G such that $x^1_{\alpha,\beta,\gamma} \in G$, $x^2_{\alpha',\beta',\gamma'} \notin G$ or there exists a τ-open NS H such that $x^1_{\alpha,\beta,\gamma} \notin H$, $x^2_{\alpha',\beta',\gamma'} \in H$. Suppose G exists such that $x^1_{\alpha,\beta,\gamma} \in G$ and $x^2_{\alpha',\beta',\gamma'} \notin G$. Since f is a neutrosophic pre-open function, so $f(G)$ is a σ-NPO set such that $y^1_{p,q,r} = f(x^1_{\alpha,\beta,\gamma}) \in f(G)$ and $y^2_{p',q',r'} = f(x^2_{\alpha',\beta',\gamma'}) \notin f(G)$. Similarly, $f(H)$ is a σ-NPO set such that $y^1_{p,q,r} = f(x^1_{\alpha,\beta,\gamma}) \notin f(H)$ and $y^2_{p',q',r'} = f(x^2_{\alpha',\beta',\gamma'}) \in f(H)$. Thus for any two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$ in Y such that $y^1 \neq y^2$, there exists a σ-NPO set $f(G)$ such that $y^1_{p,q,r} \in f(G)$, $y^2_{p',q',r'} \notin f(G)$ or there exists a σ-NPO set $f(H)$ such that $y^1_{p,q,r} \notin f(H)$, $y^2_{p',q',r'} \in f(H)$. Therefore (Y,σ) is an NPT_0-space.

Sixth Proposition: Let f be a one-one neutrosophic pre-continuous function from an NTS (X,τ) to an NTS (Y,σ). If (Y,σ) is NT_0 then (X,τ) is also an NPT_0-space.
Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since Y is NT_0, so there exists a σ-open NS G such that $y^1_{p,q,r} \in G$, $y^2_{p',q',r'} \notin G$ or there exists a σ-open NS H such that $y^1_{p,q,r} \notin H$, $y^2_{p',q',r'} \in H$. Since f is a neutrosophic pre-continuous function, so $f^{-1}(G)$ is a τ-NPO set in X. Also $y^1_{p,q,r} \in G \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(G) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$ and $y^2_{p',q',r'} \notin G \Rightarrow f^{-1}(y^2_{p',q',r'}) \notin f^{-1}(G) \Rightarrow x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$. Similarly $f^{-1}(H)$ is a τ-NPO set in X such that $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$, $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exists a τ-NPO set $f^{-1}(G)$ in X such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(G), x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$ or there exists a τ-NPO set $f^{-1}(H)$ in X such that $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$. Therefore (X,τ) is a NPT_0-space. Hence proved.

Seventh Proposition: Let f be a one-one neutrosophic pre-irresolute function from an NTS (X,τ) to an NTS (Y,σ). If (Y,σ) is NPT_0 then (X,τ) is also an NPT_0-space.
Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since Y is NPT_0, so there exists a σ-NPO set G such that $y^1_{p,q,r} \in G$, $y^2_{p',q',r'} \notin G$ or there exists a σ-NPO set H such that $y^1_{p,q,r} \notin H$, $y^2_{p',q',r'} \in H$. Since f is a neutrosophic pre-irresolute function, so $f^{-1}(G)$ is a τ-NPO set in X. Also $y^1_{p,q,r} \in G \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(G) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$ and $y^2_{p',q',r'} \notin G \Rightarrow f^{-1}(y^2_{p',q',r'}) \notin f^{-1}(G) \Rightarrow x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$. Similarly $f^{-1}(H)$ is a τ-NPO set in X such that $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$, $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exists a τ-NPO set $f^{-1}(G)$ in X such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(G), x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$ or there exists a τ-NPO set $f^{-1}(H)$ in X such that $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$. Therefore (X,τ) is a NPT_0-space.

Eighth Proposition: Let f be a one-one neutrosophic pre*-continuous function from an NTS (X,τ) to an NTS (Y,σ). If (Y,σ) is NPT_0 then (X,τ) is an NT_0-space.
Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since Y is NPT_0, so there exists a σ-NPO set G such that $y^1_{p,q,r} \in G$, $y^2_{p',q',r'} \notin G$ or there exists a σ-NPO set H such that $y^1_{p,q,r} \notin H$, $y^2_{p',q',r'} \in H$. Since f is a neutrosophic pre*-continuous function, so $f^{-1}(G)$ is a τ-open NS in X. Also $y^1_{p,q,r} \in G \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(G) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$ and $y^2_{p',q',r'} \notin G \Rightarrow f^{-1}(y^2_{p',q',r'}) \notin f^{-1}(G) \Rightarrow x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$. Similarly $f^{-1}(H)$ is a τ-open NS in X such that $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$, $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exists a $\tau-$open NS $f^{-1}(G)$ in X such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(G), x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$ or there exists a $\tau-$

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open NS $f^{-1}(H)$ in X such that $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$. So, (X,τ) is an NT_0-space.

Second Definition: An NTS (X,τ) is called a neutrosophic pre-T_1-space (NPT_1-space, for short) iff for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists an NPO set U in X such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ and there exists an NPO set V in X such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$.

Example on Second Def.: Let $X = \{a,b\}$ and $\tau = \{\widetilde{\emptyset}, \tilde{X}, A, B\}$, where $A = \{\langle a,1,0,0\rangle, \langle b,0,1,1\rangle\}$ and $B = \{\langle a,0,1,1\rangle, \langle b,1,0,0\rangle\}$. Clearly, the NTS (X,τ) is an NPT_1-space.

Ninth Proposition: Let τ and τ^* be two neutrosophic topologies on a set X such that τ^* is pre-finer than τ. If (X,τ) is an NPT_1-space then (X,τ^*) is also an NPT_1-space.
Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, be two NPs in X. Since (X,τ) is an NPT_1-space, so there exists a τ-NPO set G in X such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ and there exists a τ-NPO set H in X such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Since τ^* is pre-finer than τ, so every τ-NPO set in X is a τ^*-NPO set in X. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in X such that $x \neq y$, there exists a τ^*-NPO set G such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ and there exists a τ^*-NPO set H such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Hence (X,τ^*) is an NPT_1-space.

Tenth Proposition: Let (X,τ) be an NTS. If X is NT_1-space then X is an NPT_1-space.

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, be two NPs in X. Since (X,τ) is an NT_1-space, so there exists a τ-open NS G in X such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ and there exists τ-open NS H in X such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Since every neutrosophic open set is an NPO set, so for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists a τ-NPO set G such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ and there exists a τ-NPO set H such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Hence (X,τ) is an NPT_1-space.

Remark on Tenth Proposition: Converse of the 10th proposition "Let (X,τ) be an NTS. If X is NT_1-space then X is an NPT_1-space" is not true in general.

We establish it by the following counter example.

Let $X = \{a,b\}$ and $\tau = \{\widetilde{\emptyset}, \tilde{X}\}$. Clearly (X,τ) is not an NT_1-space.

We now show that (X,τ) is an NPT_1-space. Let $a_{\alpha,\beta,\gamma}$ and $b_{\alpha',\beta',\gamma'}$ be two NPs in X $a \neq b$. Also let $A = \{\langle a,1,0,0\rangle, \langle b,0,1,1\rangle\}$and $B = \{\langle a,0,1,1\rangle, \langle b,1,0,0\rangle\}$. Clearly A and B are two NPO sets in X. Thus for any two NPs $a_{\alpha,\beta,\gamma}$ and $b_{\alpha',\beta',\gamma'}$, $a \neq b$, there exists an NPO set A in X such that $a_{\alpha,\beta,\gamma} \in A$, $b_{\alpha',\beta',\gamma'} \notin A$ and there exists an NPO set B in X such that $a_{\alpha,\beta,\gamma} \notin B$, $b_{\alpha',\beta',\gamma'} \in B$. Therefore, (X,τ) is an NPT_1-space.

Hence the NTS (X,τ) is an NPT_1-space but not an NT_1-space.

Eleventh Proposition: Let (X,τ) be an NPT_1-space. Then every neutrosophic subspace of X is an NPT_1-space and hence the property is hereditary.
Proof: Let $(Y,\tau|_Y)$ be a neutrosophic subspace of (X,τ), where $\tau|_Y = \{G|_Y : G \in \tau\}$. We want to show $(Y,\tau|_Y)$ is an NPT_1-space. Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be two NPs in Y such that $x \neq y$. Then $x_{\alpha,\beta,\gamma}, y_{\alpha',\beta',\gamma'} \in X$, $x \neq y$. Since (X,τ) is an NPT_1-space, so there exists a τ-NPO set U such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ and there exists a τ-NPO set V such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$. Then $(x_{\alpha,\beta,\gamma} \in U|_Y$, $y_{\alpha',\beta',\gamma'} \notin U|_Y)$ or $(x_{\alpha,\beta,\gamma} \notin V|_Y$, $y_{\alpha',\beta',\gamma'} \in V|_Y)$. Also $U|_Y, V|_Y$ are $\tau|_Y$-NPO sets in Y as U and V are τ-NPO sets in X. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in Y such that $x \neq y$, there exists a $\tau|_Y$-NPO set $U|_Y$ such that $x_{\alpha,\beta,\gamma} \in U|_Y$, $y_{\alpha',\beta',\gamma'} \notin U|_Y$ and there exists a $\tau|_Y$-NPO set $V|_Y$ such that $x_{\alpha,\beta,\gamma} \notin V|_Y$, $y_{\alpha',\beta',\gamma'} \in V|_Y$. Therefore, $(Y,\tau|_Y)$ is an NPT_1-space and hence the property is hereditary.

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Twelfth Proposition: Let $(\boldsymbol{X},\boldsymbol{\tau})$ be an NTS. If every neutrosophic point in $\boldsymbol{X}$ is an NPC set then $\boldsymbol{X}$ is an $\boldsymbol{NPT_1}$-space.
Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, be two NPs in X such that $x \neq y$. Since every NP is an NPC set, so the neutrosophic crisp points $x_{1,0,0}$ and $y_{1,0,0}$ are NPC sets in X. Then $\left(x_{1,0,0}\right)^c$ and $\left(y_{1,0,0}\right)^c$ are NPO sets in X such that $x_{\alpha,\beta,\gamma} \in \left(y_{1,0,0}\right)^c, y_{\alpha',\beta',\gamma'} \notin \left(y_{1,0,0}\right)^c$ and $x_{\alpha,\beta,\gamma} \notin \left(x_{1,0,0}\right)^c, y_{\alpha',\beta',\gamma'} \in \left(x_{1,0,0}\right)^c$. Therefore (X,τ) is an NPT_1-space.

Thirteenth Proposition: Let (X,τ) be an NTS. Then every NCP in X is an NPC set iff X is an NPT_1-space.
Proof: Necessary part: Immediate from the 12th prop. of section 4.
Sufficient part: Let $x_{1,0,0}$ be an NCP in X. Also let $y_{p,q,r} \in (x_{1,0,0})^c$ be any NP. Obviously, $x \neq y$. Let us consider an NP $x_{\alpha,\beta,\gamma}$ with support x. Since X is an NPT_1-space, so for $y_{p,q,r}$ and $x_{\alpha,\beta,\gamma}$, there exists a τ-NPO set G such that $y_{p,q,r} \in G$ and $x_{\alpha,\beta,\gamma} \notin G$. Since for all α,β,γ with $0 < \alpha \leq 1, 0 \leq \beta < 1, 0 \leq \gamma < 1$, one such G exists, therefore we must have a τ-NPO set H such that $y_{p,q,r} \in H$ and $x_{1,0,0} \cap H = \widetilde{\emptyset}$. Therefore, $y_{p,q,r} \in H \subseteq (x_{1,0,0})^c$. Therefore, by the proposition 3.9, $(x_{1,0,0})^c$ is a τ-NPO set and consequently $x_{1,0,0}$ is a τ-NPC set.
Hence proved.
Fourteenth Proposition: Let (X,τ) be an NTS. If (X,τ) is an NPT_1-space then it is a NPT_0-space.
Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, be two NPs in X. Since X is NPT_1-space, so there exists a τ-NPO set U such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ and there exists a τ-NPO set such that V $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$. Hence (X,τ) is an NPT_0-space.

Remark on Fourteenth Prop.: Converse of the 14th proposition "Let (X,τ) be an NTS. If (X,τ) is an NPT_1-space then it is a NPT_0-space." is not true in general.

We establish it by the following counter example.
Let $X = \{a,b\}$ and $\tau = \{\widetilde{\emptyset}, \tilde{X}, A\}$, , where $A = \{\langle a,1,0,0\rangle, \langle b,0,1,1\rangle\}$. Clearly the NTS (X,τ) is an NPT_0-space.

We now show that (X,τ) is not an NPT_1-space.

We first establish that the NCP $a_{1,0,0}$ is a not an NPC set. We have $cl\left(int\left(a_{1,0,0}\right)\right) = cl(A) = \tilde{X}$. Therefore $cl\left(int\left(a_{1,0,0}\right)\right) \nsubseteq a_{1,0,0}$, i.e., $a_{1,0,0}$ is not an NPC set. Therefore by the proposition 4.19, (X,τ) is not an NPT_1-space.

Thus the NTS (X,τ) is an NPT_0-space but not an NPT_1-space.

Fifteenth Proposition: Let f be a bijective neutrosophic pre-open function from an NTS (X,τ) to an NTS (Y,σ). If (X,τ) is NT_1 then (Y,σ) is an NPT_1-space.

Proof: Let $y^1_{p,q,r}$ and $y^2_{p',q',r'}$ be any two NPs in Y such that $y^1 \neq y^2$. Since f is bijective, so there exist two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$, $x^1 \neq x^2$, in X such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$. Since X is NT_1, so there exists a τ-open NS G such that $x^1_{\alpha,\beta,\gamma} \in G$, $x^2_{\alpha',\beta',\gamma'} \notin G$ and there exists a τ-open NS H such that $x^1_{\alpha,\beta,\gamma} \notin H$, $x^2_{\alpha',\beta',\gamma'} \in H$. Since f is neutrosophic pre-open function, so $f(G)$ is a σ-NPO set such that $y^1_{p,q,r} = f(x^1_{\alpha,\beta,\gamma}) \in f(G)$ and $y^2_{p',q',r'} = f(x^2_{\alpha',\beta',\gamma'}) \notin f(G)$. Similarly $f(H)$ is a σ-NPO set such that $y^1_{p,q,r} = f(x^1_{\alpha,\beta,\gamma}) \notin f(H)$ and $y^2_{p',q',r'} = f(x^2_{\alpha',\beta',\gamma'}) \in f(H)$. Thus for any two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$ in Y such that $y^1 \neq y^2$, there exists a σ-NPO set $f(G)$ is such that $y^1_{p,q,r} \in f(G)$, $y^2_{p',q',r'} \notin f(G)$ and there exists a σ-NPO set $f(H)$ such that $y^1_{p,q,r} \notin f(H)$, $y^2_{p',q',r'} \in f(H)$. Therefore, (Y,σ) is an NPT_1-space.

Sixteenth Proposition: Let f be a one-one neutrosophic pre-continuous function from an NTS (X,τ) to an NTS (Y,σ). If (Y,σ) is NT_1 then (X,τ) is an NPT_1-space.
Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs

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$y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since Y is NT_1, so there exists a σ-open NS G such that $y^1_{p,q,r} \in G$, $y^2_{p',q',r'} \notin G$ and there exists a σ-open NS H such that $y^1_{p,q,r} \notin H$, $y^2_{p',q',r'} \in H$. Since f is a neutrosophic pre-continuous function, so $f^{-1}(G)$ and $f^{-1}(H)$ are τ-NPO sets in X. Also $y^1_{p,q,r} \in G \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(G) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$ and $y^2_{p',q',r'} \notin G \Rightarrow f^{-1}(y^2_{p',q',r'}) \notin f^{-1}(G) \Rightarrow x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$. Similarly, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$ and $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exists a τ-NPO set $f^{-1}(G)$ in X such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(G), x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$ and there exists a τ-NPO set $f^{-1}(H)$ in X such that $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$. Therefore (X,τ) is an NPT_1-space. Hence proved.

Seventeenth Proposition: Let f be a one-one neutrosophic pre-irresolute function from an NTS (X,τ) to an NTS (Y,σ). If (Y,σ) is NPT_1 then (X,τ) is also an NPT_1-space.

Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since Y is NPT_1, so there exists a σ-NPO set G such that $y^1_{p,q,r} \in G$, $y^2_{p',q',r'} \notin G$ and there exists a σ-NPO set H such that $y^1_{p,q,r} \notin H$, $y^2_{p',q',r'} \in H$. Since f is a neutrosophic pre-irresolute function, so $f^{-1}(G)$ is a τ-NPO set in X. Also $y^1_{p,q,r} \in G \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(G) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$ and $y^2_{p',q',r'} \notin G \Rightarrow f^{-1}(y^2_{p',q',r'}) \notin f^{-1}(G) \Rightarrow x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$. Similarly $f^{-1}(H)$ is a τ-NPO set in X such that $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$, $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exists a τ-NPO set $f^{-1}(G)$ in X such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(G), x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$ and there exists a τ-NPO set $f^{-1}(H)$ in X such that $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$. Therefore (X,τ) is an NPT_1-space.

Eighteenth Proposition: Let f be a one-one neutrosophic pre*-continuous function from an NTS (X,τ) to an NTS (Y,σ). If (Y,σ) is NPT_1 then (X,τ) is an NT_1-space.

Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since Y is NPT_1, so there exists a σ-NPO set G such that $y^1_{p,q,r} \in G$, $y^2_{p',q',r'} \notin G$ and there exists a σ-NPO set H such that $y^1_{p,q,r} \notin H$, $y^2_{p',q',r'} \in H$. Since f is a neutrosophic pre*-continuous function, so $f^{-1}(G)$ is a τ-open NS in X. Also $y^1_{p,q,r} \in G \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(G) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$ and $y^2_{p',q',r'} \notin G \Rightarrow f^{-1}(y^2_{p',q',r'}) \notin f^{-1}(G) \Rightarrow x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$. Similarly $f^{-1}(H)$ is a τ-open NS in X such that $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$, $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exists a $\tau -$ open NS $f^{-1}(G)$ in X such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(G), x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$ and there exists a $\tau -$ open NS $f^{-1}(H)$ in X such that $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$. So, (X,τ) is an NT_1-space.

Third Definition: An NTS (X,τ) is called a neutrosophic pre-T_2-space or neutrosophic Hausdorff space (NPT_2-space or N-Hausdorff space, for short) iff for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exist two NPO sets U,V in X such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \in V$ and $U \cap V = \widetilde{\emptyset}$.

Example on Third Def.: Let $X = \{a,b\}$ and $\tau = \{\widetilde{\emptyset}, \tilde{X}, A, B\}$, where $A = \{\langle a,1,0,0\rangle, \langle b,0,1,1\rangle\}$ and $B = \{\langle a,0,1,1\rangle, \langle b,1,0,0\rangle\}$. Clearly the NTS (X,τ) is an NPT_2-space.

Nineteenth Proposition: Let τ and τ^* be two neutrosophic topologies on a set X such that τ^* is finer than τ. If (X,τ) is an NPT_2-space then (X,τ^*) is also an NPT_2-space.

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, be two NPs in X. Since (X,τ) is an NPT_2-space, so there exist τ-NPO sets G,H such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \in H$ and $G \cap H = \widetilde{\emptyset}$. Since τ^* is finer than τ, so every τ-NPO set is a τ^*-NPO set. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in X such that $x \neq y$, there exist τ-NPO sets G,H such that

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$x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \in H$ and $G \cap H = \widetilde{\emptyset}$. Hence (X, τ^*) is an NPT_2-space.

Twentieth Proposition: Let (X, τ) be an NTS. If (X, τ) is an NT_2-space then it is an NPT_2-space.
Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x \neq y$. Since (X, τ) is an NT_2-space, so there exist τ-open neutrosophic sets H and K such that $x_{\alpha,\beta,\gamma} \in H$, $y_{\alpha',\beta',\gamma'} \in K$ and $H \cap K = \widetilde{\emptyset}$. Since every neutrosophic open set is an NPO set, so for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exist τ-NPO sets H and K such that $x_{\alpha,\beta,\gamma} \in H$, $y_{\alpha',\beta',\gamma'} \in K$ and $H \cap K = \widetilde{\emptyset}$. Hence (X, τ) is an NPT_2-space.

Remark on Twentieth Prop.: Converse of the 20th proposition "Let (X, τ) be an NTS. If (X, τ) is an NT_2-space then it is an NPT_2-space" is not true.

We establish it by the following counter example.
Let $X = \{a, b\}$ and $\tau = \{\widetilde{\emptyset}, \widetilde{X}\}$. Clearly (X, τ) is not an NT_2-space.

We now show that (X, τ) is an NPT_2-space. Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be any two NPs in X. Also let $A = \{\langle a, 1,0,0\rangle, \langle b, 0,1,1\rangle\}$ and $B = \{\langle a, 0,1,1\rangle, \langle b, 1,0,0\rangle\}$. Clearly A and B are two NPO sets in X such that $A \cap B = \widetilde{\emptyset}$. Thus there exist NPO sets A and B such that $x_{\alpha,\beta,\gamma} \in A$, $y_{\alpha',\beta',\gamma'} \in B$ and $A \cap B = \widetilde{\emptyset}$. Therefore (X, τ) is an NPT_2-space.

Thus the NTS (X, τ) is an NPT_2-space but not an NT_2-space.

Twenty-First Proposition: Let (X, τ) be an NPT_2-space. Then every neutrosophic subspace of X is an NPT_2-space and hence the property is hereditary.
Proof: Let $(Y, \tau|_Y)$ be a neutrosophic subspace of (X, τ), where $\tau|_Y = \{G|_Y : G \in \tau\}$. We want to show $(Y, \tau|_Y)$ is an NPT_2-space. Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be two NPs in Y such that $x \neq y$. Then $x_{\alpha,\beta,\gamma}, y_{\alpha',\beta',\gamma'} \in X$, $x \neq y$. Since (X, τ) is NPT_2-space, so there exist τ-NPO sets U, V such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \in V$ and $U \cap V = \widetilde{\emptyset}$. Then $x_{\alpha,\beta,\gamma} \in U|_Y$, $y_{\alpha',\beta',\gamma'} \in V|_Y$ and $(U|_Y) \cap (V|_Y) = (U \cap V)|_Y = \widetilde{\emptyset}|_Y = \widetilde{\emptyset}$. Also $U|_Y, V|_Y$ are $\tau|_Y$-NPO sets in Y as U and V are τ-NPO sets in X. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in Y such that $x \neq y$, there exist $\tau|_Y$-NPO sets $U|_Y$, $V|_Y$ such that $x_{\alpha,\beta,\gamma} \in U|_Y$, $y_{\alpha',\beta',\gamma'} \in V|_Y$ and $(U|_Y) \cap (V|_Y) = \widetilde{\emptyset}$. Therefore, $(Y, \tau|_Y)$ is an NPT_2-space and hence the property is hereditary.

Twenty-Second Proposition: Let (X, τ) be an NTS. If (X, τ) is an NPT_2-space then it is an NPT_1-space.
Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x \neq y$. Since (X, τ) is an NPT_2-space, so there exist τ-NPO sets H and K such that $x_{\alpha,\beta,\gamma} \in H$, $y_{\alpha',\beta',\gamma'} \in K$ and $H \cap K = \widetilde{\emptyset}$. Since $x_{\alpha,\beta,\gamma} \in H$ and $H \cap K = \widetilde{\emptyset}$, so $x_{\alpha,\beta,\gamma} \notin K$. Similarly, $y_{\alpha',\beta',\gamma'} \notin H$. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists a τ-NPO set H such that $x_{\alpha,\beta,\gamma} \in H$, $y_{\alpha',\beta',\gamma'} \notin H$ and there exists a τ-NPO K such that $x_{\alpha,\beta,\gamma} \notin K$, $y_{\alpha',\beta',\gamma'} \in K$. Hence (X, τ) is an NPT_1-space.
Remark on Twenty-Second Proposition: Converse of the 22nd proposition "Let (X, τ) be an NTS. If (X, τ) is an NPT_2-space then it is an NPT_1-space" is not true.

We establish it by the following counter example.
Let $\mathbb{N}$ be the set of all natural numbers and $\mathcal{N}(\mathbb{N})$ be the set of all neutrosophic sets over $\mathbb{N}$. Also let $\widetilde{\mathbb{N}} = \{\langle x, 1,0,0\rangle : x \in \mathbb{N}\}$ and $\widetilde{\emptyset} = \{\langle x, 0,1,1\rangle : x \in \mathbb{N}\}$. Let τ be the set containing $\widetilde{\emptyset}$ and all those neutrosophic sets over $\mathbb{N}$ whose complements are finite. Then $(\mathbb{N}, \tau)$ is a co-finite NTS. Since $\mathbb{N}$ is an infinite set and since $(\mathbb{N}, \tau)$ is a co-finite NTS, so by the 2nd lemma of section 4, $(\mathbb{N}, \tau)$ is not an NPT_2-space. We show that $(\mathbb{N}, \tau)$ is an NPT_1-space. Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be any two NPs in $\mathbb{N}$ such that $x \neq y$. Since $\left((x_{1,0,0})^c\right)^c = x_{1,0,0}$, a finite NS, so $(x_{1,0,0})^c$ is a τ-open NS and therefore, a τ-NPO set. Clearly, $y_{\alpha',\beta',\gamma'} \in (x_{1,0,0})^c$ and $x_{\alpha,\beta,\gamma} \notin (x_{1,0,0})^c$. Therefore, $(x_{1,0,0})^c$ ia a τ-NPO set such that $y_{\alpha',\beta',\gamma'} \in (x_{1,0,0})^c$ and $x_{\alpha,\beta,\gamma} \notin (x_{1,0,0})^c$. Similarly, $(y_{1,0,0})^c$ ia a τ-NPO set such that $y_{\alpha',\beta',\gamma'} \notin (y_{1,0,0})^c$ and $x_{\alpha,\beta,\gamma} \in (y_{1,0,0})^c$. Therefore, $(\mathbb{N}, \tau)$ is an NPT_1-space.

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Thus $(\mathbb{N},\tau)$ is an NPT_1-space but not an NPT_2-space.

First Lemma: In a co-finite NTS, every finite set NS is an NPC set.
Proof: Let (X,τ) be a co-finite NTS and let U be a finite NS in X. As $U=(U^c)^c$ is a finite set, so U^c is a neutrosophic open set, i.e., U is a neutrosophic closed set. Since every neutrosophic closed set is an NPC set, therefore U is an NPC set.

Remark on First Lemma: From the above lemma "In a co-finite NTS, every finite set NS is an NPC set" it is clear that in a co-finite NTS, an NPO set is a neutrosophic set whose complement is a finite neutrosophic set. Therefore, in a co-finite NTS, an NPO set is a neutrosophic open set.

Second Lemma: If X is an infinite set then the co-finite NTS (X,τ) is not an NPT_2-space. **Proof:** Suppose that (X,τ) is an NPT_2-space. Then for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in X such that $x\neq y$, there exist τ-NPO sets G,H such that $x_{\alpha,\beta,\gamma}\in G$, $y_{\alpha',\beta',\gamma'}\in H$ and $G\cap H=\widetilde{\emptyset}$. Since (X,τ) is a co-finite NTS, so G,H are neutrosophic open sets[by remark on 1st lemma] and their complements, i.e., G^c, H^c are finite neutrosophic sets. Now $G\cap H=\widetilde{\emptyset}\Rightarrow(G\cap H)^c=(\widetilde{\emptyset})^c\Rightarrow G^c\cup H^c=\tilde{X}$, which is not possible as $\tilde{X}$ is an infinite neutrosophic set and $G^c\cup H^c$ is a finite neutrosophic set being the union of two finite neutrosophic sets G^c and H^c. Therefore, the co-finite NTS (X,τ) is a not an NPT_2-space.

Twenty-Third Proposition: Let (X,τ) be an NPT_2-space. Then for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{p,q,r}$ such that $x\neq y$, there exists an NPO set G such that $x_{\alpha,\beta,\gamma}\in G$, $y_{p,q,r}\in G^c$ and $y_{p,q,r}\in[Pcl(G)]^c$.

Proof: Since X is NPT_2-space, so for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{p,q,r}$ such that $x\neq y$, there exist two NPO sets G and H in X such that $x_{\alpha,\beta,\gamma}\in G$, $y_{p,q,r}\in H$ and $G\cap H=\widetilde{\emptyset}$. Now $G\cap H=\widetilde{\emptyset}\Rightarrow H\subseteq G^c\Rightarrow y_{p,q,r}\in G^c$. Since H^c is an NPC set and $G\subseteq H^c$, so $Pcl(G)\subseteq H^c\Rightarrow H\subseteq[Pcl(G)]^c\Rightarrow y_{p,q,r}\in[Pcl(G)]^c$. Hence proved.

Twenty-Fourth Proposition: Let (X,τ) be an NPT_2-space. Then for every NP $x_{\alpha,\beta,\gamma}$ in X, $x_{\alpha,\beta,\gamma}=\cap\{Pcl(G): x_{\alpha,\beta,\gamma}\in G$ and G is an NPO set$\}$.

Proof: Let $x_{\alpha,\beta,\gamma}$ be an NP in X. Also let $y_{1,0,0}$ be an NCP in X such that $x\neq y$. Since X is NPT_2-space, so there exist two NPO sets G and H in X such that $x_{\alpha,\beta,\gamma}\in G$, $y_{1,0,0}\in H$ and $G\cap H=\widetilde{\emptyset}$. Now $G\cap H=\widetilde{\emptyset}\Rightarrow H\subseteq G^c\Rightarrow y_{1,0,0}\in G^c$. Since H^c is an NPC set and $G\subseteq H^c$, so $Pcl(G)\subseteq H^c\Rightarrow H\subseteq[Pcl(G)]^c\Rightarrow y_{1,0,0}\in[Pcl(G)]^c\Rightarrow y_{0,1,1}\in Pcl(G)$, i.e., $y_{p,q,r}\cap Pcl(G)=\widetilde{\emptyset}$ for every NP with support $y\neq x$. Thus for every NPO set G such that $x_{\alpha,\beta,\gamma}\in G$, we have $y_{p,q,r}\cap Pcl(G)=\widetilde{\emptyset}$ for all NP such that $x\neq y$. Obviously $x_{\alpha,\beta,\gamma}\in Pcl(G)$ for every NPO G with $x_{\alpha,\beta,\gamma}\in G$. Therefore $x_{\alpha,\beta,\gamma}=\cap\{Pcl(G): x_{\alpha,\beta,\gamma}\in G$ and G is an NPO set$\}$. Hence proved.

Twenty-Fifth Proposition: Let f be a bijective neutrosophic pre-open function from an NTS (X,τ) to an NTS (Y,σ). If $(X,\tau)$is NT_2 then (Y,σ) is an NPT_2-space.

Proof: Let $y^1_{p,q,r}$ and $y^2_{p',q',r'}$ be any two NPs in Y such that $y^1\neq y^2$. Since f is bijective, so there exist two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$, $x^1\neq x^2$, in X such that $f(x^1_{\alpha,\beta,\gamma})=y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'})=y^2_{p',q',r'}$. Since X is NT_2, so there exists a τ-open NSs G and H such that $x^1_{\alpha,\beta,\gamma}\in G$, $x^2_{\alpha',\beta',\gamma'}\in H$ and $G\cap H=\widetilde{\emptyset}$. Since f is neutrosophic pre-open function, so $f(G)$, $f(H)$ are σ-NPO sets such that $y^1_{p,q,r}=f(x^1_{\alpha,\beta,\gamma})\in f(G)$ and $y^2_{p',q',r'}=f(x^2_{\alpha',\beta',\gamma'})\in f(H)$. Again since f is bijective, so $f(G)\cap f(H)=f(G\cap H)=f(\widetilde{\emptyset})=\widetilde{\emptyset}$. Thus for any two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$ in Y such that $y^1\neq y^2$, there exist σ-NPO sets $f(G)$, $f(H)$ such that $y^1_{p,q,r}\in f(G)$, $y^2_{p',q',r'}\in f(H)$ and $f(G)\cap f(H)=\widetilde{\emptyset}$. Therefore (Y,σ) is an NPT_2-space.

Twenty-Sixth Proposition: Let f be a one-one neutrosophic pre-continuous function from an NTS (X,τ) to an NTS (Y,σ). If (Y,σ) is NT_2 then (X,τ) is an NPT_2-space.
Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1\neq x^2$. Since f is one-one, so there exist two NPs

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$y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since Y is NT_2, so there exist σ-open NSs G and H such that $y^1_{p,q,r} \in G$, $y^2_{p',q',r'} \in H$ and $G \cap H = \widetilde{\emptyset}$. Since f is a neutrosophic pre-continuous function, so $f^{-1}(G)$ and $f^{-1}(H)$ are τ-NPO sets in X. Now $f^{-1}(G) \cap f^{-1}(H) = f^{-1}(G \cap H) = f^{-1}(\widetilde{\emptyset}) = \widetilde{\emptyset}$. Also $y^1_{p,q,r} \in G \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(G) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$.Similarly, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exist τ-NPO sets $f^{-1}(G)$ and $f^{-1}(H)$ in X such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$ and $f^{-1}(G) \cap f^{-1}(H) = \widetilde{\emptyset}$. Therefore (X,τ) is an NPT_2-space. Hence proved.

Twenty-Seventh Proposition: Let f be a one-one neutrosophic pre-irresolute function from an NTS (X,τ) to an NTS (Y,σ). If (Y,σ) is NPT_2 then (X,τ) is also an NPT_2-space.

Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since Y is NPT_2, so there exist σ-NPO sets G and H such that $y^1_{p,q,r} \in G$, $y^2_{p',q',r'} \in G$ and $G \cap H = \widetilde{\emptyset}$. Since f is a neutrosophic pre-irresolute function, so $f^{-1}(G)$ and $f^{-1}(H)$ are τ-NPO sets in X. Now $f^{-1}(G) \cap f^{-1}(H) = f^{-1}(G \cap H) = f^{-1}(\widetilde{\emptyset}) = \widetilde{\emptyset}$. Also $y^1_{p,q,r} \in G \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(G) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$. Similarly $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exist τ-NPO sets $f^{-1}(G)$ and $f^{-1}(H)$ in X such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$ and $f^{-1}(G) \cap f^{-1}(H) = \widetilde{\emptyset}$. Therefore (X,τ) is an NPT_2-space. Hence proved.

Twenty-Eighth Proposition: Let f be a one-one neutrosophic pre*-continuous function from an NTS (X,τ) to an NTS (Y,σ). If (Y,σ) is NPT_2 then (X,τ) is an NT_2-space.

Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since Y is NPT_2, so there exist σ-NPO sets G and H such that $y^1_{p,q,r} \in G$, $y^2_{p',q',r'} \in G$ and $G \cap H = \widetilde{\emptyset}$. Since f is a neutrosophic pre*-continuous function, so $f^{-1}(G)$ and $f^{-1}(H)$ are τ-open NSs in X. Now $f^{-1}(G) \cap f^{-1}(H) = f^{-1}(G \cap H) = f^{-1}(\widetilde{\emptyset}) = \widetilde{\emptyset}$. Also $y^1_{p,q,r} \in G \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(G) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$. Similarly $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exist τ-open NSs $f^{-1}(G)$ and $f^{-1}(H)$ in X such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$ and $f^{-1}(G) \cap f^{-1}(H) = \widetilde{\emptyset}$. Therefore (X,τ) is an NT_2-space. Hence proved.

5. Conclusions:

In this article, we have defined neutrosophic pre-T_0 space, neutrosophic pre-T_1 space, and neutrosophic pre-T_2 space in connection with neutrosophic topological spaces based on single-valued neutrosophic sets and then studied their various properties with examples. We have shown that if X is a neutrosophic T_0-space (resp. T_1-space, T_2-space) then X is a neutrosophic pre-T_0 (resp. pre-T_1, pre-T_2) space but the converse is not true. We have established that a neutrosophic pre-T_2 (resp. pre-T_1) space is a neutrosophic pre-T_1 (resp. pre-T_0) space but the converse is not true. We have also proved that if Y is a neutrosophic subspace of X then for every neutrosophic pre-open (resp. pre-closed) set in X, there is a neutrosophic pre-open (resp. pre-closed) set in Y. We have proved that the property of a space of being NPT_0(resp. NPT_1, NPT_2) is a hereditary property. Lastly, we have tried to explore some results using various functions such as neutrosophic pre-open function, neutrosophic continuous function, neutrosophic pre-irresolute function, and pre*-continuous function. In the coming future, we shall study some other types of separation properties. Hope that the findings of this article will assist the research fraternity to move forward with the development of different aspects of neutrosophic topological spaces.

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