On the Classification of n-Refined Neutrosophic Rings and Its Applications in Matrix Computing Algorithms and Linear Systems

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On the Classification of n-Refined Neutrosophic Rings and Its Applications in Matrix Computing Algorithms and Linear Systems

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Abstract

This work is dedicated to classifying the general n-refined neutrosophic ring by building a ring isomorphism and the direct product of the corresponding classical rings with itself. On the other hand, we use the classification property to solve the problem of n-refined neutrosophic computing of Eigen values and Eigen vectors of an n-refined neutrosophic matrix. Also, it will be applied to solve n-refined linear systems and models.

Keywords: n-refined neutrosophic ring; n-refined neutrosophic matrix; neutrosophic eigen value; neutrosophic eigen vector.

1. Introduction

Neutrosophic logic as a new branch of mathematical philosophy has an impact in many fields of human knowledge. We see a lot of applications of Smarandache's work [7] in algebra, analysis, matrix computing, and geometry [1-6,8-15].

The structures of n-refined sets were studied by many researchers around the world, especially in n-refined matrix theory and computing [16].

In this work, we build a ring isomorphism between an n-refined neutrosophic ring and (n+1) copies of the direct product of the classical corresponding ring itself. This classification will lead to many strong solutions for computing eigen values and eigen vectors for an n-refined neutrosophic matrix, which may be very useful in engineering, and statistics [13].

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Also, we apply this isomorphisms to get a solution for the linear system\model of n-refined neutrosophic equations.

For definitions and properties of n-refined neutrosophic rings see [19].

2. **Main** Discussion

Definition

Let $R_n(I) = \{a_0 + a_1 I_1 + \cdots + a_n I_n;\ a_i \in R\} = \{a_0 + \sum_{i=1}^{n} a_i I_i;\ a_i \in R\}$ be an n-refined neutrosophic ring, we define the n-refined AH-isometry as follows:

$$f: R_n(I) \to R^{n+1} = R \times R \ldots \times R\ (n+1\ times);$$

$$f\left(a_0 + \sum_{i=1}^{n} a_i I_i\right) = (a_0, a_0 + a_1 + \cdots + a_n, a_0 + a_2 + a_3 + \cdots + a_n, \ldots, a_0 + a_n)$$

$=\left(a_0, \sum_{i=0}^{n} a_i, \sum_{i\neq 1}^{n} a_i, \sum_{i\neq 1,2}^{n} a_i, \ldots, \sum_{i\neq 1,\ldots,n-1}^{n} a_i\right)$

Theorem

Let f be the n-refined neutrosophic AH-isometry defined above, then:

a) f is well defined mapping.

b) f is a bijection.

c) f is a ring isomprphism.

d) $R_n(I) \cong R^{n+1}$.

proof.

a) Assume that $x = x_0 + \sum_{i=1}^{n} x_i I_i = y = y_0 + \sum_{i=1}^{n} y_i I_i \in R_n(I)$, hence $x_i = y_i$ for all $i = 1,2,\ldots,n$
This implies $x_0 = y_0, \sum_{i=0}^{n} x_i = \sum_{i=0}^{n} y_i, \sum_{i\neq 1}^{n} x_i = \sum_{i\neq 1}^{n} y_i, \ldots, \sum_{i\neq 1,\ldots,n-1}^{n} x_i = \sum_{i\neq 1,\ldots,n-1}^{n} y_i$.
thus $f(x) = f(y)$.

b) First of all, f is injective that is because:
Let $x = x_0 + \sum_{i=1}^{n} x_i I_i \in kerf$, hence $f(x) = 0_{R^{n+1}} = (0,0,\ldots,0)$, so that $x_0 = 0$, $\sum_{i=0}^{n} x_i = 0$, $\sum_{i\neq 1}^{n} x_i = 0,\ldots, \sum_{i\neq 1,\ldots,n-1}^{n} x_i = 0$, thus $x_i = 0$ for all $i = 1,2,\ldots,n$ and $kerf = \{0\}$.
Now, we must prove that f is surjective. For this purpose, we take an arbitrary element $(x_0, x_1, \ldots, x_n) \in R_n(I)$.
There exists $x = x_0 + (x_1 - x_2)I_1 + (x_2 - x_3)I_2 + \cdots + (x_{n-1} - x_n)I_{n-1} + +(x_n - x_0)I_n$.
Such that $f(x) = (x_0, x_1, x_2, \ldots, x_n)$, hence f is surjective.

c) f preserves addition clearly. It remains to prove that f preserves multiplication.
We use induction on n the index of $R_n(I)$.
If $n = 0$, the assumption is true easily.
Suppose that it holde for $n = k$, we must prove it for $n = k + 1$.
Let

$x = x_0 + x_1 I_1 + \cdots + x_{k+1} I_{k+1} = x_0 + \sum_{i=1}^{k} x_i I_i + x_{k+1} I_{k+1}$.

$y = y_0 + y I_1 + \cdots + y_{k+1} I_{k+1} = x_0 + \sum_{i=1}^{k} y_i I_i + y_{k+1} I_{k+1}$.

It is sufficient to check the coefficients of I_{k+1}in the product (x, y).

Doi : https://doi.org/10.54216/IJNS.180401
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The coefficient of I_{k+1} is: $x_0y_{k+1} + x_{k+1}y_0 + x_{k+1}y_{k+1}$.

This implies that the $(k+1)$ component in $f(xy)$ is equal to:

$x_0y_0 + x_0y_{k+1} + x_{k+1}y_0 + x_{k+1}y_{k+1} = (x_0 + x_{k+1}).(y_0 + y_{k+1})$ which is exactly equal to the $k+1$. Component of $f(x)$ multiplied by the corresponding $(k+1)$ Component of $f(y)$, thus by induction we get $f(xy) = f(x).f(y)$ and f is a ring isomprphism.

d) It holds directly according to (c).

Example

Let $Z_4(I) = \{a + bI_1 + cI_2 + dI_3 + eI_4, a, b, c, d, e \in Z\}$be the 4-refined neutrosophic of integers, hence $Z_4(I) \cong Z \times Z \times Z \times Z \times Z$.

Definition

$A = \begin{pmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nn} \end{pmatrix}$is called an n-refined neutrosophic square matrix if $x_{ij} \in R_n(I)$; $R_n(I)$ is an n-refined neutrosophic ring.

Remark

Any n-refined neutrosophic matrix A can be splat as follows: $A = A_0 + A_1I_1 + \cdots + A_nI_n$; A_i are square matrices defined over the classical ring R.

Example

Consider the following real. 3-refined neutrosophic matrix: $A = \begin{pmatrix} 1 + I_1 & 2 - I_2 + I_3 \\ 3 + I_1 + I_2 + I_3 & 2I_1 - I_3 \end{pmatrix}$.

A is equal to:$\begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}I_1 + \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}I_2 + \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}I_3$.

3. The applications of n-refined AH-isometry in matrix computing

In applied mathematics, it is important to compute eigen values and vectors, that is because these values can be used in statistics [13], and diagonalization [3].

In the following, we describe an algorithm to compute the eigen values and vectors of an n-refined neutrosophic real square matrix.

The description of the method:

Let $A = A_0 + A_1I_1 + \cdots + A_nI_n$ be an $m \times m$ squae n-refined neutrosophic matrix, to compute its eigen values follow these steps.

Step 1: compute the AH-isometric image of A, i.e.

$f(A) = \left(A_0, \sum_{i=0}^{n} A_i, \sum_{i\neq 1}^{n} A_i, \ldots, \sum_{i\neq 1,\ldots,n-1}^{n} A_i\right)$.

Step 2: compute the eigen values and vectors of each component by using classical matrix theory.

Step 3: Go back with the inverse isomorphism to get the corresponding n-refined neutrosophic eigen values and vectors.

Example

Consider the following 2×2 2-refined neutrosophic real matrix:

$$A = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix} + \begin{pmatrix} 3 & -3 \\ 1 & 1 \end{pmatrix}I_1 + \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}I_2 = \begin{pmatrix} 1 + 3I_1 + I_2 & 2 - 3I_1 + I_2 \\ I_1 & 2+I_1 + I_2 \end{pmatrix}$$

The AH-isommetric image is:

Doi : https://doi.org/10.54216/IJNS.180401
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$$f(A) = \big((1\ 2\ 0\ 2\ ), (5\ 0\ 1\ 4\ ), (2\ 3\ 0\ 3\ )\big)$$

The eigen values of $(1\ 2\ 0\ 2\ )$ is $\{1,2\}$.

The eigen values of $(5\ 0\ 1\ 4\ )$ is $\{5,4\}$.

The eigen values of $(2\ 3\ 0\ 3\ )$ is $\{2,3\}$.

Now, we get the following triples:

$$x_1 = (1,5,2), x_2 = (1,5,3), x_3 = (1,4,2), x_4 = (1,4,3), x_5 = (2,5,2), x_6 = (2,5,3), x_7 = (2,4,3),$$

$x_8 = (2,4,2)$.

Now, we get the corresponding n-refined neutrosophic eigen values by taking the inverse image of each $x_i; i = 1, \dots, 8$.

$$y_1 = f^{-1}(x_1) = 1 + 3I_1 + I_2, y_2 = f^{-1}(x_2) = 1 + 2I_1 + 2I_2, y_3 = f^{-1}(x_3) = 1 + 2I_1 + I_2,$$

$$y_4 = f^{-1}(x_4) = 1 + 1I_1 + 2I_2, y_5 = f^{-1}(x_5) = 2 + 3I_1, y_6 = f^{-1}(x_6) = 2 + 2I_1 + I_2,$$

$y_7 = f^{-1}(x_7) = 2 + I_1 + I_2, y_8 = f^{-1}(x_8) = 2 + 2I_1$.

4. Algorithm to Solve n-Refined Linear Systems of Equations

To solve a linear system of n-refined neutrosophic equations $A_iX_i = B_i;\ i = 1, \dots, m$, follow these steps:

Step1: Transform $A_iX_i = B_i$to a classical linear system by taking the direct AH-isometric image $f(A_i)f(X_i) = f(B_i)$.

Step2: solve the classical corresponding system and get X, Y.

Example

Consider the following 3-refined neutrosophic system of linear equations:

$$(1 + 2I_1 - I_2)X + (I_2 + I_3)Y = 1 \dots (1)$$

$$(2I_1 - I_3)X + (1 + I_3)Y = I_1 + I_2 \dots (2)$$

Where $X = x_0 + x_1I_1 + x_2I_2 + x_3I_3, Y = y_0 + y_1I_1 + y_2I_2 + y_3I_3\ ;\ x_i, y_i \in R$.

The direct AH-isometry image of equation(1)is:

$$(1,2,0,2).(x_0, x_0 + x_1 + x_2 + x_3, x_0 + x_2 + x_3, x_0 + x_3) + (0,2,2,1).(y_0, y_0 + y_1 + y_2 + y_3, y_0 + y_2 + y_3, y_0 + y_3) = (1,1,1,1)$$

Which implies: $\{x_0 = 1 \dots (a)\ 2(x_0 + x_1 + x_2 + x_3) + 2(y_0 + y_1 + y_2 + y_3) = 1 \dots (b)\ 2(y_0 + y_2 + y_3) = 1 \dots (c)\ (x_0 + x_3) + (y_0 + y_3) = 1 \dots (d)$

the direct AH-isometry image of equation (2) is:

$$(0,1,1,-1).(x_0, x_0 + x_1 + x_2 + x_3, x_0 + x_2 + x_3, x_0 + x_3) + (1,2,2,2).(y_0, y_0 + y_1 + y_2 + y_3, y_0 + y_2 + y_3, y_0 + y_3) = (0,2,1,0)$$

Which implies: $\{y_0 = 0 \dots (e)\ (x_0 + x_1 + x_2 + x_3) + 2(y_0 + y_1 + y_2 + y_3) = 2 \dots (f)\ (x_0 + x_2 + x_3) + 2(y_0 + y_2 + y_3) = 1 \dots (g)\ -(x_0 + x_3) + 2(y_0 + y_3) = 0 \dots (h)$

From (a) and (e), we get $x_0 = 1, y_0 = 0$.

From (b) and (f), we get$x_0 + x_1 + x_2 + x_3 = -1, y_0 + y_1 + y_2 + y_3 = \frac{3}{2}$.

From (c) and (g), we get $x_0 + x_2 + x_3 = 0, y_0 + y_2 + y_3 = \frac{1}{2}$.

From (d) and (h), we get $x_0 + x_3 = \frac{1}{3}, y_0 + y_3 = \frac{2}{3}$.

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This implies: $\{x_0 = 1, y_0 = 0\ x_1 = -1, y_1 = 1\ x_2 = -\frac{2}{3}, y_2 = \frac{1}{6}\ x_3 = -\frac{1}{3}, y_3 = \frac{1}{3}$

The corresponding 3-refined solutions are:

$$X = 1 - I_1 \frac{2}{3} I_2 - \frac{1}{3} I_3, Y = y_1 I_1 + \frac{1}{6} I_2 + \frac{1}{3} I_3$$

Now, we examine the matrix method to solve the previous linear system.

Example:

Consider the linear system of 3-refined neutrosophic equations in the previous example. We write it by using matrix represention as follows:

$$(1 + 2I_1 - I_2\ I_2 + I_3\ I_2 - I_3\ 1 + I_3)(X\ Y) = (1\ I_1 + I_2) \Leftrightarrow AZ = B \Rightarrow Z = A^{-1} \times B$$

Thus, we must compute $A^{-1}$by using n-refined AH-isometry.

$A = (1 + 2I_1 - I_2\ I_2 + I_3\ I_2 - I_3\ 1 + I_3)$=$(1\ 0\ 0\ 1) + (2\ 0\ 0\ 0)I_1 + (-1\ 1\ 2\ 0)I_2 + (0\ 1\ -1\ 1)I_3$

$$f(A) = \big((1\ 0\ 0\ 1), (2\ 2\ 1\ 2), (0\ 2\ 1\ 2) + (1\ 1\ -1\ 2)\big)$$

The inverse of $f(A)$ is:

$$[f(A)]^{-1} = \left((1\ 0\ 0\ 1), \left(1\ -1\ \frac{-1}{2}\ 1\right), \left(-1\ 1\ \frac{1}{2}\ 0\right) + \left(\frac{2}{3}\ \frac{-1}{3}\ \frac{1}{3}\ \frac{1}{3}\right)\right)$$

Hence, $f^{-1}\left[\left(f(A)\right)^{-1}\right] = (1\ 0\ 0\ 1) + (2\ -2\ -1\ 1)I_1 + \left(-\frac{5}{3}\ \frac{4}{3}\ \frac{1}{6}\ -\frac{1}{3}\right)I_2 + \left(-\frac{1}{3}\ -\frac{1}{3}\ \frac{1}{3}\ -\frac{2}{3}\right)I_3$

$$= \left(1 + 2I_1 - \frac{5}{3}I_2 - \frac{1}{3}I_3\ \ -2I_1 + \frac{4}{3}I_2 - \frac{1}{3}I_3\ \ -I_1 + \frac{1}{6}I_2 + \frac{1}{3}I_3\ \ 1 + I_1 - \frac{1}{3}I_2 - \frac{2}{3}I_3\right)$$

$$A^{-1} \times B = \left(1 + 2I_1 - \frac{5}{3}I_2 - \frac{1}{3}I_3\ \ -2I_1 + \frac{4}{3}I_2 - \frac{1}{3}I_3\ \ -I_1 + \frac{1}{6}I_2 + \frac{1}{3}I_3\ \ 1 + I_1 - \frac{1}{3}I_2 - \frac{2}{3}I_3\right)(1\ I_1 + I_2)$$

$\Rightarrow A^{-1} \times B = \left(1 - I_1 - \frac{2}{3}I_2 - \frac{1}{3}I_3\ I_1 + \frac{1}{6}I_2 + \frac{1}{3}I_3\right) \Rightarrow X = 1 - I_1 - \frac{2}{3}I_2 - \frac{1}{3}I_3, Y = I_1 + \frac{1}{6}I_2 + \frac{1}{3}I_3.$

5. Solving n-refined quadratic equations.

Example:

Consider the linear system of 3-refined neutrosophic quadratic equation:

$$I_2X^2 + (1 - I_3)X = 0\ ; X = x_0 + x_1I_1 + x_2I_2 + x_3I_3\ \in R_3(I)$$

To solve this equation, we can use the AH-isometry one more time as follows:

$$f(I_2)f(X^2) + f(1 - I_3).f(X) = f(0)$$

$$\Leftrightarrow (0,1,1,0).({x_0}^2, (x_0 + x_1 + x_2 + x_3)^2, (x_0 + x_2 + x_3)^2, (x_0 + x_3)^2) + (1,0,0,0).(x_0, x_0 + x_1 + x_2 + x_3, x_0 + x_2 + x_3, x_0 + x_3) = (0,0,0,0)$$

$$\Leftrightarrow \{(x_0 + x_1 + x_2 + x_3)^2 = 0 \Rightarrow x_1\ (x_0 + x_2 + x_3)^2 = 0 \Rightarrow x_2 = -x_3\ (x_0 + x_3) = 0 \Rightarrow x_3\ is\ arbitrary\ x_0 = 0$$

Thus the equation has infinite solutions with form:

$X = -x_3I_2 + x_3I_3 = x_3(-I_2 + I_3);\ x_3 \in R.$

Theorem.

Let $R_n(I)$be an n-refined neutrosophic ring, $U\big(R_n(I)\big)$ be the group of units $R_n(I)$, then

$U\big(R_n(I)\big) \cong [U(R)]^{n+1} = U(R) \times U(R) \times \ldots \times U(R)(n + 1. times).$

proof.

According to the previous results, we have $R_n(I) \cong R^{n+1}$, thus $U\big(R_n(I)\big) \cong [U(R)]^{n+1}$.

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Remark.

To compute the units in $R_n(I)$ follow these steps.

Step1. Find the units in the AH-isometric image $f\left(R_n(I)\right) = R^{n+1}$.

Step2. Use the inverse AH-isometry to find the corresponding n-refined neutrosophic units.

Example.

Let $(Z_4)_2(I)$ be the 2-refined neutrosophic ring of integers module 4, we will find its units.

$$f\left((Z_4)_2(I)\right) = Z_4 \times Z_4 \times Z_4 = (Z_4)^3$$

The unit group of Z_4 is $\{1,3\}$.

The units of $(Z_4)^3$ are:

$$e_1 = (1,1,1), e_2 = (1,3,1), e_3 = (3,1,1), e_4 = (1,1,3), e_5 = (3,3,3), e_6 = (3,3,1), e_7 = (3,1,3),$$

$e_8 = (1,3,3)$.

The units of $R_2(I)$ are:

$$f^{-1}(e_1) = 1, f^{-1}(e_2) = 1 + 2I_1, f^{-1}(e_3) = 3 - 2I_2 = 3 + 2I_2, f^{-1}(e_4) = 1 - 2I_1 + 2I_2,$$

$f^{-1}(e_5) = 3, f^{-1}(e_6) = 3 + 2I_1 + 2I_2, f^{-1}(e_7) = 3 + 2I_1, f^{-1}(e_8) = 1 + 2I_2$.

Example.

Let $Z_3(I)$ be the 3-refined neutrosophic ring of integers, $Z_3(I) \cong (Z)^5$,

The units in Z are $\{1,-1\}$, thus the units of $(Z)^5$ are:

$(1,1,1,1), (1,1,1,-1), (1,1,-1,1), (1,-1,11), (-1,1,1,1), (1,1,-1,-1), (-1,-1,1,1), (-1,1,1,-1), (-1,1,-1,1),$

$(-1,-1,-1,1), (-1,-1,-1,-1), (-1,-1,-1,-1), (-1,-1,-1,-1), (1,-1,-1,-1), (1,-1,1,-1), (1,-1,-1,1)$

The corresponding 3-refined neutrosophic units are:

$1, 1 + 2I_2 - 2I_3, 1 + 2I_1 - 2I_2, 1 - 2I_1, -1 + 2I_3, 1 + 2I_1 - 2I_3, -1 - 2I_1 + 2I_3, -1 + 2I_2, -1 + 2I_2, -1 + 2I_1 - 2I_2 + 2I_3, -1 - 2I_2 + 2I_3, -1 - 2I_1 + 2I_2, , -1 + 2I_1, -1, -1 - 2I_2 + 2I_3, -1 - 2I_1 + 2I_2, -1 + 2I_1, 1 - 2I_3, 1 - 2I_1 + 2I_2 - 2I_3, 1 - 2I_2$.

6. Applications in computing powers

Now, we are able to compute any power x^m for all $x \in R_n(I)$. By using the AH-isometry.

Example.

Consider $x = 2 + I_1 - 3I_2 + I_3 + 2I_4 \in R_4(I)$ the 4-refined neutrosophic field of real numbers.

Let's compute x^3.

The AH-isometric image of x is:

$f(x) = (2,3,2,5,4), \left(f(x)\right)^3 = (8,27,8,125,64)$, thus $x^3 = f^{-1}\left[\left(f(x)\right)^3\right] = 8 + 19I_1 - 117I_2 + 61I_3 + 56I_4$.

Example.

Consider $x = 2 + I_1 - 4I_2 \in R_2(I), y = 1 - I_1 + I_2 \in R_2(I)$, we must compute x^y.

$z = [f(x)]^{f(y)} = (2,-1,-2)^{(1,1,2)} \Rightarrow x^y = f^{-1}(z) = 2 - 5I_1 + 2I_2$.

7. Conclusion

In this paper, we have founded a novel algorithm to compute n-refined neutrosophic eigen values\ vectors of an n-refined neutrosophic matrix by building a classification isomorphism. Also, we have applied this classification

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property to solve the group of units' problem and n-refined linear system of equations. In the future, we aim that the previous problems will be discussed for n-cyclic refined neutrosophic matrices.

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