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On Neutrosophic Generalized Alpha Generalized Separation Axioms

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Abstract

The paper provided a new notion of neutrosophic separation axioms as neutrosophic gαg-R_i-space & neutrosophic gαg-T_j-space (note that the indexes i & j are natural numbers of the spaces R & T are from 0 to 1 & from 0 to 2 alternately).

Mathematical Subject Classification (2010): 54A40.

Keywords: $\mathrm{N}^{g\alpha g}$-OS; $\mathrm{N}^{g\alpha g}$-CS; $\mathrm{N}^{g\alpha g}$-R_i-space; $i = 0,1$ & $\mathrm{N}^{g\alpha g}$-T_j-space; $j = 0,1,2$.

1. Introduction

F. Smarandache [1,2] furnished the impression of a "neutrosophic set". A. Alblowi et al. [3] offered the evidence of neutrosophic topological space (or artlessly NTS). I. Arokiarani et al. [4] combined the interpretation of neutrosophic α-open subsets of neutrosophic topological spaces. Q. H. Imran et al. [5] proposed neutrosophic semi-open sets in neutrosophic topological spaces. R. Dhavaseelan et al. [6,7] offered the notion of generalized neutrosophic closed sets & neutrosophic α^m-continuity. Md. Hanif PAGE et al. [8] gave the idea of neutrosophic generalized homeomorphism. Q. H. Imran et al. [9] presented the concepts of neutrosophic generalized αg-closed sets & neutrosophic generalized αg-continuous functions. The purpose is to initiate a newfangled idea of neutrosophic separation axioms such as neutrosophic gαg-R_i-space, $i = 0,1$ & neutrosophic gαg-T_j-space, $j = 0,1,2$ & affirm some of their primary characteristics.

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2. Preliminaries

During this paper, $(\mathcal{X},\xi)$ (or artlessly $\mathcal{X}$) constantly recouple to NTS. The sequel of a neutrosophic open set (N-OS) is named the neutrosophic closed set (N-CS) in $(\mathcal{X},\xi)$. For an NS $\mathcal{M}$ in an NTS $(\mathcal{X},\xi)$, $Ncl(\mathcal{M})$, $Nint(\mathcal{M})$ & $\mathcal{M}^c = 1_N - \mathcal{M}$ signifies the neutrosophic closure of $\mathcal{M}$, the neutrosophic interior of $\mathcal{M}$ & the neutrosophic sequel of $\mathcal{M}$ alternately.

Definition 2.1:
A NS $\mathcal{M}$ of a NTS $(\mathcal{X},\xi)$ is uttered:
(i) a neutrosophic α-closed set (artlessly N^{α}-CS) if $Ncl(Nint(Ncl(\mathcal{M}))) \sqsubseteq \mathcal{M}$. The supplement of a N^{α}-CS in $\mathcal{X}$ is a neutrosophic α- open set (artlessly N^{α}-OS) in $\mathcal{X}$. The neutrosophic α-closure of a NS $\mathcal{M}$ of a NTS $(\mathcal{X},\xi)$ is the junction ∀ N^{α}-CSs that restrain $\mathcal{M}$ & is signalized by $N^{\alpha}cl(\mathcal{M})$. [4]
(ii) a neutrosophic g-closed set (artlessly N^{g}-CS) if $Ncl(\mathcal{M}) \sqsubseteq \mathcal{U}$ if $\mathcal{M} \sqsubseteq \mathcal{U}$ & $\mathcal{U}$ is a N-OS in $\mathcal{X}$. The sequel of a N^{g}-CS in $\mathcal{X}$ is a N^{g}-OS in $\mathcal{X}$. [10]
(iii) a neutrosophic αg-closed set (artlessly $N^{\alpha g}$-CS) if $N^{\alpha}cl(\mathcal{M}) \sqsubseteq \mathcal{U}$ if $\mathcal{M} \sqsubseteq \mathcal{U}$ & $\mathcal{U}$ is a N^{α}-OS in $\mathcal{X}$. The supplement of a $N^{\alpha g}$-CS in $\mathcal{X}$ is a $N^{\alpha g}$-OS in $\mathcal{X}$. [11]
(iv) a neutrosophic indiscriminate αg-closed set (artlessly $N^{g\alpha g}$-CS) if $Ncl(\mathcal{M}) \sqsubseteq \mathcal{U}$ if $\mathcal{M} \sqsubseteq \mathcal{U}$ & $\mathcal{U}$ is a $N^{\alpha g}$-OS in $\mathcal{X}$. The collection of all $N^{g\alpha g}$-CSs of a NTS $(\mathcal{X},\xi)$ is designated by $N^{g\alpha g}$-$C(\mathcal{X})$. The sequel of a $N^{g\alpha g}$-CS in $\mathcal{X}$ is a $N^{g\alpha g}$-OS in $\mathcal{X}$. The collection of all $N^{g\alpha g}$-OSs of a NTS $(\mathcal{X},\xi)$ is popularized as $N^{g\alpha g}$-$O(\mathcal{X})$. [9]

Example 2.2:
Let $\mathcal{X} = \{\mathcal{k},\ell\}$ & $\xi = \{0_N,\mathcal{A},\mathcal{B},1_N\}$, so NSs $\mathcal{A} = \langle x,(0.6,0.7),(0.1,0.1),(0.4,0.2)\rangle$ & $\mathcal{B} = \langle x,(0.1,0.2),(0.1,0.1),(0.8,0.8)\rangle$, so that $(\mathcal{X},\xi)$ is a NTS. However, the NS $\mathcal{C} = \langle x,(0.2,0.2),(0.1,0.1),(0.6,0.7)\rangle$ is a N^{α}-CS, N^{g}-CS, $N^{\alpha g}$-CS & $N^{g\alpha g}$-CS.

Remark 2.3 [9]:
In a NTS $(\mathcal{X},\xi)$, so the subsequent declaration engages, & the contrary of any confession is not true:
(i) Every N-OS (resp. N-CS) is a $N^{g\alpha g}$-OS (resp. $N^{g\alpha g}$-CS).
(ii) Every $N^{g\alpha g}$-OS (resp. $N^{g\alpha g}$-CS) is a N^{g}-OS (resp. N^{g}-CS).
(iii) Every $N^{g\alpha g}$-OS (resp. $N^{g\alpha g}$-CS) is a $N^{\alpha g}$-OS (resp. $N^{\alpha g}$-CS).

Definition 2.4 [9]:
The crossroads ∀ $N^{g\alpha g}$-CSs in a NTS $(\mathcal{X},\xi)$ comprise $\mathcal{M}$ is denominated neutrosophic gαg-closure of $\mathcal{M}$ & is designated by $N^{g\alpha g}cl(\mathcal{M})$, $N^{g\alpha g}cl(\mathcal{M}) = \sqcap\{\mathcal{N}: \mathcal{M} \sqsubseteq \mathcal{N}, \mathcal{N}$ is a $N^{g\alpha g}$-CS$\}$.

3. Neutrosophic $g\alpha g$ -R_i-Spaces, $i = 0, 1$

Definition 3.1:
The intersection of all $N^{g\alpha g}$-OSs of a NTS $(\mathcal{X},\xi)$ comprises $\mathcal{A}$ is denominated by the neutrosophic gαg-kernel of $\mathcal{A}$ ($N^{g\alpha g}ker(\mathcal{A})$), signifying $N^{g\alpha g}ker(\mathcal{A}) = \sqcap\{\mathcal{M}: \mathcal{M} \in N^{g\alpha g}$-$O(\mathcal{X})$ & $\mathcal{A} \sqsubseteq \mathcal{M}\}$.

Definition 3.2:
Let $\mathcal{k}_\lambda$ be a neutrosophic point (NP) of a NTS $(\mathcal{X},\xi)$. The $N^{g\alpha g}$-kernel of $\mathcal{k}_\lambda$, designated by $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\})$ is defined to be the NS $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\}) = \sqcap\{\mathcal{M}: \mathcal{M} \in N^{g\alpha g}$-$O(\mathcal{X})$ & $\mathcal{k}_\lambda \in \mathcal{M}\}$.

Definition 3.3:
In a NTS $(\mathcal{X},\xi)$, a NS $\mathcal{A}$ is denominated weakly ultra $N^{g\alpha g}$-separated from $\mathcal{B}$ if ∃ a $N^{g\alpha g}$-OS $\mathcal{M}$ s.t. $\mathcal{M}\sqcap\mathcal{B} = 0_N$ or $\mathcal{A}\sqcap N^{g\alpha g}cl(\mathcal{B}) = 0_N$.
By definition (3.3), got: For any two distinct NPs $\mathcal{k}_\lambda$ & ℓ_μ of a NTS $(\mathcal{X},\xi)$
(i) $N^{g\alpha g}cl(\{\mathcal{k}_\lambda\}) = \{\ell_\mu : \{\ell_\mu\}$ is not weakly ultra $N^{g\alpha g}$-separated from $\{\mathcal{k}_\lambda\}\}$.
(ii) $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\}) = \{\ell_\mu : \{\mathcal{k}_\lambda\}$ is not weakly ultra $N^{g\alpha g}$-separated from $\{\ell_\mu\}\}$.

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Lemma 3.4:
Let $(\mathcal{X},\xi)$ be a NTS, then $\ell_\mu \in N^{g\alpha g}ker(\{k_\lambda\})$ iff $k_\lambda \in N^{g\alpha g}cl(\{\ell_\mu\})$ for any $k_\lambda \neq \ell_\mu \in \mathcal{X}$.
Proof:
Suppose that $\ell_\mu \notin N^{g\alpha g}ker(\{k_\lambda\})$. Then $\exists$ $N^{g\alpha g}$-OS $\mathcal{U}$ comprise k_λ s.t. $\ell_\mu \notin \mathcal{U}$. Therefore, we have $k_\lambda \notin N^{g\alpha g}cl(\{\ell_\mu\})$. The same way can prove the contrariwise part. ▪

Definition 3.5:
A NTS $(\mathcal{X},\xi)$ is denominated neutrosophic $g\alpha g$-R_0-space ($N^{g\alpha g}$-R_0-space, for short) if for any $N^{g\alpha g}$-OS $\mathcal{U}$ & $k_\lambda \in \mathcal{U}$, then $N^{g\alpha g}cl(\{k_\lambda\}) \sqsubseteq \mathcal{U}$.

Definition 3.6:
A NTS $(\mathcal{X},\xi)$ is denominated neutrosophic $g\alpha g$-R_1-space ($N^{g\alpha g}$-R_1-space, for short) if for any two distinct NPs k_λ & ℓ_μ of $\mathcal{X}$ with $N^{g\alpha g}cl(\{k_\lambda\}) \neq N^{g\alpha g}cl(\{\ell_\mu\})$, there exist disjoint $N^{g\alpha g}$-OSs $\mathcal{U},\mathcal{V}$ s.t. $N^{g\alpha g}cl(\{k_\lambda\}) \sqsubseteq \mathcal{U}$ & $N^{g\alpha g}cl(\{\ell_\mu\}) \sqsubseteq \mathcal{V}$.

Theorem 3.7:
Let $(\mathcal{X},\xi)$ be a NTS. Then $(\mathcal{X},\xi)$ is a $N^{g\alpha g}$-R_0-space iff $N^{g\alpha g}cl(\{k_\lambda\}) = N^{g\alpha g}ker(\{k_\lambda\})$, for any $k_\lambda \in \mathcal{X}$.
Proof:
Let $(\mathcal{X},\xi)$ be a $N^{g\alpha g}$-R_0-space. If $N^{g\alpha g}cl(\{k_\lambda\}) \neq N^{g\alpha g}ker(\{k_\lambda\})$, for any $k_\lambda \in \mathcal{X}$, then there exists another NP $\ell_\mu \neq k_\lambda$ s.t. $\ell_\mu \in N^{g\alpha g}cl(\{k_\lambda\})$ & $\ell_\mu \notin N^{g\alpha g}ker(\{k_\lambda\})$ This recouple has there existed a $N^{g\alpha g}$-OS $\mathcal{U}_{k_\lambda}$, $\ell_\mu \notin \mathcal{U}_{k_\lambda}$ implies $N^{g\alpha g}cl(\{k_\lambda\}) \not\sqsubseteq \mathcal{U}_{k_\lambda}$ This ambivalence. Consequently $N^{g\alpha g}cl(\{k_\lambda\}) = N^{g\alpha g}ker(\{k_\lambda\})$.
Contrariwise, let $N^{g\alpha g}cl(\{k_\lambda\}) = N^{g\alpha g}ker(\{k_\lambda\})$, for any $N^{g\alpha g}$-OS $\mathcal{U}, k_\lambda \in \mathcal{U}$, then $N^{g\alpha g}ker(\{k_\lambda\}) = N^{g\alpha g}cl(\{k_\lambda\}) \sqsubseteq \mathcal{U}$ [by def. (3.1)]. So by def. (3.5), $(\mathcal{X},\xi)$ is a $N^{g\alpha g}$-R_0-space. ▪

Theorem 3.8:
A NTS $(\mathcal{X},\xi)$ is $N^{g\alpha g}$-R_0-space iff for any $N^{g\alpha g}$-CS $\mathcal{M}$ & $k_\lambda \in \mathcal{M}$, then $N^{g\alpha g}ker(\{k_\lambda\}) \sqsubseteq \mathcal{M}$.
Proof:
Let for any $\mathcal{M}$ $N^{g\alpha g}$-CS & $k_\lambda \in \mathcal{M}$, then $N^{g\alpha g}ker(\{k_\lambda\}) \sqsubseteq \mathcal{M}$ & $\mathcal{U}$ be a $N^{g\alpha g}$-OS, $k_\lambda \in \mathcal{U}$ then, for any $\ell_\mu \notin \mathcal{U}$ implies $\ell_\mu \in \mathcal{U}^c$ is a $N^{g\alpha g}$-CS implies $N^{g\alpha g}ker(\{\ell_\mu\}) \sqsubseteq \mathcal{U}^c$[by hypothesis]. Therefore $k_\lambda \notin N^{g\alpha g}ker(\{\ell_\mu\})$ implies $\ell_\mu \notin N^{g\alpha g}cl(\{k_\lambda\})$ [by lemma (3.4)]. So $N^{g\alpha g}cl(\{k_\lambda\}) \sqsubseteq \mathcal{U}$. Consequently, $(\mathcal{X},\xi)$ is a $N^{g\alpha g}$-R_0-space.
Contrariwise, let $(\mathcal{X},\xi)$ be a $N^{g\alpha g}$-R_0-space & $\mathcal{M}$ be a $N^{g\alpha g}$-CS & $k_\lambda \in \mathcal{M}$. Then for any $\ell_\mu \notin \mathcal{M}$ implies $\ell_\mu \in \mathcal{M}^c$ is a $N^{g\alpha g}$-OS, then $N^{g\alpha g}cl(\{\ell_\mu\}) \sqsubseteq \mathcal{M}^c$ [since $(\mathcal{X},\xi)$ is a $N^{g\alpha g}$-R_0-space], so $N^{g\alpha g}ker(\{k_\lambda\}) = N^{g\alpha g}cl(\{k_\lambda\})$. Consequently $N^{g\alpha g}ker(\{k_\lambda\}) \sqsubseteq \mathcal{M}$. ▪

Corollary 3.9:
A NTS $(\mathcal{X},\xi)$ is $N^{g\alpha g}$-R_0-space iff for any $\mathcal{U}$ $N^{g\alpha g}$-OS & $k_\lambda \in \mathcal{U}$, then $N^{g\alpha g}cl(N^{g\alpha g}ker(\{k_\lambda\})) \sqsubseteq \mathcal{U}$.
Proof:
Obviously. ▪

Theorem 3.10:
Every $N^{g\alpha g}$-R_1-space is a $N^{g\alpha g}$-R_0-space.
Proof:
Assume $(\mathcal{X},\xi)$ is $N^{g\alpha g}$-R_1-space & let $\mathcal{U}$ be a $N^{g\alpha g}$-OS, $k_\lambda \in \mathcal{U}$, then for any $\ell_\mu \notin \mathcal{U}$ then $\ell_\mu \in \mathcal{U}^c$ is a $N^{g\alpha g}$-CS & $N^{g\alpha g}cl(\{\ell_\mu\}) \sqsubseteq \mathcal{U}^c$ got $N^{g\alpha g}cl(\{k_\lambda\}) \neq N^{g\alpha g}cl(\{\ell_\mu\})$. Wherefore by def.(3.6), $N^{g\alpha g}cl(\{k_\lambda\}) \sqsubseteq \mathcal{U}$. Consequently, $(\mathcal{X},\xi)$ is a $N^{g\alpha g}$-R_0-space. ▪

Theorem 3.11:
A NTS $(\mathcal{X},\xi)$ is $N^{g\alpha g}$-R_1-space iff for any $k_\lambda \neq \ell_\mu \in \mathcal{X}$ with $N^{g\alpha g}ker(\{k_\lambda\}) \neq N^{g\alpha g}ker(\{\ell_\mu\})$, then there exist $N^{g\alpha g}$-CSs $\mathcal{M}_1$, $\mathcal{M}_2$ s.t. $N^{g\alpha g}ker(\{k_\lambda\}) \sqsubseteq \mathcal{M}_1$, $N^{g\alpha g}ker(\{k_\lambda\}) \sqcap \mathcal{M}_2 = 0_N$ & $N^{g\alpha g}ker(\{\ell_\mu\}) \sqsubseteq \mathcal{M}_2$, $N^{g\alpha g}ker(\{\ell_\mu\}) \sqcap \mathcal{M}_1 = 0_N$ & $\mathcal{M}_1 \sqcup \mathcal{M}_2 = 1_N$.

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Proof:

Let $(\mathcal{X},\xi)$ be a $\mathrm{N}^{g\alpha g}$-R_1-space. Then for any $\hbar_\lambda \neq \ell_\mu \in \mathcal{X}$ with $N^{g\alpha g}ker(\{\hbar_\lambda\}) \neq N^{g\alpha g}ker(\{\ell_\mu\})$. Such any $\mathrm{N}^{g\alpha g}$-R_1-space is a $\mathrm{N}^{g\alpha g}$-R_0-space [by Thm. (3.10)], & by Thm. (3.7), $N^{g\alpha g}cl(\{\hbar_\lambda\}) \neq N^{g\alpha g}cl(\{\ell_\mu\})$, then there exist $\mathrm{N}^{g\alpha g}$-OSs $\mathcal{U}_1$,$\mathcal{U}_2$ s.t. $N^{g\alpha g}cl(\{\hbar_\lambda\}) \sqsubseteq \mathcal{U}_1$ & $N^{g\alpha g}cl(\{\ell_\mu\}) \sqsubseteq \mathcal{U}_2$ & $\mathcal{U}_1 \sqcap \mathcal{U}_2 = 0_N$ [since $(\mathcal{X},\xi)$ is a $\mathrm{N}^{g\alpha g}$-R_1-space], then $\mathcal{U}_1^c$ & $\mathcal{U}_2^c$ are $\mathrm{N}^{g\alpha g}$-CSs s.t. $\mathcal{U}_1^c \sqcup \mathcal{U}_2^c = 1_N$. Put $\mathcal{M}_1 = \mathcal{U}_1^c$ & $\mathcal{M}_2 = \mathcal{U}_2^c$. Consequently $\hbar_\lambda \in \mathcal{U}_1 \sqsubseteq \mathcal{M}_2$ & $\ell_\mu \in \mathcal{U}_2 \sqsubseteq \mathcal{M}_1$ so $N^{g\alpha g}ker(\{\hbar_\lambda\}) \sqsubseteq \mathcal{U}_1 \sqsubseteq \mathcal{M}_2$ & $N^{g\alpha g}ker(\{\ell_\mu\}) \sqsubseteq \mathcal{U}_2 \sqsubseteq \mathcal{M}_1$.

Contrariwise, let for any $\hbar_\lambda \neq \ell_\mu \in \mathcal{X}$ with $N^{g\alpha g}ker(\{\hbar_\lambda\}) \neq N^{g\alpha g}ker(\{\ell_\mu\})$, there exist $\mathrm{N}^{g\alpha g}$-CSs $\mathcal{M}_1$, $\mathcal{M}_2$ s.t. $N^{g\alpha g}ker(\{\hbar_\lambda\}) \sqsubseteq \mathcal{M}_1$, $N^{g\alpha g}ker(\{\hbar_\lambda\}) \sqcap \mathcal{M}_2 = 0_N$ & $N^{g\alpha g}ker(\{\ell_\mu\}) \sqsubseteq \mathcal{M}_2$, $N^{g\alpha g}ker(\{\ell_\mu\}) \sqcap \mathcal{M}_1 = 0_N$ & $\mathcal{M}_1 \sqcup \mathcal{M}_2 = 1_N$, then $\mathcal{M}_1^c$ & $\mathcal{M}_2^c$ are $\mathrm{N}^{g\alpha g}$-OSs s.t. $\mathcal{M}_1^c \sqcap \mathcal{M}_2^c = 0_N$. Put $\mathcal{M}_1^c = \mathcal{U}_2$ & $\mathcal{M}_2^c = \mathcal{U}_1$. Consequently, $N^{g\alpha g}ker(\{\hbar_\lambda\}) \sqsubseteq \mathcal{U}_1$ & $N^{g\alpha g}ker(\{\ell_\mu\}) \sqsubseteq \mathcal{U}_2$ & $\mathcal{U}_1 \sqcap \mathcal{U}_2 = 0_N$, so that $\hbar_\lambda \in \mathcal{U}_1$ & $\ell_\mu \in \mathcal{U}_2$ implies $\hbar_\lambda \notin N^{g\alpha g}cl(\{\ell_\mu\})$ & $\ell_\mu \notin N^{g\alpha g}cl(\{\hbar_\lambda\})$, then $N^{g\alpha g}cl(\{\hbar_\lambda\}) \sqsubseteq \mathcal{U}_1$ & $N^{g\alpha g}cl(\{\ell_\mu\}) \sqsubseteq \mathcal{U}_2$. Consequently, $(\mathcal{X},\xi)$ is a $\mathrm{N}^{g\alpha g}$-R_1-space. ▪

Corollary 3.12:

A NTS $(\mathcal{X},\xi)$ is $\mathrm{N}^{g\alpha g}$-R_1-space iff for any $\hbar_\lambda \neq \ell_\mu \in \mathcal{X}$ with $N^{g\alpha g}cl(\{\hbar_\lambda\}) \neq N^{g\alpha g}cl(\{\ell_\mu\})$ there exist disjoint $\mathrm{N}^{g\alpha g}$-OSs $\mathcal{U},\mathcal{V}$ s.t. $N^{g\alpha g}cl(N^{g\alpha g}ker(\{\hbar_\lambda\})) \sqsubseteq \mathcal{U}$ & $N^{g\alpha g}cl(N^{g\alpha g}ker(\{\ell_\mu\})) \sqsubseteq \mathcal{V}$.

Proof:

Let $(\mathcal{X},\xi)$ be a $\mathrm{N}^{g\alpha g}$-R_1-space & let $\hbar_\lambda \neq \ell_\mu \in \mathcal{X}$ with $N^{g\alpha g}cl(\{\hbar_\lambda\}) \neq N^{g\alpha g}cl(\{\ell_\mu\})$, then there exist disjoint $\mathrm{N}^{g\alpha g}$-OSs $\mathcal{U},\mathcal{V}$ s.t. $N^{g\alpha g}cl(\{\hbar_\lambda\}) \sqsubseteq \mathcal{U}$ & $N^{g\alpha g}cl(\{\ell_\mu\}) \sqsubseteq \mathcal{V}$. Likewise, $(\mathcal{X},\xi)$ is a $\mathrm{N}^{g\alpha g}$-R_0-space [by theorem (3.10)] implies that any $\hbar_\lambda \in \mathcal{X}$, then $N^{g\alpha g}cl(\{\hbar_\lambda\}) = N^{g\alpha g}ker(\{\hbar_\lambda\})$ [by Thm. (3.7)], but $N^{g\alpha g}cl(\{\hbar_\lambda\}) = N^{g\alpha g}cl(N^{g\alpha g}cl(\{\hbar_\lambda\})) = N^{g\alpha g}cl(N^{g\alpha g}ker(\{\hbar_\lambda\}))$.

Consequently $N^{g\alpha g}cl(N^{g\alpha g}ker(\{\hbar_\lambda\})) \sqsubseteq \mathcal{U}$ & $N^{g\alpha g}cl(N^{g\alpha g}ker(\{\ell_\mu\})) \sqsubseteq \mathcal{V}$.

Contrariwise, let $\forall \hbar_\lambda \neq \ell_\mu \in \mathcal{X}$ with $N^{g\alpha g}cl(\{\hbar_\lambda\}) \neq N^{g\alpha g}cl(\{\ell_\mu\})$ there exist disjoint $\mathrm{N}^{g\alpha g}$-OSs $\mathcal{U},\mathcal{V}$ s.t. $N^{g\alpha g}cl(N^{g\alpha g}ker(\{\hbar_\lambda\})) \sqsubseteq \mathcal{U}$ & $N^{g\alpha g}cl(N^{g\alpha g}ker(\{\ell_\mu\})) \sqsubseteq \mathcal{V}$. Since $\{\hbar_\lambda\} \sqsubseteq N^{g\alpha g}ker(\{\hbar_\lambda\})$, then $N^{g\alpha g}cl(\{\hbar_\lambda\}) \sqsubseteq N^{g\alpha g}cl(N^{g\alpha g}ker(\{\hbar_\lambda\}))$ for any $\hbar_\lambda \in \mathcal{X}$. So we get $N^{g\alpha g}cl(\{\hbar_\lambda\}) \sqsubseteq \mathcal{U}$ & $N^{g\alpha g}cl(\{\ell_\mu\}) \sqsubseteq \mathcal{V}$. Consequently, $(\mathcal{X},\xi)$ is a $\mathrm{N}^{g\alpha g}$-R_1-space. ▪

4. Neutrosophic gαg-T_j-Spaces, $j = 0, 1, 2$

Definition 4.1:

Let $(\mathcal{X},\xi)$ be a NTS, $\mathcal{X}$ is denominated:

(i) neutrosophic gαg-T_0-space ($\mathrm{N}^{g\alpha g}$-T_0-space, for short) iff any couple of distinct NPs in $\mathcal{X}$, $\exists \mathrm{N}^{g\alpha g}$-OS in $\mathcal{X}$ comprises one & not the other.

(ii) neutrosophic gαg-T_1-space ($\mathrm{N}^{g\alpha g}$-T_1-space, for short) iff for any couple of distinct NPs $\hbar_\lambda$ & ℓ_μ of $\mathcal{X}$, there exist $\mathrm{N}^{g\alpha g}$-OSs $\mathcal{M},\mathcal{N}$ comprise $\hbar_\lambda$ & ℓ_μ alternately s.t. $\ell_\mu \notin \mathcal{M}$ & $\hbar_\lambda \notin \mathcal{N}$.

(iii) neutrosophic gαg-T_2-space ($\mathrm{N}^{g\alpha g}$-T_2-space, for short) iff for any couple of distinct NPs $\hbar_\lambda$ & ℓ_μ of $\mathcal{X}$, there exist disjoint $\mathrm{N}^{g\alpha g}$-OSs $\mathcal{M},\mathcal{N}$ in $\mathcal{X}$ s.t. $\hbar_\lambda \in \mathcal{M}$ & $\ell_\mu \in \mathcal{N}$.

Remark 4.2:

Every $\mathrm{N}^{g\alpha g}$-T_k-space is a $\mathrm{N}^{g\alpha g}$-T_{k-1}-space, $k = 1,2$.

Proof:

Obviously. ▪

Theorem 4.3:

A NTS $(\mathcal{X},\xi)$ is $\mathrm{N}^{g\alpha g}$-T_0-space iff either $\ell_\mu \notin N^{g\alpha g}ker(\{\hbar_\lambda\})$ or $\hbar_\lambda \notin N^{g\alpha g}ker(\{\ell_\mu\})$, for any $\hbar_\lambda \neq \ell_\mu \in \mathcal{X}$.

Proof:

Let $(\mathcal{X},\xi)$ be a $\mathrm{N}^{g\alpha g}$-T_0-space then for any $\hbar_\lambda \neq \ell_\mu \in \mathcal{X}$, there exists a $\mathrm{N}^{g\alpha g}$-OS $\mathcal{M}$ s.t. $\hbar_\lambda \in \mathcal{M}, \ell_\mu \notin \mathcal{M}$ or $\hbar_\lambda \notin \mathcal{M}$, $\ell_\mu \in \mathcal{M}$. Consequently either $\hbar_\lambda \in \mathcal{M}$, $\ell_\mu \notin \mathcal{M}$ implies $\ell_\mu \notin N^{g\alpha g}ker(\{\hbar_\lambda\})$ or $\hbar_\lambda \notin \mathcal{M}, \ell_\mu \in \mathcal{M}$ implies $\hbar_\lambda \notin N^{g\alpha g}ker(\{\ell_\mu\})$.

Contrariwise, let either $\ell_\mu \notin N^{g\alpha g}ker(\{\hbar_\lambda\})$ or $\hbar_\lambda \notin N^{g\alpha g}ker(\{\ell_\mu\})$, for any $\hbar_\lambda \neq \ell_\mu \in \mathcal{X}$. Then there exists a $\mathrm{N}^{g\alpha g}$-OS $\mathcal{M}$ s.t. $\hbar_\lambda \in \mathcal{M}, \ell_\mu \notin \mathcal{M}$ or $\hbar_\lambda \notin \mathcal{M}, \ell_\mu \in \mathcal{M}$. Consequently, $(\mathcal{X},\xi)$ is a $\mathrm{N}^{g\alpha g}$-T_0-space. ▪

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Theorem 4.4:
A NTS $(\mathcal{X},\xi)$ is $\mathrm{N^{g\alpha g}}$-T_0-space iff either $N^{g\alpha g}ker(\{k_\lambda\})$ is weakly ultra $\mathrm{N^{g\alpha g}}$-separated from $\{\ell_\mu\}$ or $N^{g\alpha g}ker(\{\ell_\mu\})$ is weakly ultra $\mathrm{N^{g\alpha g}}$-separated from $\{k_\lambda\}$ for any $k_\lambda \neq \ell_\mu \in \mathcal{X}$.
Proof:
Let $(\mathcal{X},\xi)$ be a $\mathrm{N^{g\alpha g}}$-T_0-space then for any $k_\lambda \neq \ell_\mu \in \mathcal{X}$, there exists a $\mathrm{N^{g\alpha g}}$-OS $\mathcal{M}$ s.t. $k_\lambda \in \mathcal{M}$, $\ell_\mu \notin \mathcal{M}$ or $k_\lambda \notin \mathcal{M}$, $\ell_\mu \in \mathcal{M}$. Now if $k_\lambda \in \mathcal{M}$, $\ell_\mu \notin \mathcal{M}$ implies $N^{g\alpha g}ker(\{k_\lambda\})$ is weakly ultra $\mathrm{N^{g\alpha g}}$-separated from $\{\ell_\mu\}$. Or if $k_\lambda \notin \mathcal{M}$, $\ell_\mu \in \mathcal{M}$ implies $N^{g\alpha g}ker(\{\ell_\mu\})$ is weakly ultra $\mathrm{N^{g\alpha g}}$-separated from $\{k_\lambda\}$.
Contrariwise, let either $N^{g\alpha g}ker(\{k_\lambda\})$ be weakly ultra $\mathrm{N^{g\alpha g}}$-separated from $\{\ell_\mu\}$ or $N^{g\alpha g}ker(\{\ell_\mu\})$ be weakly ultra $\mathrm{N^{g\alpha g}}$-separated from $\{k_\lambda\}$. Then there exists a $\mathrm{N^{g\alpha g}}$-OS $\mathcal{M}$ s.t. $N^{g\alpha g}ker(\{k_\lambda\}) \sqsubseteq \mathcal{M}$ & $\ell_\mu \notin \mathcal{M}$ or $N^{g\alpha g}ker(\{\ell_\mu\}) \sqsubseteq \mathcal{M}$, $k_\lambda \notin \mathcal{M}$ implies $k_\lambda \in \mathcal{M}$, $\ell_\mu \notin \mathcal{M}$ or $k_\lambda \notin \mathcal{M}$, $\ell_\mu \in \mathcal{M}$. Consequently, $(\mathcal{X},\xi)$ is a $\mathrm{N^{g\alpha g}}$-T_0-space. ▪

Theorem 4.5:
A NTS $(\mathcal{X},\xi)$ is $\mathrm{N^{g\alpha g}}$-T_1-space iff for any $k_\lambda \neq \ell_\mu \in \mathcal{X}$, $N^{g\alpha g}ker(\{k_\lambda\})$ is weakly ultra $\mathrm{N^{g\alpha g}}$-separated from $\{\ell_\mu\}$ & $N^{g\alpha g}ker(\{\ell_\mu\})$ is weakly ultra $\mathrm{N^{g\alpha g}}$-separated from $\{k_\lambda\}$.
Proof:
Let $(\mathcal{X},\xi)$ be a $\mathrm{N^{g\alpha g}}$-T_1-space, then for any $k_\lambda \neq \ell_\mu \in \mathcal{X}$, there exist $\mathrm{N^{g\alpha g}}$-OSs $\mathcal{U},\mathcal{V}$ s.t. $k_\lambda \in \mathcal{U}$, $\ell_\mu \notin \mathcal{U}$ & $k_\lambda \notin \mathcal{V}$, $\ell_\mu \in \mathcal{V}$. Implies $N^{g\alpha g}ker(\{k_\lambda\})$ is weakly ultra $\mathrm{N^{g\alpha g}}$-separated from $\{\ell_\mu\}$ & $N^{g\alpha g}ker(\{\ell_\mu\})$ is weakly ultra $\mathrm{N^{g\alpha g}}$-separated from $\{k_\lambda\}$.
Contrariwise, let $N^{g\alpha g}ker(\{k_\lambda\})$ be weakly ultra $\mathrm{N^{g\alpha g}}$-separated from $\{\ell_\mu\}$ & $N^{g\alpha g}ker(\{\ell_\mu\})$ be weakly ultra $\mathrm{N^{g\alpha g}}$-separated from $\{k_\lambda\}$. Then there exist $\mathrm{N^{g\alpha g}}$-OSs $\mathcal{U},\mathcal{V}$ s.t. $N^{g\alpha g}ker(\{k_\lambda\}) \sqsubseteq \mathcal{U}, \ell_\mu \notin \mathcal{U}$ & $N^{g\alpha g}ker(\{\ell_\mu\}) \sqsubseteq \mathcal{V}, k_\lambda \notin \mathcal{V}$ implies $k_\lambda \in \mathcal{U}$, $\ell_\mu \notin \mathcal{U}$ & $k_\lambda \notin \mathcal{V}$, $\ell_\mu \in \mathcal{V}$. Consequently, $(\mathcal{X},\xi)$ is a $\mathrm{N^{g\alpha g}}$-T_1-space. ▪

Theorem 4.6:
A NTS $(\mathcal{X},\xi)$ is $\mathrm{N^{g\alpha g}}$-T_1-space iff for any $k_\lambda \in \mathcal{X}$, $N^{g\alpha g}ker(\{k_\lambda\}) = \{k_\lambda\}$.
Proof:
Let $(\mathcal{X},\xi)$ be a $\mathrm{N^{g\alpha g}}$-T_1-space & let $N^{g\alpha g}ker(\{k_\lambda\}) \neq \{k_\lambda\}$. Then $N^{g\alpha g}ker(\{k_\lambda\})$ contains other NPs distinct from k_λ say ℓ_μ. So $\ell_\mu \in N^{g\alpha g}ker(\{k_\lambda\})$ implies $N^{g\alpha g}ker(\{k_\lambda\})$ is not weakly ultra $\mathrm{N^{g\alpha g}}$-separated from $\{\ell_\mu\}$. Wherefore by Thm. (4.5), $(\mathcal{X},\xi)$ is not a $\mathrm{N^{g\alpha g}}$-T_1-space, this is ambivalence. Consequently, $N^{g\alpha g}ker(\{k_\lambda\}) = \{k_\lambda\}$.
Contrariwise, let $N^{g\alpha g}ker(\{k_\lambda\}) = \{k_\lambda\}$, for any $k_\lambda \in \mathcal{X}$ & let $(\mathcal{X},\xi)$ be not a $\mathrm{N^{g\alpha g}}$-T_1-space. Then by Thm. (4.5), $N^{g\alpha g}ker(\{k_\lambda\})$ is not weakly ultra $\mathrm{N^{g\alpha g}}$-separated from $\{\ell_\mu\}$ for some $k_\lambda \neq \ell_\mu \in \mathcal{X}$, this recouple has that for every $\mathrm{N^{g\alpha g}}$-OS $\mathcal{M}$ contains $N^{g\alpha g}ker(\{k_\lambda\})$ then $\ell_\mu \in \mathcal{M}$ implies $\ell_\mu \in \sqcap\{\mathcal{M} \in \mathrm{N^{g\alpha g}}\text{-}O(\mathcal{X}) \colon k_\lambda \in \mathcal{M}\}$ implies $\ell_\mu \in N^{g\alpha g}ker(\{k_\lambda\})$, this is ambivalence. Consequently, $(\mathcal{X},\xi)$ is a $\mathrm{N^{g\alpha g}}$-T_1-space. ▪

Theorem 4.7:
A NTS $(\mathcal{X},\xi)$ is $\mathrm{N^{g\alpha g}}$-T_1-space iff for any $k_\lambda \neq \ell_\mu \in \mathcal{X}$, $\ell_\mu \notin N^{g\alpha g}ker(\{k_\lambda\})$ & $k_\lambda \notin N^{g\alpha g}ker(\{\ell_\mu\})$.
Proof:
Let $(\mathcal{X},\xi)$ be a $\mathrm{N^{g\alpha g}}$-T_1-space then for any $k_\lambda \neq \ell_\mu \in \mathcal{X}$, $\exists\mathrm{N^{g\alpha g}}$-OSs $\mathcal{U},\mathcal{V}$ s.t. $k_\lambda \in \mathcal{U}$, $\ell_\mu \notin \mathcal{U}$ & $\ell_\mu \in \mathcal{V}$, $k_\lambda \notin \mathcal{V}$. Implies $\ell_\mu \notin N^{g\alpha g}ker(\{k_\lambda\})$ & $k_\lambda \notin N^{g\alpha g}ker(\{\ell_\mu\})$.
Contrariwise, let $\ell_\mu \notin N^{g\alpha g}ker(\{k_\lambda\})$ & $k_\lambda \notin N^{g\alpha g}ker(\{\ell_\mu\})$, for any $k_\lambda \neq \ell_\mu \in \mathcal{X}$. Then $\exists\mathrm{N^{g\alpha g}}$-OSs $\mathcal{U},\mathcal{V}$ s.t. $k_\lambda \in \mathcal{U}$, $\ell_\mu \notin \mathcal{U}$ & $\ell_\mu \in \mathcal{V}$, $k_\lambda \notin \mathcal{V}$. Consequently, $(\mathcal{X},\xi)$ is a $\mathrm{N^{g\alpha g}}$-T_1-space. ▪

Theorem 4.8:
A NTS $(\mathcal{X},\xi)$ is $\mathrm{N^{g\alpha g}}$-T_1-space iff for any $k_\lambda \neq \ell_\mu \in \mathcal{X}$ implies $N^{g\alpha g}ker(\{k_\lambda\}) \sqcap N^{g\alpha g}ker(\{\ell_\mu\}) = 0_N$.
Proof:
Let $(\mathcal{X},\xi)$ be a $\mathrm{N^{g\alpha g}}$-T_1-space. Then $N^{g\alpha g}ker(\{k_\lambda\}) = \{k_\lambda\}$ & $N^{g\alpha g}ker(\{\ell_\mu\}) = \{\ell_\mu\}$ [by Thm. (4.6)]. Consequently, $N^{g\alpha g}ker(\{k_\lambda\}) \sqcap N^{g\alpha g}ker(\{\ell_\mu\}) = 0_N$.

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Contrariwise, let for any $\mathcal{k}_\lambda \neq \ell_\mu \in \mathcal{X}$ implies $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\}) \sqcap N^{g\alpha g}ker(\{\ell_\mu\}) = 0_N$ & let $(\mathcal{X}, \xi)$ be not $N^{g\alpha g}$-T_1-space, then for any $\mathcal{k}_\lambda \neq \ell_\mu \in \mathcal{X}$ implies $\ell_\mu \in N^{g\alpha g}ker(\{\mathcal{k}_\lambda\})$ or $\mathcal{k}_\lambda \in N^{g\alpha g}ker(\{\ell_\mu\})$ [by Thm. (4.7)], then $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\}) \sqcap N^{g\alpha g}ker(\{\ell_\mu\}) \neq 0_N$. This is ambivalence. Consequently, $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-T_1-space. ▪

Proposition 4.9:

A NTS $(\mathcal{X}, \xi)$ is $N^{g\alpha g}$-T_1-space iff $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-T_0-space & $N^{g\alpha g}$-R_0-space.

Proof:

Let $(\mathcal{X}, \xi)$ be a $N^{g\alpha g}$-T_1-space & let $\mathcal{k}_\lambda \in \mathcal{U}$ be a $N^{g\alpha g}$-OS, then for any $\mathcal{k}_\lambda \neq \ell_\mu \in \mathcal{X}$, $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\}) \sqcap N^{g\alpha g}ker(\{\ell_\mu\}) = 0_N$ [by Thm. (4.8)] implies $\mathcal{k}_\lambda \notin N^{g\alpha g}ker(\{\ell_\mu\})$ & $\ell_\mu \notin N^{g\alpha g}ker(\{\mathcal{k}_\lambda\})$, this recouple tos $N^{g\alpha g}cl(\{\mathcal{k}_\lambda\}) = \{\mathcal{k}_\lambda\}$, wherefore $N^{g\alpha g}cl(\{\mathcal{k}_\lambda\}) \sqsubseteq \mathcal{U}$. Consequently, $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-R_0-space.

Contrariwise, let $(\mathcal{X}, \xi)$ be a $N^{g\alpha g}$-T_0-space & $N^{g\alpha g}$-R_0-space, then for any $\mathcal{k}_\lambda \neq \ell_\mu \in \mathcal{X}$ $\exists N^{g\alpha g}$-OS $\mathcal{U}$ s.t. $\mathcal{k}_\lambda \in \mathcal{U}$, $\ell_\mu \notin \mathcal{U}$ or $\mathcal{k}_\lambda \notin \mathcal{U}$, $\ell_\mu \in \mathcal{U}$. Say $\mathcal{k}_\lambda \in \mathcal{U}$, $\ell_\mu \notin \mathcal{U}$ since $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-R_0-space, then $N^{g\alpha g}cl(\{\mathcal{k}_\lambda\}) \sqsubseteq \mathcal{U}$, this recouple tos $\exists N^{g\alpha g}$-OS $\mathcal{V}$ s.t. $\ell_\mu \in \mathcal{V}$, $\mathcal{k}_\lambda \notin \mathcal{V}$. Consequently, $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-T_1-space. ▪

Theorem 4.10:

A NTS $(\mathcal{X}, \xi)$ is $N^{g\alpha g}$-T_2-space iff

(i) $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-T_0-space & $N^{g\alpha g}$-R_1-space.

(ii) $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-T_1-space & $N^{g\alpha g}$-R_1-space.

Proof:

(i) Let $(\mathcal{X}, \xi)$ be a $N^{g\alpha g}$-T_2-space, then it is a $N^{g\alpha g}$-T_0-space. Wherefore $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-T_2-space, then for any $\mathcal{k}_\lambda \neq \ell_\mu \in \mathcal{X}$, there exist disjoint $N^{g\alpha g}$-OSs $\mathcal{U}, \mathcal{V}$ s.t. $\mathcal{k}_\lambda \in \mathcal{U}$ & $\ell_\mu \in \mathcal{V}$ implies $\mathcal{k}_\lambda \notin N^{g\alpha g}cl(\{\ell_\mu\})$ & $\ell_\mu \notin N^{g\alpha g}cl(\{\mathcal{k}_\lambda\})$, therefore $N^{g\alpha g}cl(\{\mathcal{k}_\lambda\}) = \{\mathcal{k}_\lambda\} \sqsubseteq \mathcal{U}$ & $N^{g\alpha g}cl(\{\ell_\mu\}) = \{\ell_\mu\} \sqsubseteq \mathcal{V}$. Consequently, $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-R_1-space.

Contrariwise, let $(\mathcal{X}, \xi)$ be a $N^{g\alpha g}$-T_0-space & $N^{g\alpha g}$-R_1-space, then for any $\mathcal{k}_\lambda \neq \ell_\mu \in \mathcal{X}$, there exists a $N^{g\alpha g}$-OS $\mathcal{U}$ s.t. $\mathcal{k}_\lambda \in \mathcal{U}$, $\ell_\mu \notin \mathcal{U}$ or $\ell_\mu \in \mathcal{U}$, $\mathcal{k}_\lambda \notin \mathcal{U}$, got $N^{g\alpha g}cl(\{\mathcal{k}_\lambda\}) \neq N^{g\alpha g}cl(\{\ell_\mu\})$, since $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-R_1-space [by hypothesis], then there exists disjoint $N^{g\alpha g}$-OSs $\mathcal{M}, \mathcal{N}$ s.t. $\mathcal{k}_\lambda \in \mathcal{M}$ & $\ell_\mu \in \mathcal{N}$. Consequently, $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-T_2-space.

(ii) Similarly to (i), $N^{g\alpha g}$-T_2-space is a $N^{g\alpha g}$-T_1-space & $N^{g\alpha g}$-R_1-space.

Contrariwise, let $(\mathcal{X}, \xi)$ be a $N^{g\alpha g}$-T_1-space & $N^{g\alpha g}$-R_1-space, then for any $\mathcal{k}_\lambda \neq \ell_\mu \in \mathcal{X}$, $\exists N^{g\alpha g}$-OSs $\mathcal{U}, \mathcal{V}$ s.t. $\mathcal{k}_\lambda \in \mathcal{U}$, $\ell_\mu \notin \mathcal{U}$ & $\ell_\mu \in \mathcal{V}$, $\mathcal{k}_\lambda \notin \mathcal{V}$ implies $N^{g\alpha g}cl(\{\mathcal{k}_\lambda\}) \neq N^{g\alpha g}cl(\{\ell_\mu\})$, since $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-R_1-space, then there exist disjoint $N^{g\alpha g}$-OSs $\mathcal{M}, \mathcal{N}$ s.t. $\mathcal{k}_\lambda \in \mathcal{M}$ & $\ell_\mu \in \mathcal{N}$. Consequently, $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-T_2-space. ▪

Corollary 4.11:

A $N^{g\alpha g}$-T_0-space is $N^{g\alpha g}$-T_2-space iff for any $\mathcal{k}_\lambda \neq \ell_\mu \in \mathcal{X}$ with $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\}) \neq N^{g\alpha g}ker(\{\ell_\mu\})$, then $\exists N^{g\alpha g}$-CSs $\mathcal{M}_1$, $\mathcal{M}_2$ s.t. $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\}) \sqsubseteq \mathcal{M}_1$, $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\}) \sqcap \mathcal{M}_2 = 0_N$ & $N^{g\alpha g}ker(\{\ell_\mu\}) \sqsubseteq \mathcal{M}_2$, $N^{g\alpha g}ker(\{\ell_\mu\}) \sqcap \mathcal{M}_1 = 0_N$ & $\mathcal{M}_1 \sqcup \mathcal{M}_2 = 1_N$.

Proof:

By Thm. (3.11) & Thm. (4.10). ▪

Corollary 4.12:

A $N^{g\alpha g}$-T_1-space is $N^{g\alpha g}$-T_2-space iff one of the following satisfies:

(i) for any $\mathcal{k}_\lambda \neq \ell_\mu \in \mathcal{X}$ with $N^{g\alpha g}cl(\{\mathcal{k}_\lambda\}) \neq N^{g\alpha g}cl(\{\ell_\mu\})$, then there exist $N^{g\alpha g}$-OSs $\mathcal{U}, \mathcal{V}$ s.t. $N^{g\alpha g}cl(N^{g\alpha g}ker(\{\mathcal{k}_\lambda\})) \sqsubseteq \mathcal{U}$ & $N^{g\alpha g}cl(N^{g\alpha g}ker(\{\ell_\mu\})) \sqsubseteq \mathcal{V}$.

(ii) for any $\mathcal{k}_\lambda \neq \ell_\mu \in \mathcal{X}$ with $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\}) \neq N^{g\alpha g}ker(\{\ell_\mu\})$, then there exist $N^{g\alpha g}$-CSs $\mathcal{M}_1$, $\mathcal{M}_2$ s.t. $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\}) \sqsubseteq \mathcal{M}_1$, $N^{g\alpha g}ker(\{\mathcal{k}_\lambda\}) \sqcap \mathcal{M}_2 = 0_N$ & $N^{g\alpha g}ker(\{\ell_\mu\}) \sqsubseteq \mathcal{M}_2$, $N^{g\alpha g}ker(\{\ell_\mu\}) \sqcap \mathcal{M}_1 = 0_N$ & $\mathcal{M}_1 \sqcup \mathcal{M}_2 = 1_N$.

Proof:

(i) By corollary (3.12) & Thm. (4.10).

(ii) By Thm. (3.11) & Thm. (4.10). ▪

Theorem 4.13:

A $N^{g\alpha g}$-R_1-space is $N^{g\alpha g}$-T_2-space iff one of the following satisfies:

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(i) for any $k_\lambda \in \mathcal{X}$, $N^{g\alpha g}ker(\{k_\lambda\}) = \{k_\lambda\}$.
(ii) for any $k_\lambda \neq \ell_\mu \in \mathcal{X}$, $N^{g\alpha g}ker(\{k_\lambda\}) \neq N^{g\alpha g}ker(\{\ell_\mu\})$ implies $N^{g\alpha g}ker(\{k_\lambda\}) \sqcap N^{g\alpha g}ker(\{\ell_\mu\}) = 0_N$.
(iii) for any $k_\lambda \neq \ell_\mu \in \mathcal{X}$, either $k_\lambda \notin N^{g\alpha g}ker(\{\ell_\mu\})$ or $\ell_\mu \notin N^{g\alpha g}ker(\{k_\lambda\})$.
(iv) for any $k_\lambda \neq \ell_\mu \in \mathcal{X}$ then $k_\lambda \notin N^{g\alpha g}ker(\{\ell_\mu\})$ & $\ell_\mu \notin N^{g\alpha g}ker(\{k_\lambda\})$.

Proof:

(i) Assume $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-T_2-space. So $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-T_1-space & $N^{g\alpha g}$-R_1-space [by Thm. (4.10)]. Wherefore by Thm. (4.6), $N^{g\alpha g}ker(\{k_\lambda\}) = \{k_\lambda\}$ for any $k_\lambda \in \mathcal{X}$.

Contrariwise, let for any $k_\lambda \in \mathcal{X}$, $N^{g\alpha g}ker(\{k_\lambda\}) = \{k_\lambda\}$, then by Thm. (4.6), $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-T_1-space. Likewise, $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-R_1-space by hypothesis. Wherefore by Thm. (4.10), $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-T_2-space.

(ii) Let $(\mathcal{X}, \xi)$ be a $N^{g\alpha g}$-T_2-space. Then $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-T_1-space [by remark (4.2)]. Wherefore by Thm. (4.8), $N^{g\alpha g}ker(\{k_\lambda\}) \sqcap N^{g\alpha g}ker(\{\ell_\mu\}) = 0_N$ for any $k_\lambda \neq \ell_\mu \in \mathcal{X}$.

Contrariwise, assume that for any $k_\lambda \neq \ell_\mu \in \mathcal{X}$, $N^{g\alpha g}ker(\{k_\lambda\}) \neq N^{g\alpha g}ker(\{\ell_\mu\})$ implies $N^{g\alpha g}ker(\{k_\lambda\}) \sqcap N^{g\alpha g}ker(\{\ell_\mu\}) = 0_N$. So by Thm. (4.8), $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-T_1-space, likewise $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-R_1-space by hypothesis. Wherefore by Thm. (4.10), $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-T_2-space.

(iii) Let $(\mathcal{X}, \xi)$ be a $N^{g\alpha g}$-T_2-space. Then $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-T_0-space [by remark (4.2)]. Wherefore by Thm. (4.3), either $k_\lambda \notin N^{g\alpha g}ker(\{\ell_\mu\})$ or $\ell_\mu \notin N^{g\alpha g}ker(\{k_\lambda\})$ for any $k_\lambda \neq \ell_\mu \in \mathcal{X}$.

Contrariwise, assume that for any $k_\lambda \neq \ell_\mu \in \mathcal{X}$, either $k_\lambda \notin N^{g\alpha g}ker(\{\ell_\mu\})$ or $\ell_\mu \notin N^{g\alpha g}ker(\{k_\lambda\})$ for any $k_\lambda \neq \ell_\mu \in \mathcal{X}$. So by Thm. (4.3), $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-T_0-space, likewise $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-R_1-space by hypothesis. Consequently, $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-T_2-space [by Thm. (4.10)].

(iv) Let $(\mathcal{X}, \xi)$ be a $N^{g\alpha g}$-T_2-space. Then $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-T_1-space & $N^{g\alpha g}$-R_1-space [by Thm. (4.10)]. Wherefore by Thm. (4.7), $k_\lambda \notin N^{g\alpha g}ker(\{\ell_\mu\})$ & $\ell_\mu \notin N^{g\alpha g}ker(\{k_\lambda\})$.

Contrariwise, let for any $k_\lambda \neq \ell_\mu \in \mathcal{X}$ then $k_\lambda \notin N^{g\alpha g}ker(\{\ell_\mu\})$ & $\ell_\mu \notin N^{g\alpha g}ker(\{k_\lambda\})$. Then by Thm. (4.7), $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-T_1-space. Likewise, $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-R_1-space by hypothesis. Wherefore by Thm. (4.10), $(\mathcal{X}, \xi)$ is a $N^{g\alpha g}$-T_2-space. ▪

Remark 4.14:

Any $N^{g\alpha g}$-separation axiom is defined as the conjunction of two weaker neutrosophic axioms: $N^{g\alpha g}$-T_k-space = $N^{g\alpha g}$-R_{k-1}-space & $N^{g\alpha g}$-T_{k-1}-space = $N^{g\alpha g}$-R_{k-1}-space & $N^{g\alpha g}$-T_0-space, $k = 1,2$.

Remark 4.15:

The relevance between $N^{g\alpha g}$-separation axioms can be illustrated as a matrix. Therefore, a_{ij} refers to this relation. As the following matrix impersonation shows:

&	$N^{g\alpha g}$-T_0	$N^{g\alpha g}$-T_1	$N^{g\alpha g}$-T_2	$N^{g\alpha g}$-R_0	$N^{g\alpha g}$-R_1
$N^{g\alpha g}$-T_0	$N^{g\alpha g}$-T_0	$N^{g\alpha g}$-T_1	$N^{g\alpha g}$-T_2	$N^{g\alpha g}$-T_1	$N^{g\alpha g}$-T_2
$N^{g\alpha g}$-T_1	$N^{g\alpha g}$-T_1	$N^{g\alpha g}$-T_1	$N^{g\alpha g}$-T_2	$N^{g\alpha g}$-T_1	$N^{g\alpha g}$-T_2
$N^{g\alpha g}$-T_2	$N^{g\alpha g}$-T_2	$N^{g\alpha g}$-T_2	$N^{g\alpha g}$-T_2	$N^{g\alpha g}$-T_2	$N^{g\alpha g}$-T_2
$N^{g\alpha g}$-R_0	$N^{g\alpha g}$-T_1	$N^{g\alpha g}$-T_1	$N^{g\alpha g}$-T_2	$N^{g\alpha g}$-R_0	$N^{g\alpha g}$-R_1
$N^{g\alpha g}$-R_1	$N^{g\alpha g}$-T_2	$N^{g\alpha g}$-T_2	$N^{g\alpha g}$-T_2	$N^{g\alpha g}$-R_1	$N^{g\alpha g}$-R_1

Matrix Representation

Figure 4.1: The relation between $N^{g\alpha g}$-separation axioms

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5. Conclusions

We have provided some new concepts of neutrosophic separation axioms, such as neutrosophic $g\alpha g$-R_i-space, $i = 0,1$ & neutrosophic $g\alpha g$-T_j-space, $= 0,1,2$. Furthermore, likewise proved some of their related attributes.

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