Bipolar Trapezoidal Neutrosophic Differential Equation and its Application

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Abstract

Neutrosophic set plays a vital role in dealing with indeterminacy in real-world problems. Differential equations represent the relation between a function and its derivatives and its applications have importance in both pedagogical and real life. In this paper, neutrosophic differential equation is proposed and solved using bipolar trapezoidal neutrosophic number and applied the concept in predicting bacterial reproduction over separate bodies with graphical representation using MATLAB. Also, comparative analysis is done with the existing method to prove the effectiveness of the proposed method.

Keywords: Trapezoidal Neutrosophic Numbers; Neutrosophic Set; Neutrosophic Differential Equations; Bipolar Trapezoidal Neutrosophic Number; Neutrosophic Numbers.

1. Introduction

To deal with an uncertain parameter there are a many methods are available like interval numbers, fuzzy number, and intuitionistic fuzzy number Neutrosophic number. But Neutrosophic number is the method to deals with indeterminacy where fuzzy number and intuitionistic fuzzy numbers fails to deal. Fuzzy numbers handle only the membership function, whereas in intuitionistic fuzzy number can handle both membership function and non-membership function but in Neutrosophic numbers only we deal both membership function and non-membership function in addition to that indeterminacy also. Any real-life problems can be modeled by differential equation, which is an equation involving the rate of change of a quantity. First time the term fuzzy differential equations introduced by Zadeh [1]. A very basic formulation of a fuzzy first-order initial value problems at first they found the crisp solution, then fuzzified it to check to see if it satisfies the fuzzy system of differential equations were presented by bukley [5]. Under generalized differentiability introduced by D.S-Le,H.Vu, and P.D-Nguyen also they introduced a new approach for approximating the fuzzy linear system of differential equations. Atanassov K [2] introduced the concept of an intuitionistic fuzzy differential equation. Ettoussi, R., et al [7] discussed the existence and uniqueness for the solution of a intuitionistic fuzzy differential equation and they proven and defined the averaging of intuitionistic fuzzy differential equation. The Romanian mathematician Florentine Smarandache [3-4] introduced the Neutrosophic ordinary differential equation of first order through neutrosophic numbers. A.EI Allaoui., et al. [8] optiomized the averaging intuitionistic fuzzy differential equations. I. R.Sumathi, I.:Mohana Priya.V et al., [9-10] demonstrated the neutrosophic linear differential equation with a new concept of neutrosophic derivatives. I.R.Sumathi, C.Antony Crispin [11] first time a second –order neutrosophic boundary value problem has been introduced with different types of first and second order derivatives. Suvankar Biswas, Sandip Moi,Smita

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Pal [12] extended the second-order linear differential equation with trapezoidal neutrosophic numbers as boundary conditions.

The special feature of using neutrosophic set is to deal with indeterminacy whereas fuzzy and intuitionistic unable to do in real world problems. It has been applied in many fields like decision-making problems, stochastic process, graph theory, optimization techniques. Recently the concept of bipolarity has been introduced in many research domains. The rest of the paper organized as follows. In section 2, literature review has been done for the past ten years. In section 3, preliminaries of neutrosophic and neutrosophic (alpha, beta, gamma)-cut are presented. In section 4, proposed the first order neutrosophic ordinary differential equation with BTpNNs. In section 5, the definition of differential equation using BTpNNs is proposed. In section 6, the solutions of the proposed equation are derived. In section 7, the proposed approach is applied in a bacterium culture model using BTpNNs with graphical representation using MATLAB. In section 8, comparative analysis has been done with the existing method to prove the soundness of the proposed concept. In section 9, the current work is concluded with the future direction.

2. Literature Review:

In [1], the author introduced first time the term fuzzy differential equations. In [14], the author presented the effect of the forcing term to the solution of a fuzzy differential equation. In [15], extended the fuzzy differential equations in two different ways that is by using a family of differential inclusions, and the extension principle for the solution of the model with considering its parameters and initial conditions given in fuzzy sets. In [16], the author investigated the numerical algorithms for solving first-order fuzzy differential equations and hybrid fuzzy differential equations. In [17], the author presented the various patterns of techniques incorporated into the fuzzy differential equations in both linear and non-linear type. In [18], introduced fuzzy differential equations of integer and fractional order and derived fuzzy derivative with help of fuzzy number valued function. In [2], introduced the concept of an intuitionistic fuzzy differential equation.

In [19], presented the solution concept for differential equations with intuitionistic fuzzy numbers. In [20], presented the first-order fuzzy differential equations with intuitionistic fuzzy initial valued problem. In [21], proposed averaging of intuitionistic fuzzy differential equation. In [22], introduced the system of intuitionistic fuzzy differential equations with intuitionistic fuzzy initial values.In [23], proposed the existence and uniqueness theorem of solution of intuitionistic fuzzy differential equation with non-linear conditions. In [3-4] the author introduced the neutrosophic differential equation. Neutrosophic ordinary differential equation of first order through neutrosophic numbers and also optimized.

In [9-10], demonstrated the neutrosophic linear differential equation with a new concept of neutrosophic derivatives. [11] First time a second –order neutrosophic boundary value problem has been introduced with different types of first and second order derivatives. [12] Extended the second-order linear differential equation with trapezoidal neutrosophic numbers as boundary conditions. From the review of literature, it is found that; neutrosophic differential is not studied using BTpNN. And hence, in this paper, neutrosophic differential equation is proposed and solved using bipolar trapezoidal neutrosophic environment.

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3. Preliminaries

In this section, basic definitions are presented for better understanding of the present work.

Definition 3.1. [9]

Let X be a universe set. A Neutrosophic set A on X is defined as $A=\{Tr_A(x), Ind_A(x), Fal_A(x) : x \in X\}$ where $Tr_A(x), Ind_A(x), Fal_A(x) : X \to {}^{-}]0,1[^{+}$ represents the degree of membership, degree of indeterministic and degree of non-membership respectively of the element $x \in X$ such that ${}^{-}0 \le Tr_A(x), Ind_A(x), Fal_A(x) \le 3^{+}$.

Definition 3.2. [13]

Let X be the universal set. A single valued Neutrosophic set A on X is defined as $A=\{< Tr_A(x), Ind_A(x), Fal_A(x) >: x \in X\}$ where $Tr_A(x), Ind_A(x), Fal_A(x) : X \to [0,1]$ represents the degree of membership, degree of indeterministic and degree of non-membership respectively of the element $x \in X$ such that $0 \le T_A(x), I_A(x), F_A(x) \le 3$.

Definition 3.3. [13]

$(\mathrm{A}_\alpha, \mathrm{B}_\beta, \Gamma_\gamma)$-cut: The $(\mathrm{A}_\alpha, \mathrm{B}_\beta, \Gamma_\gamma)$-cut Neutrosophic set is denoted by $F_{(\mathrm{A}_\alpha, \mathrm{B}_\beta, \Gamma_\gamma)}$, where $(\mathrm{A}_\alpha, \mathrm{B}_\beta, \Gamma_\gamma) \in [0,1]$ and are fixed numbers such that $(\mathrm{A}_\alpha, \mathrm{B}_\beta, \Gamma_\gamma) \le 3$ is defined as by $F_{(\mathrm{A}_\alpha, \mathrm{B}_\beta, \Gamma_\gamma)} = \{< Tr_A(x), Ind_A(x), Fal_A(x) >: x \in X, Tr_A(x) \ge \mathrm{A}_\alpha, Ind_A(x) \le \mathrm{B}_\beta, Fal_A(x) \le \Gamma_\gamma\}$.

Definition 3.4. Properties of Neutrosophic Number [13]

A Neutrosophic set 'A' is defined on the universal set of real numbers and 'R' is said to be neutrosophic number if it satisfies the following properties..

(i) A is normal it there exists $X_0 \in R$ such that $Tr_A(x_0)=1.(Ind_A(x_0)=Fal_A(x_0)=0)$.

(ii) A is convex for the truth function $Tr_A(x)$ i.e.,

$$Tr_A(\zeta x_1+(1-\zeta)x_2) \ge \min(Tr_A(x_1), Tr_A(x_2)) \forall x_1, x_2 \in R \,\&\, \zeta \in [0,1]$$

(iii) A is concave set for the indeterministic function $I_A(x)$ and false function $F_A(x)$ i.e.,

$$Ind_A(\zeta x_1+(1-\zeta)x_2) \ge \max(Ind_A(x_1), Ind_A(x_2)) \forall x_1, x_2 \in R \,\&\, \zeta \in [0,1].$$

$$Tr_A(\zeta x_1+(1-\zeta)x_2) \ge \max(Fal_A(x_1), Fal_A(x_2)) \forall x_1, x_2 \in R \,\&\, \zeta \in [0,1],,$$

Definition 3.5. Triangular Neutrosophic Number [13]

A triangular Neutrosophic number A will be a subset of Neutrosophic numbers in R with the following truth membership function, indeterministic function , and falsity function is given by

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$$Tr_A(x)=\begin{cases}\left(\dfrac{x-\kappa_1}{\kappa_2-\kappa_1}\right)\Omega_x & for \quad \kappa_1\le x\le\kappa_2\\ \Omega_x & for \quad x=\kappa_2\\ \left(\dfrac{\kappa_3-x}{\kappa_3-\kappa_2}\right)\Omega_x & for \quad \kappa_2\le x\le\kappa_3\\ 0 & otherwise\end{cases}$$

$$Ind_A(x)=\begin{cases}\left(\dfrac{\kappa_2-x}{\kappa_2-\kappa_1}\right)\Psi_x & for \quad \kappa_1\le x\le\kappa_2\\ \Psi_x & for \quad x=\kappa_2\\ \left(\dfrac{x-\kappa_1}{\kappa_3-\kappa_2}\right)\Psi_x & for \quad \kappa_2\le x\le\kappa_3\\ 1 & otherwise\end{cases}$$

$$Fal_A(x)=\begin{cases}\left(\dfrac{\kappa_2-x}{\kappa_2-\kappa_1}\right)\Upsilon_x & for \quad \kappa_1\le x\le\kappa_2\\ \Upsilon_x & for \quad x=\kappa_2\\ \left(\dfrac{x-\kappa_3}{\kappa_3-\kappa_2}\right)\Upsilon_x & for \quad \kappa_2\le x\le\kappa_3\\ 1 & otherwise\end{cases}$$

where $\kappa_1\le\kappa_2\le\kappa_3\le\kappa_4$ and a triangular neutrosophic number is represented as $A_{TN}=\langle(\kappa_1,\kappa_2,\kappa_3):\Omega_x,\Psi_x,\Upsilon_x\rangle$.

Definition 3.6. Trapezoidal Neutrosophic Number [13]

A trapezoidal Neutrosophic number ‘A’ is a subset of neutrosophic numbers in R with the truth membership function, indeterministic function, and falsity function as follows:

$$Tr_A(x)=\begin{cases}\left(\dfrac{x-\kappa_1}{\kappa_2-\kappa_1}\right)\Omega_x & for \quad \kappa_1\le x\le\kappa_2\\ \Omega_x & for \quad \kappa_2\le x\le\kappa_3\\ \left(\dfrac{\kappa_4-x}{\kappa_4-\kappa_3}\right)\Omega_x & for \quad \kappa_3\le x\le\kappa_4\\ 0 & otherwise\end{cases}$$

$$Ind_A(x)=\begin{cases}\left(\dfrac{\kappa_2-x}{\kappa_2-\kappa_1}\right)\Psi_x & for \quad \kappa_1\le x\le\kappa_2\\ \Psi_x & for \quad \kappa_2\le x\le\kappa_3\\ \left(\dfrac{x-\kappa_4}{\kappa_4-\kappa_3}\right)\Psi_x & for \quad \kappa_3\le x\le\kappa_4\\ 1 & otherwise\end{cases}$$

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$$Fal_A(x)=\begin{cases}\left(\frac{\kappa_2-x}{\kappa_2-\kappa_1}\right)\Upsilon_x & for \quad \kappa_1\le x\le\kappa_2\\ \Upsilon_x & for \quad \kappa_2\le x\le\kappa_3\\ \left(\frac{x-\kappa_4}{\kappa_4-\kappa_3}\right)\Upsilon_x & for \quad \kappa_3\le x\le\kappa_4\\ 1 & otherwise\end{cases}$$

where $\kappa_1\le\kappa_2\le\kappa_3\le\kappa_4$ and a trapezoidal Neutrosophic number is represented as $A_{TRN}=\langle(\kappa_1,\kappa_2,\kappa_3,\kappa_4):\Omega_x,\Psi_x,\Upsilon_x\rangle$.

Definition 3.7. Bipolar Trapezoidal Neutrosophic Number (BTpNN)

A BTpNN "A" will be a subset of neutrosophic numbers in R with truth membership, indeterministic, and falsityin both positive and negative interval is given by:

$$Tr^+_{BTpNN}(x)=\begin{cases}\left(\frac{x-e_1}{e_2-e_1}\right)\Omega_x & for \quad e_1\le x\le e_2\\ \Omega_x & for \quad x=e_3\\ \left(\frac{e_4-x}{e_4-e_3}\right)\Omega_x & for \quad e_3\le x\le e_4\\ 0 & otherwise\end{cases} \qquad Tr^-_{BTpNN}(x)=\begin{cases}\left(\frac{e_2-x}{e_2-e_1}\right)\Omega_x & for \quad e_1\le x\le e_2\\ -\Omega_x & for \quad x=e_3\\ \left(\frac{x-e_4}{e_4-e_3}\right)\Omega_x & for \quad e_3\le x\le e_4\\ 0 & otherwise\end{cases}$$

$$Ind^+_{BTpNN}(x)=\begin{cases}\left(\frac{f_2-x}{f_2-f_1}\right)\Psi_x & for \quad f_1\le x\le f_2\\ \Psi_x & for \quad f_2\le x\le f_3\\ \left(\frac{x-f_4}{f_4-f_3}\right)\Psi_x & for \quad f_3\le x\le f_4\\ 1 & otherwise\end{cases} \qquad Ind^-_{BTpNN}(x)=\begin{cases}\left(\frac{x-f_2}{f_2-f_1}\right)\Psi_x & for \quad f_1\le x\le f_2\\ -\Psi_x & for \quad f_2\le x\le f_3\\ \left(\frac{f_4-x}{f_4-f_1}\right)\Psi_x & for \quad f_3\le x\le f_4\\ -1 & otherwise\end{cases}$$

$$Fal^+_{BTpNN}(x)=\begin{cases}\left(\frac{g_2-x}{g_2-g_1}\right)\Upsilon_x & for \quad g_1\le x\le g_2\\ \Upsilon_x & for \quad g_2\le x\le g_3\\ \left(\frac{x-g_4}{g_4-g_3}\right)\Upsilon_x & for \quad g_3\le x\le g_4\\ 1 & otherwise\end{cases} \qquad Fal^-_{BTpNN}(x)=\begin{cases}\left(\frac{x-g_2}{g_2-g_1}\right)\Upsilon_x & for \quad g_1\le x\le g_2\\ 0 & for \quad g_2\le x\le g_3\\ \left(\frac{g_4-x}{g_4-g_3}\right)\Upsilon_x & for \quad g_3\le x\le g_4\\ -1 & otherwise\end{cases}$$

Where Tr^+_{BTpNN}, Tr^-_{BTpNN}, Ind^+_{BTpNN}, Ind^-_{BTpNN}, Fal^+_{BTpNN} and Fal^-_{BTpNN} denotes the positive part and negative part of validity membership function, the positive part and negative part of indeterminacy function respectively, and the positive and negative part of falsity respectively, of trapezoidal bipolar Neutrosophic number.

4. Proposed First Order Neutrosophic Ordinary Differential Equation

In this Section, first order linear differential equation using BTpNNs and proposed the Neutrosophic Ordinary Differential Equation with BTpNNs.

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Definition 4.1. First order Neutrosophic ordinary differential equation with BTpNNs

Let the first order linear homogeneous neutrosophic ordinary differential equation: $\frac{dP}{dx}=\eta P$ with $P(x_0)=P_0$ where η and P_0 are trapezoidal Neutrosophic numbers. Let the solution of the above neutrosophic differential equation be $P(x)$ and its $(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma)$ -cut be

$$P(x,\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma)=\left(\left[P_1(x,\mathrm{A}_\alpha),P_2(x,\mathrm{A}_\alpha)\right],\left[P_1^{'}(x,\mathrm{B}_\beta),P_2^{'}(x,\mathrm{B}_\beta)\right],P_1^{''}(x,\Gamma_\gamma),P_2^{''}(x,\Gamma_\gamma)\right)$$

The solution is effective if it is implemented correctly.

$$\frac{dP_1(x,\mathrm{A}_\alpha)}{d\mathrm{A}_\alpha}>0,\frac{dP_2(x,\mathrm{A}_\alpha)}{d\mathrm{A}_\alpha}<0,\quad \forall \mathrm{A}_\alpha\in[0,1],P_1(x,1)\le P_2(x,1)$$

$$\frac{dP_1^{'}(\mathrm{B}_\beta)}{d\mathrm{B}_\beta}<0,\frac{dP_2^{'}(\mathrm{B}_\beta)}{d\mathrm{B}_\beta}>0,\quad \forall \mathrm{B}_\beta\in[0,1],P_1^{'}(x,0)\le P_2^{'}(x,0)$$

$$\frac{dP_1^{''}(\Gamma_\gamma)}{d\Gamma_\gamma}>0,\frac{dP_2^{''}(\Gamma_\gamma)}{d\Gamma_\gamma}<0,\quad \forall \Gamma_\gamma\in[0,1],P_1^{''}(x,0)\le P_2^{''}(x,0)$$

Otherwise it is a week solution.

Definition 4.2. BTpNNs with $\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma-cut$

If A is a bipolar trapezoidal neutrosophic number, then $\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma$ -cut is defined by

$$A=\begin{cases}\left[A_1^+(\mathrm{A}_\alpha),A_2^+(\mathrm{A}_\alpha),A_3^+(\mathrm{A}_\alpha),A_4^+(\mathrm{A}_\alpha)\right] & for \quad \mathrm{A}_\alpha\in[0,1]\\ \left[A_1^-(\mathrm{A}_\alpha),A_2^-(\mathrm{A}_\alpha),A_3^-(\mathrm{A}_\alpha),A_4^-(\mathrm{A}_\alpha)\right] & for \quad \mathrm{A}_\alpha\in[-1,0]\\ \left[\left[A_1^{'}\right]^+(\mathrm{B}_\beta),\left[A_2^{'}\right]^+(\mathrm{B}_\beta),\left[A_3^{'}\right]^+(\mathrm{B}_\beta),\left[A_4^{'}\right]^+(\mathrm{B}_\beta)\right] & for\ \mathrm{B}_\beta\in[0,1]\\ \left[\left[A_1^{'}\right]^-(\mathrm{B}_\beta),\left[A_2^{'}\right]^-(\mathrm{B}_\beta),\left[A_3^{'}\right]^-(\mathrm{B}_\beta),\left[A_4^{'}\right]^-(\mathrm{B}_\beta)\right] & for\ \mathrm{B}_\beta\in[-1,0]\\ \left[\left[A_1^{''}\right]^+(\Gamma_\gamma),\left[A_2^{''}\right]^+(\Gamma_\gamma),\left[A_3^{''}\right]^+(\Gamma_\gamma),\left[A_4^{''}\right]^+(\Gamma_\gamma)\right] & for\ \Gamma_\gamma\in[0,1]\\ \left[\left[A_1^{''}\right]^-(\Gamma_\gamma),\left[A_2^{''}\right]^-(\Gamma_\gamma),\left[A_3^{''}\right]^-(\Gamma_\gamma),\left[A_4^{''}\right]^-(\Gamma_\gamma)\right] & for \quad \Gamma_\gamma\in[0,1]\end{cases} \tag{1}$$

with $\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\le 3$.

Here,

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(i)
$$\frac{d(A_1)^+(\mathrm{A}_\alpha)}{d\mathrm{A}_\alpha}>0,\frac{d(A_2)^+(\mathrm{A}_\alpha)}{d\mathrm{A}_\alpha}<0,\quad \forall \mathrm{A}_\alpha\in[0,1],A_1^+(1)\le A_2^+(1)$$
$$\frac{d(A_3)^+(\mathrm{A}_\alpha)}{d\mathrm{A}_\alpha}>0,\frac{d(A_4)^+(\mathrm{A}_\alpha)}{d\mathrm{A}_\alpha}<0,\quad \forall \mathrm{A}_\alpha\in[0,1],A_3^+(1)\le A_4^+(1)$$
$$\frac{d(A_1)^-(\mathrm{A}_\alpha)}{d\mathrm{A}_\alpha}>0,\frac{d(A_2)^-(\mathrm{A}_\alpha)}{d\mathrm{A}_\alpha}<0,\quad \forall \mathrm{A}_\alpha\in[-1,0],A_1^-(1)\le A_2^-(1)$$
$$\frac{d(A_3)^-(\mathrm{A}_\alpha)}{d\mathrm{A}_\alpha}>0,\frac{d(A_4)^-(\mathrm{A}_\alpha)}{d\mathrm{A}_\alpha}<0,\quad \forall \mathrm{A}_\alpha\in[-1,0],A_3^-(1)\le A_4^-(1)$$

(ii)
$$\frac{d(A_1^{\cdot\cdot})^+(\Gamma_\gamma)}{d\Gamma_\gamma}>0,\frac{d(A_2^{\cdot\cdot})^+(\gamma)}{d\Gamma_\gamma}<0,\quad \forall \Gamma_\gamma\in[0,1],[A_1^{\cdot\cdot}]^+(0)\le[A_2^{\cdot\cdot}]^+(0)$$
$$\frac{d(A_3^{\cdot\cdot})^+(\Gamma_\gamma)}{d\Gamma_\gamma}>0,\frac{d(A_4^{\cdot\cdot})^+(\gamma)}{d\Gamma_\gamma}<0,\quad \forall \Gamma_\gamma\in[0,1],[A_3^{\cdot\cdot}]^+(0)\le[A_4^{\cdot\cdot}]^+(0)$$
$$\frac{d(A_1^{\cdot\cdot})^-(\Gamma_\gamma)}{d\Gamma_\gamma}>0,\frac{d(A_2^{\cdot\cdot})^-(\gamma)}{d\Gamma_\gamma}<0,\quad \forall \Gamma_\gamma\in[-1,0],[A_1^{\cdot\cdot}]^-(0)\le[A_2^{\cdot\cdot}]^-(0)$$
$$\frac{d(A_3^{\cdot\cdot})^-(\Gamma_\gamma)}{d\Gamma_\gamma}>0,\frac{d(A_4^{\cdot\cdot})^-(\gamma)}{d\Gamma_\gamma}<0,\quad \forall \Gamma_\gamma\in[-1,0],[A_3^{\cdot\cdot}]^-(0)\le[A_4^{\cdot\cdot}]^-(0)$$

(iii)
$$\frac{d(\mathrm{A}_1^{\cdot})^+(\mathrm{B}_\beta)}{d\mathrm{B}_\beta}<0,\frac{d(\mathrm{A}_2^{\cdot})^+(\mathrm{B}_\beta)}{d\mathrm{B}_\beta}>0,\quad \forall \mathrm{B}_\beta\in[0,1],[A_1^{\cdot}]^+(0)\le[A_2^{\cdot}]^+(0)$$
$$\frac{d(\mathrm{A}_3^{\cdot})^+(\mathrm{B}_\beta)}{d\mathrm{B}_\beta}<0,\frac{d(\mathrm{A}_4^{\cdot})^+(\mathrm{B}_\beta)}{d\mathrm{B}_\beta}>0,\quad \forall \mathrm{B}_\beta\in[0,1],[A_3^{\cdot}]^+(0)\le[A_4^{\cdot}]^+(0)$$
$$\frac{d(\mathrm{A}_1^{\cdot})^-(\mathrm{B}_\beta)}{d\mathrm{B}_\beta}<0,\frac{d(\mathrm{A}_2^{\cdot})^-(\beta)}{d\mathrm{B}_\beta}>0,\quad \forall \mathrm{B}_\beta\in[-1,0],[A_1^{\cdot}]^-(0)\le[A_2^{\cdot}]^-(0)$$
$$\frac{d(\mathrm{A}_3^{\cdot})^-(\mathrm{B}_\beta)}{d\mathrm{B}_\beta}<0,\frac{d(\mathrm{A}_4^{\cdot})^-(\beta)}{d\mathrm{B}_\beta}>0,\quad \forall \mathrm{B}_\beta\in[-1,0],[A_3^{\cdot}]^-(0)\le[A_4^{\cdot}]^-(0)$$

If $A=((-e_1,-e_2,-e_3,-e_4:e_1,e_2,e_3,e_4),(-f_1,-f_2,-f_3,-f_4:f_1,f_2,f_3,f_4),(-g_1,-g_2,-g_3,-g_4:g_1,g_2,g_3,g_4);\Omega_x,\Psi_x,\Upsilon_x)$ then $\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma$ -cut is defined as

$$\begin{aligned}A_{(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma)}=\{&[(e_2+\mathrm{A}_\alpha(e_2-e_1))\Omega_x,(e_4-\mathrm{A}_\alpha(e_4-e_3))\Omega_x],[(e_2-\mathrm{A}_\alpha(e_2-e_1))\Omega_x,(e_4+\mathrm{A}_\alpha(e_4-e_3))\Omega_x]\\&[(f_2-\mathrm{B}_\beta(f_2-f_1))\Psi_x,(f_3+\mathrm{B}_\beta(f_4-f_3))\Psi_x],[(f_2+\mathrm{B}_\beta(f_2-f_1))\Psi_x,(f_3-\mathrm{B}_\beta(f_4-f_3))\Psi_x]\\&[(g_2-\Gamma_\gamma(g_2-g_1))\Upsilon_x,(g_3+\Gamma_\gamma(g_4-g_3))\Upsilon_x],[(g_2+\Gamma_\gamma(g_2-g_1))\Upsilon_x,(g_3-\Gamma_\gamma(g_4-g_3))\Upsilon_x]\}\end{aligned} \quad (2)$$

5. Proposed definition of Differential Equation with BTpNN

In this section, definition of differential equations using BTpNN method has been proposed.

5.1 Proposed definition for Bipolar Neutrosophic Differential Equation (BpNDE)

Let the first order linear homogeneous neutrosophic ordinary differential equation: $\frac{dP}{dx}=\eta P$ with $P(x_0)=P_0$ where η and P_0 are bipolar trapezoidal neutrosophic numbers. Let the solution of the above equation be $P(x)$ and its $\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma$ -cut be

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$$P\left(x,\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)=\begin{pmatrix}\left[P_1^+\left(x,\mathrm{A}_\alpha\right),P_1^+\left(x,\mathrm{A}_\alpha\right),P_1^-\left(x,\mathrm{A}_\alpha\right),P_1^-\left(x,\mathrm{A}_\alpha\right)\right],\left[\left(P_1^{'}\right)^+\left(x,\mathrm{B}_\beta\right),\left(P_2^{'}\right)^+\left(x,\mathrm{B}_\beta\right),\left(P_1^{'}\right)^-\left(x,\mathrm{B}_\beta\right),\left(P_2^{'}\right)^-\left(x,\mathrm{B}_\beta\right)\right],\\ \left[\left(P_1^{''}\right)^+\left(x,\Gamma_\gamma\right),\left(P_2^{''}\right)^+\left(x,\Gamma_\gamma\right),\left(P_1^{''}\right)^-\left(x,\Gamma_\gamma\right),\left(P_2^{''}\right)^-\left(x,\Gamma_\gamma\right)\right]\end{pmatrix}$$

(i)
$$\frac{dP_1^+\left(x,\mathrm{A}_\alpha\right)}{d\mathrm{A}_\alpha}>0,\frac{dP_2^+\left(x,\mathrm{A}_\alpha\right)}{d\mathrm{A}_\alpha}<0,\quad \forall \mathrm{A}_\alpha\in[0,1],P_1^+(x,1)\le P_2^+(x,1)$$
$$\frac{dP_1^-\left(x,\mathrm{A}_\alpha\right)}{d\mathrm{A}_\alpha}>0,\frac{dP_2^-\left(x,\mathrm{A}_\alpha\right)}{d\mathrm{A}_\alpha}<0,\quad \forall \mathrm{A}_\alpha\in[0,1],P_1^-(x,1)\le P_2^-(x,1)$$

(ii)
$$\frac{\left(dP_1^{'}\right)^+\left(\mathrm{B}_\beta\right)}{d\mathrm{B}_\beta}<0,\frac{\left(dP_2^{'}\right)^+\left(\mathrm{B}_\beta\right)}{d\mathrm{B}_\beta}>0,\quad \forall \mathrm{B}_\beta\in[0,1],\left(P_1^{'}\right)^+(x,0)\le\left(P_2^{'}\right)^+(x,0)$$
$$\frac{\left(dP_1^{'}\right)^-\left(\mathrm{B}_\beta\right)}{d\mathrm{B}_\beta}<0,\frac{\left(dP_2^{'}\right)^-\left(\mathrm{B}_\beta\right)}{d\mathrm{B}_\beta}>0,\quad \forall \mathrm{B}_\beta\in[0,1],\left(P_1^{'}\right)^-(x,0)\le\left(P_2^{'}\right)^-(x,0)$$

(iii)
$$\frac{\left(dP_1^{''}\right)^+\left(\Gamma_\gamma\right)}{d\Gamma_\gamma}<0,\frac{\left(dP_2^{''}\right)^+\left(\Gamma_\gamma\right)}{d\Gamma_\gamma}>0,\quad \forall \Gamma_\gamma\in[0,1],\left(P_1^{''}\right)^+(x,0)\le\left(P_2^{''}\right)^+(x,0)$$
$$\frac{\left(dP_1^{''}\right)^-\left(\Gamma_\gamma\right)}{d\Gamma_\gamma}<0,\frac{\left(dP_2^{''}\right)^-\left(\Gamma_\gamma\right)}{d\Gamma_\gamma}>0,\quad \forall \Gamma_\gamma\in[0,1],\left(P_1^{''}\right)^-(x,0)\le\left(P_2^{''}\right)^-(x,0)$$

6. Proposed Solution of BpNDE

In this section, derived a solution of neutrosophic differential equation (NDE) is proposed using BTpNNs.

6.1 Solution of BTpNDE

Consider the first order linear homogeneous neutrosophic ordinary differential equation:

Suppose $\frac{dP}{dx}=\eta P \quad with$

Initial condition

$$P(x_0)=A_{BTpNN}=\left\langle(-e_1,-e_2,-e_3,-e_4:e_1,e_2,e_3,e_4),(-f_1,-f_2,-f_3,-f_4:f_1,f_2,f_3,f_4),(-g_1,-g_2,-g_3,-g_4:g_1,g_2,g_3,g_4):\Omega_x,\Psi_x,\Upsilon_x\right\rangle. \quad (3)$$

Case 1: When $\eta>0$

Taking $\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma$ -cut of the equation (3) we have

$$\frac{dP}{dx}=\eta\begin{pmatrix}\left[(P_1)^-\left(x,\mathrm{A}_\alpha\right),(P_2)^-\left(x,\mathrm{A}_\alpha\right),(P_1)^+\left(x,\mathrm{A}_\alpha\right),(P_2)^-\left(x,\mathrm{A}_\alpha\right)\right],\left[\left(P_1^{'}\right)^-\left(x,\mathrm{B}_\beta\right),\left(P_2^{'}\right)^-\left(x,\mathrm{B}_\beta\right),\left(P_1^{'}\right)^+\left(x,\mathrm{B}_\beta\right),\left(P_2^{'}\right)^+\left(x,\mathrm{B}_\beta\right)\right],\\ \left[\left(P_1^{''}\right)^-\left(x,\Gamma_\gamma\right),\left(P_2^{''}\right)^-\left(x,\Gamma_\gamma\right),\left(P_1^{''}\right)^+\left(x,\Gamma_\gamma\right),\left(P_2^{''}\right)^+\left(x,\Gamma_\gamma\right)\right]\end{pmatrix}$$

with the initial conditions in equations (4), and initial condition for both negative and positive interval in equation (5) and equation (6) as follows:

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$$y\left(x_0;\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)=\begin{cases}\left(\left[e_1\mathrm{A}_\alpha,e_2\mathrm{A}_\alpha,e_3\mathrm{A}_\alpha,e_4\mathrm{A}_\alpha\right],\left[e_1^{'}\mathrm{B}_\beta,e_2^{'}\mathrm{B}_\beta,e_3^{'}\mathrm{B}_\beta,e_4^{'}\mathrm{B}_\beta\right],\left[e_1^{''}\Gamma_\gamma,e_2^{''}\Gamma_\gamma,e_3^{''}\Gamma_\gamma,e_4^{''}\Gamma_\gamma\right]\right),\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\le 3,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\in\left[-1,0\right]\&\left[0,1\right]\\ \left(\left[f_1\mathrm{A}_\alpha,f_2\mathrm{A}_\alpha,f_3\mathrm{A}_\alpha,f_4\mathrm{A}_\alpha\right],\left[f_1^{'}\mathrm{B}_\beta,f_2^{'}\mathrm{B}_\beta,f_3^{'}\mathrm{B}_\beta,f_4^{'}\mathrm{B}_\beta\right],\left[f_1^{''}\Gamma_\gamma,f_2^{''}\Gamma_\gamma,f_3^{''}\Gamma_\gamma,f_4^{''}\Gamma_\gamma\right]\right),\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\le 3,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\in\left[-1,0\right]\&\left[0,1\right]\\ \left(\left[g_1\mathrm{A}_\alpha,g_2\mathrm{A}_\alpha,g_3\mathrm{A}_\alpha,g_4\mathrm{A}_\alpha\right],\left[g_1^{'}\mathrm{B}_\beta,g_2^{'}\mathrm{B}_\beta,g_3^{'}\mathrm{B}_\beta,g_4^{'}\mathrm{B}_\beta\right],\left[g_1^{''}\Gamma_\gamma,g_2^{''}\Gamma_\gamma,g_3^{''}\Gamma_\gamma,g_4^{''}\Gamma_\gamma\right]\right),\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\le 3,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\in\left[-1,0\right]\&\left[0,1\right]\end{cases}\quad(4)$$

$$\left.\begin{array}{ll}(P_1)^-(x,\mathrm{A}_\alpha)=\eta(P_1)^-(x,\mathrm{A}_\alpha) & (P_2)^-(x,\mathrm{A}_\alpha)=\eta(P_2)^-(x,\mathrm{A}_\alpha)\\ \left(P_1^{'}\right)^-(x,\mathrm{B}_\beta)=\eta\left(P_1^{'}\right)^-(x,\mathrm{B}_\beta) & \left(P_2^{'}\right)^-(x,\mathrm{B}_\beta)=\eta\left(P_2^{'}\right)^-(x,\mathrm{B}_\beta)\\ \left(P_1^{''}\right)^-(x,\Gamma_\gamma)=\eta\left(P_1^{''}\right)^-(x,\Gamma_\gamma) & \left(P_2^{''}\right)^-(x,\Gamma_\gamma)=\eta\left(P_2^{''}\right)^-(x,\Gamma_\gamma)\end{array}\right\}\quad(5)$$

$$\left.\begin{array}{ll}(P_1)^+(x,\mathrm{A}_\alpha)=\eta(P_1)^+(x,\mathrm{A}_\alpha) & (P_2)^+(x,\mathrm{A}_\alpha)=\eta(P_2)^+(x,\mathrm{A}_\alpha)\\ \left(P_1^{'}\right)^+(x,\mathrm{B}_\beta)=\eta\left(P_1^{'}\right)^+(x,\mathrm{B}_\beta) & \left(P_2^{'}\right)^+(x,\mathrm{B}_\beta)=\eta\left(P_2^{'}\right)^+(x,\mathrm{B}_\beta)\\ \left(P_3^{''}\right)^+(x,\Gamma_\gamma)=\eta\left(P_1^{''}\right)^+(x,\Gamma_\gamma) & \left(P_2^{''}\right)^+(x,\Gamma_\gamma)=\eta\left(P_2^{''}\right)^+(x,\Gamma_\gamma)\end{array}\right\}\quad(6)$$

In the first row of equation (4)

i.e.,

$$\left(\left[e_1\mathrm{A}_\alpha,e_2\mathrm{A}_\alpha,e_3\mathrm{A}_\alpha,e_4\mathrm{A}_\alpha\right],\left[e_1^{'}\mathrm{B}_\beta,e_2^{'}\mathrm{B}_\beta,e_3^{'}\mathrm{B}_\beta,e_4^{'}\mathrm{B}_\beta\right],\left[e_1^{''}\Gamma_\gamma,e_2^{''}\Gamma_\gamma,e_3^{''}\Gamma_\gamma,e_4^{''}\Gamma_\gamma\right]\right),\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\le 3,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\in\left[-1,0\right]\&\left[0,1\right]$$

Apply initial conditions of equation (5) and equation (6) in the first row of equation (4) we get the below conditions for both negative and positive interval as follows:

with initial condition of negative membership function

$$\left.\begin{array}{ll}(P_1)^-(x_0,\mathrm{A}_\alpha)=e_1(\mathrm{A}_\alpha) & (P_2)^-(x_0,\mathrm{A}_\alpha)=e_2(\mathrm{A}_\alpha)\\ \left(P_1^{'}\right)^-(x_0,\mathrm{B}_\beta)=e_1^{'}(\mathrm{B}_\beta) & \left(P_2^{'}\right)^-(x_0,\mathrm{B}_\beta)=e_2^{'}(\mathrm{B}_\beta)\\ \left(P_1^{''}\right)^-(x_0,\Gamma_\gamma)=e_1^{''}(\Gamma_\gamma) & \left(P_2^{''}\right)^-(x_0,\Gamma_\gamma)=e_2^{''}(\Gamma_\gamma)\end{array}\right\}\quad(7)$$

with initial condition of positive membership function

$$\left.\begin{array}{ll}(P_1)^+(x_0,\mathrm{A}_\alpha)=e_3(\mathrm{A}_\alpha) & (P_2)^+(x_0,\mathrm{A}_\alpha)=e_4(\mathrm{A}_\alpha)\\ \left(P_1^{'}\right)^+(x_0,\mathrm{B}_\beta)=e_3^{'}(\mathrm{B}_\beta) & \left(P_2^{'}\right)^+(x_0,\mathrm{B}_\beta)=e_4^{'}(\mathrm{B}_\beta)\\ \left(P_1^{''}\right)^+(x_0,\Gamma_\gamma)=e_3^{''}(\Gamma_\gamma) & \left(P_2^{''}\right)^+(x_0,\Gamma_\gamma)=e_4^{''}(\Gamma_\gamma)\end{array}\right\}\quad(8)$$

In the second row of equation (4)

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i.e.,

$$\left(\left[f_1\mathrm{A}_\alpha, f_2\mathrm{A}_\alpha, f_3\mathrm{A}_\alpha, f_4\mathrm{A}_\alpha\right],\left[f_1'\mathrm{B}_\beta, f_2'\mathrm{B}_\beta, f_3'\mathrm{B}_\beta, f_4'\mathrm{B}_\beta\right],\left[f_1''\Gamma_\gamma, f_2''\Gamma_\gamma, f_3''\Gamma_\gamma, f_4''\Gamma_\gamma\right]\right),\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\le 3,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\in\left[-1,0\right]\&\left[0,1\right]$$

Apply initial conditions of equation (5) and equation (6) in the second row of equation (4) we get the below conditions for both negative and positive interval as follows:

with initial condition of negative membership function

$$\left.\begin{array}{ll} (P_1)^-(x_0,\mathrm{A}_\alpha)=f_1(\mathrm{A}_\alpha) & (P_2)^-(x_0,\mathrm{A}_\alpha)=f_2(\mathrm{A}_\alpha) \\ (P_1')^-(x_0,\mathrm{B}_\beta)=f_1'(\mathrm{B}_\beta) & (P_2')^-(x_0,\mathrm{B}_\beta)=f_2'(\mathrm{B}_\beta) \\ (P_1'')^-(x_0,\Gamma_\gamma)=f_1''(\Gamma_\gamma) & (P_2'')^-(x_0,\Gamma_\gamma)=f_2''(\Gamma_\gamma) \end{array}\right\} \quad (9)$$

with initial condition of positive membership function

$$\left.\begin{array}{ll} (P_1)^+(x_0,\mathrm{A}_\alpha)=f_3(\mathrm{A}_\alpha) & (P_2)^+(x_0,\mathrm{A}_\alpha)=f_4(\mathrm{A}_\alpha) \\ (P_1')^+(x_0,\mathrm{B}_\beta)=f_3'(\mathrm{B}_\beta) & (P_2')^+(x_0,\mathrm{B}_\beta)=f_4'(\mathrm{B}_\beta) \\ (P_1'')^+(x_0,\Gamma_\gamma)=f_3''(\Gamma_\gamma) & (P_2'')^+(x_0,\Gamma_\gamma)=f_4''(\Gamma_\gamma) \end{array}\right\} \quad (10)$$

In the third row

i.e.,

$$\left(\left[g_1\mathrm{A}_\alpha, g_2\mathrm{A}_\alpha, g_3\mathrm{A}_\alpha, g_4\mathrm{A}_\alpha\right],\left[g_1'\mathrm{B}_\beta, g_2'\mathrm{B}_\beta, g_3'\mathrm{B}_\beta, g_4'\mathrm{B}_\beta\right],\left[g_1''\Gamma_\gamma, g_2''\Gamma_\gamma, g_3''\Gamma_\gamma, g_4''\Gamma_\gamma\right]\right),\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\le 3,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\in\left[-1,0\right]\&\left[0,1\right]$$

initial conditions of equation (5) and equation (6) we can apply in the third row of equation (4) we get the below conditions for both negative and positive interval as follows:

with initial condition of negative membership function

$$\left.\begin{array}{ll} (P_1)^-(x_0,\mathrm{A}_\alpha)=g_1(\mathrm{A}_\alpha) & (P_2)^-(x_0,\mathrm{A}_\alpha)=g_2(\mathrm{A}_\alpha) \\ (P_1')^-(x_0,\mathrm{B}_\beta)=g_1'(\mathrm{B}_\beta) & (P_2')^-(x_0,\mathrm{B}_\beta)=g_2'(\mathrm{B}_\beta) \\ (P_1'')^-(x_0,\Gamma_\gamma)=g_1''(\Gamma_\gamma) & (P_2'')^-(x_0,\Gamma_\gamma)=g_2''(\Gamma_\gamma) \end{array}\right\} \quad (11)$$

with initial condition of positive membership function

$$\left.\begin{array}{ll} (P_1)^+(x_0,\mathrm{A}_\alpha)=g_3(\mathrm{A}_\alpha) & (P_2)^+(x_0,\mathrm{A}_\alpha)=g_4(\mathrm{A}_\alpha) \\ (P_1')^+(x_0,\mathrm{B}_\beta)=g_3'(\mathrm{B}_\beta) & (P_2')^+(x_0,\mathrm{B}_\beta)=g_4'(\mathrm{B}_\beta) \\ (P_1'')^+(x_0,\Gamma_\gamma)=g_3''(\Gamma_\gamma) & (P_2'')^+(x_0,\Gamma_\gamma)=g_4''(\Gamma_\gamma) \end{array}\right\} \quad (12)$$

Then the solution obtained for the first row of equation (4) from the above equations (7) & (8) for both negative side and positive interval as follows.

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$$\begin{array}{llll}
(P_1)^-(x,A_\alpha)=e_1(A_\alpha)e^{\eta(x-x_0)} & (P_2)^-(x,A_\alpha)=e_2(A_\alpha)e^{\eta(x-x_0)} & (P_1)^+(x,A_\alpha)=e_3(A_\alpha)e^{\eta(x-x_0)} & (P_2)^-(x,A_\alpha)=e_4(A_\alpha)e^{\eta(x-x_0)} \\
(P_1^{'})^-(x,B_\beta)=e_1^{'}(B_\beta)e^{\eta(x-x_0)} & (P_2^{'})^-(x,B_\beta)=e_2^{'}(B_\beta)e^{\eta(x-x_0)} & (P_1^{'})^+(x,B_\beta)=e_3^{'}(B_\beta)e^{\eta(x-x_0)} & (P_2^{'})^-(x,B_\beta)=e_4^{'}(B_\beta)e^{\eta(x-x_0)} \\
(P_1^{"})^-(x,\Gamma_\gamma)=e_1^{"}(\Gamma_\gamma)e^{\eta(x-x_0)} & (P_2^{"})^-(x,\Gamma_\gamma)=e_2^{"}(\Gamma_\gamma)e^{\eta(x-x_0)} & (P_1^{"})^+(x,\Gamma_\gamma)=e_3^{"}(\Gamma_\gamma)e^{\eta(x-x_0)} & (P_2^{"})^-(x,\gamma)=e_4^{"}(\Gamma_\gamma)e^{\eta(x-x_0)}
\end{array}$$

Then the solution obtained for the second row of equation (4) from the above equations (9) & (10) for both negative side and positive interval as follows.

$$\begin{array}{llll}
(P_1)^-(x,A_\alpha)=f_1(A_\alpha)e^{\eta(x-x_0)} & (P_2)^-(x,A_\alpha)=f_2(A_\alpha)e^{\eta(x-x_0)} & (P_1)^-(x,A_\alpha)=f_3(A_\alpha)e^{\eta(x-x_0)} & (P_2)^-(x,A_\alpha)=f_4(A_\alpha)e^{\eta(x-x_0)} \\
(P_1^{'})^-(x,B_\beta)=f_1^{'}(B_\beta)e^{\eta(x-x_0)} & (P_2^{'})^-(x,B_\beta)=f_2^{'}(B_\beta)e^{\eta(x-x_0)} & (P_1^{'})^-(x,B_\beta)=f_3^{'}(B_\beta)e^{\eta(x-x_0)} & (P_2^{'})^-(x,B_\beta)=f_4^{'}(B_\beta)e^{\eta(x-x_0)} \\
(P_1^{"})^-(x,\Gamma_\gamma)=f_1^{"}(\Gamma_\gamma)e^{\eta(x-x_0)} & (P_2^{"})^-(x,\Gamma_\gamma)=f_2^{"}(\Gamma_\gamma)e^{\eta(x-x_0)} & (P_1^{"})^-(x,\Gamma_\gamma)=f_3^{"}(\Gamma_\gamma)e^{\eta(x-x_0)} & (P_2^{"})^-(x,\Gamma_\gamma)=f_4^{"}(\Gamma_\gamma)e^{\eta(x-x_0)}
\end{array}$$

Then the solution obtained for the third row of equation (4) from the above equations (11) & (12) for both negative side and positive side are,

$$\begin{array}{llll}
(P_1)^-(x,A_\alpha)=g_1(A_\alpha)e^{\eta(x-x_0)} & (P_2)^-(x,A_\alpha)=g_2(A_\alpha)e^{\eta(x-x_0)} & (P_1)^+(x,A_\alpha)=g_3(A_\alpha)e^{\eta(x-x_0)} & (P_2)^+(x,A_\alpha)=g_4(A_\alpha)e^{\eta(x-x_0)} \\
(P_1^{'})^-(x,B_\beta)=g_1^{'}(B_\beta)e^{\eta(x-x_0)} & (P_2^{'})^-(x,B_\beta)=g_2^{'}(B_\beta)e^{\eta(x-x_0)} & (P_1^{'})^+(x,B_\beta)=g_3^{'}(B_\beta)e^{\eta(x-x_0)} & (P_2^{'})^+(x,B_\beta)=g_4^{'}(B_\beta)e^{\eta(x-x_0)} \\
(P_1^{"})^-(x,\Gamma_\gamma)=g_1^{"}(\Gamma_\gamma)e^{\eta(x-x_0)} & (P_2^{"})^-(x,\Gamma_\gamma)=g_2^{"}(\Gamma_\gamma)e^{\eta(x-x_0)} & (P_1^{"})^+(x,\Gamma_\gamma)=g_3^{"}(\Gamma_\gamma)e^{\eta(x-x_0)} & (P_2^{"})^+(x,\Gamma_\gamma)=g_4^{"}(\Gamma_\gamma)e^{\eta(x-x_0)}
\end{array}$$

Case 2: If $\eta < 0$

When $\eta = -ve$ Taking $A_\alpha, B_\beta, \Gamma_\gamma$ -cut of the equation (3) we get

$$\frac{dP}{dx} = -\eta\left(\begin{array}{l}\left[(P_1)^-(x,A_\alpha),(P_2)^-(x,A_\alpha),(P_1)^+(x,A_\alpha),(P_2)^-(x,A_\alpha)\right],\left[(P_1^{'})^-(x,B_\beta),(P_2^{'})^-(x,B_\beta),(P_1^{'})^+(x,B_\beta),(P_2^{'})^+(x,B_\beta)\right],\\ \left[(P_1^{"})^-(x,\Gamma_\gamma),(P_2^{"})^-(x,\Gamma_\gamma),(P_1^{"})^+(x,\Gamma_\gamma),(P_2^{"})^+(x,\Gamma_\gamma)\right]\end{array}\right)$$

with the initial conditions of equations (13), and initial condition for both negative and positive interval in equation (14) and equation (15) as follows:

$$P(x_0;A_\alpha,B_\beta,\Gamma_\gamma)=\left\{\begin{array}{l}\left(\left[e_1A_\alpha,e_2A_\alpha,e_3A_\alpha,e_4A_\alpha\right],\left[e_1^{'}B_\beta,e_2^{'}B_\beta,e_3^{'}B_\beta,e_4^{'}B_\beta\right],\left[e_1^{"}\Gamma_\gamma,e_2^{"}\Gamma_\gamma,e_3^{"}\Gamma_\gamma,e_4^{"}\Gamma_\gamma\right]\right),(A_\alpha,B_\beta,\Gamma_\gamma)\le 3,(A_\alpha,B_\beta,\Gamma_\gamma)\in[0,1]\&[-1,0]\\ \left(\left[f_1A_\alpha,f_2A_\alpha,f_3A_\alpha,f_4A_\alpha\right],\left[f_1^{'}B_\beta,f_2^{'}B_\beta,f_3^{'}B_\beta,f_4^{'}B_\beta\right],\left[f_1^{"}\Gamma_\gamma,f_2^{"}\Gamma_\gamma,f_3^{"}\Gamma_\gamma,f_4^{"}\Gamma_\gamma\right]\right),(A_\alpha,B_\beta,\Gamma_\gamma)\le 3,(A_\alpha,B_\beta,\Gamma_\gamma)\in[0,1]\&[-1,0]\\ \left(\left[g_1A_\alpha,g_2A_\alpha,g_3A_\alpha,f_4A_\alpha\right],\left[g_1^{'}B_\beta,g_2^{'}B_\beta,g_3^{'}B_\beta,f_4^{'}B_\beta\right],\left[g_1^{"}\Gamma_\gamma,g_2^{"}\Gamma_\gamma,g_3^{"}\Gamma_\gamma,g_4^{"}\Gamma_\gamma\right]\right),(A_\alpha,B_\beta,\Gamma_\gamma)\le 3,(A_\alpha,B_\beta,\Gamma_\gamma)\in[0,1]\&[-1,0]\end{array}\right\}$$

(13)

$$\left.\begin{array}{ll}(P_1)^-(x,A_\alpha)=-\eta P_1(x,A_\alpha) & (P_2)^-(x,A_\alpha)=-\eta P_2(x,A_\alpha)\\ (P_1^{'})^-(x,B_\beta)=-\eta P_1^{'}(x,B_\beta) & (P_2^{'})^-(x,B_\beta)=-\eta P_2^{'}(x,B_\beta)\\ (P_1^{"})^-(x,\Gamma_\gamma)=-\eta P_1^{"}(x,\Gamma_\gamma) & (P_2^{"})^-(x,\Gamma_\gamma)=-\eta P_2^{"}(x,\Gamma_\gamma)\end{array}\right\}$$

(14)

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$$\left.\begin{array}{ll} (P_1)^+(x,\mathrm{A}_\alpha)=-\eta\, P_1(x,\mathrm{A}_\alpha) & (P_2)^+(x,\mathrm{A}_\alpha)=-\eta P_2(x,\mathrm{A}_\alpha) \\ \left(P_1^{'}\right)^+(x,\mathrm{B}_\beta)=-\eta P_1^{'}(x,\mathrm{B}_\beta) & \left(P_2^{'}\right)^+(x,\mathrm{B}_\beta)=-\eta P_2^{'}(x,\mathrm{B}_\beta) \\ \left(P_1^{"}\right)^+(x,\Gamma_\gamma)=-\eta P_3^{"}(x,\Gamma_\gamma) & \left(P_2^{"}\right)^+(x,\Gamma_\gamma)=-\eta P_2^{"}(x,\Gamma_\gamma) \end{array}\right\} \qquad (15)$$

In the first row of equation (13)

i.e.,

$$\left(\left[e_1\mathrm{A}_\alpha,e_2\mathrm{A}_\alpha,e_3\mathrm{A}_\alpha,e_4\mathrm{A}_\alpha\right],\left[e_1^{'}\mathrm{B}_\beta,e_2^{'}\mathrm{B}_\beta,e_3^{'}\mathrm{B}_\beta,e_4^{'}\mathrm{B}_\beta\right],\left[e_1^{"}\Gamma_\gamma,e_2^{"}\Gamma_\gamma,e_3^{"}\Gamma_\gamma,e_4^{"}\Gamma_\gamma\right]\right),\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\le 3,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\in[0,1]\&[-1,0]$$

Apply initial conditions of equation (14) and equation (15) in the first row of equation (13) we get the below equations for both negative and positive interval :

with initial condition of negative membership function

$$\left.\begin{array}{ll} (P_1)^-(x_0,\mathrm{A}_\alpha)=e_1(\mathrm{A}_\alpha) & (P_2)^-(x_0,\mathrm{A}_\alpha)=e_2(\mathrm{A}_\alpha) \\ \left(P_1^{'}\right)^-(x_0,\mathrm{B}_\beta)=e_1^{'}(\mathrm{B}_\beta) & \left(P_2^{'}\right)^-(x_0,\mathrm{B}_\beta)=e_2^{'}(\mathrm{B}_\beta) \\ \left(P_1^{"}\right)^-(x_0,\Gamma_\gamma)=e_1^{"}(\Gamma_\gamma) & \left(P_2^{"}\right)^-(x_0,\Gamma_\gamma)=e_2^{"}(\Gamma_\gamma) \end{array}\right\} \qquad (16)$$

with initial condition of positive membership function

$$\left.\begin{array}{ll} (P_1)^+(x_0,\mathrm{A}_\alpha)=e_3(\mathrm{A}_\alpha) & (P_2)^+(x_0,\mathrm{A}_\alpha)=e_4(\mathrm{A}_\alpha) \\ \left(P_1^{'}\right)^+(x_0,\mathrm{B}_\beta)=e_3^{'}(\mathrm{B}_\beta) & \left(P_2^{'}\right)^+(x_0,\mathrm{B}_\beta)=e_4^{'}(\mathrm{B}_\beta) \\ \left(P_1^{"}\right)^+(x_0,\Gamma_\gamma)=e_3^{"}(\Gamma_\gamma) & \left(P_2^{"}\right)^+(x_0,\Gamma_\gamma)=e_4^{"}(\Gamma_\gamma) \end{array}\right\} \qquad (17)$$

In the second row of equation (13)

i.e.,

$$\left(\left[f_1\mathrm{A}_\alpha,f_2\mathrm{A}_\alpha,f_3\mathrm{A}_\alpha,f_4\mathrm{A}_\alpha\right],\left[f_1^{'}\mathrm{B}_\beta,f_2^{'}\mathrm{B}_\beta,f_3^{'}\mathrm{B}_\beta,f_4^{'}\mathrm{B}_\beta\right],\left[f_1^{"}\Gamma_\gamma,f_2^{"}\Gamma_\gamma,f_3^{"}\Gamma_\gamma,f_4^{"}\Gamma_\gamma\right]\right),\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\le 3,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\in[0,1]\&[-1,0]$$

Apply initial conditions of equation (14) and equation (15) in the second row of equation (13) we get the below equations for both negative and positive interval:

with initial condition of negative membership function

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$$\left.\begin{array}{ll}(P_1)^-(x_0,\mathrm{A}_\alpha)=f_1(\mathrm{A}_\alpha) & (P_2)^-(x_0,\mathrm{A}_\alpha)=f_2(\mathrm{A}_\alpha)\\ (P_1^{'})^-(x_0,\mathrm{B}_\beta)=f_1^{'}(\mathrm{B}_\beta) & (P_2^{'})^-(x_0,\mathrm{B}_\beta)=f_2^{'}(\mathrm{B}_\beta)\\ (P_1^{''})^-(x_0,\Gamma_\gamma)=f_1^{''}(\Gamma_\gamma) & (P_2^{''})^-(x_0,\Gamma_\gamma)=f_2^{''}(\Gamma_\gamma)\end{array}\right\} \quad (18)$$

with initial condition of positive membership function

$$\left.\begin{array}{ll}(P_1)^+(x_0,\mathrm{A}_\alpha)=f_3(\mathrm{A}_\alpha) & (P_2)^+(x_0,\mathrm{A}_\alpha)=f_4(\mathrm{A}_\alpha)\\ (P_1^{'})^+(x_0,\mathrm{B}_\beta)=f_3^{'}(\beta) & (P_2^{'})^+(x_0,\mathrm{B}_\beta)=f_4^{'}(\mathrm{B}_\beta)\\ (P_1^{''})^+(x_0,\Gamma_\gamma)=f_3^{''}(\Gamma_\gamma) & (P_2^{''})^+(x_0,\Gamma_\gamma)=f_4^{''}(\Gamma_\gamma)\end{array}\right\} \quad (19)$$

In the third row of equation (13)

i.e.,

$$\left(\left[g_1\mathrm{A}_\alpha,g_2\mathrm{A}_\alpha,g_3\mathrm{A}_\alpha,g_4\mathrm{A}_\alpha\right],\left[g_1^{'}\mathrm{B}_\beta,g_2^{'}\mathrm{B}_\beta,g_3^{'}\mathrm{B}_\beta,g_4^{'}\mathrm{B}_\beta\right],\left[g_1^{''}\Gamma_\gamma,g_2^{''}\Gamma_\gamma,g_3^{''}\Gamma_\gamma,g_4^{''}\Gamma_\gamma\right]\right),\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\le 3,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\in\left[-1,0\right]\&\left[0,1\right]$$

Apply initial conditions of equation (14) and equation (15) in the second row of equation (13) we get the below equations for both negative and positive interval:

with initial condition of negative membership function

$$\left.\begin{array}{ll}(P_1)^-(x_0,\mathrm{A}_\alpha)=g_1(\mathrm{A}_\alpha) & (P_2)^-(x_0,\mathrm{A}_\alpha)=g_2(\mathrm{A}_\alpha)\\ (P_1^{'})^-(x_0,\mathrm{B}_\beta)=g_1^{'}(\mathrm{B}_\beta) & (P_2^{'})^-(x_0,\mathrm{B}_\beta)=g_2^{'}(\mathrm{B}_\beta)\\ (P_1^{''})^-(x_0,\Gamma_\gamma)=g_1^{''}(\Gamma_\gamma) & (P_2^{''})^-(x_0,\Gamma_\gamma)=g_2^{''}(\Gamma_\gamma)\end{array}\right\} \quad (20)$$

with initial condition of positive membership function

$$\left.\begin{array}{ll}(P_1)^+(x_0,\mathrm{A}_\alpha)=g_3(\mathrm{A}_\alpha) & (P_2)^+(x_0,\alpha)=g_3(\mathrm{A}_\alpha)\\ (P_1^{'})^+(x_0,\mathrm{B}_\beta)=g_3^{'}(\mathrm{B}_\beta) & (P_2^{'})^+(x_0,\mathrm{B}_\beta)=g_4^{'}(\mathrm{B}_\beta)\\ (P_1^{''})^+(x_0,\Gamma_\gamma)=g_3^{''}(\Gamma_\gamma) & (P_2^{''})^+(x_0,\Gamma_\gamma)=g_4^{''}(\Gamma_\gamma)\end{array}\right\} \quad (21)$$

Then the solution obtained for the first row from the above equations (16) and (17) for both negative and positive interval as follows:

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$$(P_1)^-\left(x,\left(A_\alpha,B_\beta,\Gamma_\gamma\right)\right)=\left\{\begin{array}{l}\left(\frac{e_1\left(A_\alpha,B_\beta,\Gamma_\gamma\right)-e_2\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\+\left(\frac{e_1\left(A_\alpha,B_\beta,\Gamma_\gamma\right)+e_2\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\\+\left(\frac{e_3\left(A_\alpha,B_\beta,\Gamma_\gamma\right)-e_4\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\+\left(\frac{e_3\left(A_\alpha,B_\beta,\Gamma_\gamma\right)+e_4\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\end{array}\right\}$$

$$(P_1)^+\left(x,\left(A_\alpha,B_\beta,\Gamma_\gamma\right)\right)=\left\{\begin{array}{l}\left(\frac{e_1\left(A_\alpha,B_\beta,\Gamma_\gamma\right)+e_2\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\+\left(\frac{e_1\left(A_\alpha,B_\beta,\Gamma_\gamma\right)-e_2\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\\+\left(\frac{e_3\left(A_\alpha,B_\beta,\Gamma_\gamma\right)+e_4\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\+\left(\frac{e_3\left(A_\alpha,B_\beta,\Gamma_\gamma\right)-e_4\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\end{array}\right\}$$

$$(P_2)^-\left(x,\left(A_\alpha,B_\beta,\Gamma_\gamma\right)\right)=\left\{\begin{array}{l}\left(\frac{e_1\left(A_\alpha,B_\beta,\Gamma_\gamma\right)-e_2\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\-\left(\frac{e_1\left(A_\alpha,B_\beta,\Gamma_\gamma\right)+e_2\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\\+\left(\frac{e_3\left(A_\alpha,B_\beta,\Gamma_\gamma\right)-e_4\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\-\left(\frac{e_3\left(A_\alpha,B_\beta,\Gamma_\gamma\right)+e_4\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\end{array}\right\}$$

$$(P_2)^+\left(x,\left(A_\alpha,B_\beta,\Gamma_\gamma\right)\right)=\left\{\begin{array}{l}\left(\frac{e_1\left(A_\alpha,B_\beta,\Gamma_\gamma\right)+e_2\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\-\left(\frac{e_1\left(A_\alpha,B_\beta,\Gamma_\gamma\right)-e_2\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\\+\left(\frac{e_3\left(A_\alpha,B_\beta,\Gamma_\gamma\right)+e_4\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\-\left(\frac{e_3\left(A_\alpha,B_\beta,\Gamma_\gamma\right)-e_4\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\end{array}\right\}$$

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$$\left(P_1^{'}\right)^{+}\left(x,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\right)=\begin{Bmatrix}\left(\frac{e_1^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+e_2^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\+\left(\frac{e_1^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-e_2^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\\+\left(\frac{e_3^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+e_4^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\+\left(\frac{e_3^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-e_4^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\end{Bmatrix}$$

$$\left(P_1^{'}\right)^{-}\left(x,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\right)=\begin{Bmatrix}\left(\frac{e_1^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-e_2^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\+\left(\frac{e_1^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+e_2^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\\+\left(\frac{e_3^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-e_4^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\+\left(\frac{e_3^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+e_4^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\end{Bmatrix}$$

$$\left(P_2^{'}\right)^{+}\left(x,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\right)=\begin{Bmatrix}\left(\frac{e_1^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+e_2^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\-\left(\frac{e_1^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-e_2^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\\+\left(\frac{e_3^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+e_4^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\-\left(\frac{e_3^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-e_4^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\end{Bmatrix}$$

$$\left(P_2^{'}\right)^{-}\left(x,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\right)=\begin{Bmatrix}\left(\frac{e_1^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-e_2^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\-\left(\frac{e_1^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+e_2^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\\+\left(\frac{e_3^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-e_4^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\-\left(\frac{e_3^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+e_4^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\end{Bmatrix}$$

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$$\left(P_1^{"}\right)^{-}\left(x,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\right)=\begin{Bmatrix}\left(\dfrac{e_1^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+e_2^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\+\left(\dfrac{e_1^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-e_2^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\\+\left(\dfrac{e_3^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+e_4^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\+\left(\dfrac{e_3^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-e_4^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\end{Bmatrix}$$

$$\left(P_1^{"}\right)^{+}\left(x,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\right)=\begin{Bmatrix}\left(\dfrac{e_1^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-e_2^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\+\left(\dfrac{e_1^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+e_2^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\\+\left(\dfrac{e_3^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-e_4^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\+\left(\dfrac{e_3^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+e_4^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\end{Bmatrix}$$

$$\left(P_2^{"}\right)^{-}\left(x,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\right)=\begin{Bmatrix}\left(\dfrac{e_1^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-e_2^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\-\left(\dfrac{e_1^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+e_2^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\\+\left(\dfrac{e_3^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-e_4^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\-\left(\dfrac{e_3^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+e_4^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\end{Bmatrix}$$

$$\left(P_2^{"}\right)^{+}\left(x,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\right)=\begin{Bmatrix}\left(\dfrac{e_1^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+e_2^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\-\left(\dfrac{e_1^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-e_2^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\\+\left(\dfrac{e_3^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+e_4^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\-\left(\dfrac{e_3^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-e_4^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\end{Bmatrix}$$

Then the solution obtained for the second row from the above equations (18) and (19) for both negative and positive interval as follows:

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$$(P_1)^-\left(x,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\right)=\begin{Bmatrix}\left(\frac{f_1\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-f_2\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\+\left(\frac{f_1\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+f_2\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\\+\left(\frac{f_3\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-f_4\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\+\left(\frac{f_3\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+f_4\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\end{Bmatrix}$$

$$(P_1)^+\left(x,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\right)=\begin{Bmatrix}\left(\frac{f_1\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+f_2\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\+\left(\frac{f_1\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-f_2\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\\+\left(\frac{f_3\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+f_4\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\+\left(\frac{f_3\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-f_4\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\end{Bmatrix}$$

$$(P_2)^-\left(x,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\right)=\begin{Bmatrix}\left(\frac{f_1\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-f_2\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\-\left(\frac{f_1\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+f_2\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\\+\left(\frac{f_3\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-f_4\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\-\left(\frac{f_3\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+f_4\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\end{Bmatrix}$$

$$(P_2)^+\left(x,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\right)=\begin{Bmatrix}\left(\frac{f_1\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+f_2\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\-\left(\frac{f_1\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-f_2\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\\+\left(\frac{f_3\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+f_4\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\-\left(\frac{f_3\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-f_4\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\end{Bmatrix}$$

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$$\left(P_1^{'}\right)^{-}\left(x,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\right)=\left\{\begin{array}{l}\left(\dfrac{f_1^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-f_2^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\+\left(\dfrac{f_1^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+f_2^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\\+\left(\dfrac{f_3^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-f_4^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\+\left(\dfrac{f_3^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+f_4^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\end{array}\right\}$$

$$\left(P_1^{'}\right)^{+}\left(x,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\right)=\left\{\begin{array}{l}\left(\dfrac{f_1^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+f_2^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\+\left(\dfrac{f_1^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-f_2^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\\+\left(\dfrac{f_3^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+f_4^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\+\left(\dfrac{f_3^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-f_4^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\end{array}\right\}$$

$$\left(P_2^{'}\right)^{+}\left(x,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\right)=\left\{\begin{array}{l}\left(\dfrac{f_1^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+f_2^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\-\left(\dfrac{f_1^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-f_2^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\\+\left(\dfrac{f_3^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+f_4^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\-\left(\dfrac{f_3^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-f_4^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\end{array}\right\}$$

$$\left(P_2^{'}\right)^{-}\left(x,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\right)=\left\{\begin{array}{l}\left(\dfrac{f_1^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-f_2^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\-\left(\dfrac{f_1^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+f_2^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\\+\left(\dfrac{f_3^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-f_4^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\-\left(\dfrac{f_3^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+f_4^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\end{array}\right\}$$

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$$\left(P_1''\right)^-\left(x,\left(A_\alpha,B_\beta,\Gamma_\gamma\right)\right)=\begin{Bmatrix}\left(\dfrac{f_1''\left(A_\alpha,B_\beta,\Gamma_\gamma\right)+f_2''\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\+\left(\dfrac{f_1''\left(A_\alpha,B_\beta,\Gamma_\gamma\right)-f_2''\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\\+\left(\dfrac{f_3''\left(A_\alpha,B_\beta,\Gamma_\gamma\right)+f_4''\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\+\left(\dfrac{f_3''\left(A_\alpha,B_\beta,\Gamma_\gamma\right)-f_4''\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\end{Bmatrix}$$

$$\left(P_1''\right)^+\left(x,\left(A_\alpha,B_\beta,\Gamma_\gamma\right)\right)=\begin{Bmatrix}\left(\dfrac{f_1''\left(A_\alpha,B_\beta,\Gamma_\gamma\right)-f_2''\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\+\left(\dfrac{f_1''\left(A_\alpha,B_\beta,\Gamma_\gamma\right)+f_2''\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\\+\left(\dfrac{f_3''\left(A_\alpha,B_\beta,\Gamma_\gamma\right)-f_4''\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\+\left(\dfrac{f_3''\left(A_\alpha,B_\beta,\Gamma_\gamma\right)+f_4''\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\end{Bmatrix}$$

$$\left(P_2''\right)^-\left(x,\left(A_\alpha,B_\beta,\Gamma_\gamma\right)\right)=\begin{Bmatrix}\left(\dfrac{f_1''\left(A_\alpha,B_\beta,\Gamma_\gamma\right)-f_2''\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\-\left(\dfrac{f_1''\left(A_\alpha,B_\beta,\Gamma_\gamma\right)+f_2''\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\\+\left(\dfrac{f_3''\left(A_\alpha,B_\beta,\Gamma_\gamma\right)-f_4''\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\-\left(\dfrac{f_3''\left(A_\alpha,B_\beta,\Gamma_\gamma\right)+f_4''\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\end{Bmatrix}$$

Then the solution obtained for the third row from the above equations (20) and (21) for both negative and positive interval as follows:

$$\left(P_1\right)^-\left(x,\left(A_\alpha,B_\beta,\Gamma_\gamma\right)\right)=\begin{Bmatrix}\left(\dfrac{g_1\left(A_\alpha,B_\beta,\Gamma_\gamma\right)-g_2\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\+\left(\dfrac{g_1\left(A_\alpha,B_\beta,\Gamma_\gamma\right)+g_2\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\\+\left(\dfrac{g_3\left(A_\alpha,B_\beta,\Gamma_\gamma\right)-g_4\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\+\left(\dfrac{g_3\left(A_\alpha,B_\beta,\Gamma_\gamma\right)+g_4\left(A_\alpha,B_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\end{Bmatrix}$$

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$$(P_1)^+\left(x,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\right)=\begin{Bmatrix}\left(\dfrac{g_1\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+g_2\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\+\left(\dfrac{g_1\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-g_2\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\\+\left(\dfrac{g_3\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+g_4\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\+\left(\dfrac{g_3\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-g_4\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\end{Bmatrix}$$

$$(P_2)^-\left(x,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\right)=\begin{Bmatrix}\left(\dfrac{g_1\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-g_2\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\-\left(\dfrac{g_1\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+g_2\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\\+\left(\dfrac{g_3\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-g_4\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\-\left(\dfrac{g_3\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+g_4\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\end{Bmatrix}$$

$$(P_2)^+\left(x,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\right)=\begin{Bmatrix}\left(\dfrac{g_1\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+g_2\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\-\left(\dfrac{g_1\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-g_2\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\\+\left(\dfrac{g_3\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+g_4\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\-\left(\dfrac{g_3\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-g_4\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\end{Bmatrix}$$

$$(P_1')^-\left(x,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\right)=\begin{Bmatrix}\left(\dfrac{g_1'\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-g_2'\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\+\left(\dfrac{g_1'\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+g_2'\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\\+\left(\dfrac{g_3'\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-g_4'\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta(x-x_0)}\\+\left(\dfrac{g_3'\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+g_4'\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta(x-x_0)}\end{Bmatrix}$$

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$$\left(P_1^{'}\right)^{+}\left(x,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\right)=\begin{cases}\left(\dfrac{g_1^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+g_2^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\+\left(\dfrac{g_1^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-g_2^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\\+\left(\dfrac{g_3^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+g_4^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\+\left(\dfrac{g_3^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-g_4^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\end{cases}$$

$$\left(P_2^{'}\right)^{+}\left(x,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\right)=\begin{cases}\left(\dfrac{g_1^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+g_2^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\-\left(\dfrac{g_1^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-g_2^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\\+\left(\dfrac{g_3^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+g_4^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\-\left(\dfrac{g_3^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-g_4^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\end{cases}$$

$$\left(P_2^{'}\right)^{-}\left(x,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\right)=\begin{cases}\left(\dfrac{g_1^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-g_2^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\-\left(\dfrac{g_1^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+g_2^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\\+\left(\dfrac{g_3^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-g_4^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\-\left(\dfrac{g_3^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+g_4^{'}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\end{cases}$$

$$\left(P_1^{''}\right)^{+}\left(x,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\right)=\begin{cases}\left(\dfrac{g_1^{''}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-g_2^{''}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\+\left(\dfrac{g_1^{''}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+g_2^{''}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\\+\left(\dfrac{g_3^{''}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-g_4^{''}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\+\left(\dfrac{g_3^{''}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+g_4^{''}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\end{cases}$$

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$$\left(P_1^{"}\right)^{-}\left(x,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\right)=\left\{\begin{array}{l}\left(\dfrac{g_1^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+g_2^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\+\left(\dfrac{g_1^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-g_2^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\\+\left(\dfrac{g_3^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+g_4^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\+\left(\dfrac{g_3^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-g_4^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\end{array}\right\}$$

$$\left(P_2^{"}\right)^{-}\left(x,\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)\right)=\left\{\begin{array}{l}\left(\dfrac{g_1^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-g_2^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\-\left(\dfrac{g_1^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+g_2^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\\+\left(\dfrac{g_3^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)-g_4^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{-\eta\left(x-x_0\right)}\\-\left(\dfrac{g_3^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)+g_4^{"}\left(\mathrm{A}_\alpha,\mathrm{B}_\beta,\Gamma_\gamma\right)}{2}\right)e^{\eta\left(x-x_0\right)}\end{array}\right\}$$

7. Predicting bacterial reproduction rate using proposed BTpNDE

In this section proposed concept is applied to predict the reproduction rate of bacteria over the separate bodies and the problem is taken from [9].A group of bacteria derived from the same mother cell in a wealthy-environment and how the single parent cell divides to produce multiple copies that are genetically identical to produce clusters of bacteria to form a bacterial colony is examined. Generally the bacterial colonies arise through the binary fission. The bacteria take up the nutrients and then initiate the growth and multiplication of the microorganism. In this environment, the population of the bacteria replicating through binary fission. The rate of change in which population increases directly proportional to the number of bacteria. And hence the concept of differential equation applied under neutrosophic environment by considering the initial conditions as a neutrosophic number.

7.1. Predicting bacterial reproduction rate using proposed BTpNDE

The rate at which this population grows is proportional to the number of bacteria present. This rule can be expressed in the form of a differential equation: $\frac{dP}{dt}=\eta P$ If the initial population is a BTpNNs $\left(\left(-3,-4,-5,-6;3,4,5,6\right);\left(0.8,0.2,0.3\right)\right)$. The population of bacteria after two days can be obtained as follows: (The constant proportional $\eta=\frac{1}{3}$).

Substitute $\eta=\frac{1}{3}\Rightarrow\frac{dP}{dt}=\frac{1}{3}P\quad where\quad P\left(0\right)=\left(\left(-3,-4,-5,-6;3,4,5,6\right);0.8,0.2,0.3\right).$

$$\Rightarrow\frac{dP}{dt}=\frac{1}{3}P\quad where\quad P\left(0\right)=\left[\left(-3,-4,-5,-6;3,4,5,6\right);\left(0.8,0.2,0.3\right)\right].$$

The obtained solution from the equation (4) and equation (13) in the format of equation (2) is as follows.

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$$\left.\begin{array}{ll} P_1^+(t,A_\alpha)=(3.2+0.8A_\alpha)e^{\frac{t}{3}} & P_2^+(t,A_\alpha)=(4.8-0.8A_\alpha)e^{\frac{t}{3}} \\ P_1^-(t,A_\alpha)=(-3.2+0.8A_\alpha)e^{\frac{t}{3}} & P_2^-(t,A_\alpha)=(-4.8-0.8A_\alpha)e^{\frac{t}{3}} \end{array}\right\} \quad (22)$$

$$\left.\begin{array}{ll} (P_1^{'})^+(t,B_\beta)=(0.8-0.2B_\beta)e^{\frac{t}{3}} & (P_2^{'})^+(t,B_\beta)=(1+0.2B_\beta)e^{\frac{t}{3}} \\ (P_1^{'})^-(t,B_\beta)=(-0.8-0.2B_\beta)e^{\frac{t}{3}} & (P_2^{'})^-(t,B_\beta)=(-1+0.2B_\beta)e^{\frac{t}{3}} \end{array}\right\} \quad (23)$$

$$\left.\begin{array}{ll} (P_1^{''})^+(t,\Gamma_\gamma)=(1.2-0.3\Gamma_\gamma)e^{\frac{t}{3}} & (P_2^{''})^+(t,\Gamma_\gamma)=(1.5+0.3\Gamma_\gamma)e^{\frac{t}{3}} \\ (P_1^{''})^-(t,\Gamma_\gamma)=(-1.2-0.3\Gamma_\gamma)e^{\frac{t}{3}} & (P_2^{''})^-(t,\Gamma_\gamma)=(-1.5+0.3\Gamma_\gamma)e^{\frac{t}{3}} \end{array}\right\} \quad (24)$$

Here,

$$\frac{dP_1^+}{dA_\alpha}=0.8e^{\frac{t}{3}}>0, \quad \frac{dP_2^+}{dA_\alpha}=-0.8e^{\frac{t}{3}}<0, \quad \left(\frac{dP_1^{'}}{dB_\beta}\right)^+=-0.2e^{\frac{t}{3}}<0, \quad \left(\frac{dP_2^{'}}{dB_\beta}\right)^+=0.2e^{\frac{t}{3}}>0,$$

$$\frac{dP_1^-}{dA_\alpha}=0.8e^{\frac{t}{3}}>0, \quad \frac{dP_2^-}{dA_\alpha}=-0.8e^{\frac{t}{3}}<0, \quad \left(\frac{dP_1^{'}}{dB_\beta}\right)^-=-0.3e^{\frac{t}{3}}<0, \quad \left(\frac{dP_2^{'}}{dB_\beta}\right)^-=0.3e^{\frac{t}{3}}>0,$$

$$\left(\frac{dP_1^{''}}{d\Gamma_\gamma}\right)^+=-0.3e^{\frac{t}{3}}<0, \quad \left(\frac{dP_2^{''}}{d\Gamma_\gamma}\right)^+=0.3e^{\frac{t}{3}}>0,$$

$$\left(\frac{dP_1^{''}}{d\Gamma_\gamma}\right)^-=-0.3e^{\frac{t}{3}}<0, \quad \left(\frac{dP_2^{''}}{d\Gamma_\gamma}\right)^-=0.3e^{\frac{t}{3}}>0,$$

Subsequently the arrangement is solid. The answer for $t=2$ and various qualities for $(A_\alpha, B_\beta, \Gamma_\gamma)$ is given in the accompanying table:

Table 1: solution for different values of A_α using equation (22)

A_α	$P_1^+(t,A_\alpha)$	$P_2^+(t,A_\alpha)$	$P_1^-(t,A_\alpha)$	$P_2^-(t,A_\alpha)$
0	6.2327	9.3491	-6.2327	-9.3491
0.1	6.3886	9.1933	-6.0769	-9.5049
0.2	6.5443	9.0375	-5.9211	-9.6608
0.3	6.7002	8.8817	-5.7653	-9.8166
0.4	6.8560	8.7258	-5.6094	-9.9724
0.5	7.0118	8.5700	-5.4537	-10.128
0.6	7.1677	8.4142	-5.2978	-10.284

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0.7	7.3235	8.2584	-5.1420	-10.4399
0.8	7.4793	8.1026	-4.9862	-10.5957
0.9	7.6351	7.9468	-4.8304	-10.7515
1.0	7.7909	7.7909	-4.6746	-10.9073

Table 2: solution for different values of B_β using equation (23)

Γ_γ	$\left(P_1^{''}\right)^+(t,\Gamma_\gamma)$	$\left(P_2^{''}\right)^+(t,\Gamma_\gamma)$	$\left(P_1^{''}\right)^-(t,\Gamma_\gamma)$	$\left(P_1^{''}\right)^-(t,\Gamma_\gamma)$
0	2.3373	2.9216	-2.373	-2.9216
0.1	2.2788	2.9800	-2.3957	-2.8632
0.2	2.2204	3.0385	-2.4541	-2.8047
0.3	2.1620	3.0969	-2.5126	-2.7463
0.4	2.1036	3.1553	-2.5710	-2.6879
0.5	2.0451	3.2138	-2.6294	-2.6294
0.6	1.9867	3.2722	-2.6879	-2.5710
0.7	1.9283	3.3306	-2.7463	-2.5126
0.8	1.8698	3.3891	-2.8047	-2.4541
0.9	1.8114	3.4475	-2.8632	-2.3945
1.0	1.7530	3.5059	-2.9216	-2.3373

Table 3:solution for different values of Γ_γ using equation (24)

B_β	$\left(P_1^{'}\right)^+(t,B_\beta)$	$\left(P_2^{'}\right)^+(t,\beta)$	$\left(P_1^{'}\right)^-(t,B_\beta)$	$\left(P_2^{'}\right)^-(t,B_\beta)$
0	1.5582	1.9477	-1.5582	-1.9477
0.1	1.5192	1.9867	-1.5971	-1.9088
0.2	1.4803	2.0256	-1.6371	-1.8698
0.3	1.4413	2.0646	-1.6751	-1.8309
0.4	1.4024	2.1036	-1.7140	-1.7919
0.5	1.3634	2.1425	-1.7530	-1.7530
0.6	1.3245	2.1815	-1.7919	-1.7140
0.7	1.2855	2.2204	-1.8309	-1.6751
0.8	1.2465	2.2594	-1.8698	-1.6361
0.9	1.2076	2.2583	-1.9088	-1.5971
1.0	1.1686	2.3373	-1.9477	-1.5582

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The above table 22, table 23 and table 24 represents the different values of $\left(\mathrm{A}_{\alpha},\mathrm{B}_{\beta},\Gamma_{\gamma}\right)$ values at $t=2$ in both positive and negative interval.

The graphical representation of table 1, table 2 and table 3 are shown in figure 1, figure 2 and figure 3 respectively.

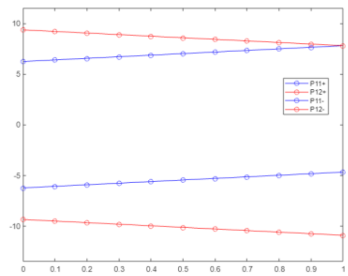

Figure 1: Graphical representation of equation (22) for different values of A_{α}

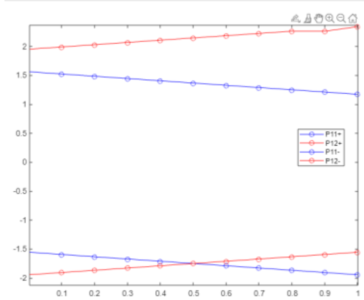

Figure 2: Graphical representation of equation (23) for different values of B_{β}

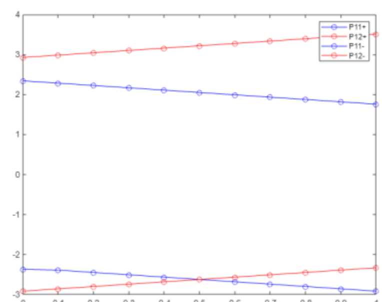

Figure 3: Graphical representation of equation (24) for different values of Γ_{γ}

The above figure 1, figure 2 and figure 3 graphically represents the different values of $\left(\mathrm{A}_{\alpha},\mathrm{B}_{\beta},\Gamma_{\gamma}\right)$ values at $t=2$ in both positive and negative interval.

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8. Comparative Analysis

In this section, comparative analysis of the proposed method with the existing method

Table 4:comparative analysis with the existing method

Methods	Neutrosophic numbers	Solution of the Problem	Conditions for solution become strong
Triangular Method [9]	(3,4,5;0.8,0.2,0.3)	$P_1(t,A_\alpha)=(2.4+0.8\alpha)e^{\frac{t}{3}}$ $P_2(t,A_\alpha)=(4-0.8\alpha)e^{\frac{t}{3}}$ $P_1(t,B_\beta)=(0.8-0.2\beta)e^{\frac{t}{3}}$ $P_2(t,B_\beta)=(0.8+0.2\beta)e^{\frac{t}{3}}$ $P_1(t,\Gamma_\gamma)=(1.2-0.3\gamma)e^{\frac{t}{3}}$ $P_2(t,\Gamma_\gamma)=(1.2+0.3\gamma)e^{\frac{t}{3}}$	$\frac{dP_1}{dA_\alpha}=0.8e^{\frac{t}{3}}>0,$ $\frac{dP_2}{dA_\alpha}=-0.8e^{\frac{t}{3}}<0,$ $\frac{dP_1^{'}}{dB_\beta}=-0.2e^{\frac{t}{3}}<0,$ $\frac{dP_2^{'}}{dB_\beta}=0.2e^{\frac{t}{3}}>0,$ $\frac{dP_1^{''}}{d\Gamma_\gamma}=-0.3e^{\frac{t}{3}}<0,$ $\frac{dP_2^{''}}{d\Gamma_\gamma}=0.3e^{\frac{t}{3}}>0,$
Bipolar Trapezoidal Method	((-3,-4,-5,-6;3,4,5,6);0.8,0.2,0.3)	$P_1^+(t,A_\alpha)=(3.2+0.8A_\alpha)e^{\frac{t}{3}}$ $P_2^+(t,A_\alpha)=(4.8-0.8A_\alpha)e^{\frac{t}{3}}$ $P_1^-(t,A_\alpha)=(-3.2+0.8A_\alpha)e^{\frac{t}{3}}$ $P_2^-(t,A_\alpha)=(-4.8-0.8A_\alpha)e^{\frac{t}{3}}$ $(P_1^{'})^+(t,B_\beta)=(0.8-0.2B_\beta)e^{\frac{t}{3}}$ $(P_2^{'})^+(t,B_\beta)=(1+0.2B_\beta)e^{\frac{t}{3}}$ $(P_1^{'})^-(t,B_\beta)=(-0.8-0.2B_\beta)e^{\frac{t}{3}}$ $(P_2^{'})^-(t,B_\beta)=(-1+0.2B_\beta)e^{\frac{t}{3}}$ $(P_1^{''})^+(t,\Gamma_\gamma)=(1.2-0.3\Gamma_\gamma)e^{\frac{t}{3}}$ $(P_2^{''})^+(t,\Gamma_\gamma)=(1.5+0.3\Gamma_\gamma)e^{\frac{t}{3}}$ $(P_1^{''})^-(t,\Gamma_\gamma)=(-1.2-0.3\Gamma_\gamma)e^{\frac{t}{3}}$ $(P_2^{''})^-(t,\Gamma_\gamma)=(-1.5+0.3\Gamma_\gamma)e^{\frac{t}{3}}$	$\frac{dP_1^+}{dA_\alpha}=0.8e^{\frac{t}{3}}>0,$ $\frac{dP_2^+}{dA_\alpha}=-0.8e^{\frac{t}{3}}<0,$ $\frac{dP_1^-}{dA_\alpha}=0.8e^{\frac{t}{3}}>0,$ $\frac{dP_2^-}{dA_\alpha}=-0.8e^{\frac{t}{3}}<0,$ $\left(\frac{dP_1^{'}}{dB_\beta}\right)^+=-0.2e^{\frac{t}{3}}<0,$ $\left(\frac{dP_2^{'}}{dB_\beta}\right)^+=0.2e^{\frac{t}{3}}>0,$ $\left(\frac{dP_1^{'}}{dB_\beta}\right)^-=-0.3e^{\frac{t}{3}}<0,$ $\left(\frac{dP_2^{'}}{dB_\beta}\right)^-=0.3e^{\frac{t}{3}}>0,$ $\left(\frac{dP_1^{''}}{d\Gamma_\gamma}\right)^+=-0.3e^{\frac{t}{3}}<0,$ $\left(\frac{dP_2^{''}}{d\Gamma_\gamma}\right)^+=0.3e^{\frac{t}{3}}>0,$ $\left(\frac{dP_1^{''}}{d\Gamma_\gamma}\right)^-=-0.3e^{\frac{t}{3}}<0,$ $\left(\frac{dP_2^{''}}{d\Gamma_\gamma}\right)^-=0.3e^{\frac{t}{3}}>0,$

Table 4, shows the effectiveness of using trapezoidal bipolar neutrosophic numbers with the existing method of triangular neutrosophic numbers. Triangular neutrosophic method shown the effectiveness of the solution only in the positive interval whereas bipolar trapezoidal neutrosophic method shown the effectiveness of the solution in both positive and negative interval. In both the method the solution become strong and satisfying the conditions for the derivatives.

9. Conclusion

Differential equations have an amazing ability to forecast what is going on in the world around us. They can be used to describe exponential growth and decay, species population expansion, or change rates through time. The Neutrosophic differential equation has been devised to address the rate of change of indeterminacy. In this research, a neutrosophic ordinary differential equation and derived its $(A_\alpha, B_\beta, \Gamma_\gamma)$-cut solution for bipolar trapezoidal neutrosophic number have been proposed. To test the viability and productivity of the proposed technique, the proposed concept has been applied to predict reproduction rate of bacteria using MATLAB with graphical portrayal. In addition, a comparative analysis has been done with existing methods to demonstrate the effectiveness of the proposed method with trapezoidal and bipolar trapezoidal neutrosophic environment. In future, this work can be extended to refined and other neutrosophic environments.

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