A Study of Neutrosophic Bernoulli's and Recati Differential Equations

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A Study of Neutrosophic Bernoulli's and Recati Differential Equations

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Abstract

In this paper, we study the neutrosophic Bernouli and Ricatti differential equations by using one dimensional AH-isometry. Also, we illustrate many examples to clarify the validity of our work.

Keywords: One-Dimensional Geometric AH-Isometry; Neutrosophi Ricatti Differential Equation.

1.Introduction

Neutrosophic logic. Neutrosophy, Neutrosophic set, Neutrosophic probability, are recently creations of Smarandache, being characterized by having the indeterminacy as component of their framework,and a notable feature of neutrosophic logic is that can be considered a generaliazationof fuzzy logics, encompassing the classical logic as well[1]. Also.F. Smarandache, has defined the concept of continuation of a neutrosophic function in year 2015 in[1],and neutrosophicmereo-limit[1], mereo-continuity.Moreover, in 2014, he has defined the concept of a neutro-oscillator differentialin [3], and mereo-derivative. Finallyin 2013 he introduced neutrosophic integration in [2], and mereo-integral, besides the classical defintions of limit, continuity, deverative, and integral respectively.
Among the recent applications there are: neutrosophic crisp set theory in image processing,neutrosophic setsmedical field [6-10], in information geographic systems and possible applications to database. Also,neutrosophic triplet group application to physics.Morever Several researches have made multiple contributions to neutrosophic topology and algebra [14-20, 34-50], Also More researches have made multiple contributions to neutrosophic analysis[21 − 33]. Finally the neutrosophic integration may have application in calculus the areas between two neutrosophic functions.

2. Preliminaries

Definition: Neutrosophic Real Number

Suppose that w is a neutrosophic number, then it takes the following standard form: $w = a + bI$ where a, b are real coefficients, and I represents the indeterminacy, where $0.I = 0$ and $I^n = I$ for all positive integers n.

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For example:

$w = 1 + 2I, w = 3 = 3 + 0I.$

Definition: Division of neutrosophic real numbers

Suppose that w_1, w_2 are two neutrosophic number,where

$$w_1 = a_1 + b_1 I, w_2 = a_2 + b_2 I$$

Then:

$$\frac{w_1}{w_2} = \frac{a_1 + b_1 I}{a_2 + b_2 I} = \frac{a_1}{a_2} + \frac{a_2 b_1 - a_1 b_2}{a_2(a_2 + b_2)} I$$

Definition :

Let $R(I) = \{a + bI\ ; a, b \in R\}$ where $I^2 = I$be the neutrosophic field of reals. The one-dimensional isometry (AH-Isometry) is defined as follows

$$T: R(I) \to R \times R$$
$$T(a + bI) = (a, a + b)$$

Remark :

T is an algebraic isomorphism between two rings, it has the following properties:

1) T is bijective.
2) T preserves addition and multiplication, i.e.:

$$T[(a + bI) + (c + dI)] = T(a + bI) + T(c + dI)$$

And

$$T[(a + bI) \cdot (c + dI)] = T(a + bI) \cdot T(c + dI)$$

3) Since T is bijective, then it is invertible by:

$$T^{-1}: R \times R \to R(I)$$
$$T^{-1}(a, b) = a + (b - a)I$$

4) T preserves distances, i.e.:

The distance on $R(I)$ can be defined as follows:

Let $A = a + bI, B = c + dI$ be two neutrosophic real numbers, then $L = \|\overrightarrow{AB}\| = d[(a + bI, c + dI)] = |a + bI - (c + dI)| = |(a - c) + I(b - d)| = |a - c| + I[|a + b - c - d| - |a - c|].$

On the other hand, we have:

$T\left(\|\overrightarrow{AB}\|\right) = (|a - c|, |(a + b) - (c + d)|) = \big(d(a, c), d(a + b, c + d)\big) = d[(a, a + b), (c, c + d)]$=$d\big(T(a + bI), T(c + dI)\big)$

=$\|T(\overrightarrow{AB})\|$.

This implies that the distance is preserved up to isometry. i.e.$\|T(AB)\| = T(\|AB\|)$

NeutrosophicBernoulli's equation.

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Inthis section is defined a Neutrosophic Bernoulli's equationby Using the One-Dimensional Geometric AH-Isometry and solutions are found for this equation.

Definition

Let $Y = y_1 + y_2 I$, $X = x_1 + x_2 I$We define the Neutrosophic Bernoulli's equationby Using the One-Dimensional Geometric AH-Isometry as form:

$$\acute{Y} + f(X)Y = g(X)Y^n$$

This equation can be written as follow:

$$\begin{aligned}(y_1' + I[(y_1 + y_2)' - y_1'])(f(x_1) + I[f(x_1 + x_2) - f(x_1)])(y_1 + y_2 I)\\ = (g(x_1) + I[g(x_1 + x_2) - g(x_1)])(y_1 + y_2 I)^n\end{aligned}$$

$$\begin{aligned}\Rightarrow y_1' + f(x_1)y_1 + I[(y_1 + y_2)' - f(x_1 + x_2)(y_1 + y_2) - (y_1' + f(x_1)y_1)]\\ = (g(x_1) + I[g(x_1 + x_2) - g(x_1)])((y_1)^n + I[(y_1 + y_2)^n - (y_1)^n])\end{aligned}$$

Method of solution.

1. Take AH-Isometry for the differential equation, we have.

$$\begin{aligned}T(y_1' + f(x_1)y_1 + I[(y_1 + y_2)' - f(x_1 + x_2)(y_1 + y_2) - (y_1' + f(x_1)y_1)])\\ = (g(x_1) + I[g(x_1 + x_2) - g(x_1)])((y_1)^n + I[(y_1 + y_2)^n - (y_1)^n])\end{aligned}$$

$$\begin{aligned}T(y_1' + f(x_1)y_1 + I[(y_1 + y_2)' - f(x_1 + x_2)(y_1 + y_2) - (y_1' + f(x_1)y_1)])\\ = T(g(x_1) + I[g(x_1 + x_2) - g(x_1)]).T((y_1)^n + I[(y_1 + y_2)^n - (y_1)^n])\end{aligned}$$

$$[y_1' + f(x_1)y_1, (y_1 + y_2)' - f(x_1 + x_2)(y_1 + y_2)] = [g(x_1), g(x_1 + x_2)].[(y_1)^n, (y_1 + y_2)^n]$$

Then.

$$\begin{cases} y_1' + f(x_1)y_1 = g(x_1)(y_1)^n \dots\dots(1) \\ (y_1 + y_2)' - f(x_1 + x_2)(y_1 + y_2) = g(x_1 + x_2)(y_1 + y_2)^n \dots\dots(2)\end{cases}$$

The equations $(1)$and $(2)$are two Bernoulli'sdifferential equation classical.

2. We find the solution to the equations classical(1) and (2), we have.

y_1 the solution to the equation (1).

$(y_1 + y_2)$ the solution to the equation (2).

3. We Take invertibleAH-Isometry, then, we have the solution of aNeutrosophic identical linear differential equation.

$$Y = y_1 + y_2 I = T^{-1}(y_1, y_1 + y_2) = y_1 + \big((y_1 + y_2) - y_1\big)I$$

Example Find a solution to the equation:

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$$\acute{Y}+\frac{1}{X}Y=XY^3$$

Solution.

Let $Y=y_1+y_2I, X=x_1+x_2I$. Then.

$$y_1'+\frac{1}{x_1}y_1+I\left[(y_1+y_2)'-\frac{1}{(x_1+x_2)}(y_1+y_2)-\left(y_1'+\frac{1}{x_1}y_1\right)\right]$$
$$=(x_1+I[(x_1+x_2)-(x_1)])((y_1)^3+I[(y_1+y_2)^3-(y_1)^3])$$

Now, Take AH-Isometry for the differential equation, we have.

$$\begin{cases} y_1'+\dfrac{1}{x_1}y_1=x_1(y_1)^3 \dots\dots (3) \\ (y_1+y_2)'-\dfrac{1}{x_1+x_2}(y_1+y_2)=(x_1+x_2)(y_1+y_2)^3 \dots\dots (4) \end{cases}$$

The solution of equation (3) written as follow:

$$y_1=\left\{\frac{1}{\mu(x_1)}\left(a+\int \mu(x_1)g(x_1)dx_1\right)\right\}^{\frac{1}{-n+1}}$$

$$y_1=\left\{\frac{1}{(x_1)^2}\left(a+\int -2(x_1)^3d(x_1)\right)\right\}^{\frac{-1}{2}}$$

$$y_1=\left\{\frac{1}{(x_1)^2}\left(a-\frac{(x_1)^4}{2}\right)\right\}^{\frac{-1}{2}}=\left\{\frac{a}{(x_1)^2}-\frac{(x_1)^2}{2}\right\}^{\frac{-1}{2}}, \text{ where } a\in R.$$

The solution of equation (4) written as follow:

$$(y_1+y_2)=\left\{\frac{1}{(x_1+x_2)^2}\left(b-\frac{(x_1+x_2)^4}{2}\right)\right\}^{\frac{-1}{2}}=\left\{\frac{b}{(x_1+x_2)^2}-\frac{(x_1+x_2)^2}{2}\right\}^{\frac{-1}{2}}, \text{ where } b\in R$$

Now, Take invertibleAH-Isometry, then, we have the solution of aNeutrosophic identical linear differential equation .

$$Y=y_1+y_2I=T^{-1}\left(\left\{\frac{a}{(x_1)^2}-\frac{(x_1)^2}{2}\right\}^{\frac{-1}{2}},\left\{\frac{b}{(x_1+x_2)^2}-\frac{(x_1+x_2)^2}{2}\right\}^{\frac{-1}{2}}\right)$$

$$Y=y_1+y_2I=\left\{\frac{a}{(x_1)^2}-\frac{(x_1)^2}{2}\right\}^{\frac{-1}{2}}+\left(\left\{\frac{b}{(x_1+x_2)^2}-\frac{(x_1+x_2)^2}{2}\right\}^{\frac{-1}{2}}-\left\{\frac{a}{(x_1)^2}-\frac{(x_1)^2}{2}\right\}^{\frac{-1}{2}}\right)I$$

By Definition 2.7, we have.

$$Y=y_1+y_2I=\left\{\frac{a}{(x_1)^2}-\frac{(x_1)^2}{2}+\left(\frac{b}{(x_1+x_2)^2}-\frac{(x_1+x_2)^2}{2}\right)I\right\}^{\frac{-1}{2}}$$

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$$Y = y_1 + y_2 I = \left\{\frac{a}{(x_1)^2} + \left(\frac{b}{(x_1+x_2)^2}\right) I - \frac{(x_1)^2}{2} - \left(\frac{(x_1+x_2)^2}{2}\right) I\right\}^{\frac{-1}{2}}$$

$$Y = y_1 + y_2 I = \left\{\frac{a}{(x_1)^2} + \left(\frac{b}{(x_1+x_2)^2}\right) I - \left[\frac{(x_1)^2}{2} + \left(\frac{(x_1+x_2)^2}{2}\right)\right] I\right\}^{\frac{-1}{2}}$$

By Definition 2.5, we have.

$$\frac{a}{(x_1)^2} + \left(\frac{b}{(x_1+x_2)^2}\right) I = \frac{a}{(x_1)^2} + \frac{b}{(x_1+x_2)^2} I = \frac{a+bI}{(x_1+x_2 I)^2}$$

Then.

$$Y = y_1 + y_2 I = \left\{\frac{a+bI}{(x_1+x_2 I)^2} + \frac{-(x_1+x_2 I)^2}{2}\right\}^{\frac{-1}{2}} = \left\{\frac{a+bI}{X^2} + \frac{-X^2}{2}\right\}^{\frac{-1}{2}}$$

So that,

$$Y = y_1 + y_2 I = \left\{\frac{a+bI}{X^2} + \frac{-X^2}{2}\right\}^{\frac{-1}{2}}$$

where $a + bI \in R(I)$.

Example Find a solution to the equation:

$$\acute{Y} + tan(X)Y = sin(X)Y^2$$

Solution.

Let $Y = y_1 + y_2 I, X = x_1 + x_2 I$. Then.

$$y_1' + tan(x_1)y_1 + I[(y_1+y_2)' + tan(x_1+x_2)(y_1+y_2) - (y_1' + tan(x_1)y_1)]$$
$$= (sin(x_1) + I[sin(x_1+x_2) - sin(x_1)])((y_1)^2 + I[(y_1+y_2)^2 - (y_1)^2])$$

Now, Take AH-Isometry for the differential equation, we have.

$$\begin{cases} y_1' + tan(x_1)y_1 = sin(x_1)(y_1)^2 \ldots\ldots (5) \\ (y_1+y_2)' + tan(x_1+x_2)(y_1+y_2) = sin(x_1+x_2)(y_1+y_2)^2 \ldots\ldots (6) \end{cases}$$

The solution of equation $(5s)$ written as follow:

$$y_1 = \left\{\frac{1}{\mu(x_1)}\left(a + \int \mu(x_1) g(x_1) dx_1\right)\right\}^{\frac{1}{-n+1}}$$

$$y_1 = \left\{\frac{1}{cos(x_1)}\left(a + \int cos(x_1).sin(x_1)d(x_1)\right)\right\}^{-1}$$

$y_1 = \left\{\frac{1}{cos(x_1)}\left(a + \frac{1}{4}cos2(x_1)\right)\right\}^{-1} = \left\{\frac{a}{cos(x_1)} + \frac{1}{4}\frac{cos2(x_1)}{cos(x_1)}\right\}^{-1}$, where $a \in R$.

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The solution of equation (6) written as follow:

$(y_1 + y_2) = \left\{\frac{1}{cos(x_1+x_2)}\left(b + \frac{1}{4}cos2(x_1 + x_2)\right)\right\}^{-1} = \left\{\frac{b}{cos(x_1+x_2)} + \frac{1}{4}\frac{cos2(x_1+x_2)}{cos(x_1)}\right\}^{-1}$, where $b \in R$

Now, Take invertibleAH-Isometry, then, we have the solution of aNeutrosophic identical linear differential equation .

$$Y = y_1 + y_2 I = T^{-1}\left(\left\{\frac{a}{cos(x_1)} + \frac{1}{4}c\frac{cos2(x_1)}{cos(x_1)}\right\}^{-1}, \left\{\frac{b}{cos(x_1+x_2)} + \frac{1}{4}\frac{cos2(x_1+x_2)}{cos(x_1)}\right\}^{-1}\right)$$

$$Y = y_1 + y_2 I = \left\{\frac{a}{cos(x_1)} + \frac{1}{4}\frac{cos2(x_1)}{cos(x_1)}\right\}^{-1} + \left(\left\{\frac{b}{cos(x_1+x_2)} + \frac{1}{4}\frac{cos2(x_1+x_2)}{cos(x_1)}\right\}^{-1} - \left\{\frac{a}{cos(x_1)} + \frac{1}{4}\frac{cos2(x_1)}{cos(x_1)}\right\}^{-1}\right)I$$

we have.

$$Y = y_1 + y_2 I = \left\{\frac{a}{cos(x_1)} + \frac{1}{4}\frac{cos2(x_1)}{cos(x_1)} + \left(\frac{b}{cos(x_1+x_2)} + \frac{1}{4}\frac{cos2(x_1+x_2)}{cos(x_1)}\right)I\right\}^{-1}$$

$$Y = y_1 + y_2 I = \left\{\frac{a}{cos(x_1)} + \left(\frac{b}{cos(x_1+x_2)}\right)I + \frac{1}{4}\frac{cos2(x_1)}{cos(x_1)} + \left(\frac{1}{4}\frac{cos2(x_1+x_2)}{cos(x_1)}\right)I\right\}^{-1}$$

we have.

$$\frac{a}{cos(x_1)} + \left(\frac{b}{cos(x_1+x_2)}\right)I = \frac{a}{cos(x_1)} + \frac{b}{cos(x_1+x_2)}I = \frac{a+bI}{cos(x_1+x_2 I)}$$

Then.

$$Y = y_1 + y_2 I = \left\{\frac{a+bI}{cos(x_1+x_2 I)} + \frac{1}{4}\frac{cos2(x_1+x_2 I)}{cos(x_1+x_2 I)}\right\}^{-1} = \left\{\frac{a+bI}{cos(X)} + \frac{1}{4}\frac{cos2X}{cosX}\right\}^{-1}$$

So that,

$$Y = y_1 + y_2 I = \left\{\frac{a+bI}{cos(X)} + \frac{1}{4}\frac{cos2X}{cosX}\right\}^{-1}$$

where $a + bI \in R(I)$.

NeutrosophicRecati equation.

In this section is defined a NeutrosophicRecatiequationby Using the One-Dimensional Geometric AH-Isometry and solutions are found for this equation.

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Definition .

Let $Y = y_1 + y_2 I$, $X = x_1 + x_2 I$ We define the NeutrosophicRecati equationby Using the One-Dimensional Geometric AH-Isometry as form:

$$\acute{Y} + f(X)Y^2 + g(X)Y + h(X) = 0$$

And takes a particular solution:

$$Z = z_1 + z_2 I = r(X) = r(x_1 + x_2 I)$$

This equation can be written as follow:

$$\begin{aligned}&({y_1}' + I[(y_1 + y_2)' - {y_1}'])(f(x_1) + I[f(x_1 + x_2) - f(x_1)])(y_1 + y_2 I)^2 \\ &\quad + (g(x_1) + I[g(x_1 + x_2) - g(x_1)])(y_1 + y_2 I) + h(x_1) + I[h(x_1 + x_2) - h(x_1)] = 0\end{aligned}$$

$$\begin{aligned}&y_1' + f(x_1)(y_1)^2 + I[(y_1 + y_2)' - f(x_1 + x_2)(y_1 + y_2) - (y_1' + f(x_1)(y_1)^2)] \\ &\quad + (g(x_1) + I[g(x_1 + x_2) - g(x_1)])\big((y_1) + I[(y_1 + y_2) - (y_1)]\big) + h(x_1) \\ &\quad + I[h(x_1 + x_2) - h(x_1)] = 0\end{aligned}$$

And:

$$Z = z_1 + z_2 I = r(x_1 + x_2 I) = r(x_1) + I[r(x_1 + x_2) - r(x_1)]$$

Method of solution.

1. Take AH-Isometry for the differential equation, and Take AH-Isometry fora particular solution, we have.

$$\begin{aligned}&T\big(y_1' + f(x_1)(y_1)^2 + I[(y_1 + y_2)' - f(x_1 + x_2)(y_1 + y_2) - (y_1' + f(x_1)(y_1)^2)] \\ &\quad + (g(x_1) + I[g(x_1 + x_2) - g(x_1)])\big((y_1) + I[(y_1 + y_2) - (y_1)]\big) + h(x_1) \\ &\quad + I[h(x_1 + x_2) - h(x_1)]\big) = T(0)\end{aligned}$$

$$\begin{aligned}&[y_1' + f(x_1)(y_1)^2 + g(x_1)y_1 + h(x_1), (y_1 + y_2)' + f(x_1 + x_2)(y_1 + y_2)^2 + g(x_1 + x_2)(y_1 + y_2) + h(x_1 + x_2)] \\ &\quad = [0,0]\end{aligned}$$

Then.

$$\begin{cases} y_1' + f(x_1)(y_1)^2 + g(x_1)y_1 + h(x_1) = 0 \ldots\ldots(7) \\ (y_1 + y_2)' + f(x_1 + x_2)(y_1 + y_2)^2 + g(x_1 + x_2)(y_1 + y_2) + h(x_1 + x_2) = 0 \ldots\ldots(8) \end{cases}$$

And:

$$T(z_1 + z_2 I) = T(r(x_1) + I[r(x_1 + x_2) - r(x_1)])$$

$$[z_1, (z_1 + z_2)] = [r(x_1), r(x_1 + x_2)]$$

$$\begin{cases} z_1 = r(x_1) \ldots\ldots(9) \\ (z_1 + z_2) = r(x_1 + x_2) \ldots\ldots(10) \end{cases}$$

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The equations (7) and (8)are two Recati differential equation classical with two a particular solution(9) and (10).

2. We find the solution to the equations classical (7) and (8), we have.

y_1 the solution to the equation (7).

$(y_1 + y_2)$ the solution to the equation (8).

3. We Take invertibleAH-Isometry, then, we have the solution of aNeutrosophic identical linear differential equation .

$Y = y_1 + y_2 I = T^{-1}(y_1, y_1 + y_2) = y_1 + \big((y_1 + y_2) - y_1\big)I$

Example Find the general solution for the following neutrosophicricati equation:

$$\acute{Y} + \left(\frac{cosX}{1-sinX\ cosX}\right)Y^2 + \left(\frac{-1}{1-sinX\ cosX}\right)Y + \frac{sinX}{1-sinX\ cosX} = 0$$

If a particular solution is:

$$Z = z_1 + z_2 I = cosX$$

Solution.

Let $Y = y_1 + y_2 I, X = x_1 + x_2 I$. Then.

$$\begin{aligned} y_1' + \frac{cosx_1}{1 - sinx_1.cosx_1}(y_1)^2 \\ &+ I\left[(y_1 + y_2)' + \frac{cos(x_1 + x_2)}{1 - sin(x_1 + x_2).cos(x_1 + x_2)}(y_1 + y_2) - \left(y_1' + \frac{cosx_1}{1 - sinx_1.cosx_1}(y_1)^2\right)\right] \\ &+ \left(\frac{-1}{1 - sinx_1.cosx_1} + I\left[\frac{-1}{1 - sin(x_1 + x_2).cos(x_1 + x_2)} - \left(\frac{-1}{1 - sinx_1.cosx_1}\right)\right]\right)((y_1) \\ &+ I[(y_1 + y_2) - (y_1)]) + \frac{sinx_1}{1 - sinx_1.cosx_1} \\ &+ I\left[\frac{sin(x_1 + x_2)}{1 - sin(x_1 + x_2).cos(x_1 + x_2)} - \frac{sinx_1}{1 - sinx_1.cosx_1}\right] = 0 \end{aligned}$$

And:

$Z = z_1 + z_2 I = cosX = cosx_1 + I[cos(x_1 + x_2) - cosx_1]$

Now, Take AH-Isometry for the differential equation, and Take AH-Isometry fora particular solution, we have..

$$\begin{cases} y_1' + \dfrac{cosx_1}{1 - sinx_1.cosx_1}(y_1)^2 - \dfrac{1}{1 - sinx_1.cosx_1}y_1 + \dfrac{sinx_1}{1 - sinx_1.cosx_1} = 0 \dots\dots (11) \\ (y_1 + y_2)' + \dfrac{cos(x_1 + x_2)}{1 - sin(x_1 + x_2).cos(x_1 + x_2)}(y_1 + y_2)^2 - \dfrac{1}{1 - sin(x_1 + x_2).cos(x_1 + x_2)}(y_1 + y_2) \\ \qquad + \dfrac{sin(x_1 + x_2)}{1 - sin(x_1 + x_2).cos(x_1 + x_2)} = 0 \dots\dots (12) \end{cases}$$

And:

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$$\begin{cases} z_1 = cosx_1 \\ (z_1 + z_2) = cos(x_1 + x_2) \end{cases}$$

The solution of equation (11) written as follow:

$y_1 = cosx_1 + \left\{\frac{1}{1-sinx_1.cosx_1}(a + sinx_1)\right\}^{-1}$,where $a \in R$.

By the method same, The solution of equation (12) written as follow:

$(y_1 + y_2) = cos(x_1 + x_2) + \left\{\frac{1}{1-sin(x_1+x_2).cos(x_1+x_2)}\big(b + sin(x_1 + x_2)\big)\right\}^{-1}$, where $b \in R$

Now, Take invertibleAH-Isometry, then, we have the solution of aNeutrosophic identical linear differential equation .

$$Y = y_1 + y_2I = T^{-1}\left(cosx_1 + \left\{\frac{1}{1 - sinx_1.cosx_1}(a + sinx_1)\right\}^{-1}, cos(x_1 + x_2) + \left\{\frac{1}{1 - sin(x_1 + x_2).cos(x_1 + x_2)}\big(b + sin(x_1 + x_2)\big)\right\}^{-1}\right)$$

$$Y = y_1 + y_2I = cosx_1 + \left\{\frac{1}{1 - sinx_1.cosx_1}(a + sinx_1)\right\}^{-1} + I\left[cos(x_1 + x_2) + \left\{\frac{1}{1 - sin(x_1 + x_2).cos(x_1 + x_2)}\big(b + sin(x_1 + x_2)\big)\right\}^{-1} - \left(cosx_1 + \left\{\frac{1}{1 - sinx_1.cosx_1}(a + sinx_1)\right\}^{-1}\right)\right]$$

$$Y = y_1 + y_2I = cosx_1 + I(cos(x_1 + x_2) - cosx_1) + \left\{\frac{1}{1 - sinx_1.cosx_1}(a + sinx_1)\right\}^{-1} + I\left[\left\{\frac{1}{1 - sin(x_1 + x_2).cos(x_1 + x_2)}\big(b + sin(x_1 + x_2)\big)\right\}^{-1} - \left(\left\{\frac{1}{1 - sinx_1.cosx_1}(a + sinx_1)\right\}^{-1}\right)\right]$$

$$Y = y_1 + y_2I = cos(x_1 + x_2I) + \left\{\frac{a}{1 - sinx_1.cosx_1} + \frac{sinx_1}{1 - sinx_1.cosx_1}\right\}^{-1} + I\left[\left\{\frac{b}{1 - sin(x_1 + x_2).cos(x_1 + x_2)} + \frac{sin(x_1 + x_2)}{1 - sin(x_1 + x_2).cos(x_1 + x_2)}\right\}^{-1} - \left\{\frac{a}{1 - sinx_1.cosx_1} + \frac{sinx_1}{1 - sinx_1.cosx_1}\right\}^{-1}\right]$$

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$$Y = y_1 + y_2 I = cos(X) + \left\{\frac{a}{1 - sinx_1 . cosx_1} + \frac{sinx_1}{1 - sinx_1 . cosx_1}\right\}^{-1}$$
$$+ I\left[\left\{\frac{b}{1 - sin(x_1 + x_2). cos(x_1 + x_2)} + \frac{sin(x_1 + x_2)}{1 - sin(x_1 + x_2). cos(x_1 + x_2)}\right\}^{-1} - \left\{\frac{a}{1 - sinx_1 . cosx_1} + \frac{sinx_1}{1 - sinx_1 . cosx_1}\right\}^{-1}\right]$$

we have.

$$\left\{\frac{a}{1 - sinx_1 . cosx_1} + \frac{sinx_1}{1 - sinx_1 . cosx_1}\right\}^{-1}$$
$$+ I\left[\left\{\frac{b}{1 - sin(x_1 + x_2). cos(x_1 + x_2)} + \frac{sin(x_1 + x_2)}{1 - sin(x_1 + x_2). cos(x_1 + x_2)}\right\}^{-1} - \left\{\frac{a}{1 - sinx_1 . cosx_1} + \frac{sinx_1}{1 - sinx_1 . cosx_1}\right\}^{-1}\right]$$
$$= \left\{\frac{a}{1 - sinx_1 . cosx_1} + \frac{sinx_1}{1 - sinx_1 . cosx_1} + \left(\frac{b}{1 - sin(x_1 + x_2). cos(x_1 + x_2)} + \frac{sin(x_1 + x_2)}{1 - sin(x_1 + x_2). cos(x_1 + x_2)}\right) I\right\}^{-1}$$

$$\left\{\frac{a}{1 - sinx_1 . cosx_1} + \frac{sinx_1}{1 - sinx_1 . cosx_1} + \frac{b}{1 - sin(x_1 + x_2). cos(x_1 + x_2)} I + \frac{sin(x_1 + x_2)}{1 - sin(x_1 + x_2). cos(x_1 + x_2)} I\right\}^{-1}$$

=

$$\left\{\frac{a}{1 - sinx_1 . cosx_1} + \frac{b}{1 - sin(x_1 + x_2). cos(x_1 + x_2)} I + \frac{sinx_1}{1 - sinx_1 . cosx_1} + \frac{sin(x_1 + x_2)}{1 - sin(x_1 + x_2). cos(x_1 + x_2)} I\right\}^{-1}$$

By Definition 2.5, we have.

$$\frac{a}{1 - sinx_1 . cosx_1} + \frac{b}{1 - sin(x_1 + x_2). cos(x_1 + x_2)} I = \frac{a + bI}{1 - sin(x_1 + x_2 I). cos(x_1 + x_2 I)} = \frac{a + bI}{1 - sin(X). cos(X)}$$

And.

$$\frac{sinx_1}{1 - sinx_1 . cosx_1} + \frac{sin(x_1 + x_2)}{1 - sin(x_1 + x_2). cos(x_1 + x_2)} I = \frac{sin(x_1 + x_2 I)}{1 - sin(x_1 + x_2 I). cos(x_1 + x_2 I)} == \frac{sin(X)}{1 - sin(X). cos(X)}$$

So that,

$$Y = y_1 + y_2 I = cos(X) + \left\{\frac{a + bI}{1 - sin(X). cos(X)} + \frac{sin(X)}{1 - sin(X). cos(X)}\right\}^{-1}$$

where $a + bI \in R(I)$.

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