

Agriculture Production Decision Making using Generalized q-Rung Neutrosophic Soft Set Method

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Abstract

This paper introduces the generalized *q*-rung neutrosophic soft set (GqRNSSS) theory and its use to solve actual problems. We also define a few operations that make use of the GqRNSSS. The GqRNSSS is constructed by generalizing both the Pythagorean neutrosophic soft set (PyNSSS) and Pythagorean fuzzy soft set (PyFSS). We give a method for agricultural output that is based on the proposed similarity measure of GqRNSSS. If two GqRNSSS are compared, it can be determined whether or not a person produces good agricultural output. We support a strategy for dealing with the decision-making (DM) problem that makes use of the generalized *q*-rung soft set model. In this article, we discuss the application of a similarity measure between two GqRNSSS in agricultural output. Show how they can be successfully applied to challenges with uncertainty.

Keywords: GqRNSSS; PyFSS; decision making problem

1 Introduction

Following the fuzzy set (FS) demonstration by Zadeh, they have become incredibly popular in almost all scientific fields. This demonstrates that when decision making (DM) about ambiguous topics, decision makers should consider their membership grade (MG), according to.²⁶ The concept of an intuitionistic fuzzy set (IFS) proposed by Atanassov, is characterized by an MG and non membership grade (NMG) that satisfy the total of MG and NMG does not exceed 1.³ We might run into trouble when DM, though, if the total of MG and NMG for a certain attribute is greater than 1. Yager²⁴ formulation of the Pythagorean FS (PFS) concept. The constraint that it does not exceed 1 indicates that the square total of its MG and NMG has been extended from IFSs. The concept of image FSs is based on extensions of FSs and IFSs.⁴ In 2015, Cuong produced picture fuzzy set (PicFS). When dealing with human opinions that involve many types of responses, such as yes, abstain, no, and refuse, models based on PicFSs may be appropriate. Human voters can be categorized into four groups: those who support a candidate, those who abstain from voting, those who vote against a candidate, and those who refuse to cast a ballot. Voting is a good illustration. Decision makers can allocate MG, NMG, and reluctance grades over a larger area using these data. The creator of FS, Cuong et al., ⁵ used

https://doi.org/10.54216/IJNS.190112 Received: April 19, 2022 Accepted: August 13, 2022 positive MG (PMG), neutral MG (neuMG), and negative MG (NegMG) pointers, with the total of these three grades not exceeding 1. Finally, it delivers more benefits than IFS and PFS for a select applications.

Smarandache²² founded the neutosophy movement to address the problems of unclear and contradictory information because this collection has a number of application related challenges. Recently, a novel theory called neutrosophic set (NSS) has been proposed. The study of neutral cognition is known as neutosophy, and this neutrality is what separates FS from IFS. Smarandache is credited for creating NSS reasoning, 22 which has a range of [0,1], is another generalization of the FS and IFS. It has been proven that an NSS generalizes a FS and IVFS. Jansi et al. recently discussed the Pythagorean NSS.⁷

The theory of soft set (SS) was the main contribution of Molodtsov.¹² SSs better convey the objectivity and complexity of DM in real world circumstances as compared to other uncertain theories. The incorporation of SSs with other mathematical models is also a vital area for research. Maji proposed the ideas of fuzzy soft set (FSS)⁹ and intuitionistic FSS (IFSS).¹⁰ A variety of DM concerns are addressed using these two approaches, PicFSS.²⁵ In recent years, Peng²⁰ has extended FSS to include PyFSS. A group of MADM issues where the total of the MG and NMG is larger than 1, but the total of the squares is equal to or not greater than 1 were solved using this methodology. Majumdara discussed FSS in general,.¹¹ The concept of a generalized interval valued FSS was discussed by Shawkat et al.²¹ The possibility FSS is a unique idea with real world applications, according to Alkhazaleh et al.² After that, Karaaslan offered the justification for the possibility of NSSSs using a DM.⁶ This paper broadens the concept of the GSS model in order to parameterize the GqRNSSS. We will then develop a similarity measure based on this GSS model. After relations on GqRNSSSs are identified, their properties are investigated, and their relations are defined, a DM problem is handled as an application. Numerous scientists talked about practical applications based on the NSS. 1,8,23 Examined several ideal structures and their applications. 13-19

The Section 1 contains an introduction. Section 2 contains the NSS and the GSS. Section 3 presents a conceptualization of the GqRNSSS. Section 4 describes the GqRNSSS similarity measure method. In Section 5, it is suggested that a GqRNSSS model might be used to solve a agriculture production problem. The conclusion is given in section 6. Give some numerical examples if you are evaluating the GqRNSSS model.

2 Preliminaries

The Pythagorean NSS and generalized FSS, two well-known literary notions, are reviewed and a few new concepts are introduced in this section.

⁷ Let $\mathcal X$ be the universal, Pythagorean NSIV (PyNSIV) set P in $\mathcal X$ is $\overrightarrow{F} = \{c \overrightarrow{\zeta}_F(\varsigma), \overrightarrow{\eta}_F(\varsigma), \overrightarrow{\sigma}_F(\varsigma) \mid \varsigma \in \mathcal X\}$, where $\overrightarrow{\zeta}_F(\varsigma) = [\zeta_F^{\mathcal{L}}(\varsigma), \zeta_F^{\mathcal{T}}(\varsigma)]$ and $\overrightarrow{\eta}_F(\varsigma) = [\eta_F^{\mathcal{L}}(\varsigma), \eta_F^{\mathcal{T}}(\varsigma)]$ and $\overrightarrow{\sigma}_F(\varsigma) = [\sigma_F^{\mathcal{L}}(\varsigma), \sigma_F^{\mathcal{T}}(\varsigma)]$ represents the degree of truth membership (DTM), indeterminacy membership (DIM) and falsity membership (DFM) of F, respectively, $\overrightarrow{\zeta}_F : \mathcal{X} \to D[0,1], \overrightarrow{\eta}_F : \mathcal{X} \to D[0,1], \overrightarrow{\sigma}_F : \mathcal{X} \to D[0,1]$ and $0 \le (\overrightarrow{\zeta}_F(\varsigma))^2 + (\overrightarrow{\eta}_F(\varsigma))^2 + (\overrightarrow{\sigma}_F(\varsigma))^2 \le 2$ implies $0 \le (\zeta_F^{\mathcal{T}}(\varsigma))^2 + (\eta_F^{\mathcal{T}}(\varsigma))^2 + (\sigma_F^{\mathcal{T}}(\varsigma))^2 \le 2$. Hence, $\overrightarrow{F} = \left\langle [\zeta_F^{\mathcal{L}}, \zeta_F^{\mathcal{T}}], [\eta_F^{\mathcal{L}}, \eta_F^{\mathcal{T}}], [\sigma_F^{\mathcal{L}}, \sigma_F^{\mathcal{T}}] \right\rangle$ is called a Pythagorean NSIV number (PyNSIVN).

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<sup>7</sup> Given that \overrightarrow{\kappa_1} = \langle \zeta_{\overrightarrow{\kappa_1}}, \eta_{\overrightarrow{\kappa_1}}, \sigma_{\overrightarrow{\kappa_1}} \rangle, \overrightarrow{\kappa_2} = \langle \zeta_{\overrightarrow{\kappa_2}}, \eta_{\overrightarrow{\kappa_2}}, \sigma_{\overrightarrow{\kappa_2}} \rangle and \overrightarrow{\kappa_3} = \langle \zeta_{\overrightarrow{\kappa_3}}, \eta_{\overrightarrow{\kappa_3}}, \sigma_{\overrightarrow{\kappa_3}} \rangle are any three PyNSIVNs
over (\mathcal{X}, E). Prove that
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(i)
$$\overrightarrow{\kappa_1^c} = \langle \sigma_{\overrightarrow{\kappa_1}}, \eta_{\overrightarrow{\kappa_1}}, \zeta_{\overrightarrow{\kappa_1}} \rangle$$

(ii)
$$\overrightarrow{\kappa_1} \sqcup \overrightarrow{\kappa_2} = \left\langle \max(\zeta_{\overrightarrow{\kappa_1}}, \zeta_{\overrightarrow{\kappa_2}}), \min(\eta_{\overrightarrow{\kappa_1}}, \eta_{\overrightarrow{\kappa_2}}), \min(\sigma_{\overrightarrow{\kappa_1}}, \sigma_{\overrightarrow{\kappa_2}}) \right\rangle$$

(iii)
$$\overrightarrow{\kappa_1} \sqcap \overrightarrow{\kappa_2} = \left\langle \min(\zeta_{\overrightarrow{\kappa_1}}, \zeta_{\overrightarrow{\kappa_2}}), \min(\eta_{\overrightarrow{\kappa_1}}, \eta_{\overrightarrow{\kappa_2}}), \max(\sigma_{\overrightarrow{\kappa_1}}, \sigma_{\overrightarrow{\kappa_2}}) \right\rangle$$

(iv)
$$\overrightarrow{\kappa_1} \leq \overrightarrow{\kappa_2}$$
 if and only if $\zeta_{\overrightarrow{\kappa_1}} \leq \zeta_{\overrightarrow{\kappa_2}}$ and $\eta_{\overrightarrow{\kappa_1}} \leq \eta_{\overrightarrow{\kappa_2}}$ and $\sigma_{\overrightarrow{\kappa_1}} \succeq \sigma_{\overrightarrow{\kappa_2}}$ (v) $\overrightarrow{\kappa_1} = \overrightarrow{\kappa_2}$ if and only if $\zeta_{\overrightarrow{\kappa_1}} = \zeta_{\overrightarrow{\kappa_2}}$ and $\eta_{\overrightarrow{\kappa_1}} = \eta_{\overrightarrow{\kappa_2}}$ and $\sigma_{\overrightarrow{\kappa_1}} = \sigma_{\overrightarrow{\kappa_2}}$.

Let $\mathcal{X}=\{\varsigma_1,\varsigma_2,...,\varsigma_n\}$ and $\mathbb{E}=\{\epsilon_1,\epsilon_2,...,\epsilon_m\}$ be the universal and set of parameter, respectively and $(\mathcal{X}, \mathbb{E})$ is a soft universe. The mapping $\overrightarrow{\mathcal{I}}: \mathbb{E} \to D(I)^{\mathcal{X}}$ and $\overrightarrow{\xi}$ be an IVF subset of \mathbb{E} , i.e., $\overrightarrow{\xi}: \mathbb{E} \to I = D[0,1]$. Let $\overrightarrow{\mathcal{I}}_{\xi}: \mathbb{E} \to D(I)^{\mathcal{X}} \times D(I)$ is defined as $\overrightarrow{\mathcal{I}}_{\xi}(\epsilon) = \left[\overrightarrow{\mathcal{I}}(\epsilon)(\varsigma), \overrightarrow{\xi}(\epsilon)\right], \forall \varsigma \in \mathbb{E}$

 \mathcal{X} . Then $\overrightarrow{\mathcal{I}}_{\xi}$ is called as generalized IVF soft (GIVFS) set on $(\mathcal{X}, \mathbb{E})$. For each parameter ϵ_i , $\overrightarrow{\mathcal{I}}_{\xi}(\epsilon_i) = \left[\overrightarrow{\mathcal{I}}(\epsilon_i)(\varsigma), \overrightarrow{\xi}(\epsilon_i)\right], \forall \varsigma \in \mathcal{X}$ not simply the degree to which the constituent parts of \mathcal{X} in $\overrightarrow{\mathcal{I}}(\epsilon_i)$. Moreover the IV likelihood of such belongingness, which is given by $\overrightarrow{\xi}(\epsilon_i)$. We write $\overrightarrow{\mathcal{I}}_{\xi}(\epsilon_i)$ as $\overrightarrow{\mathcal{I}}_{\xi}(\epsilon_i) = \left[\left\{\frac{\varsigma_1}{\overrightarrow{\mathcal{I}}(\epsilon_i)(\varsigma_1)}, \frac{\varsigma_2}{\overrightarrow{\mathcal{I}}(\epsilon_i)(\varsigma_2)}, ..., \frac{\varsigma_n}{\overrightarrow{\mathcal{I}}(\epsilon_i)(\varsigma_n)}\right\}, \overrightarrow{\xi}(\epsilon_i)\right]$, where $\overrightarrow{\mathcal{I}}(\epsilon_i)(\varsigma_1), \overrightarrow{\mathcal{I}}(\epsilon_i)(\varsigma_2), ..., \overrightarrow{\mathcal{I}}(\epsilon_i)(\varsigma_n)$ are the degrees of belongingness and degree of such belongingness IV possibility is $\overrightarrow{\xi}(\epsilon_i)$.

Let $\mathcal{X} = \{\varsigma_1, \varsigma_2, ..., \varsigma_n\}$ and $\mathbb{E} = \{\epsilon_1, \epsilon_2, ..., \epsilon_m\}$ be the universe and set of parameter, respectively. The mapping $\overrightarrow{\mathcal{I}} : \mathbb{E} \to \overrightarrow{\mathcal{I}}(\mathcal{X})$ and $\overrightarrow{\xi}$ be an IVF subset of \mathbb{E} , i.e., $\overrightarrow{\xi} : \mathbb{E} \to \overrightarrow{\mathcal{I}}(\mathcal{X})$. Let $\overrightarrow{\mathcal{I}}_{\xi} : \mathbb{E} \to \overrightarrow{\mathcal{I}}(\mathcal{X}) \times \overrightarrow{\mathcal{I}}(\mathcal{X})$ is defined as $\overrightarrow{\mathcal{I}}_{\xi}(\epsilon) = (\overrightarrow{\mathcal{I}}(\epsilon)(\varsigma), \overrightarrow{\xi}(\epsilon)(\varsigma)), \forall \varsigma \in \mathcal{X}$. Then $\overrightarrow{\mathcal{I}}_{\xi}$ is called a possibility IVF soft (PIVFS) set on $(\mathcal{X}, \mathbb{E})$. For each parameter ϵ_i , $\overrightarrow{\mathcal{I}}_{\xi}(\epsilon_i) = (\overrightarrow{\mathcal{I}}(\epsilon_i)(\varsigma), \overrightarrow{\xi}(\epsilon_i)(\varsigma))$ not merely how much the components of \mathcal{X} belong together in $\overrightarrow{\mathcal{I}}(\epsilon_i)$, also the degree of IV possibility of such belongingness which is $\overrightarrow{\xi}(\epsilon_i)$. Hence, $\overrightarrow{\mathcal{I}}_{\xi}(\epsilon_i) = \left\{ \begin{bmatrix} \underline{\varsigma_1} \\ \overline{\mathcal{I}}(\epsilon_i)(\varsigma_1) \end{bmatrix}, \overrightarrow{\xi}(\epsilon_i)(\varsigma_1) \right\}, \begin{bmatrix} \underline{\varsigma_2} \\ \overline{\mathcal{I}}(\epsilon_i)(\varsigma_2) \end{bmatrix}, ..., \begin{bmatrix} \underline{\varsigma_n} \\ \overline{\mathcal{I}}(\epsilon_i)(\varsigma_n) \end{bmatrix}, \overrightarrow{\xi}(\epsilon_i)(\varsigma_n) \end{bmatrix} \right\}$.

3 Generalized q-rung neutrosophic soft set (GqRNSSS)

Let $\mathcal{X} = \{o_1, o_2, ..., o_n\}$ be the universe and $E = \{\Omega 1, \Omega 2, ..., \Omega m\}$ be the set of parameter. Suppose that $\overline{\mathcal{I}}: E \to \overline{S\mathcal{I}(\mathcal{X})}$ and f is a NS subset of \mathbb{E} . That is $f: \mathbb{E} \to \overline{[0,1]}$, where $\overline{S\mathcal{I}(\mathcal{X})}$ denotes the collection of all NS subsets of \mathcal{X} . If $\overline{\mathcal{I}_f}: \mathbb{E} \to \overline{S\mathcal{I}(\mathcal{X})} \times \overline{[0,1]}$ is defined as $\overline{\mathcal{I}_f(\epsilon)} = \left[\overline{\mathcal{I}(\epsilon)(x)}, \overline{f(\epsilon)}\right], x \in \mathcal{X}$, then $\overline{\mathcal{I}_f}$ is a

GqRNSSS on
$$(\mathcal{X}, \mathbb{E})$$
. For each parameter ϵ ,
$$\overline{\mathcal{I}_f(\Omega i)} = \left[\left\{ \frac{o_1}{(\Xi_{\mathcal{I}(\epsilon)}(o_1), \Lambda_{\mathcal{I}(\epsilon)}(o_1), \Omega_{\mathcal{I}(\epsilon)}(o_1))}, ..., \frac{o_n}{(\Xi_{\mathcal{I}(\epsilon)}(o_n), \Lambda_{\mathcal{I}(\epsilon)}(o_n), \Omega_{\mathcal{I}(\epsilon)}(o_n))} \right\}, (f_1(\Omega i), f_2(\Omega i), f_3(\Omega i)) \right].$$

To illustrate the Definition 3, we give the following numerical example:

Let $\mathcal{X} = \{o_1, o_2, o_3\}$ be the universal, $\mathbb{E} = \{\Omega 1, \Omega 2, \Omega 3\}$ is a set of parameters. Suppose that $\overline{\mathcal{I}_f} : \mathbb{E} \to \mathbb{E}$ $\overline{SI(\mathcal{X})} \times \overline{[0,1]}$ is given by

$$\overline{\mathcal{I}_{p}(\Omega 1)} = \left[\begin{cases} \frac{o_{1}}{(0.65, 0.30, 0.80)} \\ \frac{o_{2}}{(0.75, 0.35, 0.65)} \\ \frac{o_{3}}{(0.55, 0.45, 0.70)} \end{cases} \right\}, (0.70, 0.65, 0.35) \right]; \ \overline{\mathcal{I}_{p}(\Omega 2)} = \left[\begin{cases} \frac{o_{1}}{(0.55, 0.35, 0.80)} \\ \frac{o_{2}}{(0.65, 0.50, 0.70)} \\ \frac{o_{3}}{(0.60, 0.40, 0.80)} \end{cases} \right\}, (0.50, 0.40, 0.60) \right];$$

$$\overline{\mathcal{I}_{p}(\Omega 3)} = \left[\begin{cases} \frac{o_{1}}{(0.30, 0.55, 0.80)} \\ \frac{o_{2}}{(0.40, 0.65, 0.60)} \\ \frac{o_{3}}{(0.30, 0.45, 0.70)} \end{cases} \right\}, (0.60, 0.50, 0.60) \right].$$

Similarity measure between two GqRNSSSs

Finding a similarity measure between GqRNSSSs is provided below in this section.

Let $\mathcal{X} = \{o_1, o_2, ..., o_m\}$ be the universal and $\mathbb{E} = \{\Omega 1, \Omega 2, ..., \Omega n\}$ be the set of parameters. Suppose that $\overline{\mathcal{I}_f}$ and $\overline{\mathcal{J}_g}$ are two GqRNSSSs on $(\mathcal{X}, \mathbb{E})$. The similarity measure between two GqRNSSSs $\overline{\mathcal{I}_f}$ and $\overline{\mathcal{J}_g}$ is defined as $Sim(\overline{\mathcal{I}_f}, \overline{\mathcal{J}_g}) = \Phi(\overline{\mathcal{I}}, \overline{\mathcal{J}}) \cdot \Psi(f, g)$ such that $\Phi(\overline{\mathcal{I}}, \overline{\mathcal{J}}) = \frac{1}{m} \sum_{z=1}^m \min \left\{ T_1 \left[\overline{\mathcal{I}(\epsilon)(o_z)}, \overline{\mathcal{J}(\epsilon)(o_z)} \right], T_2 \left[\overline{\mathcal{I}(\epsilon)(o_z)}, \overline{\mathcal{J}(\epsilon)(o_z)} \right], S\left[\overline{\mathcal{I}(\epsilon)(o_z)}, \overline{\mathcal{J}(\epsilon)(o_z)} \right] \right\}$

$$\Phi(\overline{\mathcal{I}}, \overline{\mathcal{J}}) = \frac{1}{m} \sum_{z=1}^{m} \min \left\{ T_1 \left[\overline{\mathcal{I}(\epsilon)(o_z)}, \overline{\mathcal{J}(\epsilon)(o_z)} \right], T_2 \left[\overline{\mathcal{I}(\epsilon)(o_z)}, \overline{\mathcal{J}(\epsilon)(o_z)} \right], S \left[\overline{\mathcal{I}(\epsilon)(o_z)}, \overline{\mathcal{J}(\epsilon)(o_z)} \right] \right\}$$

$$\begin{split} & \text{and } \Psi(f,g) = 1 - \frac{\displaystyle\sum_{y=1}^{n} |f(\Omega y) - g(\Omega y)|}{\displaystyle\sum_{y=1}^{n} |f(\Omega y) + g(\Omega y)|}, \text{ where} \\ & \displaystyle T_1\left[\overline{\mathcal{I}(\epsilon)(o_z)}, \overline{\mathcal{J}(\epsilon)(o_z)}\right] = \frac{\displaystyle\sum_{y=1}^{n} (\Xi_{\mathcal{I}(\Omega y)}(o_z) \, \cdot \, \Xi_{\mathcal{J}(\Omega y)}(o_z))}{\displaystyle\sum_{y=1}^{n} (1 - \sqrt[q]{(1 - \Xi_{\mathcal{I}(\Omega y)}^{q}(o_z)) \, \cdot \, (1 - \Xi_{\mathcal{J}(\Omega y)}^{q}(o_z))}}, \\ & \displaystyle T_2\left[\overline{\mathcal{I}(\epsilon)(o_z)}, \overline{\mathcal{J}(\epsilon)(o_z)}\right] = \frac{\displaystyle\sum_{y=1}^{n} (\Lambda_{\mathcal{I}(\Omega y)}^{q}(o_z) \, \cdot \, \Lambda_{\mathcal{J}(\Omega y)}^{q}(o_z)) \, \cdot \, (1 - \Lambda_{\mathcal{J}(\Omega y)}^{2q}(o_z))}{\displaystyle\sum_{y=1}^{n} (1 - \sqrt[q]{(1 - \Lambda_{\mathcal{I}(\Omega y)}^{2q}(o_z)) \, \cdot \, (1 - \Lambda_{\mathcal{J}(\Omega y)}^{2q}(o_z))})}, \\ & S\left[\overline{\mathcal{I}(\epsilon)(o_z)}, \overline{\mathcal{J}(\epsilon)(o_z)}\right] = \sqrt[q]{1 - \frac{\displaystyle\sum_{y=1}^{n} |\Omega_{\mathcal{I}(\Omega y)}^{q}(o_z) - \Omega_{\mathcal{J}(\Omega y)}^{q}(o_z)|}{\displaystyle\sum_{y=1}^{n} 1 + ((\Omega_{\mathcal{I}(\Omega y)}^{q}(o_z)) \, \cdot \, (\Omega_{\mathcal{J}(\Omega y)}^{q}(o_z)))}, \text{ for } z = 1, 2, ..., m. \end{split}$$

Let $\overline{\mathcal{I}_f}$, $\overline{\mathcal{J}_q}$ and $\overline{\mathcal{K}_h}$ be the any three GqRNSSSs over $(\mathcal{X}, \mathbb{E})$. Then the following statements hold:

(a) $Sim(\overline{\mathcal{I}_f}, \overline{\mathcal{J}_g}) = Sim(\overline{\mathcal{J}_g}, \overline{\mathcal{I}_f}),$ (b) $0 \leq Sim(\overline{\mathcal{I}_f}, \overline{\mathcal{J}_g}) \leq 1,$ (c) $\overline{\mathcal{I}_f} = \overline{\mathcal{J}_g} \Longrightarrow Sim(\overline{\mathcal{I}_f}, \overline{\mathcal{J}_g}) = 1,$ (d) $\overline{\mathcal{I}_f} \sqsubseteq \overline{\mathcal{J}_g} \sqsubseteq \overline{\mathcal{K}_h} \Longrightarrow Sim(\overline{\mathcal{I}_f}, \overline{\mathcal{K}_h}) \leq Sim(\overline{\mathcal{J}_g}, \overline{\mathcal{K}_h}),$ (e) $\overline{\mathcal{I}_f} \sqcap \overline{\mathcal{J}_g} = \{\varphi\} \Leftrightarrow Sim(\overline{\mathcal{I}_f}, \overline{\mathcal{J}_g}) = 0.$

Proof. The proof (a), (b) and (e) are trivial. (c) Suppose that $\overline{\mathcal{I}_f} = \overline{\mathcal{J}_g}$ implies that $\Xi_{\mathcal{I}(\Omega_y)}(o_z) = \Xi_{\mathcal{J}(\Omega_y)}(o_z)$, $\Lambda_{_{\mathcal{I}(\Omega y)}}(o_z) = \Lambda_{_{\mathcal{I}(\Omega y)}}(o_z), \, \Omega_{_{\mathcal{I}(\Omega y)}}(o_z) = \Omega_{_{\mathcal{I}(\Omega y)}}(o_z) \text{ and } f(\Omega y) = g(\Omega y).$

Now,
$$T_1\left[\overline{\mathcal{I}(\epsilon)(o_1)}, \overline{\mathcal{J}(\epsilon)(o_1)}\right] = \frac{\displaystyle\sum_{y=1}^n \Xi_{\mathcal{I}(\Omega y)}^q(o_1)}{\displaystyle\sum_{y=1}^n (1-1+\Xi_{\mathcal{I}(\Omega y)}^q(o_1))} = \frac{\displaystyle\sum_{y=1}^n \Xi_{\mathcal{I}(\Omega y)}^q(o_1)}{\displaystyle\sum_{y=1}^n (1-1+\Xi_{\mathcal{I}(\Omega y)}^q(o_1))} = \frac{\displaystyle\sum_{y=1}^n \Xi_{\mathcal{I}(\Omega y)}^q(o_1)}{\displaystyle\sum_{y=1}^n \Lambda_{\mathcal{I}(\Omega y)}^{2q}(o_1)} = 1$$
 and $T_2\left[\overline{\mathcal{I}(\epsilon)(o_1)}, \overline{\mathcal{J}(\epsilon)(o_1)}\right] = \frac{\displaystyle\sum_{y=1}^n \Lambda_{\mathcal{I}(\Omega y)}^{2q}(o_1)}{\displaystyle\sum_{y=1}^n (1-1+\Lambda_{\mathcal{I}(\Omega y)}^{2q}(o_1))} = \frac{\displaystyle\sum_{y=1}^n \Lambda_{\mathcal{I}(\Omega y)}^{2q}(o_1)}{\displaystyle\sum_{y=1}^n \Lambda_{\mathcal{I}(\Omega y)}^{2q}(o_1)} = 1$ and $S\left[\overline{\mathcal{I}(\epsilon)(o_1)}, \overline{\mathcal{J}(\epsilon)(o_1)}\right] = \sqrt[q]{(1-0)} = 1$. Now,min $\left\{T_1\left[\overline{\mathcal{I}(\epsilon)(o_1)}, \overline{\mathcal{J}(\epsilon)(o_1)}\right], T_2\left[\overline{\mathcal{I}(\epsilon)(o_1)}, \overline{\mathcal{J}(\epsilon)(o_1)}\right], S\left[\overline{\mathcal{I}(\epsilon)(o_1)}, \overline{\mathcal{J}(\epsilon)(o_1)}\right]\right\} = 1$. Also, if we replace o_1 by $\{o_2, o_2, \dots, o_m\}$, we get the sequence $\{1, 1, 1, \dots, 1(m-1) \}$.

Also, if we replace o_1 by $\{o_2,o_3,...,o_m\}$, we get the sequence $\{1,1,1,...,1(\mathsf{m}-1\ times)\}$. Thus, $\Phi(\overline{\mathcal{I}},\overline{\mathcal{J}}) = \frac{1}{m}\{1+1+1+...+1(\mathsf{m}\ times)\} = \frac{m}{m} = 1$ and $\Psi(f,g) = 1$.

Hence $Sim(\overline{\mathcal{I}_f}, \overline{\mathcal{J}_g}) = 1$.

(d) For z = 1, 2, ..., m

$$\left\{
\begin{array}{l}
\overline{\mathcal{I}_{f}} \sqsubseteq \overline{\mathcal{J}_{g}} \implies \Xi_{\mathcal{I}(\Omega y)}(o_{z}) \leq \Xi_{\mathcal{J}(\Omega y)}(o_{z}), \Lambda_{\mathcal{I}(\Omega y)}(o_{z}) \leq \Lambda_{\mathcal{J}(\Omega y)}(o_{z}), \\
\Omega_{\mathcal{I}(\Omega y)}(o_{z}) \succeq \Omega_{\mathcal{J}(\Omega y)}(o_{z}), f(\Omega y) \leq g(\Omega y), \\
\overline{\mathcal{I}_{f}} \sqsubseteq \overline{\mathcal{K}_{h}} \implies \Xi_{\mathcal{I}(\Omega y)}(o_{z}) \leq \Xi_{\mathcal{K}(\Omega y)}(o_{z}), \Lambda_{\mathcal{I}(\Omega y)}(o_{z}) \leq \Lambda_{\mathcal{K}(\Omega y)}(o_{z}), \\
\Omega_{\mathcal{I}(\Omega y)}(o_{z}) \succeq \Omega_{\mathcal{K}(\Omega y)}(o_{z}), f(\Omega y) \leq h(\Omega y), \\
\overline{\mathcal{J}_{g}} \sqsubseteq \overline{\mathcal{K}_{h}} \implies \Xi_{\mathcal{J}(\Omega y)}(o_{z}) \leq \Xi_{\mathcal{K}(\Omega y)}(o_{z}), \Lambda_{\mathcal{J}(\Omega y)}(o_{z}) \leq \Lambda_{\mathcal{K}(\Omega y)}(o_{z}), \\
\Omega_{\mathcal{J}(\Omega y)}(o_{z}) \succeq \Omega_{\mathcal{K}(\Omega y)}(o_{z}), g(\Omega y) \leq h(\Omega y)
\end{array}\right\}$$

$$(1)$$

Clearly, $\Xi_{\mathcal{I}(\Omega u)}(o_z) \cdot \Xi_{\mathcal{K}(\Omega u)}(o_z) \leq \Xi_{\mathcal{I}(\Omega u)}(o_z) \cdot \Xi_{\mathcal{K}(\Omega u)}(o_z)$ implies that

$$\sum_{y=1}^{n} (\Xi_{\mathcal{I}(\Omega_y)}(o_z) \cdot \Xi_{\mathcal{K}(\Omega_y)}(o_z)) \le \sum_{y=1}^{n} (\Xi_{\mathcal{I}(\Omega_y)}(o_z) \cdot \Xi_{\mathcal{K}(\Omega_y)}(o_z)), \text{ for } z = 1, 2, ..., m.$$
 (2)

Clearly,
$$\Xi^q_{\mathcal{I}(\Omega y)}(o_z) \leq \Xi^q_{\mathcal{I}(\Omega y)}(o_z)$$
 implies that $-\Xi^q_{\mathcal{I}(\Omega y)}(o_z) \succeq -\Xi^q_{\mathcal{I}(\Omega y)}(o_z)$ and $(1-(\Xi^q_{\mathcal{I}(\Omega y)}(o_z))) \cdot (1-(\Xi^q_{\mathcal{K}(\Omega y)}(o_z))) \succeq (1-(\Xi^q_{\mathcal{I}(\Omega y)}(o_z))) \cdot (1-(\Xi^q_{\mathcal{K}(\Omega y)}(o_z)))$ and

$$\sqrt[q]{(1-(\Xi^q_{\mathcal{I}(\Omega y)}(o_z)))\cdot(1-(\Xi^q_{\mathcal{K}(\Omega y)}(o_z)))}\succeq\sqrt[q]{(1-(\Xi^q_{\mathcal{I}(\Omega y)}(o_z)))\cdot(1-(\Xi^q_{\mathcal{K}(\Omega y)}(o_z)))} \text{ and } \\ 1-\sqrt[q]{(1-(\Xi^q_{\mathcal{I}(\Omega y)}(o_z)))\cdot(1-(\Xi^q_{\mathcal{K}(\Omega y)}(o_z)))}\leq 1-\sqrt[q]{(1-(\Xi^q_{\mathcal{I}(\Omega y)}(o_z)))\cdot(1-(\Xi^q_{\mathcal{K}(\Omega y)}(o_z)))} \text{ and } \\ 1-\sqrt[q]{(1-(\Xi^q_{\mathcal{I}(\Omega y)}(o_z)))\cdot(1-(\Xi^q_{\mathcal{K}(\Omega y)}(o_z)))} \leq 1-\sqrt[q]{(1-(\Xi^q_{\mathcal{I}(\Omega y)}(o_z)))\cdot(1-(\Xi^q_{\mathcal{K}(\Omega y)}(o_z)))} \text{ and } \\ 1-\sqrt[q]{(1-(\Xi^q_{\mathcal{I}(\Omega y)}(o_z)))\cdot(1-(\Xi^q_{\mathcal{K}(\Omega y)}(o_z)))} \leq 1-\sqrt[q]{(1-(\Xi^q_{\mathcal{I}(\Omega y)}(o_z)))\cdot(1-(\Xi^q_{\mathcal{K}(\Omega y)}(o_z)))} \text{ and } \\ 1-\sqrt[q]{(1-(\Xi^q_{\mathcal{I}(\Omega y)}(o_z)))\cdot(1-(\Xi^q_{\mathcal{K}(\Omega y)}(o_z)))} \leq 1-\sqrt[q]{(1-(\Xi^q_{\mathcal{I}(\Omega y)}(o_z)))\cdot(1-(\Xi^q_{\mathcal{K}(\Omega y)}(o_z)))} \text{ and } \\ 1-\sqrt[q]{(1-(\Xi^q_{\mathcal{I}(\Omega y)}(o_z)))\cdot(1-(\Xi^q_{\mathcal{K}(\Omega y)}(o_z)))} \leq 1-\sqrt[q]{(1-(\Xi^q_{\mathcal{I}(\Omega y)}(o_z)))\cdot(1-(\Xi^q_{\mathcal{K}(\Omega y)}(o_z)))} \text{ and } \\ 1-\sqrt[q]{(1-(\Xi^q_{\mathcal{I}(\Omega y)}(o_z)))\cdot(1-(\Xi^q_{\mathcal{K}(\Omega y)}(o_z)))} \leq 1-\sqrt[q]{(1-(\Xi^q_{\mathcal{I}(\Omega y)}(o_z)))\cdot(1-(\Xi^q_{\mathcal{K}(\Omega y)}(o_z)))}$$

$$\sum_{y=1}^{n} 1 - \sqrt[q]{\left(1 - \left(\Xi_{\mathcal{I}(\Omega y)}^{q}(o_{z})\right)\right) \cdot \left(1 - \left(\Xi_{\mathcal{K}(\Omega y)}^{q}(o_{z})\right)\right)} \leq \sum_{y=1}^{n} 1 - \sqrt[q]{\left(1 - \left(\Xi_{\mathcal{I}(\Omega y)}^{q}(o_{z})\right)\right) \cdot \left(1 - \left(\Xi_{\mathcal{K}(\Omega y)}^{q}(o_{z})\right)\right)}. \tag{3}$$

Equations (2) and (3), we get

$$\frac{\sum_{y=1}^{n} (\Xi_{\mathcal{I}(\Omega y)}(o_z) \cdot \Xi_{\mathcal{K}(\Omega y)}(o_z))}{\sum_{y=1}^{n} 1 - \sqrt[q]{(1 - (\Xi_{\mathcal{I}(\Omega y)}^{q}(o_z))) \cdot (1 - (\Xi_{\mathcal{K}(\Omega y)}^{q}(o_z)))}} \le \frac{\sum_{y=1}^{n} (\Xi_{\mathcal{J}(\Omega y)}(o_z) \cdot \Xi_{\mathcal{K}(\Omega y)}(o_z))}{\sum_{y=1}^{n} 1 - \sqrt[q]{(1 - (\Xi_{\mathcal{J}(\Omega y)}^{q}(o_z))) \cdot (1 - (\Xi_{\mathcal{K}(\Omega y)}^{q}(o_z)))}}.$$
(4)

Clearly, $\Lambda^q_{\mathcal{I}(\Omega u)}(o_z) \cdot \Lambda^q_{\mathcal{K}(\Omega u)}(o_z) \leq \Lambda^q_{\mathcal{I}(\Omega u)}(o_z) \cdot \Lambda^q_{\mathcal{K}(\Omega u)}(o_z)$ implies that

$$\sum_{y=1}^{n} (\Lambda_{\mathcal{I}(\Omega y)}^{q}(o_z) \cdot \Lambda_{\mathcal{K}(\Omega y)}^{q}(o_z)) \le \sum_{y=1}^{n} (\Lambda_{\mathcal{J}(\Omega y)}^{q}(o_z) \cdot \Lambda_{\mathcal{K}(\Omega y)}^{q}(o_z)), \text{ for } z = 1, 2, ..., m.$$
 (5)

Clearly,
$$\Lambda^{2q}_{_{\mathcal{I}(\Omega y)}}(o_z) \leq \Lambda^{2q}_{_{\mathcal{I}(\Omega y)}}(o_z)$$
 implies that $-\Lambda^{2q}_{_{\mathcal{I}(\Omega y)}}(o_z) \succeq -\Lambda^{2q}_{_{\mathcal{I}(\Omega y)}}(o_z)$ and $(1-(\Lambda^{2q}_{_{\mathcal{I}(\Omega y)}}(o_z))) \cdot (1-(\Lambda^{2q}_{_{\mathcal{K}(\Omega y)}}(o_z))) \succeq (1-(\Lambda^{2q}_{_{\mathcal{I}(\Omega y)}}(o_z))) \cdot (1-(\Lambda^{2q}_{_{\mathcal{K}(\Omega y)}}(o_z)))$ and

$$\sqrt[q]{\left(1-(\Lambda_{\mathcal{I}(\Omega y)}^{2q}(o_z))\right)\cdot\left(1-(\Lambda_{\mathcal{K}(\Omega y)}^{2q}(o_z))\right)}\succeq\sqrt[q]{\left(1-(\Lambda_{\mathcal{I}(\Omega y)}^{2q}(o_z))\right)\cdot\left(1-(\Lambda_{\mathcal{K}(\Omega y)}^{2q}(o_z))\right)}\text{ and }\\1-\sqrt[q]{\left(1-(\Lambda_{\mathcal{I}(\Omega y)}^{2q}(o_z))\right)\cdot\left(1-(\Lambda_{\mathcal{K}(\Omega y)}^{2q}(o_z))\right)}\leq1-\sqrt[q]{\left(1-(\Lambda_{\mathcal{I}(\Omega y)}^{2q}(o_z))\right)\cdot\left(1-(\Lambda_{\mathcal{K}(\Omega y)}^{2q}(o_z))\right)}\text{ and }$$

$$\sum_{y=1}^{n} 1 - \sqrt[q]{\left(1 - \left(\Lambda_{\mathcal{I}(\Omega y)}^{2q}(o_z)\right)\right) \cdot \left(1 - \left(\Lambda_{\mathcal{K}(\Omega y)}^{2q}(o_z)\right)\right)} \le \sum_{y=1}^{n} 1 - \sqrt[q]{\left(1 - \left(\Lambda_{\mathcal{I}(\Omega y)}^{2q}(o_z)\right)\right) \cdot \left(1 - \left(\Lambda_{\mathcal{K}(\Omega y)}^{2q}(o_z)\right)\right)}.$$
(6)

Equations (5) and (6), we get

$$\frac{\sum_{y=1}^{n} \left(\Lambda_{\mathcal{I}(\Omega y)}^{q}(o_z) \cdot \Lambda_{\mathcal{K}(\Omega y)}^{q}(o_z)\right)}{\sum_{y=1}^{n} 1 - \sqrt[q]{\left(1 - \left(\Lambda_{\mathcal{I}(\Omega y)}^{2q}(o_z)\right)\right) \cdot \left(1 - \left(\Lambda_{\mathcal{K}(\Omega y)}^{2q}(o_z)\right)\right)}} \le \frac{\sum_{y=1}^{n} \left(\Lambda_{\mathcal{J}(\Omega y)}^{q}(o_z) \cdot \Lambda_{\mathcal{K}(\Omega y)}^{q}(o_z)\right)}{\sum_{y=1}^{n} 1 - \sqrt[q]{\left(1 - \left(\Lambda_{\mathcal{J}(\Omega y)}^{2q}(o_z)\right)\right) \cdot \left(1 - \left(\Lambda_{\mathcal{K}(\Omega y)}^{2q}(o_z)\right)\right)}}.$$

Clearly, $\Omega^q_{\mathcal{I}(\Omega y)}(o_z) \succeq \Omega^q_{\mathcal{I}(\Omega y)}(o_z)$ and $\Omega^q_{\mathcal{I}(\Omega y)}(o_z) - \Omega^q_{\mathcal{K}(\Omega y)}(o_z) \succeq \Omega^q_{\mathcal{I}(\Omega y)}(o_z) - \Omega^q_{\mathcal{K}(\Omega y)}(o_z)$. Hence

$$\sum_{y=1}^{n} \left| \Omega_{\mathcal{I}(\Omega y)}^{q}(o_z) - \Omega_{\mathcal{K}(\Omega y)}^{q}(o_z) \right| \succeq \sum_{y=1}^{n} \left| \Omega_{\mathcal{I}(\Omega y)}^{q}(o_z) - \Omega_{\mathcal{K}(\Omega y)}^{q}(o_z) \right|. \tag{8}$$

Also, $\Omega^q_{\mathcal{I}(\Omega y)}(o_z) \cdot \Omega^q_{\mathcal{K}(\Omega y)}(o_z) \succeq \Omega^q_{\mathcal{J}(\Omega y)}(o_z) \cdot \Omega^q_{\mathcal{K}(\Omega y)}(o_z)$ implies that

$$\sum_{y=1}^{n} 1 + (\Omega_{\mathcal{I}(\Omega_y)}^q(o_z) \cdot \Omega_{\mathcal{K}(\Omega_y)}^q(o_z)) \succeq \sum_{y=1}^{n} 1 + (\Omega_{\mathcal{J}(\Omega_y)}^q(o_z) \cdot \Omega_{\mathcal{K}(\Omega_y)}^q(o_z)), \text{ for } z = 1, 2, ..., m.$$
 (9)

Equations (8) and (9), we get

$$\frac{\sum\limits_{y=1}^{n}|\Omega^{q}_{\mathcal{I}(\Omega y)}(o_z)-\Omega^{q}_{\mathcal{K}(\Omega y)}(o_z)|}{\sum\limits_{y=1}^{n}1+(\Omega^{q}_{\mathcal{I}(\Omega y)}(o_z)\cdot\Omega^{q}_{\mathcal{K}(\Omega y)}(o_z))}\succeq\frac{\sum\limits_{y=1}^{n}|\Omega^{q}_{\mathcal{I}(\Omega y)}(o_z)-\Omega^{q}_{\mathcal{K}(\Omega y)}(o_z)|}{\sum\limits_{y=1}^{n}1+(\Omega^{q}_{\mathcal{I}(\Omega y)}(o_z)\cdot\Omega^{q}_{\mathcal{K}(\Omega y)}(o_z))}\text{ and }$$

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$$1 - \frac{\displaystyle\sum_{y=1}^{n} |\Omega^q_{\mathcal{I}(\Omega y)}(o_z) - \Omega^q_{\mathcal{K}(\Omega y)}(o_z)|}{\displaystyle\sum_{y=1}^{n} 1 + (\Omega^q_{\mathcal{I}(\Omega y)}(o_z) \cdot \Omega^q_{\mathcal{K}(\Omega y)}(o_z))} \leq 1 - \frac{\displaystyle\sum_{y=1}^{n} |\Omega^q_{\mathcal{I}(\Omega y)}(o_z) - \Omega^q_{\mathcal{K}(\Omega y)}(o_z)|}{\displaystyle\sum_{y=1}^{n} 1 + (\Omega^q_{\mathcal{I}(\Omega y)}(o_z) \cdot \Omega^q_{\mathcal{K}(\Omega y)}(o_z))} \text{ and }$$

$$\sqrt{1 - \frac{\sum_{y=1}^{n} |\Omega_{\mathcal{I}(\Omega y)}^{q}(o_z) - \Omega_{\mathcal{K}(\Omega y)}^{q}(o_z)|}{\sum_{y=1}^{n} 1 + (\Omega_{\mathcal{I}(\Omega y)}^{q}(o_z) \cdot \Omega_{\mathcal{K}(\Omega y)}^{q}(o_z))}} \leq \sqrt{1 - \frac{\sum_{y=1}^{n} |\Omega_{\mathcal{I}(\Omega y)}^{q}(o_z) - \Omega_{\mathcal{K}(\Omega y)}^{q}(o_z)|}{\sum_{y=1}^{n} 1 + (\Omega_{\mathcal{I}(\Omega y)}^{q}(o_z) \cdot \Omega_{\mathcal{K}(\Omega y)}^{q}(o_z))}}.$$
(10)

Equations (4), (7) and (10), we get

$$\Phi(\overline{\mathcal{I}}, \overline{\mathcal{K}}) \le \Phi(\overline{\mathcal{J}}, \overline{\mathcal{K}}). \tag{11}$$

By Equation (1), clearly $f(\Omega y) \leq g(\Omega y) \leq h(\Omega y)$ and $f(\Omega y) - h(\Omega y) \leq g(\Omega y) - h(\Omega y)$. Since $f(\Omega y), g(\Omega y), h(\Omega y)$ are numerical values, we have $\Big|g(\Omega y) - h(\Omega y)\Big| \leq \Big|f(\Omega y) - h(\Omega y)\Big|$ and

$$-\left|f(\Omega y) - h(\Omega y)\right| \le -\left|g(\Omega y) - h(\Omega y)\right|$$
 and

$$-\sum_{y=1}^{n} \left| f(\Omega y) - h(\Omega y) \right| \le -\sum_{y=1}^{n} \left| g(\Omega y) - h(\Omega y) \right|. \tag{12}$$

Since $\left|f(\Omega y)+h(\Omega y)\right|\leq \left|g(\Omega y)+h(\Omega y)\right|$ implies that

$$\sum_{y=1}^{n} \left| f(\Omega y) + h(\Omega y) \right| \le \sum_{y=1}^{n} \left| g(\Omega y) + h(\Omega y) \right|. \tag{13}$$

Equations (12) and (13), we get

$$- \frac{\sum\limits_{y=1}^{n} \left| f(\Omega y) - h(\Omega y) \right|}{\sum\limits_{y=1}^{n} \left| f(\Omega y) + h(\Omega y) \right|} \leq - \frac{\sum\limits_{y=1}^{n} \left| g(\Omega y) - h(\Omega y) \right|}{\sum\limits_{y=1}^{n} \left| g(\Omega y) + h(\Omega y) \right|} \text{ implies}$$

$$1 - \frac{\sum_{y=1}^{n} |f(\Omega y) - h(\Omega y)|}{\sum_{y=1}^{n} |f(\Omega y) + h(\Omega y)|} \le 1 - \frac{\sum_{y=1}^{n} |g(\Omega y) - h(\Omega y)|}{\sum_{y=1}^{n} |g(\Omega y) + h(\Omega y)|}.$$

Thus,
$$1 - \frac{\sum\limits_{y=1}^{n} \left| f(\Omega y) - h(\Omega y) \right|}{\sum\limits_{y=1}^{n} \left| f(\Omega y) + h(\Omega y) \right|} \le 1 - \frac{\sum\limits_{y=1}^{n} \left| g(\Omega y) - h(\Omega y) \right|}{\sum\limits_{y=1}^{n} \left| g(\Omega y) + h(\Omega y) \right|}.$$

Hence.

$$\Psi(f,h) \le \Psi(g,h). \tag{14}$$

Equations (11) and (14), we get, $\Phi(\overline{\mathcal{I}}, \overline{\mathcal{K}}) \cdot \Psi(f, h) \leq \Phi(\overline{\mathcal{J}}, \overline{\mathcal{K}}) \cdot \Psi(g, h)$. Hence, $Sim(\overline{\mathcal{I}_f}, \overline{\mathcal{K}_h}) \leq Sim(\overline{\mathcal{J}_q}, \overline{\mathcal{K}_h})$.

Calculate the similarity measure between the two GqRNSSSs such as $\overline{\mathcal{I}_f}$ and $\overline{\mathcal{J}_g}$. We choose the universal set $\mathcal{X} = \{o_1, o_2\}$ and parameter $\mathbb{E} = \{\Omega 1, \Omega 2, \Omega 3, \Omega 4\}$ can be defined as

$$\begin{array}{c|ccccc} \overline{\mathcal{J}_g(\epsilon)} & \Omega 1 & \Omega 2 & \Omega 3 & \Omega 4 \\ \hline \overline{\mathcal{J}(\epsilon)(o_1)} & (0.45, 0.35, 0.55) & (0.5, 0.55, 0.50) & (0.60, 0.65, 0.35) & (0.50, 0.25, 0.40) \\ \hline \overline{\mathcal{J}(\epsilon)(o_2)} & (0.50, 0.45, 0.55) & (0.45, 0.35, 0.65) & (0.55, 0.15, 0.45) & (0.45, 0.45, 0.55) \\ g(\epsilon) & (0.35, 0.40, 0.50) & (0.45, 0.40, 0.30) & (0.55, 0.35, 0.30) & (0.45, 0.45, 0.50) \\ \hline \end{array}$$

Using Definition 4, we get
$$T_1\left[\overline{\mathcal{I}(\epsilon)(o_1)},\overline{\mathcal{J}(\epsilon)(o_1)}\right]=0.938723,\,T_2\left[\overline{\mathcal{I}(\epsilon)(o_1)},\overline{\mathcal{J}(\epsilon)(o_1)}\right]=0.650332$$
 and $S\left[\overline{\mathcal{I}(\epsilon)(o_1)},\overline{\mathcal{J}(\epsilon)(o_1)}\right]=0.920783.$ Now, $\min\left\{T_1\left[\overline{\mathcal{I}(\epsilon)(o_1)},\overline{\mathcal{J}(\epsilon)(o_1)}\right],T_2\left[\overline{\mathcal{I}(\epsilon)(o_1)},\overline{\mathcal{J}(\epsilon)(o_1)}\right],S\left[\overline{\mathcal{I}(\epsilon)(o_1)},\overline{\mathcal{J}(\epsilon)(o_1)}\right]\right\}=0.650332$ and $\Psi(f,g)=1-\frac{1.3}{9.6}=0.864583.$ Similarly, $T_1\left[\overline{\mathcal{I}(\epsilon)(o_2)},\overline{\mathcal{J}(\epsilon)(o_2)}\right]=0.870205,\,T_2\left[\overline{\mathcal{I}(\epsilon)(o_2)},\overline{\mathcal{J}(\epsilon)(o_2)}\right]=0.733187$ and $S\left[\overline{\mathcal{I}(\epsilon)(o_2)},\overline{\mathcal{J}(\epsilon)(o_2)}\right]=0.914072.$ Now, $\min\left\{T_1\left[\overline{\mathcal{I}(\epsilon)(o_2)},\overline{\mathcal{J}(\epsilon)(o_2)}\right],T_2\left[\overline{\mathcal{I}(\epsilon)(o_2)},\overline{\mathcal{J}(\epsilon)(o_2)}\right],S\left[\overline{\mathcal{I}(\epsilon)(o_2)},\overline{\mathcal{J}(\epsilon)(o_2)}\right]\right\}=0.733187.$ Thus, $\Phi(\overline{\mathcal{I}},\overline{\mathcal{J}})=\frac{0.650332+0.733186}{2}=0.691759.$ Hence, $Sim(\overline{\mathcal{I}_f},\overline{\mathcal{J}_g})=0.691759\times0.864583=0.598084.$

5 Similarity measure in agriculture production using GqRNSSS model

One of the key sectors of the economy is agriculture, as is well known. It involves using natural resources like water and land to produce things like crops, animals, fiber, fuel, and more. Forestry, fishing, animal husbandry, and mainly agricultural output are all included. Throughout the history of agriculture, one of the problems that most interested farmers was how difficult it was to increase crop productivity. What are the most straightforward methods for increasing crop yield per acre? What elements have the greatest impact on crop yield? This problem has become increasingly important recently as a result of the world population's continued expansion. However, new approaches and technology are emerging that are referred to as a response to the most recent problems facing agrarians. What growers can do to increase crop output on their farmlands and how new technologies will help farmers in this regard are the subjects of this article.

How important is crop yield, and what does it mean?

Crop yield is the quantity of seeds or grains produced on a particular land tract. The standard units of measurement are kilos per hectare or bushels per acre. Given that it captures the outcomes of all the resources and labor invested by agrarians in the growth of plants on their fields, it is regarded as most likely the most important metric of a farmers performance. Given this, it is not surprising that the majority of farmers constantly ask themselves. How can we increase the average crop production per acre? Lets look at the variables that affect crop output the most.

How can agricultural output be increased?

The advancements in science and technology will delight the agriculture sector. Lets look at the main strategies used by farmers to improve crop.

1. Good seed quality and zoning of productivity on the land:

The key component in the production of agriculture is seeds. For enhanced cultivars, it is thought that high-quality seed can boost productivity by about 20-25 %. The development of high yielding varieties and hybrids is greatly aided by the introduction of modern plant breeding techniques and biotechnology advancements in the seed industry. A farmer must first understand the productivity of the area to be sown and, if necessary, identify specific areas where plants thrive before sowing. Productivity zoning is the name of this technique. A cultivator can plant seeds more densely in the areas with higher productivity in this manner, likely resulting in an increased crop output.

2. Surveillance of crop growth:

Its crucial to keep an eye on plant health from the beginning of growth through blossoming and up until harvest in order to quickly identify any problems that might develop on a given acreage (such as pest

infestations, plant diseases, weeds, etc.) and affect crop yield. For instance, regular satellite monitoring of land plots enables farmers to easily monitor plant development and estimate crop yield using remote sensing.

3. Proper irrigation:

Weather forecasts and efficient irrigation of agricultural land are closely related. Access to hyper-local weather forecasts is possible because to modern technology. It provides a pathway for precision irrigation and enables farmers to prepare earlier and plan irrigation of their crops in the most precise and cost-effective way.

4. Efficient application of fertilizers:

Although fertilizers are intended to nourish the various types of soil, encourage plant development, and raise yields, their application should be moderate and careful. Overuse of fertilizers will have a negative impact on soil quality and, consequently, agricultural productivity. The most practical option in this case is to apply fertilizers selectively, taking into account the need for them in various zones. This effective method of field fertilizing helps maintain healthy soil, which raises the average crop output per acre.

5. Methods of crop conservation:

There are several different crop protection techniques in general. Weed and pest management is one of the most popular. Due to their high adaptability and rapid reproduction, weeds and pests pose a threat to a particular farmlands produce and, if they are not controlled in a timely manner, can significantly reduce yields. Farmers should always be able to respond to pest infestation problems very quickly.

In this application, we present a method for a agriculture production problem based on the proposed similarity measure of GqRNSSSs. This technique of similarity measure between two GqRNSSSs can be applied to agriculture production work. We first give the following definition: Let $\overline{\mathcal{I}_f}$ and $\overline{\mathcal{J}_g}$ be two GqRNSSSs over the same soft universe $(\mathcal{X}, \mathbb{E})$. We call the two GqRNSSSs to be significantly similar if $Sim(\overline{\mathcal{I}_f}, \overline{\mathcal{J}_g}) > 1/2$. With the assistance of an agriculturalist, we first create a GqRNSSS for the pest and a GqRNSSS for the surrounding area. The similarity between two GqRNSSSs is then calculated. If they are remarkably comparable, we deduce that the person might produce well even if they don't otherwise.

5.1 Algorithms based on the similarity measures for GqRNSSS Model

An algorithm for DM problem using GqRNSSS model is explained.

- **Step 1.** Enter the GqRNSSS in tabular form.
- **Step 2.** The set of choice parameters $A \subseteq \mathbb{E}$.
- **Step 3.** Compute $T_1(o_z)$, $T_2(o_z)$ and $S(o_z)$ and $1 \le z \le m$.

Step 4. Determine
$$\Phi = \frac{1}{m} \sum_{z=1}^{m} \min\{T_1(o_z), T_2(o_z), S(o_z)\}$$

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$$\Phi = \frac{1}{m} \sum_{z=1}^{m} \min\{T_1(o_z), T_2(o_z), S(o_z)\}.$$
Step 5. Find $\Psi(f,g) = 1 - \frac{\sum_{z=1}^{m} |f(\Omega y) - g(\Omega y)|}{\sum_{z=1}^{m} |f(\Omega y) + g(\Omega y)|}$ and $1 \le i \le n$.

- **Step 6.** Compute the similarity measure = $\Phi \cdot \Psi$
- Step 7. When appropriate criteria for considerably similarity exist, choose the similarity measure.
- **Step 8.** Decision to the problem.
- Step 9. End.

5.2 Data Analysis

Suppose that there are five farmers \mathcal{F}_1 , \mathcal{F}_2 , \mathcal{F}_3 , \mathcal{F}_4 and \mathcal{F}_5 in a agriculture land production. Let the universal set contain only 3 elements. Here, $\mathcal{X} = \{o_1 : \text{Good work}, o_2 : \text{fair}, o_3 : \text{not work}\}$ and the set of parameters \mathbb{E} is the set of parameters such as $\mathbb{E} = \{\Omega 1 : \text{Good seed quality and zoning of productivity on the land., } \Omega 2:$ Surveillance of crop growth, $\Omega 3$: Proper irrigation, $\Omega 4$: Efficient application of fertilizers, $\Omega 5$: Methods of crop conservation.

Table 1 is represented by the agriculturist with the assistance of farmers.

 $\begin{tabular}{l} \textbf{Table 1} \\ \textbf{GqRNSSS model for agriculturist help of a farmer.} \\ \end{tabular}$

$\overline{\mathcal{L}_{f(\epsilon)}}$	$\Omega 1$	$\Omega 2$	$\Omega 3$	$\Omega 4$	$\Omega 5$
$\overline{\mathcal{L}(\epsilon)(o_1)}$	(0.75, 0.35, 0.45)	(0.65, 0.35, 0.40)	(0.55, 0.40, 0.55)	(0.65, 0.30, 0.55)	(0.55, 0.60, 0.40)
$\overline{\mathcal{L}(\epsilon)(o_2)}$	(0.75, 0.25, 0.45)	(0.65, 0.30, 0.40)	(0.55, 0.45, 0.30)	(0.65, 0.35, 0.40)	(0.55, 0.65, 0.50)
$\overline{\mathcal{L}(\epsilon)(o_3)}$	(0.75, 0.35, 0.15)	(0.65, 0.35, 0.45)	(0.50, 0.45, 0.35)	(0.65, 0.35, 0.25)	(0.50, 0.65, 0.40)
$\overline{f(\epsilon)}$	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)	(1, 1, 1)

We construct the GqRNSSSs for five farmers under consideration as in Table 2 to Table 6.

$\overline{\mathcal{F}1_{p_1(\epsilon)}}$	$\Omega 1$	$\Omega 2$	$\Omega 3$	$\Omega 4$	$\Omega 5$
$\overline{\mathcal{F}_1(\epsilon)(o_1)}$	(0.15, 0.45, 0.50)	(0.25, 0.65, 0.45)	(0.55, 0.35, 0.35)	(0.35, 0.55, 0.30)	(0.45, 0.65, 0.55)
$\overline{\mathcal{F}_1(\epsilon)(o_2)}$	(0.65, 0.65, 0.55)	(0.55, 0.45, 0.45)	(0.50, 0.45, 0.65)	(0.55, 0.35, 0.60)	(0.55, 0.45, 0.75)
$\overline{\mathcal{F}_1(\epsilon)(o_3)}$	(0.65, 0.35, 0.40)	(0.55, 0.55, 0.50)	(0.45, 0.45, 0.75)	(0.30, 0.65, 0.55)	(0.45, 0.35, 0.75)
$\overline{p_1(\epsilon)}$	(0.30, 0.25, 0.15)	(0.50, 0.35, 0.20)	(0.60, 0.15, 0.40)	(0.60, 0.50, 0.35)	(0.50, 0.30, 0.25)

$\begin{tabular}{ll} \textbf{Table 3} \\ \textbf{GqRNSSS for the farmer \mathcal{F}_2.} \end{tabular}$

$\overline{\mathcal{F}2_{p_2(\epsilon)}}$	$\Omega 1$	$\Omega 2$	$\Omega 3$	$\Omega 4$	$\Omega 5$
$\overline{\mathcal{F}_2(\epsilon)(o_1)}$	(0.35, 0.35, 0.25)	(0.50, 0.55, 0.45)	(0.65, 0.45, 0.35)	(0.55, 0.45, 0.15)	(0.25, 0.50, 0.25)
$\overline{\mathcal{F}_2(\epsilon)(o_2)}$	(0.45, 0.45, 0.65)	(0.55, 0.30, 0.60)	(0.50, 0.45, 0.65)	(0.35, 0.50, 0.50)	(0.50, 0.55, 0.50)
$\overline{\mathcal{F}_2(\epsilon)(o_3)}$	(0.40, 0.20, 0.40)	(0.50, 0.40, 0.65)	(0.50, 0.35, 0.60)	(0.45, 0.60, 0.50)	(0.65, 0.25, 0.45)
$\overline{p_2(\epsilon)}$	(0.30, 0.10, 0.30)	(0.40, 0.35, 0.45)	(0.50, 0.25, 0.25)	(0.30, 0.15, 0.55)	(0.40, 0.35, 0.65)

$\overline{\mathcal{F}3_{p_3(\epsilon)}}$	$\Omega 1$	$\Omega 2$	$\Omega 3$	$\Omega 4$	$\Omega 5$
$\overline{\mathcal{F}_3(\epsilon)(o_1)}$	(0.80, 0.60, 0.55)	(0.50, 0.65, 0.40)	(0.70, 0.35, 0.70)	(0.35, 0.65, 0.35)	(0.40, 0.70, 0.65)
$\overline{\mathcal{F}_3(\epsilon)(o_2)}$	(0.25, 0.75, 0.50)	(0.35, 0.60, 0.45)	(0.55, 0.55, 0.65)	(0.45, 0.60, 0.60)	(0.35, 0.75, 0.45)
$\overline{\mathcal{F}_3(\epsilon)(o_3)}$	(0.65, 0.15, 0.45)	(0.50, 0.25, 0.65)	(0.70, 0.35, 0.45)	(0.45, 0.45, 0.25)	(0.35, 0.55, 0.65)
$\overline{p_3(\epsilon)}$	(0.70, 0.25, 0.65)	(0.80, 0.50, 0.70)	(0.55, 0.15, 0.65)	(0.40, 0.65, 0.75)	(0.50, 0.50, 0.45)

$\begin{tabular}{ll} \textbf{Table 5} \\ \textbf{GqRNSSS for the farmer \mathcal{F}_4.} \end{tabular}$

$\overline{\mathcal{F}4_{p_4(\epsilon)}}$	$\Omega 1$	$\Omega 2$	$\Omega 3$	$\Omega 4$	$\Omega 5$
$\overline{\mathcal{F}_4(\epsilon)(o_1)}$	(0.50, 0.45, 0.75)	(0.40, 0.55, 0.65)	(0.55, 0.25, 0.55)	(0.35, 0.50, 0.45)	(0.15, 0.65, 0.35)
$\overline{\mathcal{F}_4(\epsilon)(o_2)}$	(0.40, 0.45, 0.55)	(0.45, 0.30, 0.60)	(0.50, 0.45, 0.50)	(0.35, 0.40, 0.45)	(0.40, 0.30, 0.50)
$\overline{\mathcal{F}_4(\epsilon)(o_3)}$	(0.30, 0.25, 0.40)	(0.35, 0.35, 0.55)	(0.45, 0.25, 0.60)	(0.35, 0.45, 0.50)	(0.25, 0.10, 0.45)
$\overline{p_4(\epsilon)}$	(0.45, 0.25, 0.15)	(0.25, 0.15, 0.35)	(0.35, 0.45, 0.20)	(0.25, 0.30, 0.40)	(0.45, 0.50, 0.10)

$\overline{\mathcal{F}5_{p_5(\epsilon)}}$	$\Omega 1$	$\Omega 2$	$\Omega 3$	$\Omega 4$	$\Omega 5$
$\overline{\mathcal{F}_5(\epsilon)(o_1)}$	(0.75, 0.45, 0.45)	(0.55, 0.40, 0.40)	(0.65, 0.30, 0.35)	(0.30, 0.45, 0.30)	(0.25, 0.40, 0.25)
$\overline{\mathcal{F}_5(\epsilon)(o_2)}$	(0.45, 0.30, 0.40)	(0.35, 0.55, 0.50)	(0.40, 0.30, 0.50)	(0.20, 0.20, 0.40)	(0.10, 0.40, 0.30)
$\overline{\mathcal{F}_5(\epsilon)(o_3)}$	(0.30, 0.30, 0.40)	(0.45, 0.40, 0.50)	(0.30, 0.50, 0.60)	(0.20, 0.60, 0.55)	(0.10, 0.40, 0.75)
$\overline{p_5(\epsilon)}$	(0.45, 0.50, 0.45)	(0.55, 0.30, 0.35)	(0.35, 0.25, 0.45)	(0.60, 0.35, 0.40)	(0.50, 0.25, 0.45)

https://doi.org/10.54216/IJNS.190112 Received: April 19, 2022 Accepted: August 13, 2022 Based on their evaluation of the options in relation to the criteria being taken into consideration, the experts offered the values in Tables 2-6. In this example, using Definition 4, we should determine how similar the GqRNSSSs in Tables 2 through 6 are to the one in Table 1. Below the table is a formula for calculating the similarity measure for farmers \mathcal{F}_1 to \mathcal{F}_5 .

	$T_1(o_1)$	$T_2(o_1)$	$S(o_1)$	$T_1(o_2)$	$T_2(o_2)$	$S(o_2)$	$T_1(o_3)$	$T_2(o_3)$	$S(o_3)$
$\overline{(\mathcal{L}, \mathcal{F}_1)}$	0.723344	0.778215	0.937928	0.984929	0.645344	0.902621	0.936530	0.632158	0.870484
$\overline{({\cal L},{\cal F}_2)}$	0.875757	0.861235	0.924592	0.911070	0.891885	0.916813	0.905152	0.529629	0.916937
$\overline{({\cal L},{\cal F}_3)}$	0.939197	0.688839	0.929134	0.807473	0.644589	0.933743	0.941723	0.896391	0.925655
$\overline{({\cal L},{\cal F}_4)}$	0.834112	0.882329	0.926308	0.868241	0.630441	0.950807	0.772472	0.381026	0.929135
$\overline{(\mathcal{L},\mathcal{F}_{\scriptscriptstyle{E}})}$	0.914989	0.796360	0.951150	0.715102	0.633538	0.954995	0.673696	0.733431	0.891895

	Φ	Ψ	Similarity
$\overline{(\mathcal{L}, \mathcal{F}_1)}$	0.666949	0.514851	0.343379
$\overline{({\cal L},{\cal F}_2)}$	0.760916	0.522167	0.397326
$\overline{({\cal L},{\cal F}_3)}$	0.743273	0.706897	0.525417
$\overline{(\mathcal{L},{\mathcal F}_4)}$	0.615193	0.469388	0.288764
$\overline{(\mathcal{L}, \mathcal{F}_5)}$	0.701198	0.584906	0.410135

From the above results, we find that the similarity measure of first two and last agriculture productions are < 1/2, but the similarity measure of third agriculture production \mathcal{F}_3 is $\overline{(\mathcal{L},\mathcal{F}_3)} = 0.525417 > 1/2$. Hence these two GqRNSSS's are significantly similar. Agriculturist has advanced significantly thanks to technological innovation. It extends beyond only cultivating land and raising livestock. Anyone interested in entering the agriculture industry might choose to concentrate in one of the many alternative subjects that it offers.

6 Conclusion

The main goal of this paper is to present a GqRNSSS and analyzes some of its properties. The method for using the similarity of two GqRNSSSs to agricultural productivity is described. Future applications will involve the notion of generalized q-rung cubic NSSSs and generalized q-rung bipolar NSSSs.

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