



## Neutrosophic Near-Subtraction Semigroups and its Application

F. Viscaino Naranjo \*, A. R. León Yacelga, B. E. Villalta Jadan

Docente de la Carrera de Software de la Universidad Regional Autónoma de los Andes (UNIANDES)

Emails: [ua.fasutoviscaino@uniandes.edu.ec](mailto:ua.fasutoviscaino@uniandes.edu.ec); [ui.andresleon@uniandes.edu.ec](mailto:ui.andresleon@uniandes.edu.ec); [us.bolivarvillalta@uniandes.edu.ec](mailto:us.bolivarvillalta@uniandes.edu.ec)

### Abstract

Potential mechanisms for dealing with water resources are provided by water resource management (WRM). To determine agricultural water resources, a new multi-criteria water resource management approach was created in this work. The management of agricultural water resources has become a major issue in the current circumstances. To create this circumstance, a multi-criteria approach is required. We were able to address a real-world water resource management issue using the suggested multi-criteria decision-making technique. The neutrosophic TOPSIS environment has been taken into account in this decision-making dilemma. Summertime water demand is high, and towards the end of summer, when monsoon season arrives, water demand is low in the agricultural field. During the monsoon season, it is not uncommon for rain to fall just in parts. In agriculture, growing crops during the monsoon season was difficult at the time. As a result, the nature of the water shortage in this area is non-linear and unclear. Because of this, we suggested an MCDM technique for WRM issues in a neutrosophic TOPSIS -environment context.

**Keywords:** Neutrosophic algebra; TOPSIS; Near-Subtraction; Semigroups;

### 1. Introduction

There is a lack of river flow in the summers when seasonal water substances are depleted, making it difficult for the ecosystem to function properly[1]. It's impossible to grow a decent crop in this India's farm area without water in the current circumstances. Farmers in the Indian area are unable to grow crops because of a lack of water. The lack of water has damaged a large portion of India's agricultural land. This August, the producers in this area were out of work as a result of a disrupted crop pattern. This district's rural residents are very concerned about water conservation. They've used a variety of methods and ways to deal with the water shortage. However, they are unable to deal with this predicament as a result of poor management. The existing water resources need management expertise. For WRM, a decision-making approach is recommended. It is possible to use a multi-criterion decision-making (MCDM) method to figure out the optimal water resources strategy (alternatives).

The challenge of water resource management necessitates the use of increasingly difficult decision-making techniques. To deal with the water resource management issue, several scholars are working on decision-making (DM) approaches Tecle pioneered a new approach to water resource management decision-making based on many factors selected by the user[2]. In Romania's dry areas, researchers[3] investigated environmental multi-attribute decision-making (MADM).

According to [4], an assessment method for water resource management issues was developed based on criteria. To solve the WRM model proposed by [5], the MCDM approach was used.

At [6] developed the fuzzy MADM approach for water resource projects. provided a fuzzy compromise strategy to the WMR with ambiguity by [7]. Using the MCA approach, [8] was able to solve the WRM issue. In the Purulia area of West Bengal, [9] conducted research on water conservation and management. An MCA method was devised by [10]. to evaluate WRM techniques. In Lake Karla Basin, Greece, [11] developed an integrated WRM issue using linear programming. Every genuine situation has an element of uncertainty about it. With the introduction of fuzzy sets (FSs), which also are defined by the level of membership function and accept a value between [0, 1], [12] was the first to address this issue.

When Smarandache [13] developed the idea of neutrosophic sets, he assessed them by the three membership functions, which are truth, uncertainty, and falsehood. Single-valued neutrosophic sets are defined by [14]. In the FSs, all types of membership functions accept a value between 0 and 1. Bipolar fuzzy sets, on the other hand, have a membership function that has positive and negative areas in [0, 1] and [1, 0]. For the first time, bipolar fuzzy sets were proposed by [15]. LR bipolar fuzzy numbers comparison analysis was first suggested by [16] in 1998. (2018). An application of bipolar fuzzy sets to decision-making issues was described by [17], [18]. In the following year, [19] published the bipolar neutrosophic sets, which are an extension of bipolar fuzzy sets, and [20] proposed a discontinuous formulation of bipolar neutrosophic numbers.

The following is the purpose of our manuscript:

1. To investigate new concepts in Neutrosophic fuzzy Near-subtraction semigroups of stated bi-ideals and strong bi-ideals.
2. To investigate a few fundamental characteristics and concepts.
3. A Near-Subtraction Semigroups (NFSBI) further enhance the direct product and regularity of Neutrosophic fuzzy strong bi-ideals (NFSBI).

To sum up, here's how the document is structured: On this page, we'll go over the basics of the generalized near-subtraction semigroup. In Section 3, we presented the discussion. Section 4 considers the conclusion.

## 2. Basic preliminaries

We introduce some basic definitions and neutrosophic algebra equations [21]–[24]

### Definition 1:

Right-to-left ' $-$ ' and ' $\bullet$ ' near-subtraction semigroups are defined by the following conditions for every non-empty set  $Y$ :

There are three ways to look at this:

1.  $(Y, -)$  is a subtraction algebra
2.  $(Y, \bullet)$  is a semigroup
3.  $(a - b)c = ac - bc$  for every  $(a, b, c$  in  $Y)$ .

In  $Y$ , there are an infinite number of  $a$ , hence  $0a = 0$  is obvious. For the left near-subtraction semigroup, we can do the same.

### Definition 2:

$X$  is a Neutrosophic Fuzzy Set on the  $Y$  universe. Truth membership function  $Tr_X(a)$ , an indeterminacy function  $Ind_X(a)$ , and a non-membership function  $FL_X(a)$  are defined as  $X = a, Tr_X(a), Ind_X(a), FL_X(a) > /aY$ , where  $Tr_X(a), Ind_X(a), FL_X(a): Y$  is in the range  $[0,1]$  and  $0 \leq Tr_X(a) + Ind_X(a) + FL_X(a) \leq 3$ .

**Definition 3:**

A Neutrosophic fuzzy subalgebra  $N$  of a near Subtraction Semigroup  $Y$  must meet the following requirements:

The smallest of the two  $Tr_N$ s ( $a$  and  $b$ ) is referred to as the "mini"  $Tr_N$ .

1.  $Ind_N(a - b) \leq \max\{Ind_N(a), Ind_N(b)\}$
2. All  $a$  and  $b$  in  $N$  are covered by  $FL_N(a - b) \leq \max FL_N(a), FL_N(b)$ .

**Definition 4:**

A close subtraction If  $abc$

$= bac$  for any  $a, b, c$  in  $Y$ , then a semigroup  $Y$  is said to be left permutable.

**Definition 5:**

Allow any two Neutrosophic Fuzzy Sets of  $Y$  and  $a \in Y$  to serve as  $X$  and  $N$ , respectively. Hence,

$$X \cup N = \langle a, Tr_{X \cup N}(p), Ind_{X \cup N}(p), FL_{X \cup N}(a) \rangle / a \in Y$$

It is  $Tr_{X \cup N}(p) = \max Tr_X(p), Tr_N(p)$

As a result of this, we may calculate

The minimum value of  $Ind_X(p), Ind_N(p), FL_X(p)$ , and  $FL_N(p)$

There are two ways to look at this:

It is  $Tr_{X \cap N}(p) = \min Tr_X(p), Tr_N(p)$

As a result of this, we may calculate

The maximum value of  $Ind_X(p), Ind_N(p), FL_X(p)$ , and  $FL_N(p)$

**Definition 6:**

If  $(abc) \leq \min \beta(a), \beta(c)$ ,

then a fuzzy sub algebra is said to be a fuzzy bi-ideal of  $a, b, c$  in  $Y$ .

**Definition 7:**

It is only possible for a Neutrosophic Fuzzy Sub-algebra  $X$  to be Neutrosophic Fuzzy Bi-ideal if the following requirements are met:

To begin with, the  $Tr_X(abc) \geq \min Tr_X(a), Tr_X(c)$

$Ind_X(abc) \leq \max Ind_X(a), Ind_X(c)$

$FL_X(abc) \leq \max FL_X(a), FL_X(c)$  for each of the values of  $a, b$ , and  $c \in Y$

**Definition 8:**

Set  $X$  of  $Y$  is referred to be Neutrosophic fuzzy right(left) $Y$ -sub-algebra  $X$  of  $Y$  when

$Tr_X(a - b)$  is less than or equal to the minimum value of  $Tr_X(a) Tr_X(b)$

$Tr_X(ab) \geq Tr_X(a)$

It is also possible that  $Ind_X(a - b)$  is equal to or greater than the maximum value of  $Ind_X(a) Ind_X(b)$

$Ind_X(ab) \leq Ind_X(a)$

However, this is unlikely since  $Ind_X(ab)$  is equal or greater than  $Ind_X(a)$ .

Finally,  $FL_X(a - b)$  is equal to the maximum value of the max  $FL_X(a) FL_X(b)$  is equal to the maximum value of  $FL_X(ab) \leq FL_X(a)$  In the context of  $Y$ .

**Definition 9:**

We can think of  $X$  and  $N$  as any 2 Neutrosophic Subspaces of the Near Subtraction Semigroups ( $Y$  and  $Z$ ) that we want to consider. This is followed by the definition of the direct product

$$X \times N = \left\{ \begin{array}{l} \langle (a, b), Tr_X \times N(a, b), \\ Ind_X \times N(a, b), \\ FL_X \times N(a, b) \rangle / a \in Y, b \in Z \end{array} \right\}$$

where,

$$Tr_X \times N(a, b) = \min\{Tr_X(a), Tr_N(b)\};$$

$$Ind_X \times N(a, b) = \max\{Ind_X(a), Ind_N(b)\};$$

$$FL_X \times N(a, b) = \max\{FL_X(a), FL_N(b)\}$$

#### Definition 10:

For a *Neutrosophic Fuzzy Strong Bi – Ideal X of Y*, it must meet the following criteria to be labeled an NFSBI:

$$\begin{aligned} Tr_X(abc) &\geq \min\{Tr_X(b), Tr_X(c)\} \\ Ind_X(abc) &\leq \max\{Ind_X(b), Ind_X(c)\} \\ FL_X(abc) &\leq \max\{FL_X(b), FL_X(c)\} \\ &\text{for all } a, b, c \in Y. \end{aligned}$$

#### Example 1

The ' – ' and ' • ' can be defined as

–	0	a	b	c
0	0	0	0	0
a	A	0	A	0
b	B	B	0	0
c	C	B	A	0

•	0	a	b	c
0	0	0	0	0
a	0	B	0	b
B	0	0	0	0
C	0	B	0	b

$$\begin{aligned} Tr_X(0) &= 0.9, \\ Tr_X(a) &= 0.7, \\ Tr_X(b) &= 0.3, \\ Tr_X(c) &= 0.2, \end{aligned}$$

$$\begin{aligned} Ind_X(0) &= 0.2, \\ Ind_X(a) &= 0.3, \\ Ind_X(b) &= 0.7, \\ Ind_X(c) &= 0.9, \end{aligned}$$

$$\begin{aligned} FL_X(0) &= 0.1, \\ FL_X(a) &= 0.2, \\ FL_X(b) &= 0.5, \\ FL_X(c) &= 0.7, \end{aligned}$$

#### Theorem 1

Suppose

$X = (Tr_X, Ind_X, FL_X)$  to be a NFSBI where  $YTrTr \sqsubseteq Tr(YIndInd \sqsupseteq Ind, YFLFL \sqsupseteq FL)$

**Proof:** Accept that  $X$  is a NFSBI of  $Y$ . Let  $a, b, d, e, f \in Y$ .

Study  $f = ab$  and  $a = df$ .

that  $Tr$  is a NFBI  $Y$ .

Therefore

$$\begin{aligned} YTrTr(f) &= \sup_{f=ab} \{\min\{(YTr)(a), Tr(b)\}\} \\ &= \sup_{f=ab} \{\min\{\sup_{a=de} \{\min\{Y(d), Tr(e)\}, Tr(b)\}\} \\ &= \sup_{f=ab} \{\min\{\sup_{a=de} \{Tr(e)\}, Tr(b)\}\} \end{aligned}$$

$Tr$  is a NFBlof  $Y$

$$\begin{aligned} &= \sup_{f=ab} \min\{Tr(e), Tr(b)\} \leq \sup_{a=deb} \{Tr(deb)\} \\ &= Tr(dbe) = Tr(f) \end{aligned}$$

$YTrTr \subseteq Tr$ . Suppose that  $YTrTr \subseteq Tr$

If  $a$  cannot expressed as  $f = ab$  then,  $YTrTr(f) = 0 \leq Tr(f)$ . In both cases  $yTrTr \subseteq Tr$ . Choose  $a, b, c, f, t, g \in Y$  such that  $f = abc$ . Then

$$\begin{aligned} Tr(abc) &= Tr(f) \geq YTrTr(f) \\ &= \sup_{f=tg} \min\{(YTr)(t), Tr(g)\} \geq \min\{Y(a), \\ &Tr(b), Tr(c)\} = \min\{Tr(b), Tr(c)\} \\ YIndInd(f) &= \inf_{f=ab} \{\max\{(YInd)(a), Ind(b)\}\} \\ &= \inf_{f=ab} \{\max\{\inf_{a=ed} \{\max\{Y(d), Ind(e)\}, Ind(b)\}\} \\ &= \inf_{f=ab} \{\max\{\inf_{a=de} \{Ind(e)\}, Ind(b)\}\} \end{aligned}$$

Since  $Ind$  is a NFSBI of  $Y$ .

$$\begin{aligned} &= \inf_{f=ab} \max\{Ind(e), Ind(b)\} \geq \inf_{a=deb} \{Ind(deb)\} \\ &= Ind(deb) = Ind(f) \end{aligned}$$

We have,  $IndYInd \supseteq Ind$ .

If  $a$  cannot expressed as  $f = ab$  then  $YIndInd(f) = 0 \geq Ind(f)$ .

In both cases,  $YIndInd \supseteq Ind$

Suppose that  $YIndInd \supseteq Ind$ .

Choose  $a, b, c, f, t, g \in Y$  such that  $f = anc$ . Then

$$\begin{aligned} Ind(abc) &= Ind(f) \leq YIndInd(f) \\ &= \inf_{f=tg} \max\{(YInd)(t), \\ &Ind(g)\} \leq \max\{Y(a), Ind(b), \\ &Ind(c)\} = \max\{Ind(b), Ind(c)\} \\ FLYFL(f) &= \inf_{f=ab} \{\max\{(YFL)(a), FL(b)\}\} \\ &= \inf_{f=ab} \{\max\{\inf_{a=de} \{\max\{Y(d), FL(e)\}, FL(b)\}\} \\ &= \inf_{f=ab} \{\max\{\inf_{a=de} \{FL(e)\}, FL(b)\}\} \end{aligned}$$

Since  $FL$  is a Neutrosophic Fuzzy strong bi – ideal of  $Y$ .

$$\begin{aligned} &= \inf_{f=pq} \max\{FL(e), FL(b)\} \geq \inf_{a=deb} \{FL(deb)\} \\ &= FL(deb) = FL(f) \end{aligned}$$

Hence  $FLYFL \supseteq FL$  If  $f$  cannot expressed as

$$f = ab \text{ then } YFLFL(f) = 0 \geq FL(f).$$

$$\text{In both cases, } YFLFL \supseteq FL$$

Suppose  $FLYFL \supseteq FL$ . Choose  $a, b, c, f, t, g \in Y$

such that  $f = abc$ . Then

$$\begin{aligned} FL(abc) &= FL(f) \leq YFLFL(f) \\ &= \inf_{f=tg} \max\{(YFL)(t), FL(g)\} \\ &\leq \max\{Y(a), FL(b), FL(c)\} \\ &= \max\{FL(b), FL(c)\} \end{aligned}$$

## Theorem 2

The Direct Product of any 2 NFSBI of a Near – Subtraction Semigroups is again a NFSBI of  $Y \times Z$ .

**Proof:**

Assume  $X$  and  $N$  be any 2 NFSBI of  $Y$  and  $Z$  respectively.

We already prove that  $X \times B$  is a NFBI of  $Y \times Z$ .

$$\text{Now } a = (a_1, a_2)$$

$$b = (b_1, b_2)$$

$$c = (c_1, c_2) \in Y \times Z \text{ respectively.}$$

$$\begin{aligned} Tr_{X \times N}((a_1, a_2), (b_1, b_2), (c_1, c_2)) &= Tr_{X \times N}(a_1 b_1 c_1, a_2 b_2 c_2) \\ &= \min\{Tr_X(a_1 b_1 c_1), Tr_N(a_2 b_2 c_2)\} \\ &\geq \min\{\min\{Tr_X(b_1), Tr_X(c_1)\}, \\ &\quad \min\{Tr_N(b_2), Tr_N(c_2)\}\} \\ &= \min\{Tr_{X \times N}(b_1, b_2), \\ &\quad Tr_{X \times N}(c_1, c_2)\} \\ Ind_{X \times N}((a_1, a_2), (b_1, b_2), (c_1, c_2)) &= Ind_{X \times N}(a_1 b_1 c_1, a_2 b_2 c_2) \\ &= \max\{Ind_X(a_1 b_1 c_1), Ind_N(a_2 b_2 c_2)\} \\ &\leq \max\{\max\{Ind_X(b_1), Ind_X(c_1)\}, \\ &\quad \min\{Ind_N(b_2), Ind_N(c_2)\}\} \\ &= \max\{Ind_{X \times N}(b_1, b_2), \\ &\quad Ind_{X \times N}(c_1, c_2)\} \\ FL_{X \times N}((a_1, a_2), (b_1, b_2), (c_1, c_2)) &= FL_{X \times N}(a_1 b_1 c_1, a_2 b_2 c_2) \\ &= \max\{FL_X(a_1 b_1 c_1), FL_N(a_2 b_2 c_2)\} \\ &\leq \max\{\max\{FL_X(b_1), FL_X(c_1)\}, \\ &\quad \min\{FL_N(b_2), FL_N(c_2)\}\} \\ &= \max\{FL_{X \times N}(b_1, b_2), \\ &\quad FL_{X \times N}(c_1, c_2)\} \end{aligned}$$

So,  $X \times N$  is a NFSBI of  $Y \times Z$ .

## Theorem 3

$$\text{If } X \times N = (Tr_{X \times N}, Ind_{X \times N}, FL_{X \times N})$$

be a NFSBI of  $Y \times Z$ . Then  $X \times N = (Tr_{X \times N}, Ind_{X \times N}, FL_{X \times N})$

is a NFSBI of  $Y \times Z$ .

**Proof:**

Assume  $X \times N = (Tr_{X \times N}, Ind_{X \times N}, FL_{X \times N})$  be a NFSBI of  $Y \times Z$ .

Now Now  $a = (a_1, a_2)$

$b = (b_1, b_2)$

$c = (c_1, c_2)$

$\in Y \times Z$

$Tr_{X \times N}, Ind_{X \times N}, FL_{X \times N}$  are NFSBI of  $Y \times Z$ .

prove  $Tr_{X \times N}(a_1, a_2) (b_1, b_2)(c_1, c_2)$

$\leq \max\{Tr_{X \times N}(a_1, a_2) (b_1, b_2)(c_1, c_2)\}$

$Tr_{X \times N}(a_1, a_2) (b_1, b_2)(c_1, c_2)$

$= 1 - Tr_{X \times N}(a_1, a_2) (b_1, b_2)(c_1, c_2)$

$\leq 1 - \min\{Tr_{X \times N}(a_1, a_2) (b_1, b_2)(c_1, c_2)\}$

$= \max\{1 - Tr_{X \times N}(b_1, b_2), 1 - Tr_{X \times N}(c_1, c_2)\}$

$= \max\{Tr_{X \times N}(b_1, b_2) Tr_{X \times N}(c_1, c_2)\}$

So,  $X \times N = (Tr_{X \times N}, Ind_{X \times N}, FL_{X \times N})$

is a NFSBI of  $Y \times Z$ .

#### Theorem 4

Let  $Y$  be a Strong regular Near – Subtraction Semigroup.

Let  $X = (Tr_x, Ind_x, FL_x)$  be a NFSBI of  $Y$ ,

then  $YTrTr = Tr, YIndInd = Ind$

and  $YFLFL = FL$

#### Proof:

Assume  $X = (Tr_x, Ind_x, FL_x)$  be a NFSBI of  $Y$ .

Choose  $a \in Y$ .

$Y$  is a strong regular near subtraction semigroup there exists  $f \in Y$

such that  $a = fa^2$ .

$YTrTr(a) = YTrTr(fa^2)$ .

$(YTrTr(a) = \sup_{a=faa}\{\min\{(YTr)(fa), Tr(a)\}\})$

$\geq \min\{YTr(fa), Tr(a)\}$

$= \min\{\sup_{fa=de}\{\min\{Y(d), Tr(e)\}, Tr(a)\}\}$

$\geq \min\{\min\{Y(f), Tr(a)\}, Tr(pa)\}$

$= \min\{Tr(a), Tr(a)\} = Tr(a)$

$YTrTr \subseteq Tr$ .

From that,  $YTrTr = Tr$

$YIndInd(a) = \inf_{a=faa}\{\max\{(YInd)(fa), Ind(a)\}\}$

$$\begin{aligned}
&\leq \max\{Y\text{Ind}(fa), \text{Ind}(a)\} \\
&= \max\{\inf_{fa=de}\{\min\{Y(d), \text{Ind}(e)\}, \text{Ind}(a)\}\} \\
&\leq \max\{\max\{Y(f), \text{Ind}(a)\}, \text{Ind}(pa)\} \\
&= \max\{\text{Ind}(a), \text{Ind}(a)\} = \text{Ind}(a)
\end{aligned}$$

$$Y\text{IndInd} \subseteq \text{Ind}.$$

From that,  $Y\text{IndInd} = \text{Ind}$

$$\begin{aligned}
Y\text{FLFL}(a) &= \inf_{fa=aa}\{\max\{(Y\text{FL})(fa), \text{FL}(a)\}\} \\
&\leq \max\{Y\text{FL}(fa), \text{FL}(a)\} \\
&= \max\{\inf_{fa=de}\{\min\{Y(d), \text{FL}(e)\}, \text{FL}(a)\}\} \\
&\leq \max\{\max\{Y(f), \text{FL}(a)\}, \text{FL}(pa)\} \\
&= \max\{\text{FL}(a), \text{FL}(a)\} = \text{FL}(a)
\end{aligned}$$

$$Y\text{FLFL} \subseteq \text{FL}.$$

From that,  $Y\text{FLFL} = \text{FL}$

#### Theorem 4

Every left permutable fuzzy right  $Y$   
sub algebra of  $Y$  is a NFSBI of  $Y$ .

**Proof:**

Assume  $X = (Tr_x, \text{Ind}_x, \text{FL}_x)$

be a Neutrosophic fuzzy right  $Y$

sub algebra of  $Y$ .

First we prove  $X$  is a NFBI of  $Y$ .

Choose  $f, a, b, d, e \in Y$ .

Also  $f = ab, a = de$

$$\begin{aligned}
TrYTr(f) &= \sup_{f=ab}\{\min\{(TrY)(a), Tr(b)\}\} \\
&= \sup_{f=ab}\{\min\{\sup_{a=de}\{\min\{Tr(d), Y(e)\}, Tr(b)\}\} \\
&= \sup_{f=ab}\{\min\{\sup_{a=de}\{Tr(d)\}, Tr(b)\}\} \\
&= \sup_{f=ab}\min\{Tr(d), Tr(b)\}
\end{aligned}$$

Since  $Tr$  is a Neutrosophic fuzzy right  $Y$

sub algebra  $Tr(ab) = Tr((de)b) \geq Tr(d)$

$$\leq \sup_{f=ab}\min\{Tr(ab), Y(b)\}$$

$$Y(b) = 1 = Tr(ab) = Tr(f)$$

Therefore,  $TrYTr \subseteq Tr$



$$\begin{aligned}
IndYInd(f) &= \inf_{f=ab} \{\max\{(IndY)(a), Ind(b)\}\} \\
&= \inf_{f=ab} \{\max\{\inf_{a=de} \{\max\{Ind(d), Y(e)\}, Ind(b)\}\} \\
&= \inf_{f=ab} \{\max\{\inf_{a=de} \{Ind(d)\}, Ind(b)\}\} \\
&= \inf_{f=ab} \max\{Ind(d), Ind(b)\}
\end{aligned}$$

Since  $Ind$  is a Neutrosophic fuzzy right  $Y$

sub algebra  $Ind(ab) = Ind((de)b) \leq Ind(d)$

$$\leq \inf_{f=ab} \max\{Ind(ab), Y(b)\}$$

$$Y(b) = 1 = Ind(ab) = Ind(f)$$

Therefore,  $IndYInd \subseteq Ind$

$$\begin{aligned}
FLYFL(f) &= \inf_{f=ab} \{\max\{(FLY)(a), FL(b)\}\} \\
&= \inf_{f=ab} \{\max\{\inf_{a=de} \{\max\{FL(d), Y(e)\}, FL(b)\}\} \\
&= \inf_{f=ab} \{\max\{\inf_{a=de} \{FL(d)\}, FL(b)\}\} \\
&= \inf_{f=ab} \max\{FL(d), FL(b)\}
\end{aligned}$$

Since  $FL$  is a Neutrosophic fuzzy right  $Y$

sub algebra  $FL(ab) = FL((de)b) \leq FL(d)$

$$\leq \inf_{f=ab} \max\{FL(ab), Y(b)\}$$

$$Y(b) = 1 = FL(ab) = FL(f)$$

Therefore,  $FLYFL \subseteq FL$

$$\begin{aligned}
YTrTr(f) &= \sup_{f=ab} \{\min\{(YTr)(a), Tr(b)\}\} \\
&= \sup_{f=ab} \{\min\{\sup_{a=de} \{\min\{Y(d), Tr(e)\}, Tr(b)\}\} \\
&= \sup_{f=ab} \{\min\{\sup_{a=de} \{Tr(e)\}, Tr(b)\}\}
\end{aligned}$$

Since  $Tr$  is a left permutable Neutrosophic Fuzzy right  $Y$

Sub algebra of  $Y$ .  $Tr(ab) = Tr((de)b) = Tr(edb) \geq Tr(e)$

$$\leq \sup_{a=deb} \{\min\{Tr(ab), Y(b)\}\}.$$

$$\text{Since } Y(b) = 1 = Tr(ab) = Tr(f)$$

$$\begin{aligned}
YIndInd(f) &= \inf_{f=ab} \{\max\{(YInd)(a), Ind(b)\}\} \\
&= \inf_{f=ab} \{\max\{\inf_{a=de} \{\max\{Y(d), Ind(e)\}, Ind(b)\}\} \\
&= \inf_{f=ab} \{\max\{\inf_{a=de} \{Ind(e)\}, Ind(b)\}\}
\end{aligned}$$

Since  $Ind$  is a left permutable Neutrosophic Fuzzy right  $Y$

sub algebra of  $Y$ .

$$\begin{aligned} Ind(ab) &= Ind((de)b) = Ind(deb) \leq Ind(e) \\ &\geq \inf_{f=ab} \max\{Ind(ab), Y(b)\}. \text{ Since } Y(b) = 0 = Ind(ab) = Ind(f) \end{aligned}$$

We have,  $YIndInd \supseteq Ind$

$$\begin{aligned} YFLFL(f) &= \inf_{f=ab} \{\max\{(YFL)(a), FL(b)\}\} \\ &= \inf_{f=ab} \{\max\{\inf_{a=de} \{\max\{Y(d), FL(e)\}, FL(b)\}\} \\ &= \inf_{f=ab} \{\max\{\inf_{a=de} \{FL(e)\}, FL(b)\}\} \end{aligned}$$

Since  $FL$  is a left permutable Neutrosophic Fuzzy right  $Y$  – sub algebra of  $Y$ .

$$\begin{aligned} FL(ab) &= FL((de)b) = FL(deb) \leq FL(e) \\ &\geq \inf_{f=ab} \max\{FL(ab), Y(b)\}. \text{ Since } Y(b) = 0 = FL(ab) = FL(f) \end{aligned}$$

We have,  $YFLFL \supseteq FL$

### 3. Discussion

India in West Bengal suffers from severe droughts. India's farmers confront water shortages regularly. As a result of the scarcity of water, the district's agricultural lands are unusable. Water conservation technology has become a need in this region. The year-round importance of appropriate water management cannot be overstated.

1. O\_1(Drip drip irrigation system by wood reed),
  2. O\_2 (water preservation and stockpiling by trying to dig a pit on agricultural land),
  3. O\_3(embankment on the river),
  4. O\_4 (rainwater preservation)
  5. are the 4 options considered in the problem formulation with 3 standards
  6. S\_1 (feasibility of the techniques),
  7. S\_2 (cost execution of the methods),
  8. S\_3 (surface water and groundwater in the river basin techniques).
- . O3 is ranked higher than O2, O1, and O4.

### 4. Conclusion

Fuzzy set theory has been extended to include Neutrosophy fuzzy sets. Neutrosophic fuzzy Strong Biideal in Near Subtraction Semi Group Homomorphism Union Direct Product Intersection Homomorphism We'll go at the basics of Neutrosophy's fuzzy ideals in the future.

Agricultural fields in India have been more water-starved in recent years. To understand the purpose of dynamic water resource management, you need to have a firm grasp of management principles. India's water resource management challenge may be caused by the use of multicriteria decision-making procedures. According to this research, MCDM techniques for WRM in the various block regions are being developed.

### References

- [1] B. Das, S. C. Pal, S. Malik, and R. Chakraborty, "Modeling groundwater potential zones of Puruliya district, West Bengal, India using remote sensing and GIS techniques," *Geology, Ecology, and Landscapes*, vol. 3, no. 3, pp. 223–237, 2019.
- [2] A. Tecle, *Choice of multicriterion decision-making techniques for watershed management*. The University of Arizona, 1988.
- [3] I. A. STĂNESCU and F. G. FILIP, "ENVIRONMENTAL MULTI-ATTRIBUTE DECISION-MAKING: ACCESS TO WATER RESOURCES IN ARID REGIONS OF ROMANIA."

- [4] Z. A. Mimi and M. D. Smith, "Selection of water resources management options," 1999.
- [5] B. Yilmaz and N. Harmancioglu, "Multi-criteria decision making for water resource management: a case study of the Gediz River Basin, Turkey," *Water SA*, vol. 36, no. 5, 2010.
- [6] M. Zarghaami, R. Ardakanian, and A. Memariani, "Fuzzy multiple attribute decision making on water resources projects case study: Ranking water transfers to Zayanderud basin in Iran," *Water International*, vol. 32, no. 2, pp. 280–293, 2007.
- [7] M. J. Bender and S. P. Simonovic, "A fuzzy compromise approach to water resource systems planning under uncertainty," *Fuzzy sets and Systems*, vol. 115, no. 1, pp. 35–44, 2000.
- [8] S. Hajkowicz and K. Collins, "A review of multiple criteria analysis for water resource planning and management," *Water resources management*, vol. 21, no. 9, pp. 1553–1566, 2007.
- [9] K. Bauri, P. Gorai, and B. K. Modak, "Indigenous knowledge and practices on water conservation and management in purulia district, west bengal," *J. Environ. & Sociobiol*, vol. 17, no. 1, pp. 89–98, 2020.
- [10] A. Alamanos, N. Mylopoulos, A. Loukas, and D. Gaitanaros, "An integrated multicriteria analysis tool for evaluating water resource management strategies," *Water*, vol. 10, no. 12, p. 1795, 2018.
- [11] A. Alamanos, S. Xenarios, N. Mylopoulos, and P. Staltnacke, "Integrated water resources management in agro-economy using linear programming: The case of lake Karla basin, Greece," in *10th World Congress of EWRA*, 2017, pp. 5–9.
- [12] L. A. Zadeh, "Fuzzy sets," in *Fuzzy sets, fuzzy logic, and fuzzy systems: selected papers by Lotfi A Zadeh*, World Scientific, 1996, pp. 394–432.
- [13] F. Smarandache, "Neutrosophy: neutrosophic probability, set, and logic: analytic synthesis & synthetic analysis," 1998.
- [14] H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, *Single valued neutrosophic sets*. Infinite study, 2010.
- [15] W.-R. Zhang, "Bipolar fuzzy sets and relations: a computational framework for cognitive modeling and multiagent decision analysis," in *NAFIPS/IFIS/NASA '94. Proceedings of the First International Joint Conference of The North American Fuzzy Information Processing Society Biannual Conference. The Industrial Fuzzy Control and Intelligence*, 1994, pp. 305–309.
- [16] R. Ghanbari, K. Ghorbani-Moghadam, and N. Mahdavi-Amiri, "A direct method to compare bipolar LR fuzzy numbers," *Advances in Fuzzy Systems*, vol. 2018, 2018.
- [17] M. Sarwar and M. Akram, "Certain algorithms for computing strength of competition in bipolar fuzzy graphs," *International journal of uncertainty, fuzziness and knowledge-based systems*, vol. 25, no. 06, pp. 877–896, 2017.
- [18] M. Sarwar and M. Akram, "Novel concepts of bipolar fuzzy competition graphs," *Journal of Applied Mathematics and Computing*, vol. 54, no. 1, pp. 511–547, 2017.
- [19] I. Deli, M. Ali, and F. Smarandache, "Bipolar neutrosophic sets and their application based on multi-criteria decision making problems," in *2015 International conference on advanced mechatronic systems (ICAMEchS)*, 2015, pp. 249–254.
- [20] A. Chakraborty *et al.*, "Disjunctive representation of triangular bipolar neutrosophic numbers, de-bipolarization technique and application in multi-criteria decision-making problems," *Symmetry*, vol. 11, no. 7, p. 932, 2019.
- [21] V. Mahalakshmi, S. Maharasi, and S. Jayalakshmi, "Bi-ideals of near subtraction semigroup," *Indian Advances in Algebra*, vol. 6, no. 1, pp. 35–48, 2013.
- [22] J. SIVARANJINI and V. MAHALAKSHMI, "Neutrosophic Fuzzy bi-ideals of Near-Subtraction Semigroups." Springer Publications., Fater India Publications, Volume6 (i), 2021.
- [23] A. Solairaju and S. Thiruvani, *Neutrosophic Fuzzy Ideals of Near-Rings*. Infinite Study, 2018.
- [24] R. SUMITHA, P. A. SELVI, and S. JAYALAKSHMI, "Intuitionstic fuzzy Strong bi-ideals in near-subtraction semigroups." JETIR, 2019.