# Further Algebraic Operations on Interval Critical Valued Neutrosophic Soft Sets with Their Application

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# Further Algebraic Operations on Interval Critical Valued Neutrosophic Soft Sets with Their Application

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**Abstract**

Deli developed the idea of interval-valued neutrosophic soft set (IVNSS) as an extension of soft set (SS) theory. The interval-valued neutrosophic soft set (IVNSS) plays a critical role in handling indeterminacy and inconsistent information during the decision-making process. Similar to other models, this newly proposed model has to fulfil some algebraic operations. The aim of this paper is to present further algebraic operations for the IVNSSs. Some algebraic operations on IVNSSs are introduced. Specifically, algebraic operations of addition, multiplication, scalar multiplication, and power for the IVNSSs are presented. As well, many examples are also presented, along with supporting proofs.In addition, we explained the mechanism of using these algebraic operations in solving decision-making problems.

**Kewords**: Fuzzy Set; neutrosophic set; neutrosophic soft set; interval valued neutrosophic soft set; algebraic operations.

## 1 introduction

Algebraic structures play a great importance in building many algorithms used in solving daily life problems. It also plays an important role in many applications in economics, engineering, medicine, and other sciences. As a result, many algebraic concepts have been studied and applied to and from fuzzy structures like neutrosophic multiplication modules,[1] fuzzy polish groups,[2] mapping structures,[3] neutrosophic BCK-algebra,[4] and topological group structures,[5].[6]

On the other side,dealing with the uncertainties associated with many issues in our world requires more precision and understanding. And it should exchange researchers around the world to update concepts and theories in a way that facilitates dealing with this data.

As a new mathematical idea, Smarandache[7] introduced a neutrosophic set when he made an extension of a fuzzy set.[8] Numerous researchers around the world take care of it and work to merge it with other branches of mathematics. For example, Broumi[9] put forward the notion of a neutrosophic soft set, which is considered a subclass of the neutrosophic set and includes a soft set.[10] Follow him Deli[11] repurposed neutrosophic soft set in interval form. Deli and Broumi[12] studied the relationships between IVNSSs. A. Al Quran et al.,[13][14] compared the fuzzy sets and IVNSSs with complex and bipolar values, Al-Sharqi et al[15–18] organized neutrosophic soft set under complex interval-valued with several mathematical ideas like.[19–21]

In order to elevate this concept of IVNSSs and make it more usable in this article, we will apply more algebraic operations to IVNSSs. This article consists of two parts: the first is a re-definition of the idea of IVNSSs and the second part we will apply some algebraic operations to IVNSSs, in addition to some numerical examples and proposition.

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## 2 Preliminaries

In this part, we re-definition of the idea of NSs and IVNSSs suitable to this work is proposed.

**Definition 2.1.** [7] A NSs over a fixed non empty set $\hat{\mathcal{Z}}$ is defined as the following structure: $\hat{\mathcal{S}} = (\hat{\mathcal{S}}, \mathcal{A}) = \left\{ \alpha, \left\langle \mathcal{T}_{\mathcal{S}(\alpha)}(v), \mathcal{I}_{\mathcal{S}(\alpha)}(v), \mathcal{F}_{\mathcal{S}(\alpha)}(v) \right\rangle \alpha \in \mathcal{A}, v \in \mathcal{V} \right\}$
So that it fulfills the condition $0 \leq \mathcal{T}_{\mathcal{S}(\alpha)}(v) + \mathcal{I}_{\mathcal{S}(\alpha)}(v) + \mathcal{F}_{\mathcal{S}(\alpha)}(v) \leq 3$

**Definition 2.2.** [7] $\hat{\mathcal{S}} = (\hat{\mathcal{S}}, \mathcal{A}) = \left\{ \alpha, \left\langle \mathcal{T}_{\mathcal{S}(\alpha)}(v), \mathcal{I}_{\mathcal{S}(\alpha)}(v), \mathcal{F}_{\mathcal{S}(\alpha)}(v) \right\rangle \alpha \in \mathcal{A}, v \in \mathcal{V} \right\}$ and $\hat{\mathcal{G}} = (\hat{\mathcal{G}}, \mathcal{A}) = \left\{ \alpha, \left\langle \mathcal{T}_{\mathcal{G}(\alpha)}(v), \mathcal{I}_{\mathcal{G}(\alpha)}(v), \mathcal{F}_{\mathcal{G}(\alpha)}(v) \right\rangle \alpha \in \mathcal{A}, v \in \mathcal{V} \right\}$ be two NSs. then the operations are defined:

1.$S \oplus G = \{T_1 + T_2 - T_1.T_2, I_1.I_2, F_1.F_2\}$.

2.$S \otimes G = \{T_1.T_2, I_1 + I_2 - I_1.I_2, F_1 + F_2 - F_1.F_2\}$

3.$\lambda S = \left\{1 - (1 - T_1)^\lambda, I_1^\lambda, F_1^\lambda\right\}$

4.$S^\lambda = \left\{T_1^\lambda, 1 - \left(1 - I_1^\lambda\right), 1 - \left(1 - F_1^\lambda\right)\right\}$

**Definition 2.3.** [11] An IVNSSs over a fixed non empty set $\hat{\mathcal{Z}}$ is defined as the following structure:
$\hat{\mathcal{S}} = (\hat{\mathcal{S}}, \mathcal{A}) = \left\{ \alpha, \left\langle \mathcal{T}_{\mathcal{S}(\alpha)}(v), \mathcal{I}_{\mathcal{S}(\alpha)}(v), \mathcal{F}_{\mathcal{S}(\alpha)}(v) \right\rangle \alpha \in \mathcal{A}, v \in \mathcal{V} \right\}$
$\mathcal{T}_{\mathcal{S}(\alpha)}(v) = \left[m^l_{S(\alpha)}(v), m^u_{S(\alpha)}(v)\right]$,
$\mathcal{I}_{\mathcal{S}(\alpha)}(v) = \left[n^l_{S(\alpha)}(v), n^u_{S(\alpha)}(v)\right]$,
$\mathcal{F}_{\mathcal{S}(\alpha)}(v) = \left[r^l_{S(\alpha)}(v), r^u_{S(\alpha)}(v)\right]$.

where $\mathcal{T}_{\mathcal{S}(\alpha)}(v), \mathcal{I}_{\mathcal{S}(\alpha)}(v)$ and $\mathcal{F}_{\mathcal{S}(\alpha)}(v)$ are truthiness, indeterminacy, and falsity membership IVNSS functions respectively.

**Example 2.4.** Let $\mathcal{V} = \{v_1, v_2\}$ then the IVNSS $\hat{A}, \hat{B}$ defined on $\mathcal{V}$ as following:

$\hat{A} = \left\{ \frac{\langle [0.3,0.6],[0.4,0.8].,[0.37,0.46] \rangle}{v_1}, \frac{\langle [0.26,0.39],[0.65,0.86],[0.53,0.9] \rangle}{v_2} \right\}$
and
$\hat{B} = \left\{ \frac{\langle [0.2,0.5],[0.5,0.7],[0.2,0.8] \rangle}{v_1}, \frac{\langle [0.1,0.3],[0.31,0.7],[0.4,0.73] \rangle}{v_2} \right\}$

## 3 Some Algebraic Operations on Interval Valued Neutrosophic Soft Sets

In this section, we perform some algebraic operations on the IVNSSs, in particular, addition, multiplication, scalar multiplication, and power of the IVNSSs. Furthermore, several related examples are also present.

**Definition 3.1.** Let $\hat{\mathcal{S}} = (\hat{\mathcal{S}}, \mathcal{A}) = \left\{ \alpha, \left\langle \mathcal{T}_{\mathcal{S}(\alpha)}(v), \mathcal{I}_{\mathcal{S}(\alpha)}(v), \mathcal{F}_{\mathcal{S}(\alpha)}(v) \right\rangle \alpha \in \mathcal{A}, v \in \mathcal{V} \right\}$ and $\hat{\mathcal{G}} = (\hat{\mathcal{G}}, \mathcal{B}) = \left\{ \alpha, \left\langle \mathcal{T}_{\mathcal{G}(\alpha)}(v), \mathcal{I}_{\mathcal{G}(\alpha)}(v), \mathcal{F}_{\mathcal{G}(\alpha)}(v) \right\rangle \alpha \in \mathcal{B}, v \in \mathcal{V} \right\}$ be two IVNSSs over $V$ which are defined by
$\mathcal{T}_{\mathcal{S}(\alpha)}(v) = \left[m^l_{S(\alpha)}(v), m^u_{S(\alpha)}(v)\right]$,
$\mathcal{I}_{\mathcal{S}(\alpha)}(v) = \left[n^l_{S(\alpha)}(v), n^u_{S(\alpha)}(v)\right]$,
$\mathcal{F}_{\mathcal{S}(\alpha)}(v) = \left[r^l_{S(\alpha)}(v), r^u_{S(\alpha)}(v)\right]$.
and
$\mathcal{T}_{\mathcal{G}(\alpha)}(v) = \left[m^l_{G(\alpha)}(v), m^u_{G(\alpha)}(v)\right]$,
$\mathcal{I}_{\mathcal{G}(\alpha)}(v) = \left[n^l_{G(\alpha)}(v), n^u_{G(\alpha)}(v)\right]$,
$\mathcal{F}_{\mathcal{S}(\alpha)}(v) = \left[r^l_{S(\alpha)}(v), r^u_{S(\alpha)}(v)\right]$. respectively.

Then the addition of $\hat{\mathcal{S}}$ and $\hat{\mathcal{G}}$ denoted as $\hat{\mathcal{S}} \oplus \hat{\mathcal{G}}$ and defined as follows:

$\hat{\mathcal{S}} \oplus \hat{\mathcal{G}} = \left\{ \alpha \left\langle \hat{\mathcal{T}}_{\hat{S} \oplus \hat{G}(\alpha)}(v), \hat{\mathcal{I}}_{\hat{S} \oplus \hat{G}(\alpha)}(v), \hat{\mathcal{F}}_{\hat{S} \oplus \hat{G}(\alpha)}(v) \right\rangle v \in V \right\}$,
such that,


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$\hat{\mathcal{T}}_{\hat{S}\oplus\hat{G}(\alpha)}(v) =$
$\left[m^l_{S(\alpha)}(v) + m^l_{G(\alpha)}(v) - m^l_{S(\alpha)}(v).m^l_{G(\alpha)}(v), m^u_{S(\alpha)}(v) + m^u_{G(\alpha)}(v) - m^u_{S(\alpha)}(v).m^u_{G(\alpha)}(v)\right]$,
$\hat{\mathcal{I}}_{\hat{S}\oplus\hat{G}(\alpha)}(v) =$
$\left[n^l_{S(\alpha)}(v) + n^l_{G(\alpha)}(v) - n^l_{S(\alpha)}(v).n^l_{G(\alpha)}(v), n^u_{S(\alpha)}(v) + n^u_{G(\alpha)}(v) - n^u_{S(\alpha)}(v).n^u_{G(\alpha)}(v)\right]$,

$\hat{\mathcal{F}}_{\hat{S}\oplus\hat{G}(\alpha)}(v) =$
$\left[r^l_{S(\alpha)}(v) + r^l_{G(\alpha)}(v) - r^l_{S(\alpha)}(v).r^l_{G(\alpha)}(v), r^u_{S(\alpha)}(v) + r^u_{G(\alpha)}(v) - r^u_{S(\alpha)}(v).r^u_{G(\alpha)}(v)\right]$,

**Definition 3.2.** Let
$\hat{\mathcal{S}} = (\hat{\mathcal{S}}, \mathcal{A}) = \left\{ \alpha, \left\langle \mathcal{T}_{\mathcal{S}(\alpha)}(v), \mathcal{I}_{\mathcal{S}(\alpha)}(v), \mathcal{F}_{\mathcal{S}(\alpha)}(v)\right\rangle \alpha \in \mathcal{A}, v \in \mathcal{V}\right\}$
and $\hat{\mathcal{G}} = (\hat{\mathcal{G}}, \mathcal{B}) = \left\{ \alpha, \left\langle \mathcal{T}_{\mathcal{G}(\alpha)}(v), \mathcal{I}_{\mathcal{G}(\alpha)}(v), \mathcal{F}_{\mathcal{G}(\alpha)}(v)\right\rangle \alpha \in \mathcal{B}, v \in \mathcal{V}\right\}$ be two IVNSSs over $V$

Then the subtraction of $\hat{\mathcal{S}}$ and $\hat{\mathcal{G}}$ denoted as $\hat{\mathcal{S}} \ominus \hat{\mathcal{G}}$ and defined as follows:

$\hat{\mathcal{S}} \ominus \hat{\mathcal{G}} = \left\{\alpha \left\langle \hat{\mathcal{T}}_{\hat{S}\oplus\hat{G}(\alpha)}(v), \hat{\mathcal{I}}_{\hat{S}\oplus\hat{G}(\alpha)}(v), \hat{\mathcal{F}}_{\hat{S}\oplus\hat{G}(\alpha)}(v)\right\rangle v \in V\right\},.$
Where,

$\hat{\mathcal{T}}_{\hat{S}\ominus\hat{G}(\alpha)}(v) = \left[\frac{m^l_{S(\alpha)}(v) - m^l_{G(\alpha)}(v)}{1 - m^L_{G(\alpha)}(v)}, \frac{m^U_{S(\alpha)}(v) - m^u_{G(\alpha)}(v)}{1 - m^u_{G(\alpha)}(v)}\right],$

$\mathcal{I}_{\hat{S}\ominus\hat{G}(\alpha)}(v) = \left[\frac{n^l_{S(\alpha)}(v)}{n^l_{G(\alpha)}(v)}, \frac{n^u_{S(\alpha)}(v)}{n^u_{G(\alpha)}(v)}\right],$
and

$\mathcal{F}_{\hat{S}\ominus\hat{G}(\alpha)}(v) = \left[\frac{r^l_{S(\alpha)}(v)}{r^l_{G(\alpha)}(v)}, \frac{r^u_{S(\alpha)}(v)}{r^u_{G(\alpha)}(v)}\right],$

And these formals are proper under the following situations $\hat{S} \geq \hat{G}$ and $m^l_{\hat{G}(\alpha)}(v), m^u_{\hat{G}(\alpha)}(v)$, $\neq 1$,$n^l_{\hat{G}(\alpha)}(v), n^u_{\hat{G}(\alpha)}(v), r^l_{\hat{G}(\alpha)}(v), r^u_{\hat{G}(\alpha)}(v) \neq 0$.

**Definition 3.3.** Let
$\hat{\mathcal{S}} = (\hat{\mathcal{S}}, \mathcal{A}) = \left\{ \alpha, \left\langle \mathcal{T}_{\mathcal{S}(\alpha)}(v), \mathcal{I}_{\mathcal{S}(\alpha)}(v), \mathcal{F}_{\mathcal{S}(\alpha)}(v)\right\rangle \alpha \in \mathcal{A}, v \in \mathcal{V}\right\}$
and $\hat{\mathcal{G}} = (\hat{\mathcal{G}}, \mathcal{B}) = \left\{ \alpha, \left\langle \mathcal{T}_{\mathcal{G}(\alpha)}(v), \mathcal{I}_{\mathcal{G}(\alpha)}(v), \mathcal{F}_{\mathcal{G}(\alpha)}(v)\right\rangle \alpha \in \mathcal{B}, v \in \mathcal{V}\right\}$ be two IVNSSs over $V$

Then the multiplication of $\hat{\mathcal{S}}$ and $\hat{\mathcal{G}}$ denoted as $\hat{\mathcal{S}} \otimes \hat{\mathcal{G}}$ and defined as follows:

$\hat{\mathcal{S}} \otimes \hat{\mathcal{G}} = \left\{\alpha \left\langle \hat{\mathcal{T}}_{\hat{S}\otimes\hat{G}(\alpha)}(v), \hat{\mathcal{I}}_{\hat{S}\otimes\hat{G}(\alpha)}(v), \hat{\mathcal{F}}_{\hat{S}\otimes\hat{G}(\alpha)}(v)\right\rangle v \in V\right\},.$
where,

$\hat{\mathcal{T}}_{\hat{S}\otimes\hat{G}(\alpha)}(v) = \left[m^l_{S(\alpha)}(v).m^l_{G(\alpha)}(v), m^u_{S(\alpha)}(v).m^u_{G(\alpha)}(v)\right]$,

$I_{\hat{S}\otimes\hat{G}(\alpha)}(v) =$
$\left[n^l_{S(\alpha)}(v) + n^l_{G(\alpha)}(v) - n^l_{S(\alpha)}(v).n^l_{G(\alpha)}(v), n^u_{S(\alpha)}(v) + n^u_{G(\alpha)}(v) - q^u_{S(\alpha)}(v).n^u_{G(\alpha)}(v)\right]$, and

$F_{\hat{S}\otimes\hat{G}(\alpha)}(v) =$
$\left[r^l_{S(\alpha)}(v) + n^l_{G(\alpha)}(v) - r^l_{S(\alpha)}(v).r^l_{G(\alpha)}(v), r^u_{S(\alpha)}(v) + r^u_{G(\alpha)}(v) - r^u_{S(\alpha)}(v).r^u_{G(\alpha)}(v)\right]$,

**Definition 3.4.** Let
$\hat{\mathcal{S}} = (\hat{\mathcal{S}}, \mathcal{A}) = \left\{ \alpha, \left\langle \mathcal{T}_{\mathcal{S}(\alpha)}(v), \mathcal{I}_{\mathcal{S}(\alpha)}(v), \mathcal{F}_{\mathcal{S}(\alpha)}(v)\right\rangle \alpha \in \mathcal{A}, v \in \mathcal{V}\right\}$
and $\hat{\mathcal{G}} = (\hat{\mathcal{G}}, \mathcal{B}) = \left\{ \alpha, \left\langle \mathcal{T}_{\mathcal{G}(\alpha)}(v), \mathcal{I}_{\mathcal{G}(\alpha)}(v), \mathcal{F}_{\mathcal{G}(\alpha)}(v)\right\rangle \alpha \in \mathcal{B}, v \in \mathcal{V}\right\}$ be two IVNSSs over $V$

Then the division operation of$\hat{\mathcal{S}}$ and $\hat{\mathcal{G}}$ denoted as $\hat{\mathcal{S}} \oslash \hat{\mathcal{G}}$ and defined as follows:

$\hat{\mathcal{S}} \oslash \hat{\mathcal{G}} = \left\{\alpha \left\langle \hat{\mathcal{T}}_{\hat{S}\oslash\hat{G}(\alpha)}(v), \hat{\mathcal{I}}_{\hat{S}\oslash\hat{G}(\alpha)}(v), \hat{\mathcal{F}}_{\hat{S}\oslash\hat{G}(\alpha)}(v)\right\rangle v \in V\right\},.$

where

$\mathcal{T}_{\hat{S}\oslash\hat{G}(\alpha)}(v) = \left[\frac{m^L_{S(\alpha)}(v)}{m^L_{G(\alpha)}(v)}, \frac{m^u_{S(\alpha)}(v)}{m^u_{G(\alpha)}(v)}\right],$


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$\mathcal{I}_{\hat{S}\oslash\hat{G}(\alpha)}(v) = \left[\frac{n^l_{S(\alpha)}(v)-n^l_{G(\alpha)}(v)}{1-n^l_{G(\alpha)}(v)}, \frac{n^u_{S(\alpha)}(v)-n^u_{G(\alpha)}(v)}{1-n^U_{G(\alpha)}(v)}\right]$,and

$\mathcal{F}_{\hat{S}\oslash\hat{G}(\alpha)}(v) = \left[\frac{r^l_{S(\alpha)}(v)-r^l_{G(\alpha)}(v)}{1-r^l_{G(\alpha)}(v)}, \frac{r^U_{S(\alpha)}(v)-r^u_{G(\alpha)}(v)}{1-r^u_{G(\alpha)}(v)}\right]$.

And these formals are proper under the following situations $\hat{S} \geq \hat{G}$ and $m^l_{\hat{G}(\alpha)}(v), m^u_{\hat{G}(\alpha)}(v)$, $\neq 0$,$n^l_{\hat{G}(\alpha)}(v), n^u_{\hat{G}(\alpha)}(v), r^l_{\hat{G}(\alpha)}(v), r^u_{\hat{G}(\alpha)}(v) \neq 1$.

**Definition 3.5.** Let
$\hat{\mathcal{S}} = (\hat{\mathcal{S}}, \mathcal{A}) = \left\{\alpha, \left\langle \mathcal{T}_{\mathcal{S}(\alpha)}(v), \mathcal{I}_{\mathcal{S}(\alpha)}(v), \mathcal{F}_{\mathcal{S}(\alpha)}(v) \right\rangle \alpha \in \mathcal{A}, v \in \mathcal{V}\right\}$
and $\hat{\mathcal{G}} = (\hat{\mathcal{G}}, \mathcal{B}) = \left\{\alpha, \left\langle \mathcal{T}_{\mathcal{G}(\alpha)}(v), \mathcal{I}_{\mathcal{G}(\alpha)}(v), \mathcal{F}_{\mathcal{G}(\alpha)}(v) \right\rangle \alpha \in \mathcal{B}, v \in \mathcal{V}\right\}$ be two IVNSSs over $V$

Then the scalar multiplication of $\hat{\mathcal{S}}$ denoted as $\hat{\mathcal{C}} = \lambda\hat{\mathcal{S}}$ and defined as follows:

$\hat{C} = \lambda\hat{S} = \left\{\alpha, \left\langle \mathcal{T}_{\hat{C}(\alpha)}(v), \mathcal{I}_{\hat{C}(\alpha)}(v), \mathcal{F}_{\hat{C}(\alpha)}(v) \right\rangle v \in \mathcal{V}\right\}$,
where,

$$\mathcal{T}_{\hat{C}(\alpha)}(v) = \left[1 - \left(1 - m^l_{S(\alpha)}(v)\right)^{\lambda}, 1 - \left(1 - m^u_{S(\alpha)}(v)\right)^{\lambda}\right],$$

$$I_{\hat{C}(\alpha)}(v) = \left[\left(n^l_{S(\alpha)}(v)\right)^{\lambda}, \left(n^u_{S(\alpha)}(v)\right)^{\lambda}\right],$$
and

$$\mathcal{F}_{\hat{C}(\alpha)}(v) = \left[\left(r^l_{S(\alpha)}(v)\right)^{\lambda}, \left(r^u_{S(\alpha)}(v)\right)^{\lambda}\right].$$

To clarify the above operations, we give the following example, whose solution depends on above Definitions.

**Example 3.6.** Let

$\hat{A} = \left\{\frac{\langle[0.3,0.6],[0.4,0.8].,[0.37,0.46]\rangle}{v_1}, \frac{\langle[0.26,0.39],[0.65,0.86],[0.53,0.9]\rangle}{v_2}\right\}$
and
$\hat{B} = \left\{\frac{\langle[0.2,0.5],[0.5,0.7],[0.2,0.8]\rangle}{v_1}, \frac{\langle[0.1,0.3],[0.31,0.7],[0.4,0.73]\rangle}{v_2}\right\}$

be two IVNSSs, then algebraic operations of two IVNSSs finding as below:

(i) The addition operation.

$$\hat{S} \oplus \hat{G} = \left\{\frac{\langle[0.44,0.8],[0.7,0.94],[0.5,0.89]\rangle}{v_1}, \frac{\langle[0.33,0.57],[0.76,0.96],[0.72,0.97]\rangle}{v_2}\right\}$$

(ii) The subtraction operation.

$$\hat{S} \ominus \hat{G} = \left\{\frac{\langle[0.06,0.3],[0.7,0.94]\ [0.5,0.89]\rangle}{v_1}, \frac{\langle[0.026,0.18],[0.76,0.96],[0.72,0.97]\rangle}{v_2}\right\}$$

(iii) The multiplication operation.

$$\hat{S} \otimes \hat{G} = \left\{\frac{\langle[0.06,0.3],[0.7,0.94].e^{j2\Pi[0.39,0.50]},[0.5,0.89]\rangle}{v_1}, \frac{\langle[0.026,0.18],[0.76,0.96],[0.72,0.97]\rangle}{v_2}\right\}$$

(iv) The division operation.

$$\hat{S} \oslash \hat{G} = \left\{\frac{\langle[0.06,0.3],[0.7,0.94],[0.5,0.89]\rangle}{v_1}, \frac{\langle[0.026,0.18],[0.76,0.96],[0.72,0.97]\rangle}{v_2}\right\}$$

(v) The scalar multiplication if $\lambda = 2$.

$$\lambda\hat{A} = \left\{\frac{\langle[0.51,0.84],[0.49,0.64],[0.13,0.21]\rangle}{v_1}, \frac{\langle[0.45,0.63],[0.42,0.74],[0.28,0.81]\rangle}{v_2}\right\}$$

These operations can achieve some mathematical properties as given below.

**Proposition 3.7.** *Let $\mathcal{S}, \mathcal{G}, \mathcal{C}$ be three IVNSSs over a nonempty soft universe $\mathcal{V}$. Then the next properties come true:*


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1. $\mathcal{S} \oplus \mathcal{G} = \mathcal{G} \oplus \mathcal{T}$.

2. $\mathcal{S} \otimes \mathcal{G} = \mathcal{G} \otimes \mathcal{T}$.

3. $\mathcal{S}\bar{\oplus}\left(\mathcal{G}\bar{\oplus}\mathcal{C}\right) = \left(\mathcal{S}\bar{\oplus}\mathcal{G}\right)\bar{\oplus}\mathcal{C}$

4. $\mathcal{S}\bar{\otimes}\left(\mathcal{G}\bar{\otimes}\mathcal{C}\right) = \left(\mathcal{S}\bar{\otimes}\mathcal{G}\right)\bar{\otimes}\mathcal{C}$

5. $\lambda(\mathcal{S} \oplus \mathcal{G}) = (\lambda\mathcal{S} \oplus \lambda\mathcal{G})$

6. $\lambda(\mathcal{S} \otimes \mathcal{G}) \neq (\lambda\mathcal{S} \otimes \lambda\mathcal{G})$

7. $S^{\lambda_1} \otimes S^{\lambda_2} = S^{\lambda_1 + \lambda_2}$

8. $S^{\lambda_1} \otimes S^{\lambda_2} = S^{\lambda_1 + \lambda_2}$

*Proof.* For three IVNSSs $\mathcal{S}, \mathcal{G}, \mathcal{C}$ and $\lambda \succ 0$ then we can obtain :

1.$\hat{\mathcal{S}} \oplus \hat{\mathcal{G}} = \left\{\alpha \left\langle \hat{\mathcal{T}}_{\hat{S}\oplus\hat{G}(\alpha)}(v), \hat{\mathcal{I}}_{\hat{S}\oplus\hat{G}(\alpha)}(v), \hat{\mathcal{F}}_{\hat{S}\oplus\hat{G}(\alpha)}(v) \right\rangle v \in V\right\},$

$$= \left[m^l_{S(\alpha)}(v) + m^l_{G(\alpha)}(v) - m^l_{S(\alpha)}(v).m^l_{G(\alpha)}(v), m^u_{S(\alpha)}(v) + m^u_{G(\alpha)}(v) - m^u_{S(\alpha)}(v).m^u_{G(\alpha)}(v)\right],$$
$$\left[n^l_{S(\alpha)}(v) + n^l_{G(\alpha)}(v) - n^l_{S(\alpha)}(v).n^l_{G(\alpha)}(v), n^u_{S(\alpha)}(v) + n^u_{G(\alpha)}(v) - n^u_{S(\alpha)}(v).n^u_{G(\alpha)}(v)\right],$$
$$\left[r^l_{S(\alpha)}(v) + r^l_{G(\alpha)}(v) - r^l_{S(\alpha)}(v).r^l_{G(\alpha)}(v), r^u_{S(\alpha)}(v) + r^u_{G(\alpha)}(v) - r^u_{S(\alpha)}(v).r^u_{G(\alpha)}(v)\right]$$

$$= \left[m^l_{G(\alpha)}(v) + m^l_{S(\alpha)}(v) - m^l_{G(\alpha)}(v).m^l_{S(\alpha)}(v), m^u_{G(\alpha)}(v) + m^u_{S(\alpha)}(v) - m^u_{G(\alpha)}(v).m^u_{S(\alpha)}(v)\right],$$
$$\left[n^l_{G(\alpha)}(v) + n^l_{S(\alpha)}(v) - n^l_{G(\alpha)}(v).n^l_{S(\alpha)}(v), n^u_{G(\alpha)}(v) + n^u_{S(\alpha)}(v) - n^u_{G(\alpha)}(v).n^u_{S(\alpha)}(v)\right],$$
$$\left[r^l_{G(\alpha)}(v) + r^l_{S(\alpha)}(v) - r^l_{G(\alpha)}(v).r^l_{S(\alpha)}(v), r^u_{G(\alpha)}(v) + r^u_{S(\alpha)}(v) - r^u_{G(\alpha)}(v).r^u_{S(\alpha)}(v)\right]$$

$$= \left\{\alpha \left\langle \hat{\mathcal{T}}_{\hat{G}\oplus\hat{S}(\alpha)}(v), \hat{\mathcal{I}}_{\hat{G}\oplus\hat{S}(\alpha)}(v), \hat{\mathcal{F}}_{\hat{G}\oplus\hat{S}(\alpha)}(v) \right\rangle v \in V\right\},$$
$$= \hat{\mathcal{G}} \oplus \hat{\mathcal{S}}$$

2. $\hat{\mathcal{S}} \otimes \hat{\mathcal{G}} = \left\{\alpha \left\langle \hat{\mathcal{T}}_{\hat{S}\otimes\hat{G}(\alpha)}(v), \hat{\mathcal{I}}_{\hat{S}\otimes\hat{G}(\alpha)}(v), \hat{\mathcal{F}}_{\hat{S}\otimes\hat{G}(\alpha)}(v) \right\rangle v \in V\right\},$

$$= \left[m^l_{S(\alpha)}(v) + m^l_{G(\alpha)}(v) - m^l_{S(\alpha)}(v).m^l_{G(\alpha)}(v), m^u_{S(\alpha)}(v) + m^u_{G(\alpha)}(v) - m^u_{S(\alpha)}(v).m^u_{G(\alpha)}(v)\right],$$
$$\left[n^l_{S(\alpha)}(v) + n^l_{G(\alpha)}(v) - n^l_{S(\alpha)}(v).n^l_{G(\alpha)}(v), n^u_{S(\alpha)}(v) + n^u_{G(\alpha)}(v) - n^u_{S(\alpha)}(v).n^u_{G(\alpha)}(v)\right],$$
$$\left[r^l_{S(\alpha)}(v) + r^l_{G(\alpha)}(v) - r^l_{S(\alpha)}(v).r^l_{G(\alpha)}(v), r^u_{S(\alpha)}(v) + r^u_{G(\alpha)}(v) - r^u_{S(\alpha)}(v).r^u_{G(\alpha)}(v)\right]$$

$$= \left[m^l_{G(\alpha)}(v) + m^l_{S(\alpha)}(v) - m^l_{G(\alpha)}(v).m^l_{S(\alpha)}(v), m^u_{G(\alpha)}(v) + m^u_{S(\alpha)}(v) - m^u_{G(\alpha)}(v).m^u_{S(\alpha)}(v)\right],$$
$$\left[n^l_{G(\alpha)}(v) + n^l_{S(\alpha)}(v) - n^l_{G(\alpha)}(v).n^l_{S(\alpha)}(v), n^u_{G(\alpha)}(v) + n^u_{S(\alpha)}(v) - n^u_{G(\alpha)}(v).n^u_{S(\alpha)}(v)\right],$$
$$\left[r^l_{G(\alpha)}(v) + r^l_{S(\alpha)}(v) - r^l_{G(\alpha)}(v).r^l_{S(\alpha)}(v), r^u_{G(\alpha)}(v) + r^u_{S(\alpha)}(v) - r^u_{G(\alpha)}(v).r^u_{S(\alpha)}(v)\right]$$

$$= \left\{\alpha \left\langle \hat{\mathcal{T}}_{\hat{G}\otimes\hat{S}(\alpha)}(v), \hat{\mathcal{I}}_{\hat{G}\otimes\hat{S}(\alpha)}(v), \hat{\mathcal{F}}_{\hat{G}\otimes\hat{S}(\alpha)}(v) \right\rangle v \in V\right\},$$
$$= \hat{\mathcal{G}} \otimes \hat{\mathcal{S}}$$

The proof of the rest of the facts is clear based on the previous definitions □


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## 4 One application on these operations

In this part of the manuscript, we will show the importance of these algebraic tools in solving one of the problems addressed in,[14] by presenting a proposed algorithm based on these algebraic tools. Where we will integrate the opinions of decision-makers by multiplying them or finding their quota.

**Algorithm**
1.Put forward two models (IVNSSs) that represent the opinions of decision makers.
2.Consolidation of opinions into a single model using one of the processes suggested in this article.

**Example 4.1.** Let $\hat{A} = \left\{ \frac{\langle[0.3,0.6],[0.4,0.8].,[0.37,0.46]\rangle}{v_1}, \frac{\langle[0.26,0.39],[0.65,0.86],[0.53,0.9]\rangle}{v_2} \right\}$
and
$\hat{B} = \left\{ \frac{\langle[0.2,0.5],[0.5,0.7],[0.2,0.8]\rangle}{v_1}, \frac{\langle[0.1,0.3],[0.31,0.7],[0.4,0.73]\rangle}{v_2} \right\}$
be two IVNSSs represents two opinions of decision makers.

(i) The addition operation.

$$\hat{S} \oplus \hat{G} = \left\{ \frac{\langle[0.44,0.8],[0.7,0.94],[0.5,0.89]\rangle}{v_1}, \frac{\langle[0.33,0.57],[0.76,0.96],[0.72,0.97]\rangle}{v_2} \right\}$$

(ii) The subtraction operation.

$$\hat{S} \ominus \hat{G} = \left\{ \frac{\langle[0.06,0.3],[0.7,0.94]\ [0.5,0.89]\rangle}{v_1}, \frac{\langle[0.026,0.18],[0.76,0.96],[0.72,0.97]\rangle}{v_2} \right\}$$

(iii) The multiplication operation.

$$\hat{S} \otimes \hat{G} = \left\{ \frac{\langle[0.06,0.3],[0.7,0.94].e^{j2\Pi[0.39,0.50]},[0.5,0.89]\rangle}{v_1}, \frac{\langle[0.026,0.18],[0.76,0.96],[0.72,0.97]\rangle}{v_2} \right\}$$

(iv) The division operation.

$$\hat{S} \oslash \hat{G} = \left\{ \frac{\langle[0.06,0.3],[0.7,0.94],[0.5,0.89]\rangle}{v_1}, \frac{\langle[0.026,0.18],[0.76,0.96],[0.72,0.97]\rangle}{v_2} \right\}$$

Here we notice that we were able to merge two models of IVNSSs into one model of IVNSS.

## 5 Conclusion

In this work, we defined some new operations on IVNSSs under a neutrosophic environment. The basic algebraic operations on IVNSSs namely addition, multiplication, scalar multiplication, and power along with illustrative examples were presented. Subsequently, the basic properties of these operations such as commutative law and relevant laws are mathematically proven. This new development will broaden the fundamental knowledge of existing set theories and then can be applied to real-life problems where three NSs :truthiness, indeterminacy, and falsity could be dealt with.For more future work, we suggest applying these results in building many useful algorithms in solving soft computing problems, economics applications, and many fields.In addition, we explained the mechanism of using these algebraic operations in solving decision-making problems.

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