N-Cylindrical Fuzzy Neutrosophic Sets

Sarannya Kumari R[1] *, Sunny Kalayathankal[2], Mathews George[3], Florentin Smarandache[4]

[1]Research Scholar, Catholicate College, Pattanamthitta, Kerala, India.

ORCID ID-0000-0002-0050-6218

[2]Principal (Professor & Dean of Research), Jyothi College of Engineering, Thrissur, Kerala,India.
ORCID ID-0000-0002-0680-1031
[3]Department of Mathematics, Providence College of Engineering, Alappuzha, Kerala, India.
[4] Department of Mathematics, University of New Mexico 705 Gurley Ave. Gallup, NM 87301, USA.
ORCID ID-0000-0002-5560-5926

Emails: saranya6685@gmail.com; sunnyjoseph2014@yahoo.com,;mathews.g@providence.edu.in; smarand@unm.edu

* Correspondence: saranya6685@gmail.com

Abstract

In this paper, we introduce a new type of fuzzy Neutrosophic set called n-Cylindrical fuzzy Neutrosophic set (n-CyFNS), with I as independent neutrosophic component. The n-CyFNS can be claimed as the largest extension of fuzzy sets. In n-CyFNS, the degree of positive, neutral and negative membership functions are satisfying the condition, $0 \leq \beta_A(\mathbf{x}) \leq 1$ and $0 \leq \alpha_A{}^{\mathbf{n}}(\mathbf{x}) + \gamma_A{}^{\mathbf{n}}(\mathbf{x}) \leq 1$, n>1, is an integer. Also the distance between two n- CyFNS and its properties are also defined. Along with basic operations on n- CyFNSs, we put forward two concepts- Neutrosophic affinity degree & Neutrosophic similarity index which is used to compare and correlate n-CyFNSs respectively. A comparison is made in the n-CyFNS environment using the existing correlation measures to check its reliability.

Keyword: n- Cylindrical fuzzy neutrosophic sets (n-CyFNSs), height of n-CyFNS; peak of n-CyFNS, right cylindrical fuzzy Neutrosophic set (n-RCyFNS); neutosophic affinity degree; neutrosophic similarity index.

1. Introduction

In 1965,L.A. Zadeh [24] pioneered fuzzy sets. Zadeh's concept of fuzzy set emerged as a new tool for handling uncertainties in real-world problems, and he only discussed the membership function. Following the extensions of fuzzy set theory, Atanassov [6] generalised this concept and introduced a new set called intuitionistic fuzzy set (IFS) in 1986, which can be described as an ambiguous event's non-membership grade plus its membership grade, with the restriction that the sum of both acceptance degree and rejection degree grades does not exceed 1. The sum of the membership and non-membership degrees to which a decision maker (DM) provides an appropriate satisfying attribute may be greater than one in some practical problems. IFS was unable to deal with the belief system's ambiguous and inconsistent information. As a result, Smarandache [8] created a new concept known as the Neutrosophic set (NS), which, among other things, extrapolates fuzzy sets and intuitionistic fuzzy

Doi : https://doi.org/10.54216/IJNS.180430
Received: March 30, 2022 Accepted: July 14, 2022

sets. Truth, falsity, and indeterminacy membership are all part of a Neutrosophic set. As a result, Neutrosophic sets have reached various heights in all fields of science and engineering.

F.Smarandache introduced the dependence degree of (also, the independence degree of) the fuzzy components, as well as the neutrosophic components, for the first time in 2006[9]. I. Arokiarani and et.al,[5] initiated the notion of fuzzy neutrosophic set as the sum of all the three membership functions does not exceed 3. In 2019, Jhansi and Mohana developed pythagorean Neutrosophic fuzzy sets and its correlation measure, here dependent Neutrosophic components are T and F [18]. Here membership functions are satisfying the condition $0 \leq ((x))^2 + (\zeta_A\ (x))^2 + (\nu_A(x))^2 \leq 2$. In 2021, C.Antony Crispin Sweety and R.Jhansi introduced Fermatean Neutrosophic sets[FN-Set] for the first time. In this case the membership values satisfy $0 \leq ((x))^3 + (\zeta_A\ (x))^3 + (\nu_A(x))^3 \leq 2$ [2]. Here FN space is greater than PFN space.

Correlation coefficients are used to assess the strength of a relationship between two variables. The significance of correlation coefficients in the environment of fuzzy set stems from the matter that these can be applied to clustering, MADM, pattern recognition problems as well as medical diagnosis and so on. Hanafy et al. [11] proposed neutrosophic set correlation coefficients and investigated some of their fundamental properties. Using the centroid method, for the first time, Hanafy et al. [11] studied the concepts of correlation and correlation coefficient in neutrosophic sets, as well as some well-known properties. Broumi and Smarandache expounded the Haudroff distance among neutrosophic sets. The correlation coefficient between interval neutrosphic sets was also proposed by Broumi and Smarandache [19][20]

The paper present a new type of Fuzzy Neutrosophic set –n- Cylindrical Fuzzy Neutrosophic Set (n-CyFNS). Here we put forward two constrains in the definition of n-CyFNS so as to broaden the membership value range. We defined basic operations of n-CyFNS along with Neutrosophic affinity degree and Neutrosophic similarity index which describe the comparison and correlation among two n-CyFNSs. A comparison is made in the n-CyFNS environment using the existing correlation coefficients

2. Basic Concepts

Throughout this paper, U denotes the universe of discourse.

Definition: 2.1: [24]

A fuzzy set A in U is defined by membership function μ_A: A→ [0, 1] whose membership value μ_A (x) shows the degree to which x∈ U includes in the fuzzy set A, for all x∈ U.

Definition: 2.2: [6]

An intuitionistic fuzzy set A on U is an object of the form

A = {(x, α_A (x), γ_A (x) |x ∈ U)} where α_A (x) ∈ [0,1] is called the degree of membership of x in A, γ_A (x) ∈ [0,1] is called the degree of non-membership of x in A, and where α_A and γ_A satisfy $(\forall x \in U)\ (\alpha_A(x) + \gamma_A(x) \leq 1)$

IFS (U) denote the set of all the intuitionistic fuzzy sets (IFSs) on a universe U.

Let X and Y be ordinary non-empty sets.

Definition: 2.3: [8]

A Neutrosophic set A on U is A = < x, $T_A(x)$, $I_A(x)$, $F_A(x)$ > ; x∈ U , where T_A, , I_A , F_A: A→ $]^-0,1^+[$ and $^-0 < T_A(x) + I_A(x) + F_A(x) < 3^+$

Definition: 2.4: [5]

A fuzzy Neutrosophic set A on U is A = < x, $T_A(x)$, $I_A(x)$, $F_A(x)$ > ; x∈ U, where T_A, , I_A , F_A: A→ [0,1] and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$

Definition: 2.5: [9]

A neutrosophic set A on U is an object of the form:

Doi : https://doi.org/10.54216/IJNS.180430
Received: March 30, 2022 Accepted: July 14, 2022

$A = \{(x, u_A(x), \zeta_A(x), v_A(x)): x \in \text{U}\}$, where $u_A(x), \zeta_A(x), v_A(x) \in [0,1]$, $0 \le u_A(x) + \zeta_A(x) + v_A(x) \le 3$, for all $x \in \text{U}$. $u_A(x)$ is the degree of truth membership, $\zeta_A(x)$ is the degree of indeterminacy and $v_A(x)$ is the degree of non-membership. Here $u_A(x)$ *and* $v_A(x)$ are dependent components and $\zeta_A(x)$ is an independent component.

Definition: 2.6: [18]

A Pythagorean neutrosophic set with T and F are dependent neutrosophic components [PNS] A on U is an object of the form $A = \{(x, u_A(x), \zeta_A(x), v_A(x)): x \in \text{U}\}$,where $u_A(x), \zeta_A(x), v_A(x) \in [0,1]$, $0 \le (u_A(x))^2 + (\zeta_A(x))^2 + (v_A(x))^2 \le 2$, *for all x in* U. $u_A(x)$ is the degree of membership, $\zeta_A(x)$ is the degree of indeterminacy and $v_A(x)$ is the degree of non-membership. Here $u_A(x)$ *and* $v_A(x)$ are dependent components and $\zeta_A(x)$ is the independent component.

Definition: 2.7: [2]

A Fermatean neutrosophic set [FN Set] A on U is an object of the form: $A = \{(x, (x), (x), (x))/x \in \text{U}\}$,
Where $u_M(x), \zeta_M(x), v_M(x) \in [0,1]$, $0 \le (u_M(x))^3 + (v_M(x))^3 \le 1$ and $0 \le (\zeta_M(x))^3 \le 1$.
Then $0 \le (u_M(x))^3 + (\zeta_M(x))^3 + (v_M(x))^3 \le 2$, *for all x in* U.

3. n-Cylindrical Fuzzy Neutrosophic Sets.

Definition: 3.1:

An n- cylindrical fuzzy neutrosophic set (n-CyFNS) A on U is an object of the form
A= $\{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle \mid x \in \text{U}\}$ where $\alpha_A(x) \in [0, 1]$, called the degree of **positive** membership of x in A, $\beta_A(x) \in [0,1]$, called the degree of **neutral** membership of x in A and $\gamma_A(x) \in [0, 1]$, called the degree of **negative** membership of x in A, which satisfies the condition, $(\forall x \in \text{U})$ $(0 \le \beta_A(x) \le 1$ and $0 \le \alpha_A{}^n(x) + \gamma_A{}^n(x) \le 1$. , n>1, is an integer. Here T and F are dependent Neutrosophic components and I is 100 % independent.

For the convenience, $\langle \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle$ is called as n-Cylindrical fuzzy Neutrosophic Number (n-CyFNN) and is denoted as A= $\langle \alpha_A, \beta_A, \gamma_A\rangle$

Example: 3.2: Let X=$\{x_1, x_2\}$, P =$\{\langle x_1, 0.69, 1, 0\rangle, \langle x_2, 0.1, 0.99, 0.09\rangle\}$

Q =$\{\langle x_1, 0.6, 0.9, 0.008\rangle, \langle x_2, 0.3, 0.99, 0.09\rangle\}$.

It is clearly n-CyFNS.

Definition: 3.3: Height of an n- CyFNSs:

Height of an n- CyFNS, A is denoted as H (A) and is defined as

H (A) = max $\{\beta_A(x) \mid x \in \text{U}\}$.

Thus height of an element x ∈ U is h(x) and is equal to the degree of **neutral** membership of x in U.

Example: 3.4:

Let A={ (a; 0.3,0.5,0.6), (b;0.5, 0.6,0.6),(c;0.8, 0.9,0.5) }, Clearly A is n-CyFNS.

h(a) = it's degree of neutral membership or degree of indeterminacy = 0.5

h(b) =0.6 , h(c) =0.9

Then H(A) = max{h(x); for all x in A}= max {0.5,0.6,0.9} = 0.9

Remark: 3.5:

Geometrically, we can see that when n=2, the n-CyFNSs membership values form the interior points of the part of a cylinder of radius 1 that lies in the first octant with height, maximum neutral membership value. As a result, we named it "n-cylindrical fuzzy Neutrosophic set."

Doi : https://doi.org/10.54216/IJNS.180430
Received: March 30, 2022 Accepted: July 14, 2022

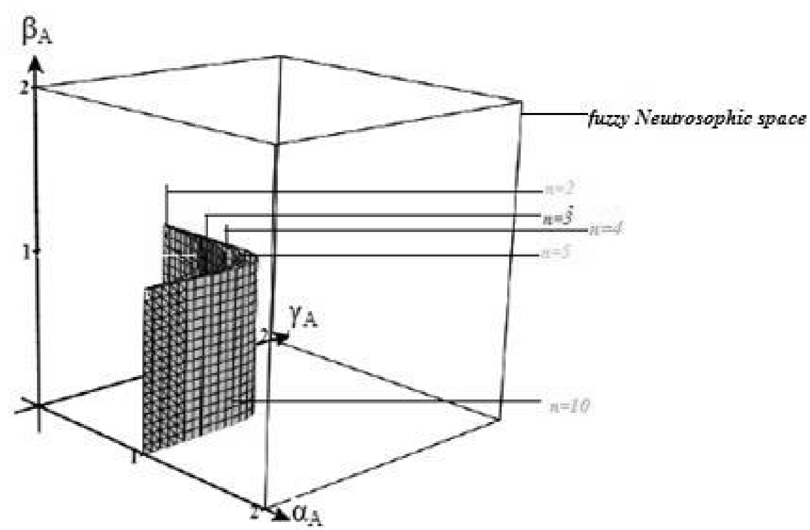

Figure: 1: n-Cylindrical fuzzy Neutrosophic space.

Figure:1 shows the n-CyFNS space for n=2, 3, 4, 5 & 10. It also gives a comparison of n-CyFNS with fuzzy neutrosophic sets.

Let $\square_N$ **(U)** denote the family of all n- cylindrical fuzzy neutrosophic sets on U.

Now we define peak of an n- CyFNS which is the key feature of n-CyFNS.

Definition 3.6:

The **peak of an element, x**∈A, where A={⟨ x, $\alpha_A(x)$, $\beta_A(x)$, $\gamma_A(x)$⟩ | x∈ U} is

$\wp_A$**(x)** =max {$\alpha_A(x)$, $\beta_A(x)$, $\gamma_A(x)$/ **x**∈A}

Now we define the Peak of an n- CyFNS set.

Definition 3.7:

Let A={⟨ x, $\alpha_A(x)$, $\beta_A(x)$, $\gamma_A(x)$⟩ | x∈ U}, then peak of A is defined as $\wp(A)$ =max { $\wp_A$(x)/ **x**∈A}.

Example: 3.8:

Let X={a, b, c, d} and A, B ∈ □ (U), where A={⟨ a,0.5,1,0.3⟩,⟨ b,0.4,0.3,0.5⟩, ⟨ c,0.3,0.8,0.6⟩,⟨ d, 0.2,0.1,0.8⟩} and B={⟨ a,0.4,0.35,0.5⟩,⟨ b,0.7,0.3,0.3⟩,⟨ c, 0.8,0.5,0.2⟩,⟨ d, 0.2,0.9,0.4⟩}

Clearly A & B are n- cylindrical fuzzy neutrosophic sets.

Here, $\wp_A$(a)= 1, $\wp_A$(b)= 0.5 , $\wp_A$(c)=0.8 , $\wp_A$(d) =0.8 and

$\wp_B$(a)= 0.5 , $\wp_B$(b)= 0.7 , $\wp_B$(c) =0.8 , $\wp_B$(d)= 0.9

The peak of A is $\wp$ **(A)** = max{ 1 ,0.5,0.8, 0.8} =1 and that of B is $\wp$ **(B)** = max {0.5, 0.7, 0.8,0.9}= 0.9

Remark: 3.9:

The difference between peak of an n- CyFNS and Height of an n-CyFNS is that, the

Peak of an n-CyFNS is the maximum membership value of an n-CyFNS. Peak of an n-CyFNS can be the highest membership value of either positive, neutral or negative degree of membership. But Height of an n-CyFNS is the highest value of degree of indeterminacy or degree of neutral membership.

Doi : https://doi.org/10.54216/IJNS.180430
Received: March 30, 2022 Accepted: July 14, 2022

Definition 3.10:

Let A={⟨ x, α_A(x), β_A(x), γ_A(x)⟩ | x∈ U}, if $\wp_A(x) = h(x)$ for all x∈ A, then A is called n-**Right cylindrical fuzzy neutrosophic set (n-RCyFNS)**

Remark: 3.11:

If A is a n-RCyFNS, then $\wp(A) = H(A)$

Example: 3.12:

Let A = {(p; 0.3,0.5, 0.4), (q; 0.6,0.8, 0.5), (r; 0.4,0.6,0.5,)} Clearly A is CyFNS.

Here we can see that**,** $\wp_A(p) = h(p) = 0.5$, $\wp_A(q) = 0.8 = h(q)$, $\wp_A(r)$ =0.6 = h(r).

Thus $\wp_A(x) = h(x)$ for all x∈ A, Then A is n- RCyFNS.

Definition 3.13:

Distance between two CyFNS

Let A={⟨ x, α_A(x), β_A(x), γ_A(x)⟩ | x∈ U} and B={⟨ x, α_B(x), β_B(x), γ_B(x)⟩ | x∈ U} are two n-CyFNSs, then the **distance** between A and B is denoted as $\mathbf{d_N}$ (A, B) and is defined as

$\mathbf{d_N}$ (A, B) = $\frac{1}{n}\sum \sqrt[n]{\max\{[|\alpha_A(x) - \alpha_B(x)|]^n, [|\beta_A(x) - \beta_B(x)|]^n, [\,|\gamma_A(x) - \gamma_B(x)|]^n\}}$ for all x∈ U.

Properties of $\mathbf{d_N}$ (A, B)

i) $0 \leq \mathbf{d_N}(A, B) \leq 1$
ii) $\mathbf{d_N}$ (A, B)= $\mathbf{d_N}$ (B,A)
iii) $\mathbf{d_N}$ (A, B)=0 if and only if A =B

Proof:

i) $0 \leq \mathbf{d_N}(A, B) \leq 1$

We know that α_A(x), β_A(x), γ_A(x) ∈ [0, 1]

Then $\max\{[|\alpha_A(x) - \alpha_B(x)|]^n, [|\beta_A(x) - \beta_B(x)|]^n, [\,|\gamma_A(x) - \gamma_B(x)|]^n\} \in [0, 1]$

And $\frac{1}{n}\sum \sqrt[n]{\max\{[|\alpha_A(x) - \alpha_B(x)|]^n, [|\beta_A(x) - \beta_B(x)|]^n, [\,|\gamma_A(x) - \gamma_B(x)|]^n\}} \in [0, 1]$

Hence, $\mathbf{d_N}$ (A, B) ∈ [0, 1]

That is $0 \leq \mathbf{d_N}(A, B) \leq 1$.

ii) $\mathbf{d_N}$ (A, B)= $\mathbf{d_N}$ (B,A), iff, n= even integer.

From the definition of $\mathbf{d_N}$ (A, B),

$\mathbf{d_N}$ (A, B) = $\frac{1}{n}\sum \sqrt[n]{\max\{[|\alpha_A(x) - \alpha_B(x)|]^n, [|\beta_A(x) - \beta_B(x)|]^n, [\,|\gamma_A(x) - \gamma_B(x)|]^n\}}$

= $\frac{1}{n}\sum \sqrt[n]{\max\{[|\alpha_A(x) - \alpha_B(x)|]^n, [|\beta_A(x) - \beta_B(x)|]^n, [\,|\gamma_A(x) - \gamma_B(x)|]^n\}}$

$$= \frac{1}{n}\sum \sqrt[n]{\max\{[|\alpha_B(x) - \alpha_A(x)|]^n, [|\beta_B(x) - \beta_A(x)|]^n, [\,|\gamma_B(x) - \gamma_A(x)|]^n}$$

= $\mathbf{d_N}$ (B,A)

iii) $\mathbf{d_N}$ (A, B)=0 if and only if A =B
From the definition of $\mathbf{d_N}$ (A, B),
$\mathbf{d_N}$ (A, B) == $\frac{1}{n}\sum \sqrt[n]{\max\{[|\alpha_A(x) - \alpha_B(x)|]^n, [|\beta_A(x) - \beta_B(x)|]^n, [\,|\gamma_A(x) - \gamma_B(x)|]^n\}}$

Doi : https://doi.org/10.54216/IJNS.180430
Received: March 30, 2022 Accepted: July 14, 2022

When A=B
$\mathbf{d_N}$ (A, B)=0

Suppose $\mathbf{d_N}$ (A, B) =0

$= \frac{1}{n}\sum \sqrt[n]{\max\{[|\alpha_A(x) - \alpha_B(x)|]^n, [|\beta_A(x) - \beta_B(x)|]^n, [|\gamma_A(x) - \gamma_B(x)|]^n}\}=0$
Ie, $\sqrt[n]{\max\{[|\alpha_B(x) - \alpha_A(x)|]^n, [|\beta_B(x) - \beta_A(x)|]^n, [|\gamma_B(x) - \gamma_A(x)|]^n} = 0$ for all x
Ie, $\max\{[|\alpha_A(x) - \alpha_B(x)|]^n, [|\beta_A(x) - \beta_B(x)|]^n, [|\gamma_A(x) - \gamma_B(x)|]^n\}=0$ for all x
Ie, $[|\alpha_A(x) - \alpha_B(x)|]^n = 0$, $[|\beta_A(x) - \beta_B(x)|]^n = 0$, $[|\gamma_A(x) - \gamma_B(x)|]^n=0$ for all x
ie, $\alpha_A(x)= \alpha_B(x)$, $\beta_A(x)=$, $\beta_B(x)$, $\gamma_A(x)= \gamma_B(x)$ for all x
ie, A=B.

Example 3.14:

Let X={a, b, c, d} and A, B ∈ □ (U), where A={⟨a,0.5,1,0.3⟩,⟨b,0.4,0.3,0.5⟩,⟨c,0.3,0.8,0.6⟩,⟨d, 0.2,0.1,0.8⟩} and B={⟨a, 0.4,0.3,0.5⟩,⟨b, 0.7,0.3,0.4⟩,⟨c, 0.8,0.5,0.1⟩,⟨d, 0.2,0.9,0.4⟩}, then

H (A) =1 and H (B) = 0.9
$\mathbf{d_N}$**(A, B) = 0.575**

4: Comparison with other fuzzy sets.

4.1: Comparison between n-CyFNS, PFS & T-SFS

It can be claimed that the n-CyFN space is larger than the picture fuzzy space (PFS), T-spherical fuzzy space (T-SFS).From the example, A= {(p, 0.6,1, 0.6), (q, 0.5, 0.7, 0.4)}. Clearly A is n-CyFNS with n=2. But A is neither PFS nor T-SFS.

In general, we can say that the set of all n-CyFNSs with height equal to 1 are neither PFS nor T-SFS.

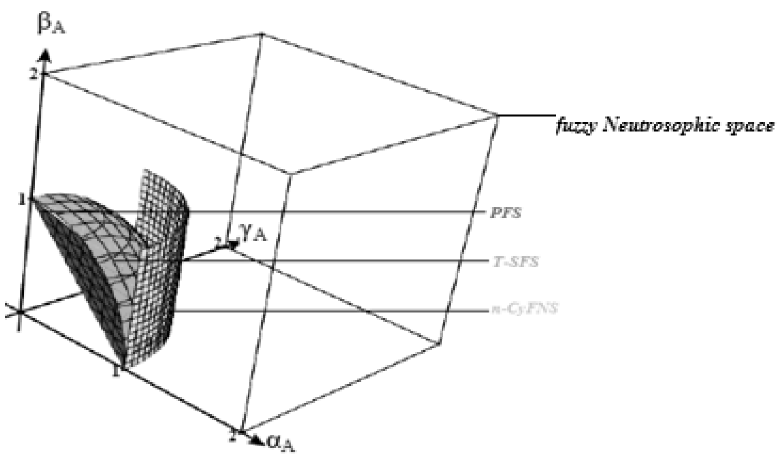

Figure: 2: Comparison between n-CyFNS, T-SFS & PFS spaces.

4.2: Comparison of n-CyFNS with Pythagorean Neutrosophic sets (PNS) and Fermatean Neutrosophic set (FN Set).

Although the degree of indeterminacy is completely independent in all the three cases, the sets are completely different, as is the corresponding space. Two constraints are defined in n-CyFNS, while a single constraint is defined in PNS and FN-Set. The n-Cylindrical fuzzy Neutrosophic sets is clearly larger than PN space and FN space.

Example: A= {(a; 0.7,1, 0.9), (b; 0.9, 0.9, 0.3)}, clearly the set A is n-CyFNS for n=4. But for the element a in A, $(0.7)^2$+1+ $(0.9)^2$>2, $(0.7)^3$+1+ $(0.9)^3$ >2, it is neither PNS nor FN-Set.

Doi : https://doi.org/10.54216/IJNS.180430
Received: March 30, 2022 Accepted: July 14, 2022

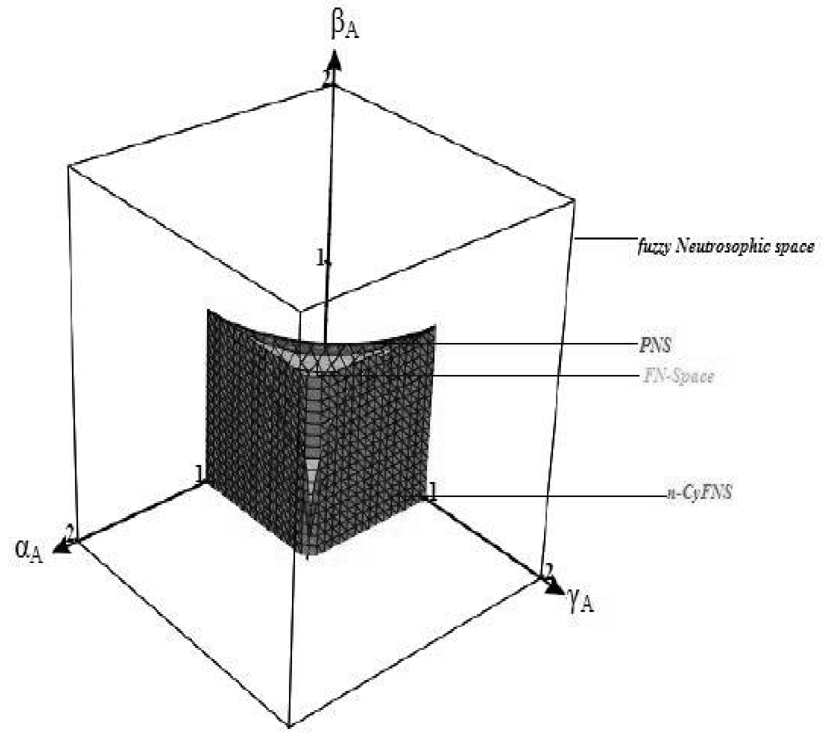

Figure: 3: Comparison of n-CyFNS with PNS and FN-Set

Hence we can see that n-CyFNS space is greater than PNS, PFS, T-SFS etc. The following figure: 4, shows the hierarchy of main fuzzy extensions.

Figure: 4: Hierarchy of Fuzzy extensions.

5: The Basic Connectives:

Let $\square_N$ **(U)** denote the family of all n-cylindrical fuzzy neutrosophic sets on U.

Doi : https://doi.org/10.54216/IJNS.180430
Received: March 30, 2022 Accepted: July 14, 2022

Definition: 5.1: Inclusion: For every two A, B ∈ □N (Ʊ),

A ⊆ B iff (∀x ∈ Ʊ, $\alpha_A(x) \le \alpha_B(x)$ and $\beta_A(x) \le \beta_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$) and

A = B iff (A ⊆ B and B ⊆ A)

Definition: 5.2: Union: For every two A, B ∈ □N (Ʊ), the union of two n-CyFNSs A and B is

A∪B(x) = { ⟨ x, max ($\alpha_A(x)$, $\alpha_B(x)$), mini ($\beta_A(x)$, $\beta_B(x)$), min ($\gamma_A(x)$, $\gamma_B(x)$) ⟩ |x ∈ Ʊ}

Definition: 5.3: Intersection: For every two A, B ∈ □N (Ʊ), the intersection of two n- CyFNSs A and B is
A∩B(x) = { ⟨ x, min ($\alpha_A(x)$, $\alpha_B(x)$), min ($\beta_A(x)$, $\beta_B(x)$), max ($\gamma_A(x)$, $\gamma_B(x)$) ⟩ |x ∈ Ʊ}

Definition: 5.4: Complementation: For every A ∈ □N (Ʊ), the complement of an n-CyFNS A is

A^c = { ⟨ x, $\gamma_A(x)$, $\beta_A(x)$, $\alpha_A(x)$ ⟩ |x ∈ Ʊ}

Definition: 5.5: Sum: For every two A, B ∈ □N (Ʊ), the sum of two n- CyFNSs A and B is

A⊕B(x) = { ⟨ x, ($\frac{\alpha_A(x).\alpha_B(x)}{\alpha_A(x)+\alpha_B(x)}$, max ($\beta_A(x)$, $\beta_B(x)$), min ($\gamma_A(x)$, γB(x)) ⟩ |x ∈ Ʊ}

Definition: 5.6: Difference: For every two A, B ∈ □N (Ʊ), the difference of two n-CyFNSs A and B is

A⊖ B(x) = { ⟨ x, max ($\alpha_A(x)$, $\alpha_B(x)$), mini ($\beta_A(x)$, $\beta_B(x)$), $\frac{\gamma_A(x).\gamma_B(x)}{\gamma_A(x)+\gamma_B(x}$ ⟩ |x ∈ Ʊ}

Definition: 5.7: Product: For every two A, B ∈ □N (Ʊ), the product of two n-CyFNSs A and B is

A⊗ B(x) = { ⟨ x, ($\alpha_A(x).\alpha_B(x)$, $\beta_A(x).\ \beta_B(x)$, $\gamma_A(x).\gamma_B(x)$) ⟩ |x ∈ Ʊ}

Definition: 5.8: Division: For every two A, B ∈ □N (Ʊ), A⊘ B is A⊘ B(x) = { ⟨ x, min ($\alpha_A(x)$, $\alpha_B(x)$), $\beta_A(x)$. $\beta_B(x)$, max ($\gamma_A(x).\gamma_B(x)$) ⟩ |x ∈ Ʊ}

Result: 5.9:

H(A∪B) = ***H***(A∩B)

Proof: From the definition, it follows trivially.

Result: 5.10:

For an n-RCyFNS, A, as $\wp(A) = \wp(A^c)$

Proof: From the definition of A^c and n-RCyFNSs, it is evident that

$$\wp(A) = H(A)$$

But, $\wp(A) = H(A)$= H(A^c) = $\wp(A^c)$

Hence proved.

Theorem: 5.11:

For any n-CyFNS A, the distance between A and A^c
$\mathbf{d_N}(A, A^c) = \frac{1}{n}\sum[\,|\alpha_A(x) - \gamma_A(x)|]$

Proof: From the definition of the distance between two n-CyFNSs and complement of an n- CyFNS,

$\mathbf{d_N}(A, B) = \frac{1}{n}\sum \sqrt[n]{\max\{[|\alpha_A(x) - \alpha_B(x)|]^n, [|\beta_A(x) - \beta_B(x)|]^n, [\,|\gamma_A(x) - \gamma_B(x)|]^n\}}$ and

A^c = { ⟨ x, $\gamma_A(x)$, $\beta_A(x)$, $\alpha_A(x)$ ⟩ |x ∈ Ʊ}

$\mathbf{d_N}(A, A^c) = \frac{1}{n}\sum \sqrt[n]{\max\{[|\alpha_A(x) - \alpha_{A^c}(x)|]^n, [|\beta_A(x) - \beta_{A^c}(x)|]^n, [\,|\gamma_A(x) - \gamma_{A^c}(x)|]^n\}}$

Doi : https://doi.org/10.54216/IJNS.180430
Received: March 30, 2022 Accepted: July 14, 2022

$=\frac{1}{n}\sum \sqrt[n]{\max\{[|\alpha_A(x)-\gamma_A(x)|]^n, 0, [\,|\gamma_A(x)-\ \alpha_A(x)|]^n\}}$

$=\frac{1}{n}\sum[|\alpha_A(x)-\gamma_A(x)|]$

Hence proved.

6. Some Operations On n-Cyfns

Now, some properties of the defined operations on n-CyFNSs are considered.

Proposition 6.1:

The following relations (mostly equalities) are valid for every three n-CyFNSs A, B and C:

a) If A ⊆ B and B ⊆ C then A ⊆ C;
b) A∪B=B∪A & A∩B=B∩A
c) (A∪B) ∪C=A∪ (B∪C) & (A∩B) ∩C=A∩ (B∩C)
d) (A∪B) ∩C= (A∩C) ∪ (B∩C) & (A∩B) ∪C= (A∪C) ∩ (B∪C)
e) A∩A=A & A∪A=A
f) De Morgan's Law for A& B ie, $(A\cup B)^{c}=A^{c}\cap B^{c}$ & $(A\cap B)^{c} = A^{c} \cup B^{c}$
g) (A⊕B) = (B⊕A)
h) (A⊗ B) = (B ⊗A)

Proof: (a) A ⊆ B means ($\forall x \in U$, $\alpha_A(x) \le \alpha_B(x)$ and $\beta_A(x) \le \beta_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$) and B ⊆ C means ($\forall x \in U$, $\alpha_B(x) \le \alpha_C(x)$ and $\beta_B(x) \le \beta_C(x)$ and $\gamma_B(x) \ge \gamma_C(x)$).

ie, $\forall x \in U$, $\alpha_A(x) \le \alpha_B(x) \le \alpha_C(x)$ and $\beta_A(x) \le \beta_B(x) \le \beta_C(x)$ and $\gamma_A(x) \ge \gamma_B(x) \ge \gamma_C(x)$.

Proof of b, c, d, e &f hold trivially.

g) $(A\oplus B)(x) = \{\langle x, (\frac{\alpha_A(x).\alpha_B(x)}{\alpha_A(x)+\alpha_B(x)}, \max(\beta_A(x), \beta_B(x)), \min(\gamma_A(x),\gamma_B(x))\rangle\}$

$= (B\oplus A)(x)$

Thus (A⊕B) = (B⊕A)

h) $(A\otimes B)(x) = \{\langle x, (\alpha_A(x).\alpha_B(x), \beta_A(x).\ \beta_B(x), \gamma_A(x).\gamma_B(x))\rangle\}$

$=\{\langle x, (\alpha_B(x).\alpha_A(x), \beta_B(x).\ \beta_A(x), \gamma_B(x).\gamma_A(x))\rangle\} = (B\otimes A)(x)$

Thus (A⊗ B) = (B ⊗A)

Example: 6.2: Consider the n- cylindrical fuzzy neutrosophic sets A, B, C in U= {x}.

A=⟨ x, 0.5, 0.5, 0.5⟩ , B=⟨ x, 0.1, 0.6, 0.5⟩ , C=⟨ x, 0.6, 0.7, 0.5⟩

A∪B=⟨ x, 0.5, 0.5, 0.5⟩ (by definition)

A∩B= ⟨ x, 0.1, 0.5, 0.5⟩

(A∪B) ∪C=⟨ x, 0.6, 0.5, 0.5⟩ = A∪ (B∪C)

(A∩B) ∩C=⟨ x, 0.1, 0.5, 0.5⟩ = A∩ (B∩C)

(A∪B) ∩C=⟨ x, 0.5, 0.5, 0.5⟩ = (A∩C) ∪ (B∩C)

(A∩B)∪C= ⟨ x, 0.6, 0.5, 0.5⟩ = (A∪C) ∩ (B∪C)

$(A\cup B)^{c}$=⟨ x, 0.5, 0.5, 0.5⟩ = $A^{c}\cap B^{c}$

Doi : https://doi.org/10.54216/IJNS.180430
Received: March 30, 2022 Accepted: July 14, 2022

$(A \cap B)^{c}$ = ⟨ x, 0.5 0.5, 0.1 ⟩ = $A^{c} \cup B^{c}$

Definition 6.3:

Let U_1 and U_2 be two universes and let A= {⟨ x, $\alpha_A(x)$, $\beta_A(x)$, $\gamma_A(x)$⟩ | x∈ U_1} and B= {⟨ y, $\alpha_B(y)$, $\beta_B(y)$, $\gamma_B(y)$⟩ | y∈ U_2} be two n- CyFNSs. The **product** of these two n-CyFNSs is defined as follows

A×B = {⟨ (x, y): ($\alpha_A(x) + \alpha_B(y) - \alpha_A(x).\alpha_B(y)$), $\beta_A(x).\beta_B(y)$, $\gamma_A(x).\gamma_B(y)$⟩ | x∈ U_1, y∈ U_2}

Example 6.4: Let X= {a, b} and Y = {p, q} be two non-empty sets, and

A ⊆ X, A= {⟨ a, 0.5,1, 0.5⟩,⟨ b, 0.4,0.3,0.6⟩}and

B⊆ Y, B = { ⟨ p, 0.3,0.6,0.7⟩,⟨ q, 0.5,1,0.5⟩}. Then

A×B = {⟨ (a,p),0.65,0.6,0.35⟩,⟨ (a,q),0.75,1,0.25⟩,

⟨ (b,p),0.58,0.18,0.42⟩,⟨ (b,q),0.7,0.3,0.3⟩}

Height of A×B is H (A×B) = 1,

Preposition: 6.5:
A×B=B×A iff A=B
(A×B) × C = A× (B×C)
(A∪B) ×C= (A×C) ∪ (B×C)
(A∩B) ×C= (A×C) ∩ (B×C)
Proof: Trivial

Example: 6.6: Consider the n-cylindrical fuzzy neutrosophic sets A, B, C in U= {x}.

A=⟨ x, 0.5, 0.5, 0.5⟩, B=⟨ x, 0.1, 0.6, 0.9⟩, C=⟨ x, 0.6, 0.7, 0.4⟩

A×B = ⟨ (x, x), 0.55, 0.3, 0.45⟩

(A×B)×C=⟨ (x, x, x), 0.82, 0.21, 0.18⟩ = A× (B×C)

(A∪B)×C=⟨ (x, x), 0.8, 0.35, 0.2⟩ = (A×C) ∪ (B×C)

(A∩B)×C= ⟨ (x, x), 0.64, 0.35, 0.36⟩ = (A×C) ∩ (B×C)

Definition: 6.7: Score function of an n-CyFNS

For any n-CyFNN A=⟨ α_A, β_A, γ_A⟩, the score function of A ($\varphi(A)$) is defined as $\varphi(A) = \frac{[\alpha_A^n + \beta_A^n + \gamma_A^n - \wp_A^n]}{\wp_A}$, where $\varphi(A) \in [0,1]$.

Result: 6.8

If $\varphi(A) < \varphi(B)$, then A≺ B and

If $\varphi(A) = \varphi(B)$, then A∼ B

Example: 6.9

Let A = ⟨ x, 1, 1, 0 ⟩ and B=⟨ x, 0, 0, 1 ⟩

$\varphi(A)$=1

$\varphi(B) = 0$

Here, $\varphi(A) < \varphi(B)$ then A≺ B.

Doi : https://doi.org/10.54216/IJNS.180430
Received: March 30, 2022 Accepted: July 14, 2022

7. Neutrosophic Affinity Degree & Neutrosophic Similarity Index

One of the important feature in the fuzzy set theory is the comparison rules. These rules play a key role in finding the difference or strength between two fuzzy numbers. Comparison rules for n-CyFNSs are defined below.

7.1 Neutrosophic Affinity Degree

Definition 7.1.1: The Neutrosophic affinity degree of an n- CyFNS, A, is a real valued function, denoted as $\mathbf{\Gamma_N}$, defined as $\Gamma_N(A) = \frac{\sum[\wp(x_i)-\overline{\wp}(x_i)]}{1+\sum[\wp(x_i)-\overline{\wp}(x_i)]}$, $x_i \in$A; $\overline{\wp}(x_i)$ is the mean peak value of x_i

Remark 7.1.2:

The Neutrosophic affinity degree of a n-RCyFNS, A, is a real valued function, is $\Gamma_N(A) = \frac{\sum[h(x_i)-\overline{h}(x_i)]}{1+\sum[h(x_i)-\overline{h}(x_i)]}$, x_i $\in$A and $\overline{h}(x_i)$ is the mean height of x_i.

Definition 7.1.3 The Neutrosophic affinity degree between two n-CyFNSs, A & B is defined as

$$\mathbf{\Gamma_N(A, B)} = \frac{\sum[\wp(x_i)-\overline{\wp}(x_i)][\wp(y_i)-\overline{\wp}(y_i)]}{1+\sum[\wp(x_i)-\overline{\wp}(x_i)[\wp(y_i)-\overline{\wp}(y_i)]}, x_i \in A, y_i \in B$$

Remark: 7.1.4:

The Neutrosophic affinity degree between two n- RCyFNSs, A & B is

$$\mathbf{\Gamma_N(A, B)} = \frac{\sum[h(x_i)-\overline{h}(x_i)][h(y_i)-\overline{h}(y_i)]}{1+\sum[h(x_i)-\overline{h}(x_i)[h(y_i)-\overline{h}(y_i)]}, x_i \in A, y_i \in B$$

Result 7.1.5:

Let A, B are two n-CyFNSs, and if $\mathbf{\Gamma_N(A) \leq \Gamma_N(B)}$, then A is coarser than B or B is finer than A

Example: 7.1.6:

Let X={p,q,r} and A, B ∈ □$_N$ (U), where A={⟨p,0.6,1,0.3⟩,⟨q,0.4,0.3,0.5⟩,⟨r,0.3,0.8,0.6⟩} and B={⟨p, 0.4,0.3,0.5⟩,⟨q, 0.7,0.3,0.4⟩,⟨r, 0.8,0.5,0.1⟩}

$$\mathbf{\Gamma_N(A)} = \frac{\sum[\wp(x_i)-\overline{\wp}(x_i)]}{1+\sum[\wp(x_i)-\overline{\wp}(x_i)]} = \mathbf{0.01}$$

$$\mathbf{\Gamma_N(B)} = \frac{\sum[\wp(x_i)-\overline{\wp}(x_i)]}{1+\sum[\wp(x_i)-\overline{\wp}(x_i)]} = \mathbf{-0.01}$$

Here $\Gamma_N(B) \leq \Gamma_N(A)$, here B is coarser than A.

Definition 7.1.7:

The Neutrosophic weighted affinity degree of an n-CyFNS, A, is a real valued function denotes as $\Gamma_{N\omega}$ and is defined as $\Gamma_{N\omega}(A) = \frac{\sum \omega_i\{[\wp(x_i)-\overline{\wp}(x_i)]\}}{1+\sum \omega_i\{[\wp(x_i)-\overline{\wp}(x_i)]\}}$ $x_i \in$A, where $(\omega_1, \omega_2, \ldots \omega_n)^T$ be the corresponding weight vector of n-CyFNS A and $\omega_i > 0$ and $\sum \omega_i = 1$

Definition 7.1.8:

The Neutrosophic weighted affinity degree between two n- CyFNSs, A &B is defined as

$$\Gamma_{N\omega}(A,B) = \frac{\sum \omega_i\{[\wp(x_i)-\overline{\wp}(x_i)][\wp(y_i)-\overline{\wp}(y_i)]\}}{1+\sum \omega_i\{[\wp(x_i)-\overline{\wp}(x_i)][\wp(y_i)-\overline{\wp}(y_i)]\}}, x_i \in A, y_i \in B$$

where $(\omega_1, \omega_2, \ldots \omega_n)^T$ be the corresponding weight vector of n-CyFNS A and $\omega_i > 0$ and $\sum \omega_i = 1$

Doi : https://doi.org/10.54216/IJNS.180430
Received: March 30, 2022 Accepted: July 14, 2022

Result: 7.1.9:

Let A, B are two n-CyFNSs, and if $\Gamma_{N\omega}$**(A)** ≤ $\Gamma_{N\omega}$**(B),** then A is coarser than B or B is finer than A.

Result: 7.1.10:

Properties of Neutrosophic affinity degree

i)a) $-1 \leq \Gamma_N(A) \leq 1$
b) $-1 \leq \Gamma_{N\omega}(A) \leq 1$
ii)a) $\Gamma_N(A,B) = \Gamma_N(B,A)$
b) $\Gamma_{N\omega}(A,B) = \Gamma_{N\omega}(B,A)$
iii)a) $\Gamma_N(A,B) \neq \Gamma_N(A).\Gamma_N(B)$
b) $\Gamma_{N\omega}(A,B) \neq \Gamma_{N\omega}(A).\Gamma_{N\omega}(B)$

Proof: From the definition of Neutrosophic affinity degree, it follows.

7.2 Neutrosophic Similarity Index

Correlation is important in statistics and engineering sciences because it allows the joint relationship of two variables to be examined using a measure of interdependence of the two variables. Here a different approach is made in measuring the correlation using the highest membership value.

Definition: 7.2.1: The Neutrosophic similarity index (NSI) of two n-CyFNS, A & B is given by

$$\zeta_N(A,B) = \frac{\Gamma_N(A,B)}{\sqrt{\Gamma_N(A,A).\Gamma_N(B,B)}}.$$

Remark: 7.2.2:

Using Neutrosophic similarity index of n-CyFNSs, we can determine the correlation among objects.

Preposition: 7.2.3: The Neutrosophic similarity index (NSI) between A& B holds the following conditions

i) $-1 \leq \zeta_N(A,B) \leq 1$
ii) $\zeta_N(A,B) = \zeta_N(B,A)$

iii) $\zeta_N(A,B)=1$ **if only if A=B &** $\wp(x_i) = h(x_i)$ & $\wp(y_i) = h(y_i)$ for all $x_i \in A$, $y_i \in B$

In other words, $\zeta_N(A,B)=1$ **if only if A=B and A &B are n-RCyFNS.**

Proof: i) Clearly from the definition of $\zeta_N(A,B)$**, it holds**

$$\zeta_N(A,B) = \frac{\Gamma_N(A,B)}{\sqrt{\Gamma_N(A,A).\Gamma_N(B,B)}}.$$

$$= \frac{\Gamma_N(B,A)}{\sqrt{\Gamma_N(A,A).\Gamma_N(B,B)}}.$$

$$= \zeta_N(B,A)$$

$$\zeta_N(A,B) = \frac{\Gamma_N(A,B)}{\sqrt{\Gamma_N(A,A).\Gamma_N(B,B)}}$$

$$= \frac{\sum[\wp_A(x_i) - \bar{\wp}(x_i)].[\wp_B(x_i) - \bar{\wp}(x_i)]}{1 + \sum[\wp_A(x_i) - \bar{\wp}(x_i)].[\wp_B(x_i) - \bar{\wp}(x_i)]} \Bigg/ \sqrt{\frac{\sum[\wp_A(x_i) - \bar{\wp}(x_i)]^2}{1 + \sum[\wp_A(x_i) - \bar{\wp}(x_i)]^2} \times \frac{\sum[\wp_B(x_i) - \bar{\wp}(x_i)]^2}{1 + \sum[\wp_B(x_i) - \bar{\wp}(x_i)]^2}}$$

Doi : https://doi.org/10.54216/IJNS.180430
Received: March 30, 2022 Accepted: July 14, 2022

if A=B then,

$$= \frac{\sum[\wp_A(x_i) - \bar{\wp}(x_i)].[\wp_A(x_i) - \bar{\wp}(x_i)]}{1 + \sum[\wp_A(x_i) - \bar{\wp}(x_i)].[\wp_A(x_i) - \bar{\wp}(x_i)]} \Big/ \sqrt{\frac{\sum[\wp_A(x_i) - \bar{\wp}(x_i)]^2}{1 + \sum[\wp_A(x_i) - \bar{\wp}(x_i)]^2} \times \frac{\sum[\wp_A(x_i) - \bar{\wp}(x_i)]^2}{1 + \sum[\wp_A(x_i) - \bar{\wp}(x_i)]^2}}$$

$$= \frac{\Gamma_N\ (A,A)}{\sqrt{\Gamma_N\ (A,A).\Gamma_N\ (A,A)}}$$

$$= 1$$

Hence proved.

Conversely if ζ_N (A,B)=1 then

$$\frac{\sum[\wp_A(x_i) - \bar{\wp}(x_i)].[\wp_B(x_i) - \bar{\wp}(x_i)]}{1 + \sum[\wp_A(x_i) - \bar{\wp}(x_i)].[\wp_B(x_i) - \bar{\wp}(x_i)]} \Big/ \sqrt{\frac{\sum[\wp_A(x_i) - \bar{\wp}(x_i)]^2}{1 + \sum[\wp_A(x_i) - \bar{\wp}(x_i)]^2} \times \frac{\sum[\wp_B(x_i) - \bar{\wp}(x_i)]^2}{1 + \sum[\wp_B(x_i) - \bar{\wp}(x_i)]^2}} = 1$$

$$\frac{\sum[\wp_A(x_i)-\bar{\wp}(x_i)].[\wp_B(x_i)-\bar{\wp}(x_i)]}{1+\sum[\wp_A(x_i)-\bar{\wp}(x_i)].[\wp_B(x_i)-\bar{\wp}(x_i)]} = \sqrt{\frac{\sum[\wp_A(x_i)-\bar{\wp}(x_i)]^2}{1+\sum[\wp_A(x_i)-\bar{\wp}(x_i)]^2} \times \frac{\sum[\wp_B(x_i)-\bar{\wp}(x_i)]^2}{1+\sum[\wp_B(x_i)-\bar{\wp}(x_i)]^2}} \quad ....1$$

Equation 1 is possible only when $\wp(x_i) = h(x_i)$ & $\wp(y_i) = h(y_i)$ for all $x_i \in$ A, $y_i \in$B and A=B.

That is A & B are n-RCyFNSs. Hence proved.

Definition: 7.2.4:

Let A & B are two n-CyFNSs, ω be the corresponding weight vector then the Neutrosophic weighted similarity index between A & B is given by

$\zeta_N\omega$ (A, B) = $\frac{\Gamma_{N\omega}\ (A,B)}{\sqrt{\Gamma_{N\omega}\ (A,A).\Gamma_{N\omega}\ (B,B)}}$

Preposition:**7.2.5 :** The Neutrosophic similarity index (NSI) between A& B follows the conditions

i) $-1 \le \zeta_N\omega$ **(A,B)** ≤ 1

ii) $\zeta_N\omega$ (A,B) = $\zeta_N\omega$ (B,A)

iii)$\zeta_N\omega$ (A,B)= 1 if and only if A=B and $\wp(x_i) = h(x_i)$ & $\wp(y_i) = h(y_i)$ for all $x_i \in$A , $y_i \in$B

or **$\zeta_N\omega$ (A,B)= 1 if and only if A=B and A & B are n-RCyFNS.**

Proof: i Clearly it is evident from the definition of **$\zeta_N\omega$ (A,B)**

Proof: ii) $\zeta_N\omega$ (A,B) = $\frac{\Gamma_{N\omega}\ (A,B)}{\sqrt{\Gamma_{N\omega}\ (A,A).\Gamma_{N\omega}\ (B,B)}}$

$= \frac{\Gamma_{N\omega}\ (B,A)}{\sqrt{\Gamma_{N\omega}\ (A,A).\Gamma_{N\omega}\ (B,B)}}$

= **$\zeta_N\omega$ (B,A)**

Proof: iii) $\zeta_N\omega$ (A,B) = $\frac{\Gamma_{N\omega}\ (A,B)}{\sqrt{\Gamma_{N\omega}\ (A,A).\Gamma_{N\omega}\ (B,B)}}$

Doi : https://doi.org/10.54216/IJNS.180430
Received: March 30, 2022 Accepted: July 14, 2022

$$= \frac{\sum \omega_i\{[\wp_A(x_i)-\bar{\wp}(x_i)].[\wp_B(x_i)-\bar{\wp}(x_i)]\}}{1+\sum \omega_i\{[\wp_A(x_i)-\bar{\wp}(x_i)].[\wp_B(x_i)-\bar{\wp}(x_i)]\}} \Bigg/ \sqrt{\frac{\sum \omega_i\{[\wp_A(x_i)-\bar{\wp}(x_i)]^2\}}{1+\sum \omega_i\{[\wp_A(x_i)-\bar{\wp}(x_i)]^2\}} \times \frac{\sum \omega_i\{[\wp_B(x_i)-\bar{\wp}(x_i)]^2\}}{1+\sum \omega_i\{[\wp_B(x_i)-\bar{\wp}(x_i)]^2\}}}$$

if A=B then,

$$= \frac{\sum \omega_i\{[\wp_A(x_i)-\bar{\wp}(x_i)].[\wp_A(x_i)-\bar{\wp}(x_i)]\}}{1+\sum \omega_i\{[\wp_A(x_i)-\bar{\wp}(x_i)].[\wp_A(x_i)-\bar{\wp}(x_i)]\}} \Bigg/ \sqrt{\frac{\sum \omega_i\{[\wp_A(x_i)-\bar{\wp}(x_i)]^2\}}{1+\sum \omega_i\{[\wp_A(x_i)-\bar{\wp}(x_i)]^2\}} \times \frac{\sum \omega_i\{[\wp_A(x_i)-\bar{\wp}(x_i)]^2\}}{1+\sum \omega_i\{[\wp_A(x_i)-\bar{\wp}(x_i)]^2\}}}$$

$$= \frac{\Gamma_{N\omega}\,(A,A)}{\sqrt{\Gamma_{N\omega}\,(A,A).\Gamma_{N\omega}\,(A,A)}}$$

= 1

Hence proved.

Conversely suppose $\zeta_N\omega$ (A,B)= 1, then

$$\frac{\sum \omega_i\{[\wp_A(x_i)-\bar{\wp}(x_i)].[\wp_B(x_i)-\bar{\wp}(x_i)]\}}{1+\sum \omega_i\{[\wp_A(x_i)-\bar{\wp}(x_i)].[\wp_B(x_i)-\bar{\wp}(x_i)]\}} \Bigg/ \sqrt{\frac{\sum \omega_i\{[\wp_A(x_i)-\bar{\wp}(x_i)]^2\}}{1+\sum \omega_i\{[\wp_A(x_i)-\bar{\wp}(x_i)]^2\}} \times \frac{\sum \omega_i\{[\wp_B(x_i)-\bar{\wp}(x_i)]^2\}}{1+\sum \omega_i\{[\wp_B(x_i)-\bar{\wp}(x_i)]^2\}}} = 1$$

$$\frac{\sum \omega_i\{[\wp_A(x_i)-\bar{\wp}(x_i)].[\wp_B(x_i)-\bar{\wp}(x_i)]\}}{1+\sum \omega_i\{[\wp_A(x_i)-\bar{\wp}(x_i)].[\wp_B(x_i)-\bar{\wp}(x_i)]\}} = \sqrt{\frac{\sum \omega_i\{[\wp_A(x_i)-\bar{\wp}(x_i)]^2\}}{1+\sum \omega_i\{[\wp_A(x_i)-\bar{\wp}(x_i)]^2\}} \times \frac{\sum \omega_i\{[\wp_B(x_i)-\bar{\wp}(x_i)]^2\}}{1+\sum \omega_i\{[\wp_B(x_i)-\bar{\wp}(x_i)]^2\}}} \quad \ldots\ldots\ldots\ldots 2$$

The equation 2 is possible only when when $\wp(x_i) = h(x_i)$ & $\wp(y_i) = h(y_i)$ for all $x_i \in$ A, $y_i \in$B and A=B.

That is A & B are n-RCyFNSs.

Hence proved.

8: Application:

Here we take two examples from [16] to measure the similarity between variables using Neutrosophic similarity index.

8.1: Medical Diagnosis Problem [16]

Consider a diagnosis set, *G* = {*V* (*Viral fever*), M (*Malaria*), *T* (*Typhoid*),

S (*Stomach Upset*)} and symptoms S = {Temperature- s1, Head Ache-s2, Stomach Ache-s3, Cough-s4, Chest-congestion- s5}. Suppose that a patient, having symptoms, is depicted as follows.

Patient = {[s1; (0.7, 0.3, 0.4)], [s2 ;(0.6, 0.5, 0.2)], [s3; (0.6, 0.1, 0.3)], [s4 ;(0.6, 0.4, 0.2)], [s5 ;(0.7, 0.4, 0.2)]}

Table 1: Relation between symptoms and diseases.

	S1-temperature	**S2-Head Ache**	**S3-stomach ache**	**S4-cough**	**S5-chest congestion**
Viral fever-V	**(0.6, .4, .1)**	**(0.6, 0.3,0.4)**	**(0.5,0.4,0.2)**	**(0.4,0.2,0.3)**	**(0.8,0.4,0.3)**
Malaria-M	**(0.7,0.3, 0.2)**	**(0.9.0.3,0.1)**	**(0.7 ,0.6, 0.2)**	**(0.5 , 0.4, 0.2)**	**(0.4,0.3,0.3)**
Typhoid-T	**(0.8,0.4,0.3)**	**(0.5,0.3,0.4)**	**(0.7,0.5,0.4)**	**(0.8,0.3,0.5)**	**(0.7,0.3,0.2)**
Stomach Upset-S	**(0.8,0.3,0.5)**	**(0.6,0.3,0.2)**	**(0.8,0.5,0.2)**	**(0.6,0.6,0.1)**	**(0.8,0.4,0.4)**

Doi : https://doi.org/10.54216/IJNS.180430
Received: March 30, 2022 Accepted: July 14, 2022

Solution:

Clearly all values belongs to n-CyFNSs

The weight of symptoms S_i, i={1,2,3,4,5}, is ω_i = [0.124, 0.216, 0.274, 0.154, 0.232]^T.

Step: 1 Finding peak of each variable of the set and its mean peak.

Step: 2 Finding Neutrosophic affinity degree between patient and each infection.

Step: 3 Calculate Neutrosophic similarity index and compare the results.

Let d(X) = $\wp(X) - \overline{\wp(X)}$

Table 2: Finding peak of each variable of the set and its mean peak.

℘ (V)	℘ (M)	℘ (T)	℘ (S)	℘ (P)	**d(V)**	**d(M)**	**d(T)**	**d(S)**	**d(P)**
0.6	0.7	0.8	0.8	0.7	**0.02**	**0.06**	**0.1**	**0.08**	**0.06**
0.6	0.9	0.5	0.6	0.6	**0.02**	**0.26**	**-0.2**	**-0.12**	**-0.04**
0.5	0.7	0.7	0.8	0.6	**-0.08**	**0.06**	**0**	**0.08**	**-0.04**
0.4	0.5	0.8	0.6	0.6	**-0.18**	**-0.14**	**0.1**	**-0.12**	**-0.04**
0.8	0.4	0.7	0.8	0.7	**0.28**	**-0.24**	**0**	**0.08**	**0.06**

Table 3: Calculate Neutrosophic similarity index and compare the results.

	d(V).d(P)	**d(M).d(P)**	**d(T).d(P)**	**d(S).d(P)**	**$[d(V)]^2$**	**$[d(M)]^2$**	**$[d(T)]^2$**	**$[d(S)]^2$**	**$[d(P)]^2$**
	0.0012	**0.0036**	**0.006**	**0.0048**	**0.0004**	**0.0036**	**0.01**	**0.0064**	**0.0036**
	-0.0008	**-0.0104**	**0.008**	**0.0048**	**0.0004**	**0.0676**	**0.04**	**0.0144**	**0.0016**
	0.0032	**0.0024**	**0**	**-0.0032**	**0.0064**	**0.0036**	**0**	**0.0064**	**0.0016**
	0.0072	**0.0056**	**-0.004**	**0.0048**	**0.0324**	**0.0196**	**0.01**	**0.0144**	**0.0016**
	0.0132	**-0.0144**	**0**	**0.0048**	**0.0484**	**0.0576**	**0**	**0.0064**	**0.0036**
sum	**0.024**	**- 0.0132**	**0.01**	**0.016**	**0.088**	**0.152**	**0.06**	**0.048**	**0.012**

Table 4: Neutrosophic affinity degree between variables

Γ_N (P, V)	Γ_N(P, M)	Γ_N(P, T)	Γ_N(P, S)	Γ_N (V, V)	Γ_N(M, M)	Γ_N(T, T)	Γ_N(S, S)	Γ_N (P, P)
0.0234	**- 0.01337**	**0.0099**	**0.0157**	**0.081**	**0.1319**	**0.0566**	**0.046**	**0.0118**

Neutrosophic similarity index between variables is given below

Doi : https://doi.org/10.54216/IJNS.180430
Received: March 30, 2022 Accepted: July 14, 2022

Table 5: Neutrosophic similarity index between variables

ζ_N (P,V)	ζ_N(P,M)	ζ_N(P, T)	ζ_N(P, S)
0.757	**- 0.1072**	**0.383**	**0.674**

Here ζ_N (P,V) > ζ_N(P,S) > ζ_N(P,T) > ζ_N(P,M)

Hence we can conclude that the patient has viral fever.

Next, we find Neutrosophic weighted similarity index, ($\zeta_N\omega$) between variables,

Table 6 :Finding Neutrosophic weighted similarity index, ($\zeta_N\omega$) between variables,

	ω_id(V).d(P)	ω_i d(M).d(P)	ω_i d(T).d(P)	ω_i d(S).d(P)	ω_i [d(V)]^2	ω_i [d(M)]^2	ω_i [d(T)]^2	ω_i [d(S)]^2	ω_i [d(P)]^2
	0.000149	0.000446	0.000744	0.000595	0.0000496	0.000446	0.00124	0.000794	0.000446
	-0.00017	-0.00225	0.001728	0.001037	0.0000864	0.014602	0.00864	0.00311	0.000346
	0.000877	0.000658	**0**	-0.00088	0.0017536	0.000986	0	0.001754	0.000438
	0.001109	0.000862	-0.00062	0.000739	0.0049896	0.003018	0.00154	0.002218	0.000246
	0.003062	-0.00334	**0**	0.001114	0.0112288	0.013363	0	0.001485	0.000835
sum	0.005024	-0.00362	0.001856	0.002608	0.018108	0.032416	0.01142	0.00936	0.002312

Table 7: Neutrosophic weighted affinity degree, ($\zeta_N\omega$) between variables,

$\Gamma_{N\omega}$ (P, V)	$\Gamma_{N\omega}$ (P, M)	$\Gamma_{N\omega}$ (P, T)	$\Gamma_{N\omega}$ (P, S)	$\Gamma_{N\omega}$ (V, V)	$\Gamma_{N\omega}$ (M, M)	$\Gamma_{N\omega}$ (T, T)	$\Gamma_{N\omega}$ (S, S)	$\Gamma_{N\omega}$(P, P)
0.005	**- 0.00363**	**0.00185**	**0.0026**	**0.0178**	**0.0314**	**0.0113**	**0.0093**	**0.00229**

Table 8: Neutrosophic weighted similarity index, ($\zeta_N\omega$) between variables

$\zeta_N\omega$ (P,V)	$\zeta_N\omega$ (P,M)	$\zeta_N\omega$(P, T)	$\zeta_N\omega$ (P, S)
0.783	**- 0.428**	**0.364**	**0.563**

Here also $\zeta_N\omega$ (P,V) > $\zeta_N\omega$ (P, S) > $\zeta_N\omega$(P, T) > $\zeta_N\omega$ (P,M) , we reached at the same conclusion that the patient has viral fever.

8.2 Selection of optimal gold furnace problem [16]

Assume a company intends to invest in a gold furnace. When compared to other furnaces, any furnace with a properly functioning cleaner production unit will profit. The cleaner production units G1, G2, G3, and G4 are evaluated by the company based on five criteria, which are as follows:

S = {S1 (Administration), S2 (Manufacturing), S3 (Assets), S4 (Waste sustainability), S5 (Environment)}

Doi : https://doi.org/10.54216/IJNS.180430
Received: March 30, 2022 Accepted: July 14, 2022

The weight vector of Si [i = 1, 2, 3,4, 5] is ω_i = [0.124, 0.216, 0.274, 0.154, 0.232]^T.

There is an unknown production furnace, G, with data as shown in Table 10. The data, in Tables 9 and 10, is utilized to calculate the Neutrosophic similarity index and Neutrosophic weighted similarity index, to determine which type the unknown production furnace G belongs to.

Table 9: Ratings given to cleaner production unit based on the given criteria.

	G1	G2	G3	G4
Administration	[0.5,0.3, 0.4]	[0.7,0.6,0.2]	[0.8,0.4,0.4]	[0.7,0.2,0.1]
Manufacturing	[0.8,0.4,0.3]	[0.9,0.3,0.1]	[0.8,0.5,0.2]	[0.6,0.3,0.4]
Assets	[0.7,0.5,0.4]	[0.7,0.3,0.2]	[0.6,0.6,0.1]	[0.5,0.4,0.2]
Waste sustainability	[0.7,0.3,0.2]	[0.4,0.3,0.3]	[0.6,0.3,0.2]	[0.4,0.2,0.3]
Environment	[0.8,0.1,0.1]	[0.5,0.4,0.2]	[0.8,0.3,0.5]	[0.8,0.4,0.3]

Table10: Data for unknown gold furnace G

	G
Administration	(0.7, 0.3, 0.4)
Manufacturing	(0.6, 0.5, 0.2)
Assets	(0.6, 0.1, 0.3)
Waste sustainability	(0.6, 0.4, 0.2)
Environment	(0.7, 0.4, 0.2)

SOLUTION

Clearly all the given values follows n- CyFNSs with n=2.

Step: 1 Finding peak of each variable of the set and its mean peak.

Step: 2 Finding Neutrosophic affinity degree between G and each Gi.

Step: 3 Calculate Neutrosophic similarity index and compare the results.

Let d(X) = $\wp(X) - \overline{\wp(X)}$

Table 11 : Finding peak of each variable of the set and its mean peak.

$\wp$ (G1)	$\wp$ (G2)	$\wp$ (G3)	$\wp$ (G4)	$\wp$ (G)	d(G1)	d(G2)	d(G3)	d(G4)	d(G)
0.5	0.7	0.8	0.7	0.7	-0.2	0.06	0.08	0.1	0.06
0.8	0.9	0.8	0.6	0.6	0.1	0.16	0.08	0	-0.04
0.7	0.7	0.6	0.5	0.6	0	0.06	-0.12	-0.1	-0.04
0.7	0.4	0.6	0.4	0.6	0	-0.24	-0.12	-0.2	-0.04
0.8	0.5	0.8	0.8	0.7	0.1	-0.14	0.08	0.2	0.06

Doi : https://doi.org/10.54216/IJNS.180430
Received: March 30, 2022 Accepted: July 14, 2022

Table 12: Finding Neutrosophic affinity degree between G and each Gi.

d(G1).d(G)	d(G2).d(G)	d(G3).d(G)	d(G4).d(G)	d^2(G1)	d^2(G2)	d^2(G3)	d^2(G4)	d^2(G)
-0.012	0.0036	0.0048	0.006	0.04	0.0036	0.0064	0.01	0.0036
-0.004	-0.0024	0.0048	0.004	0.01	0.0256	0.0064	0	0.0016
0	-0.0048	-0.0032	0	0	0.0036	0.0144	0.01	0.0016
0	0.0096	0.0048	0.008	0	0.0576	0.0144	0.04	0.0016
0.006	0.0048	0.0048	0.012	0.01	0.0196	0.0064	0.04	0.0036
-0.01	0.0108	0.016	0.03	0.06	0.11	0.048	0.1	0.012

Table 13: Neutrosophic affinity degree between variables

Γ_N (G1, G)	Γ_N(G2, G)	Γ_N(G3, G)	Γ_N(G4, G)	Γ_N (G1, G1)	Γ_N(G2, G2)	Γ_N(G3, G3)	Γ_N(G4, G4)	Γ_N (G, G)
-0.010	0.011	0.016	0.029	0.057	0.099	0.046	0.091	0.012

The Neutrosophic similarity index between variables is given below.

Table 14: Neutrosophic similarity index between variables

ζ_N (G1,G)	**ζ_N(G2,G)**	**ζ_N(G3, G)**	**ζ_N(G4, G)**
0.382	**0.319**	**0.681**	**0.878**

Here **ζ_N(G4, G)** > **ζ_N(G3, G)** > **ζ_N(G2,G)** > **ζ_N (G1,G)**

Hence we obtain the same conclusion as in [16],that the unknown production furnace G belongs to G4.

Next calculate, the Neutrosophic weighted similarity index of the same.

Given weight vectors are ω_i= (0.124, 0.216, 0.274, 0.154, 0.232)^T.

Table 15: the Neutrosophic weighted similarity index of the same.

	ω_i[d(G1).d(G)]	ω_i [d(G2).d(G)]	ω_i[d(G3).d(G)]	ω_i[d(G4).d(G)]	ω_i[d^2(G]1)]	ω_i[d^2(G2)]	ω_i[d^2(G3)]	ω_i[d^2(G4)]	ω_i[d^2(G)]
	-0.00149	0.0004464	0.0005952	0.000744	0.00496	0.000446	0.000794	0.00124	0.000446
	-0.00086	-0.000518	0.0010368	0.000864	0.00216	0.00553	0.001382	0	0.000346
	0	-0.001315	-0.000877	0	0	0.000986	0.003557	0.00274	0.000438
	0	0.0014784	0.0007392	0.001232	0	0.00887	0.002218	0.00616	0.000246
	0.001392	0.0011136	0.0011136	0.002784	0.00232	0.004547	0.001485	0.00928	0.000835
sum	-0.00096	0.0012048	0.0032032	0.005624	0.00944	0.02038	0.009435	0.01942	0.00231

Table:16 : Neutrosophic weighted affinity degree between variables.

$\Gamma_{N\omega}$ (G1, G)	$\Gamma_{N\omega}$(G2, G)	$\Gamma_{N\omega}$(G3, G)	$\Gamma_{N\omega}$(G4, G)	$\Gamma_{N\omega}$(G1, G1)	$\Gamma_{N\omega}$(G2, G2)	$\Gamma_{N\omega}$(G3, G3)	$\Gamma_{N\omega}$(G4, G4)	$\Gamma_{N\omega}$ (G, G)

Doi : https://doi.org/10.54216/IJNS.180430
Received: March 30, 2022 Accepted: July 14, 2022

-0.001	0.001	0.003	0.006	0.009	0.020	0.009	0.019	0.002

Then the Neutrosophic weighted similarity index of the same is calculated as

Table 17: Neutrosophic weighted similarity index between variables.

$\zeta_N\omega$ (G1,G)	**$\zeta_N\omega$ (G2,G)**	**$\zeta_N\omega$(G3, G)**	**$\zeta_N\omega$ G4, G)**
-0.236	**0.158**	**0.707**	**0.973**

From the table it is clear that the unknown production furnace G is G4.

Conclusion:

There is a degree of dependence (also the degree of independence) between the fuzzy components, as well as the neutrosophic components. In this paper we introduced a new type of set called n-Cylindrical fuzzy Neutrosophic set with I as independent Neutrosophic component. It can be claimed to be the largest of all the fuzzy extensions except fuzzy neutrosophic sets. Along with basic operations on n- CyFNS, distance between two n-CyFNSs, correlation between two n-CyFNSs are also introduced. The scope of this study is promising and can be extended it to many areas.

Funding: "This research received no external funding"

Conflicts of Interest: "The authors declare no conflict of interest."

References

[1]. Ashraf S, Abdullah S, Spherical aggregation operators and their application in multi attribute group decision-making, Int J Intell Syst 34(3):493–523 (2019)

[2]. C. Antony Crispin Sweety and R.Jhansi, Fermatean Neutrosophic sets, International Journal of Advanced Research in Computer and Communication Engineering, Vol. 10, Issue 6 (2021) DOI 10.17148/IJARCCE.2021.10605

[3]D.Ajay, P.Chellamani, Pythagorean Neutrosophic Fuzzy Graphs, International journal of neutrosophic science, Vol. 11, No. 2, PP. 108-114, 2020

[4]. I.Arokiarani, J.Martina Jency, More on fuzzy Neutrosophic sets and fuzzy Neutrosophic topological spaces, International Journal of innovative research & studies, 3(5),642-652, 2014.

[5] .I. Arokiarani, I.R Sumati, J.Martina Jency,Fuzzy Neutrosophic soft topological spaces, International Journal of Mathematical Archives, 4(10), 225-238, 2013.

[6]. Atanassov,K. Intuitionistic fuzzy sets, Fuzzy Sets and Systems, Vol.20, 87-96, (1986).

[7]. A. Kharal, A Neutrosophic Multicriteria Decision Making Method, New Mathematics and Natural Computation, Creighton University, USA, 2013.

[8]. F.Smarandache, A Unifying Field in Logics: Neutrosophic Logic, Neutrosophy, Neutrosophic Set, Neutrosophic Probability; American Research Press: Rehoboth, DE, USA, 1999.

[9]. F.Smarandache, Degree of Dependence and Independence of Neutrosophic Logic Components Applied in Physics, 2016 Annual Spring Meeting of the American Physical Society (APS) Ohio-Region Section, Dayton, Ohio, USA, 08-09 April 08 2016.

[10]. F. Smarandache ,Degree of Dependence and Independence of the (Sub)Components of Fuzzy Set and Neutrosophic Set, *Neutrosophic Sets and Systems, Vol. 11,95-97,2016.*

[11]. I. M. Hanafy, A. A. Salama and K. Mahfouz, Correlation Coefficients of Neutrosophic Sets by Centroid Method, International Journal of Probability and Statistics, 2(1), 9–12, 2013.

Doi : https://doi.org/10.54216/IJNS.180430
Received: March 30, 2022 Accepted: July 14, 2022

[12]. J. Ye, Vector Similarity Measures of Simplified Neutrosophic Sets and Their Application in Multi criteria Decision Making International Journal of Fuzzy Systems, Vol. 16, No. 2, 204–215, 2014.

[13]. J. Ye, Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment, International Journal of General Systems, 42(4), 386–394, 2013.

[14] Mohamed Bisher Zeina , Omar Zeitouny , Fatina Masri , Fatima Kadoura , Said Broumi, Operations on Single-Valued Trapezoidal Neutrosophic Numbers using (α,β,γ)-Cuts “Maple Package”, International Journal of Neutrosophic Science, Vol. 15 , No. 2 , (2021) : 113-122 (**Doi :** https://doi.org/10.54216/IJNS.150205)

[15] M. Mullai , Said Broumi, Dominating Energy in Neutrosophic graphs, International Journal of Neutrosophic Science, Vol. 5 , No. 1 , (2020) : 38-58 (**Doi :** https://doi.org/10.54216/IJNS.050104)

[16]. M. Rafiq et al. / The cosine similarity measures of spherical fuzzy sets and their applications, Journal of Intelligent & Fuzzy Systems, DOI:10.3233/JIFS-181922

[17] Ngan Thi Roan , Florentin Smarandache, H-Max Single-Valued Neutrosophic Distance Measure in Medical Diagnosis, International Journal of Neutrosophic Science, Vol.15, No. 2 , (2021) :62-76 (**Doi :** https://doi.org/10.54216/IJNS.150201)

[18]. R.Jansi, K.Mohana and F. Smarandache, Correlation Measure for Pythagorean Neutrosophic Fuzzy Sets with T and F as Dependent Neutrosophic Components, Neutrosophic sets and systems, 30,202-212, 2019

[19]. S. Broumi, F. Smarandache, Correlation Coefficient of Interval Neutrosophic set, Proceedings of the International Conference ICMERA, Bucharest, October 2013.

[20] S. Broumi, F. Smarandache, Several Similarity Measures of Neutrosophic Sets, Neutrosophic Sets and Systems, 1, 54–62, 2013.

[21]. Tahir Muhammed, An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets, Neural Computing andApplications,(2018)https://doi.org/10.1007/s00521-018-3521-2(012,-volV)(0123

[22]. Yager, R. R, Pythagorean membership grades in multi criteria decision making, IEEE Trans Fuzzy Syst 22(4), 958–965, (2014).

[23]. Yager, R. R, Generalized Ortho pair fuzzy sets, IEEE Trans Fuzzy Systems,(2016)

[24]. Zadeh, L.A, Fuzzy Sets, Information and Control, Vol.8, 338-353,(1965)

[25]. ZhangX, XuZ ,Extension of TOPSIS to multiple criteria decision making with pythagorean fuzzy sets, International Journal of Intelligent Systems , 29(12),1061–1078. (2014)

Doi : https://doi.org/10.54216/IJNS.180430
Received: March 30, 2022 Accepted: July 14, 2022