On n-Refined Neutrosophic Vector Spaces For Some Special Values $3\leq n \leq 6$

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Abstract

This work is dedicated to study some different types of n-refined neutrosophic vector spaces for different values of n between 3 and 6. Where we present some related algebraic concepts such as 3-refined neutrosophic homomorphism, 4-refined neutrosophic homomorphism, 5-refined neutrosophic homomorphism, and 6-refined neutrosophic homomorphism. Also, we provide some theorems to clarify the algebraic behaviour of 3-refined, 4-refined, 5-refined, and 6-refined neutrosophic subspaces.

Keywords: 3-refined neutrosophic vector space; 4-refined neutrosophic vector space; 5-refined neutrosophic vector space; 6-refined neutrosophic vector space; n-refined homomorphism.

1. Introduction

Algebraic structures as sets with specific algebraic properties related to the operations defined on them play a huge role in the study of modern mathematical concepts, as well as in their wide applications [1-3]. The concept of a neutrosophic algebraic structure was defined by Smarandache et. al [9-10], where he proposed the concept of a ring and a group based on the existence of an element with logical properties within the classical algebraic structure.

Neutrosophic algebraic structures have been extensively studied by many researchers, studying matrices, ideals, and also modules [4-7]. The theory of neutrosophic vector spaces began in [8], where the neutrosophic vector space was defined over a neutrosophic field, and then this concept was generalized through many different varieties, such as refined spaces [17-20], n-refined spaces [22-24], and n-refined structures [14-16].

Previous studies have been based on the study of partial spaces, algebraic bases, and also the inner products associated with these spaces [11-13].

In this research, we will study some different types of n-refined neutrosophic vector spaces for different values of n between 3 and 6. Where we present some related algebraic concepts such as 3-refined neutrosophic homomorphism, 4-refined neutrosophic homomorphism, 5-refined neutrosophic homomorphism, and 6-refined neutrosophic homomorphism. Also, we provide some theorems to clarify the algebraic behaviour of 3-refined, 4-refined, 5-refined, and 6-refined neutrosophic subspaces.

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2. Main results

Definition:

Let (M,+,·) be a field, we say that $M_3(I) = M + MI_1 + \cdots + MI_3 = \{m_0 + m_1I_1 + m_2I_2 + m_3I_3; m_i \in M\}$ is a 3-refined neutrosophic field.

Definition:

Let (C,+,·) be a vector space over the field M, we say that $C_n(I) = C + CI_1 + CI_2 + CI_3 = \{c_0 + c_1I_1 + c_2I_2 + c_3I_3;\ c_i \in C\}$ is a 3-refined neutrosophic vector space over the field M.

Addition on $C_n(I)$ is defined as:

$$\left[c_0 + \sum_{i=1}^{3} c_iI_i\right] + \left[d_0 + \sum_{i=1}^{3} d_iI_i\right] = (c_0 + d_0) + \sum_{i=1}^{3}(c_i + d_i)I_i.$$

Multiplication by a 3-refined neutrosophic scalar $m = m_0 + \sum_{i=1}^{3} m_iI_i \in M_3(I)$ is defined as:

$$(m_0 + \sum_{i=1}^{3} m_iI_i) \cdot (c_0 + \sum_{i=1}^{3} c_iI_i) = m_0c_0 + \sum_{i,j=0}^{3}(m_i. c_j)I_iI_j,$$

where $m_i \in M, c_i \in C, I_iI_j = I_{\min(i,j)}$.

Theorem :

Let (M,+,·) be a vector space over the field C. A 3-refined neutrosophic vector space is not a vector space but a module over the 3-refined neutrosophic field C_3(I).

Definition:

Let $M_n(I)$ be a 3-refined neutrosophic vector space over the 3-refined neutrosophic field $C_3(I)$; a nonempty subset $T_3(I)$ is called a 3-refined neutrosophic subspace of $M_3(I)$ if $T_3(I)$ is a submodule of $M_3(I)$.

Theorem:

Let $M_3(I)$ be a 3-refined neutrosophic vector space over a 3-refined neutrosophic field $C_3(I)$, $T_3(I)$ be a nonempty subset of $M_3(I)$. Then $T_3(I)$ is a 3-refined neutrosophic subspace if and only if:

$x + y \in T_3(I), m \cdot x \in T_3(I)$ for all $x, y \in T_3(I), m \in C_n(I)$.

Definition:

Let $M_3(I)$ be a 3-refined neutrosophic vector space over a 3-refined neutrosophic field $C_3(I)$, x be an arbitrary element of $M_3(I)$, we say that x is a linear combination of $\{x_1, x_2, \ldots, x_m\} \subseteq M_3(I)$ is $x = a_1x_1 + a_2x_2 + \cdots + a_mx_m$: $a_i \in C_3(I), x_i \in M_3(I)$.

Definition:

Let $M_3(I), N_3(I)$ be two 3-refined neutrosophic vector spaces over the 3-refined neutrosophic field $T_3(I)$, let $f: M_3(I) \to N_3(I)$ be a mapping, it is called 3-refined neutrosophic homomorphism if:

$f(a.x + b.y) = a.f(x) + b.f(y)$ for all $x, y \in M_3(I), a, b \in T_3(I)$.

Definition:

Let $f: M_3(I) \to N_3(I)$ be a 3-refined neutrosophic homomorphism, we define:

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(a) $Ker(f) = \{x \in M_3(I); f(x) = 0\}$.

(b) $Im(f) = \{y \in N_3(I);\ \exists x \in M_3;\ y = f(x)\}$.

Theorem:

Let $f: M_3(I) \to N_3(I)$ be a 3-refined neutrosophic homomorphism. Then

(a) $Ker(f)$ is a 3-refined neutrosophic subspace of $M_3(I)$

(b) $Im(f)$ is a 3-refined neutrosophic subspace of $N_3(I)$.

Proof:

(a) f is a module homomorphism since $M_3(I), N_3(I)$ are modules over the 3-refined neutrosophic field $T_3(I)$, hence $Ker(f)$ is a submodule of the vector space $M_3(I)$, thus $Ker(f)$ is 3-refined neutrosophic subspace of $M_3(I)$.

(b) Holds by similar argument.

Definition:

Let (M,+,·) be a field, we say that $M_4(I) = M + MI_1 + \cdots + MI_4 = \{m_0 + m_1I_1 + m_2I_2 + m_3I_3 + m_4I_4; m_i \in M\}$ is a 4-refined neutrosophic field.

Definition:

Let (C,+,·) be a vector space over the field M, we say that $C_4(I) = C + CI_1 + CI_2 + CI_3 + CI_4 = \{c_0 + c_1I_1 + c_2I_2 + c_3I_3 + c_4I_4;\ c_i \in C\}$ is a 4-refined neutrosophic vector space over the field M.

Addition on $C_4(I)$ is defined as:

$$\left[c_0 + \sum_{i=1}^{4} c_iI_i\right] + \left[d_0 + \sum_{i=1}^{4} d_iI_i\right] = (c_0 + d_0) + \sum_{i=1}^{4}(c_i + d_i)I_i.$$

Multiplication by a 4-refined neutrosophic scalar $m = m_0 + \sum_{i=1}^{4} m_iI_i \in M_4(I)$ is defined as:

$$(m_0 + \textstyle\sum_{i=1}^{4} m_iI_i) \cdot (c_0 + \sum_{i=1}^{4} c_iI_i) = m_0c_0 + \sum_{i,j=0}^{4}(m_i.c_j)I_iI_j,$$

where $m_i \in M, c_i \in C, I_iI_j = I_{\min(i,j)}$.

Theorem :

Let (M,+,·) be a vector space over the field C. A 4-refined neutrosophic vector space is not a vector space but a module over the 4-refined neutrosophic field C_4(I).

Definition:

Let $M_4(I)$ be a 4-refined neutrosophic vector space over the 4-refined neutrosophic field $C_4(I)$; a nonempty subset $T_4(I)$ is called a 4-refined neutrosophic subspace of $M_4(I)$ if $T_4(I)$ is a submodule of $M_4(I)$.

Theorem:

Let $M_4(I)$ be a 4-refined neutrosophic vector space over a 4-refined neutrosophic field $C_4(I)$, $T_4(I)$ be a nonempty subset of $M_4(I)$. Then $T_4(I)$ is a 4-refined neutrosophic subspace if and only if:

$x + y \in T_4(I), m \cdot x \in T_4(I)$ for all $x, y \in T_4(I), m \in C_4(I)$.

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Definition:

Let $M_4(I)$ be a 4-refined neutrosophic vector space over a 4-refined neutrosophic field $C_4(I)$, x be an arbitrary element of $M_4(I)$, we say that x is a linear combination of $\{x_1, x_2, \dots, x_4\} \subseteq M_4(I)$ is $x = a_1x_1 + a_2x_2 + \dots + a_mx_m: a_i \in C_4(I), x_i \in M_4(I)$.

Definition:

Let $M_4(I), N_4(I)$ be two 4-refined neutrosophic vector spaces over the 4-refined neutrosophic field $T_4(I)$, let $f: M_4(I) \to N_4(I)$ be a mapping, it is called 4-refined neutrosophic homomorphism if:

$f(a.x + b.y) = a.f(x) + b.f(y)$ for all $x, y \in M_4(I), a, b \in T_4(I)$.

Definition:

Let $f: M_4(I) \to N_4(I)$ be a 4-refined neutrosophic homomorphism, we define:

(a) $Ker(f) = \{x \in M_4(I); f(x) = 0\}$.

(b) $Im(f) = \{y \in N_4(I);\ \exists x \in M_4;\ y = f(x)\}$.

Theorem:

Let $f: M_4(I) \to N_4(I)$ be a 4-refined neutrosophic homomorphism. Then

(a) $Ker(f)$ is a 4-refined neutrosophic subspace of $M_4(I)$

(b) $Im(f)$ is a 4-refined neutrosophic subspace of $N_4(I)$.

Proof:

(a) f is a module homomorphism since $M_4(I), N_4(I)$ are modules over the 4-refined neutrosophic field $T_4(I)$, hence $Ker(f)$ is a submodule of the vector space $M_4(I)$, thus $Ker(f)$ is 4-refined neutrosophic subspace of $M_4(I)$.

(b) Holds by similar argument.

Definition:

Let (M,+,·) be a field, we say that $M_5(I) = M + MI_1 + \dots + MI_4 + MI_5 = \{m_0 + m_1I_1 + m_2I_2 + m_3I_3 + m_4I_4 + m_5I_5; m_i \in M\}$ is a 5-refined neutrosophic field.

Definition:

Let (C,+,·) be a vector space over the field M, we say that $C_5(I) = C + CI_1 + CI_2 + CI_3 + CI_4 = \{c_0 + c_1I_1 + c_2I_2 + c_3I_3 + c_4I_4 + c_5I_5;\ c_i \in C\}$ is a 5-refined neutrosophic vector space over the field M.

Addition on $C_5(I)$ is defined as:

$$\left[c_0 + \sum_{i=1}^{5} c_iI_i\right] + \left[d_0 + \sum_{i=1}^{5} d_iI_i\right] = (c_0 + d_0) + \sum_{i=1}^{5}(c_i + d_i)I_i.$$

Multiplication by a 5-refined neutrosophic scalar $m = m_0 + \sum_{i=1}^{5} m_iI_i \in M_5(I)$ is defined as:

$$\left(m_0 + \sum_{i=1}^{5} m_iI_i\right) \cdot \left(c_0 + \sum_{i=1}^{5} c_iI_i\right) = m_0c_0 + \sum_{i,j=0}^{5}(m_i.c_j)I_iI_j,$$

where $m_i \in M, c_i \in C, I_iI_j = I_{\min(i,j)}$.

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Theorem :

Let (M,+,·) be a vector space over the field C. A 5-refined neutrosophic vector space is not a vector space but a module over the 5-refined neutrosophic field C_5(I).

Definition:

Let $M_5(I)$ be a 5-refined neutrosophic vector space over the 5-refined neutrosophic field $C_5(I)$; a nonempty subset $T_5(I)$ is called a 5-refined neutrosophic subspace of $M_5(I)$ if $T_5(I)$ is a submodule of $M_5(I)$.

Theorem:

Let $M_5(I)$ be a 5-refined neutrosophic vector space over a 5-refined neutrosophic field $C_5(I)$, $T_5(I)$ be a nonempty subset of $M_5(I)$. Then $T_5(I)$ is a 5-refined neutrosophic subspace if and only if:

$x + y \in T_5(I), m \cdot x \in T_5(I)$ for all $x, y \in T_5(I), m \in C_5(I)$.

Definition:

Let $M_5(I)$ be a 5-refined neutrosophic vector space over a 5-refined neutrosophic field $C_5(I)$, x be an arbitrary element of $M_4(I)$, we say that x is a linear combination of $\{x_1, x_2, \dots, x_5\} \subseteq M_5(I)$ is $x = a_1x_1 + a_2x_2 + \cdots + a_mx_m$: $a_i \in C_5(I), x_i \in M_5(I)$.

Definition:

Let $M_5(I), N_5(I)$ be two 5-refined neutrosophic vector spaces over the 5-refined neutrosophic field $T_5(I)$, let $f: M_5(I) \to N_5(I)$ be a mapping, it is called 5-refined neutrosophic homomorphism if:

$f(a.x + b.y) = a.f(x) + b.f(y)$ for all $x, y \in M_5(I), a, b \in T_5(I)$.

Definition:

Let $f: M_5(I) \to N_5(I)$ be a 5-refined neutrosophic homomorphism, we define:

(a) $Ker(f) = \{x \in M_5(I); f(x) = 0\}$.

(b) $Im(f) = \{y \in N_5(I);\ \exists x \in M_5;\ y = f(x)\}$.

Theorem:

Let $f: M_5(I) \to N_5(I)$ be a 5-refined neutrosophic homomorphism. Then

(a) $Ker(f)$ is a 5-refined neutrosophic subspace of $M_5(I)$

(b) $Im(f)$ is a 5-refined neutrosophic subspace of $N_5(I)$.

Proof:

(a) f is a module homomorphism since $M_5(I), N_5(I)$ are modules over the 5-refined neutrosophic field $T_5(I)$, hence $Ker(f)$ is a submodule of the vector space $M_5(I)$, thus $Ker(f)$ is 5-refined neutrosophic subspace of $M_5(I)$.

(b) Holds by similar argument.

Definition:

Let (M,+,·) be a field, we say that $M_6(I) = M + MI_1 + \cdots + MI_6 = \{m_0 + m_1I_1 + m_2I_2 + m_3I_3 + m_4I_4 + m_5I_5 + m_6I_6; m_i \in M\}$ is a 6-refined neutrosophic field.

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Addition on $C_6(I)$ is defined as:

$$\left[c_0 + \sum_{i=1}^{6} c_iI_i\right] + \left[d_0 + \sum_{i=1}^{6} d_iI_i\right] = (c_0 + d_0) + \sum_{i=1}^{6}(c_i + d_i)I_i.$$

Multiplication by a 6-refined neutrosophic scalar $m = m_0 + \sum_{i=1}^{6} m_iI_i \in M_6(I)$ is defined as:

$$(m_0 + \sum_{i=1}^{6} m_iI_i) \cdot (c_0 + \sum_{i=1}^{6} c_iI_i) = m_0c_0 + \sum_{i,j=0}^{6}(m_i.c_j)I_iI_j,$$

where $m_i \in M, c_i \in C, I_iI_j = I_{\min(i,j)}$.

Theorem :

Let (M,+,·) be a vector space over the field C. A 6-refined neutrosophic vector space is not a vector space but a module over the 6-refined neutrosophic field C_6(I).

Definition:

Let $M_6(I)$ be a 6-refined neutrosophic vector space over the 6-refined neutrosophic field $C_6(I)$; a nonempty subset $T_6(I)$ is called a 6-refined neutrosophic subspace of $M_6(I)$ if $T_6(I)$ is a submodule of $M_6(I)$.

Theorem:

Let $M_6(I)$ be a 6-refined neutrosophic vector space over a 6-refined neutrosophic field $C_6(I)$, $T_6(I)$ be a nonempty subset of $M_6(I)$. Then $T_6(I)$ is a 6-refined neutrosophic subspace if and only if:

$x + y \in T_6(I), m \cdot x \in T_6(I)$ for all $x, y \in T_6(I), m \in C_6(I)$.

Definition:

Let $M_6(I)$ be a 6-refined neutrosophic vector space over a 6-refined neutrosophic field $C_6(I)$, x be an arbitrary element of $M_6(I)$, we say that x is a linear combination of $\{x_1, x_2, \ldots, x_6\} \subseteq M_6(I)$ is $x = a_1x_1 + a_2x_2 + \cdots + a_mx_m$: $a_i \in C_6(I), x_i \in M_6(I)$.

Definition:

Let $M_6(I), N_6(I)$ be two 6-refined neutrosophic vector spaces over the 6-refined neutrosophic field $T_6(I)$, let $f: M_6(I) \to N_6(I)$ be a mapping, it is called 6-refined neutrosophic homomorphism if:

$f(a.x + b.y) = a.f(x) + b.f(y)$ for all $x, y \in M_6(I), a, b \in T_6(I)$.

Definition:

Let $f: M_6(I) \to N_6(I)$ be a 6-refined neutrosophic homomorphism, we define:

(a) $Ker(f) = \{x \in M_6(I); f(x) = 0\}$.

(b) $Im(f) = \{y \in N_6(I);\ \exists x \in M_6;\ y = f(x)\}$.

Theorem:

Let $f: M_6(I) \to N_6(I)$ be a 6-refined neutrosophic homomorphism. Then

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(a) $Ker(f)$ is a 6-refined neutrosophic subspace of $M_6(I)$

(b) $Im(f)$ is a 6-refined neutrosophic subspace of $N_6(I)$.

Proof:

(a) f is a module homomorphism since $M_6(I), N_6(I)$ are modules over the 6-refined neutrosophic field $T_6(I)$, hence $Ker(f)$ is a submodule of the vector space $M_6(I)$, thus $Ker(f)$ is 6-refined neutrosophic subspace of $M_6(I)$.

(b) Holds by similar argument.

3. Conclusion

In this paper, we have studied some different types of n-refined neutrosophic vector spaces for different values of n between 3 and 6. Where we present some related algebraic concepts such as 3-refined neutrosophic homomorphism, 4-refined neutrosophic homomorphism, 5-refined neutrosophic homomorphism, and 6-refined neutrosophic homomorphism. Also, we provide some theorems to clarify the algebraic behaviour of 3-refined, 4-refined, 5-refined, and 6-refined neutrosophic subspaces.

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