On neutrosophic $\alpha\psi$ -supra open sets

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Abstract

Neutrosophic topological space is an extension of fuzzy topology. Neutrosophic topological space addresses each element's membership, indeterminacy and non-membership grades. Dropping an axiom in the neutrosophic topological space produces a new topological space called neutrosophic supra topological space. Elements in this neutrosophic supra topology are neutrosophic sets. We established the neutrosophic $\alpha\psi$-supra open set in neutrosophic supra topological spaces in this paper. Also, we investigate the properties of the newly defined set. Neutrosophic $\alpha\psi$ -supra continuity is introduced and studied subsequently.
Keywords: Neutrosophic supra topology; α -supra open set;$\alpha\psi$-supra open set;$\alpha\psi$-supra continuous map.

1 introduction

Many generalization of Zadeh's[14] fuzzy sets such as intuitionistic fuzzy, neutrosophic set(in short, NS), Pythagorean,Picture fuzzy, etc can be seen. In his theory, membership grade of an element alone studied. Atanassov's[1] IFS theory discussed failed to address neutrality of an element where as F. Smarandache[12,13] addressed three values of each element in the set namely the membership, indeterminacy and non-membership. D. Coker[2] include the FS theory in topology and derived new notion. As NS gained visibility, researchers incorporated the theory in topology,[7–9] algebra and graph theory, etc. A. A. Salama[11] derived neutrosophic topology. Later,week open and continuous function, homeomorphism on weak open sets are defined on neutrosophic topological spaces. Abbas dropped an axiom in intuitionistic fuzzy topological space to developed the new notion called intuitionistic supra fuzzy topological spaces. Some weak open sets on intuitionistic topological space have been introduced and studied by the researchers. Neutrosophic supra topological space (in short, NSTS) developed by chopping off an axiom in neutrosophic topological space. A subcollection τ on non-empty set $\mathfrak{W}$ is called NSTS then τ need not be closed under finite intersection.

Dhavaseelan et al.[3,4] introduced and investigated the notion of NSSOS, $N_\alpha SCS$ and their continuity on NSTS. The various supra-pre map on NSTS developed by Parimala et al.[8] Neutrosophic $\alpha\psi$-closed sets established by Parimala et al.[10] and its connected space studied by Parimala et al.[5]

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1.1 Motivation and Objective

Many extension of fuzzy supra topological space are developed by the researchers. NSTSs is one of the extensions. By dropping an axiom in the conventional neutrosophic topological space to make new notion in the research era. Various open sets have been introduced on the NSTS.
Our objective is to establish another open set called $\alpha\psi$- supra open set under neutrosophic environment and investigate the properties of the new notion. Also, Continuous map on the new notion is introduced and studied.

1.2 Basic definitions

Basic definitions such as NS, NSTS, basic operations, interior and closure are presented here.

Definition 1.1. [12,13] Let $\mathfrak{W} \neq \emptyset$. A NS S is defined by

$$S = \{\langle \mathfrak{a}, \mathfrak{T}_S(\mathfrak{a}), \mathfrak{I}_S(\mathfrak{a}), \mathfrak{F}_S(\mathfrak{a})\rangle : \mathfrak{a} \in \mathfrak{W}\}$$

where $\mathfrak{T}_S$, $\mathfrak{I}_S$, $\mathfrak{F}_S$ maps from $\mathfrak{W}$ to [0,1] for each $\mathfrak{a} \in \mathfrak{W}$ to S and $0 \leq \mathfrak{T}_S(\mathfrak{a}) + \mathfrak{I}_S(\mathfrak{a}) + \mathfrak{F}_S(\mathfrak{a}) \leq 3$ for each $\mathfrak{a} \in \mathfrak{W}$. Here $\mathfrak{T}_S$, $\mathfrak{I}_S$ and $\mathfrak{F}_S$ are MF, INDF and NMF respectively

Definition 1.2. [11] Let $S_1 = \{\langle \mathfrak{a}, \mathfrak{T}_{S_1}(\mathfrak{a}), \mathfrak{I}_{S_1}(\mathfrak{a}), \mathfrak{F}_{S_1}(\mathfrak{a})\rangle : \mathfrak{a} \in \mathfrak{W}\}$ and $S_2 = \{\langle \mathfrak{a}, \mathfrak{T}_{S_2}(\mathfrak{a}), \mathfrak{I}_{S_2}(\mathfrak{a}), \mathfrak{F}_{S_2}(\mathfrak{a})\rangle : \mathfrak{a} \in \mathfrak{W}\}$ be NSs. Then

(i) $S_1$is a subset of S_2 if and only if $\mathfrak{T}_{S_1}(\mathfrak{a}) \leq \mathfrak{T}_{S_2}((\mathfrak{a}))$, $\mathfrak{I}_{S_1}(\mathfrak{a}) \geq \mathfrak{I}_{S_2}(\mathfrak{a})$ and $\mathfrak{F}_{S_1}(\mathfrak{a}) \geq \mathfrak{F}_{S_2}(\mathfrak{a})$;

(ii) The complement of NS S_1, ${S_1}^C$ is $\{\langle \mathfrak{a}, \mathfrak{F}_{S_1}(\mathfrak{a}), 1 - \mathfrak{I}_{S_1}(\mathfrak{a}), \mathfrak{T}_{S_1}(\mathfrak{a})\rangle : \mathfrak{a} \in \mathfrak{W}\}$;

(iii) Intersection of S_1 and S_2 is $\{\langle \mathfrak{a}, \mathfrak{T}_{S_1}(\mathfrak{a}) \wedge \mathfrak{T}_{S_2}(\mathfrak{a}), \mathfrak{I}_{S_1}(\mathfrak{a}) \vee \mathfrak{I}_{S_2}(\mathfrak{a}), \mathfrak{F}_{S_1}(\mathfrak{a}) \vee \mathfrak{F}_{S_2}(\mathfrak{a})\rangle : \mathfrak{a} \in \mathfrak{W}\}$;

(iv) Union of S_1 and S_2 is $\{\langle \mathfrak{a}, \mathfrak{T}_{S_1}(\mathfrak{a}) \vee \mathfrak{T}_{S_2}(\mathfrak{a}), \mathfrak{I}_{S_1}(\mathfrak{a}) \wedge \mathfrak{I}_{S_2}(\mathfrak{a}), \mathfrak{F}_{S_1}(\mathfrak{a}) \wedge \mathfrak{F}_{S_2}(\mathfrak{a})\rangle : \mathfrak{a}h \in \mathfrak{W}\}$.

The symbols $\vee, \wedge$ denotes the maximum and minimum operator.

Definition 1.3. [6] A subcollection τ of neutrosophic sets on $\mathfrak{W} \neq \emptyset$ is said to be a NSTS on $\mathfrak{W}$ if the sets $\emptyset, \mathfrak{W} \in \tau$ and $\bigcup_{j=i}^{\infty} \mathfrak{S}_j \in \tau, for \mathfrak{S}_{j=1}^{\infty} \in \tau$. Then $(\mathfrak{W}, \tau)$ is called NSTS on $\mathfrak{W}$.

Definition 1.4. [6] $NSint(\mathfrak{S})$ and $NScl(\mathfrak{S})$ respectively represents the neutrosophic supra interior and neutrosophic supra closure of a NS $\mathfrak{S}$ in NSTS and they are defined by $NSint(\mathfrak{S}) = \cup\{\mathfrak{G} : neutrosophic\ supra\ open\ set\ \mathfrak{G} \subseteq \mathfrak{S}\ and\ \mathfrak{G} \in \tau\}$ and $NScl(\mathfrak{S}) = \cap\{\mathfrak{F} : neutrosophic\ supra\ closed\ set\ \mathfrak{F} \supseteq \mathfrak{S}\ and\ \mathfrak{F}^c \in \tau\}$.

Definition 1.5. [4] A NS $\mathfrak{S}$ is called a NSSOS in NSTS $(\mathfrak{W}, \tau)$ if $\mathfrak{S} \subseteq NScl(NSint\mathfrak{S})$. The complement of a NSSOS is called a NSSCS.

Definition 1.6. [3] A NS $\mathfrak{S}$ is called a $N_\alpha SOS$ in NSTS $(\mathfrak{W}, \tau)$ if $\mathfrak{S} \subseteq NSint(NScl(NSint\mathfrak{S}))$. The complement of a $N_\alpha SOS$ is called a $N_\alpha SCS$.

Definition 1.7. [3] Neutrosophic α- supra interior and neutrosophic α- supra closure of NS $\mathfrak{S}$ in NSTS is denoted and defined by $N_\alpha Sint(\mathfrak{S})$ and $N_\alpha Scl(\mathfrak{S})$ respectively and $N_\alpha Sint(\mathfrak{S}) = \cup\{\mathfrak{G} : neutrosophic\ \alpha - supra\ open\ set\ \mathfrak{G} \subseteq \mathfrak{S}\ and\ \mathfrak{G} \in \tau\}$ and $N_\alpha Scl(\mathfrak{S}) = \cap\{\mathfrak{F} : neutrosophic\ \alpha - supra\ closed\ set\ \mathfrak{F} \supseteq \mathfrak{S}\ and\ \mathfrak{F}^c \in \tau\}$.

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2 On neutrosophic $\alpha\psi$- supra open set

The definition of $\alpha\psi$- supra open set is introduced, interior, closure and their properties are studied.

Definition 2.1. Let $\mathfrak{W}$ be a NSTS. A NS $\mathfrak{S}$ is said to be a neutrosophic semi generalized supra open set if $\mathfrak{G} \subseteq NSint(\mathfrak{S})$ whenever $\mathfrak{G} \subseteq \mathfrak{S}$ and $\mathfrak{G}$ is a NSSCS in $\mathfrak{W}$.

Definition 2.2. Neutrosophic semi generalized interior and neutrosophic semi generalized closure of NS $\mathfrak{S}$ in NSTS is denoted and defined by $NSgint(\mathfrak{S})$ and $NSgcl(\mathfrak{S})$ respectively and $NSgint(\mathfrak{S}) = \cup\{\mathfrak{G} : neutrosophic\ semi\ generalized\ supra\ open\ set\ \mathfrak{G} \subseteq \mathfrak{S}\ and\ \mathfrak{G} \in \tau\}$ and $NSgcl(\mathfrak{S}) = \cap\{\mathfrak{F} : neutrosophic\ semi\ genera$ $\mathfrak{S}\ and\ \mathfrak{F}^c \in \tau\}$.

Definition 2.3. Let $\mathfrak{W}$ be a NSTS. A NS $\mathfrak{S}$ is said to be a neutrosophic ψ-supra open set, if $\mathfrak{G} \subseteq NSgint(\mathfrak{S})$ whenever $\mathfrak{G} \subseteq \mathfrak{S}$ and $\mathfrak{G}$ is a neutrosophic semi generalized - supra closed in $\mathfrak{W}$.

Definition 2.4. Neutrosophic ψ-supra interior and neutrosophic ψ-supra closure of NS $\mathfrak{S}$ in NSTS is denoted and defined by $N\psi Sint(\mathfrak{S})$ and $N\psi Scl(\mathfrak{S})$ respectively and $N\psi Sint(\mathfrak{S}) = \cup\{\mathfrak{G} : neutrosophic\ \psi\ -supra\ open\ set\ \mathfrak{G} \subseteq \mathfrak{S}\ and\ \mathfrak{G} \in \tau\}$ and $N\psi Scl(\mathfrak{S}) = \cap\{\mathfrak{F} : neutrosophic\ \psi\ -supra\ closed\ set\ \mathfrak{F} \supseteq \mathfrak{S}\ and\ \mathfrak{F}^c \in \tau\}$.

Definition 2.5. if $\mathfrak{G} \subseteq N\psi Sint(\mathfrak{S})$ whenever $\mathfrak{G} \subseteq \mathfrak{S}$ and $\mathfrak{G}$ is a $N_\alpha SCS$. Then the NS $\mathfrak{S}$ is called a $N_{\alpha\psi}SOS$. The complement of a $N_{\alpha\psi}SOS$ is called a $N_{\alpha\psi}SCS$.

Definition 2.6. Let $\mathfrak{S}$ be a NS. Neutrosophic $\alpha\psi$ supra interior and closure of $\mathfrak{S}$ in NSTS is denoted and defined by $N_{\alpha\psi}Sint(\mathfrak{S})$ and $N_{\alpha\psi}Scl(\mathfrak{S})$ respectively and $N_{\alpha\psi}Sint(\mathfrak{S}) = \cup\{\mathfrak{G} : neutrosophic\ \alpha\psi\ -supra\ open\ set\ \mathfrak{G} \subseteq \mathfrak{S}\ and\ \mathfrak{G} \in \tau\}$ and $N_{\alpha\psi}Scl(\mathfrak{S}) = \cap\{\mathfrak{F} : neutrosophic\ \alpha\psi\ -supra\ closed\ set\ \mathfrak{S} \subseteq \mathfrak{F}\ and\ \mathfrak{F}^c \in \tau\}$.

Theorem 2.7. *Every NSOS is $N_{\alpha\psi}SOS$.*

Proof:
Let $\mathfrak{S}$ be a NSOS in $\mathfrak{W}$.Then $NSint(\mathfrak{S}) = \mathfrak{S}$ and we know that $\mathfrak{S} \subseteq NScl(\mathfrak{S})$. This implies that $NSint(\mathfrak{S}) \subseteq NSint(NScl(NSint(\mathfrak{S})))$. Every NSOS is a $N_\alpha SOS$. Let $\mathfrak{G} \subseteq \mathfrak{S}$ where $\mathfrak{G}$ is $N_\alpha SCS$. Since $\mathfrak{S}$ is $N_\alpha SOS$. $\mathfrak{G} \subseteq NSint(\mathfrak{S}) \subseteq N\psi Sint(\mathfrak{S})$. Hence $\mathfrak{S}$ is a $N_{\alpha\psi}SOS$.

Example 2.8. Let $\mathfrak{W} = \{\mathfrak{a}, \mathfrak{b}\}$ be the universal set. The NS $\mathfrak{A} = \{< a, 0.3, 0.3, 0.4 >, < b, 0.4, 0.4, 0.5 >\}$, $\mathfrak{B} = \{< a, 0.4, 0.4, 0.5 >, < b, 0.2, 0.3, 0.3 >\}$. Let $\tau = \{\tilde{0}, \tilde{1}, \mathfrak{A}, \mathfrak{B}, \mathfrak{A} \cup \mathfrak{B}\}$. Let $\mathfrak{C} = \{< a, 0.4, 0.4, 0.3 >, < b, 0.5, 0.5, 0.3 >\}$. then $\mathfrak{C}$ is $N_{\alpha\psi}SOS$ but $\mathfrak{C}$ is not a NSOS.

Theorem 2.9. *Every $N_\alpha SOS$ is $N_{\alpha\psi}SOS$.*

Proof:
Let A be a N_αSOS. Since every $N_\alpha SOS$ is NSSOS. This implies $\mathfrak{S}$ is a NSSOS. Let $\mathfrak{G} \subseteq \mathfrak{S}$ where $\mathfrak{G}$ is a NSSCS. Then $\mathfrak{G} \subseteq NsSint(\mathfrak{S})$. Therefore, A is neutrosophic semi-generalized supra open set for some $\mathfrak{G} \subseteq \mathfrak{S}$ where $\mathfrak{G}$ is a neutrosophic semi-generalized supra closed set. Thus $\mathfrak{G} \subseteq N_\alpha Sint(\mathfrak{S}) \subseteq N\psi Sint(\mathfrak{S})$. Hence $\mathfrak{S}$ is a $N_{\alpha\psi}SOS$.

Theorem 2.10.
1. *Arbitrary union of $N_{\alpha\psi}SOS$ is always $N_{\alpha\psi}SOS$.*
2. *Finite intersection of $N_{\alpha\psi}SOS$ fails to be $N_{\alpha\psi}SOS$.*

Proof:

i. Let $\{\mathfrak{S}_i : i \in J\}$ be a family of $N_{\alpha\psi}SOS$ in NSTS $\mathfrak{W}$. Then $\mathfrak{G}_i \subseteq N\psi Sint(\mathfrak{S}_i)$ whenever $\mathfrak{G}_i \subseteq \mathfrak{S}_i$ for any $i \in J$. $\cup_{i\in J}\mathfrak{G}_i \subseteq \cup_{i\in J}N\psi Sint(\mathfrak{S}_\mathfrak{i}) \subseteq N\psi Sint(\cup_{i\in J}\mathfrak{S}_\mathfrak{i})$ whenever $\cup_{i\in J}\mathfrak{G}_i \subseteq \cup_{i\in J}\mathfrak{S}_i$ where $\cup_{i\in J}\mathfrak{G}_i$ is a $N_\alpha SCS$-. Hence $\cup_{i\in J}\mathfrak{S}_i$ is a $N_{\alpha\psi}SOS$.

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ii. Below numeric example evident to show that finite intersection of $N_{\alpha\psi}SOS$ fails to be $N_{\alpha\psi}SOS$. Let $\mathfrak{W} = \{a, b\}$ and NSs $\mathfrak{A} = \{< a, 0.3, 0.3, 0.4 >, < b, 0.4, 0.4, 0.5 >\}$ and $\mathfrak{B} = \{< a, 0.2, 0.2, 0.5 >, < b, 0.4, 0.4, 0.3 >\}$ and the NSTS $\tau = \{\tilde{0}, \tilde{1}, \mathfrak{A}, \mathfrak{B}, \mathfrak{A} \cup \mathfrak{B}\}$. Let $\mathfrak{C} = \{< a, 0.4, 0.4, 0.3 >, < b, 0.6, 0.6, 0.4 >\}$ be a NS. $\mathfrak{B}$ and $\mathfrak{C}$ are $N_{\alpha\psi}SOS$ but the intersection of $\mathfrak{B}$ and $\mathfrak{C}$ failed to be $N_{\alpha\psi}SOS$.

Theorem 2.11. *Arbitrary intersection of $N_{\alpha\psi}SCS$ is always $N_{\alpha\psi}SCS$*

Proof:
Let $\{\mathfrak{S}_i : i \in J\}$ be a family of $N_{\alpha\psi}SCS$ in NSTS $\mathfrak{W}$. Then $N\psi Scl(\mathfrak{S}_i) \subseteq \mathfrak{G}_i$ whenever $\mathfrak{S}_i \subseteq \mathfrak{G}_i$ for some $i \in J$. $\cap_{i\in J} N\psi Scl(\mathfrak{S}_i) \subseteq \cap_{i\in J}\mathfrak{G}_i$ implies $N\psi Scl(\cap_{i\in J}\mathfrak{S}_i) \subseteq \mathfrak{G}_i$ whenever $\cap_{i\in J}\mathfrak{G}_i \supseteq \cap_{i\in J}\mathfrak{S}_i$ where $\cap_{i\in J}\mathfrak{G}_i$ is a $N_\alpha SOS$. Hence $\cap_{i\in J}\mathfrak{S}_i$ is a $N_{\alpha\psi}SCS$.

Theorem 2.12. *Let $(\mathfrak{W}, \tau)$ be a NSTS. If $\mathfrak{A}, \mathfrak{B} \in \mathfrak{W}$, then*

1. $(N_{\alpha\psi}Sint(\mathfrak{A}))^c = N_{\alpha\psi}Scl(\mathfrak{A}^c)$.
2. $(N_{\alpha\psi}Scl(\mathfrak{A}))^c = N_{\alpha\psi}Sint(\mathfrak{A}^c)$.
3. *If $\mathfrak{A} \subseteq \mathfrak{B}$ then $N_{\alpha\psi}Scl(\mathfrak{A}) \subseteq N_{\alpha\psi}Scl(\mathfrak{B})$ and $N_{\alpha\psi}Sint(\mathfrak{A}) \subseteq N_{\alpha\psi}Sint(\mathfrak{B})$.*
4. $N_{\alpha\psi}Sint(\mathfrak{A}) \cup N_{\alpha\psi}Sint(\mathfrak{B}) \subseteq N_{\alpha\psi}Sint(\mathfrak{A} \cup \mathfrak{B})$.
5. $N_{\alpha\psi}Sint(\mathfrak{A} \cap \mathfrak{B}) \subseteq N_{\alpha\psi}Sint(\mathfrak{A}) \cap N_{\alpha\psi}Sint(\mathfrak{B})$

Theorem 2.13. *Intersection of NSOS and $N_{\alpha\psi}SOS$ is $N_{\alpha\psi}SOS$*

Theorem 2.14. *If $\mathfrak{A}$ is a $N_{\alpha\psi}SOS$ and $N\psi Sint(\mathfrak{A}) \subseteq \mathfrak{B} \subseteq \mathfrak{A}$, then $\mathfrak{B}$ is a $N_{\alpha\psi}SOS$.*

Proof: Given $\mathfrak{S}$ is $N_{\alpha\psi}SOS$ in $\mathfrak{W}$ and $\mathfrak{G} \subseteq N\psi Sint(\mathfrak{S})$ and $\mathfrak{G} \subseteq \mathfrak{S}$. Since $N\psi Sint\mathfrak{S} \subseteq \mathfrak{E}$, $\mathfrak{G} \subseteq \mathfrak{E}$ and $\mathfrak{G}$ is $N_\alpha SCS$ therefore $\mathfrak{G} \subseteq N\psi Sint\mathfrak{E}$. Hence $\mathfrak{E}$ is a $N_{\alpha\psi}SOS$.

3 Neutrosophic $\alpha\psi$ -supra continuous map

Neutrosophic $\alpha\psi$ supra continuous function is introduced and investigated the properties of neutrosophic $\alpha\psi$ supra continuous function.

Definition 3.1. Let $(\mathfrak{W}, \tau)$ and $(\mathfrak{X}, \kappa)$ be NSTS. A map $\Lambda : (\mathfrak{W}, \tau) \longrightarrow (\mathfrak{X}, \kappa)$ is said to be $N_{\alpha\psi}SCM$ if $\Lambda^{-1}(U)$ is $N_{\alpha\psi}SOS$ in τ for all NSOS U in κ.

Lemma 3.2. *Let $(\mathfrak{W}, \tau), (\mathfrak{X}, \kappa), (\mathfrak{Y}, \omega)$ be NSTS and $N_{\alpha\psi}SCM$ map $\Lambda : (\mathfrak{W}, \tau) \longrightarrow (\mathfrak{X}, \kappa)$ and neutrosophic supra continuous map $g : (\mathfrak{X}, \kappa) \longrightarrow (\mathfrak{Y}, \omega)$. Then the composition $g \circ \Lambda : (\mathfrak{W}, \tau) \longrightarrow (\mathfrak{Y}, \omega)$ is a $N_{\alpha\psi}SCM$.*

Proof:
Let U be a NSOS in $\mathfrak{Y}$. We know that $(g \circ \Lambda)^{-1}(U) = \Lambda^{-1}(g^{-1}(U))$. Since g is a neutrosophic supra continuous map then $g^{-1}(U)$ is a NSOS in $\mathfrak{X}$. Since Λ is a $N_{\alpha\psi}SCM$ then $\Lambda^{-1}(V)$ is a $N_{\alpha\psi}SOS$ in $\mathfrak{W}$ for all NSOS V in $\mathfrak{X}$. Let $V = g^{-1}(U)$ is a NSOS in $\mathfrak{X}$ then $\Lambda^{-1}(g^{-1}(U))$ is a $N_{\alpha\psi}SOS$ in $\mathfrak{W}$. Hence Then the composition $g \circ \Lambda$ is a $N_{\alpha\psi}SCM$.

Theorem 3.3. *Let $\Lambda : (\mathfrak{W}, \tau) \longrightarrow (\mathfrak{X}, \kappa)$ be a map from NSTS on $\mathfrak{W}$ to NSTS on $\mathfrak{X}$. Then the following are equivalent*

1. *Λ is a $N_{\alpha\psi}SCM$*
2. *$\Lambda^{-1}(U)$ is a $N_{\alpha\psi}SCS$ in $\mathfrak{W}$ for all NSCS in U in $\mathfrak{X}$.*
3. *Every neighborhood V of $\Lambda(x)$, there is a neighborhood U of $x \in \mathfrak{W}$ such that $\Lambda(U) \subseteq V$.*

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4. $\Lambda(\overline{A}) \subseteq \overline{\Lambda(A)}$ *for every subset A of* $\mathfrak{W}$.

5. $\overline{\Lambda^{-1}(B)}$ *is a subset of* $\Lambda^{-1}(\overline{B})$ *for every subset B of* $\mathfrak{X}$.

Proof:
$(1) \longrightarrow (2)$. Let $C \subseteq \mathfrak{X}$ be NSCS. Then $\mathfrak{X}\backslash C$ is a $N_{\alpha\psi}SOS$, $\Lambda^{-1}(\mathfrak{X}\backslash C) = \Lambda^{-1}(\mathfrak{X})\backslash\Lambda^{-1}(C) = \mathfrak{W}\backslash\Lambda^{-1}(C)$ is a $N_{\alpha\psi}SOS$ in $\mathfrak{W}$. Hence $\Lambda^{-1}(C)$ is a $N_{\alpha\psi}SCS$ in $\mathfrak{W}$.
$(2) \longrightarrow (1)$ is similar to $(1) \longrightarrow (2)$.
$(1) \longrightarrow (3)$. Since $\Lambda^{-1}(U)$ is a neutrosophic $\alpha\psi$-supra neighborhood of x, choose $\Lambda^{-1}(U) = V$.
$(3) \longrightarrow (4)$. Let $A \subseteq \mathfrak{W}$ and $x \in \overline{A}$. Let U and V be a neutrosophic supra neighborhood of x and f(x) respectively such that $\Lambda(U) \subseteq V$. Since $x \in \overline{A}$, $U \cap A \neq \emptyset$. Therefore $\emptyset \neq \Lambda(U \cap A) \subseteq \Lambda(U) \cap \Lambda(A) \subseteq V \cap \Lambda(A)$. Since V is a neighborhood of $\Lambda(x)$, $V \cap F(A) \neq \emptyset$. Hence $\Lambda(x) \in \overline{\Lambda(A)}$ and $\Lambda(\overline{A}) \subseteq \overline{\Lambda(A)}$.
$(4) \longrightarrow (5)$. Let $A = \Lambda^{-1}(B)$ then from (4),$\Lambda(\overline{\Lambda^{-1}(B)})$ is a subset of $\overline{\Lambda(\Lambda^{-1}(B))}$ and $\overline{\Lambda(\Lambda^{-1}(B))}$ is equal to $\overline{B}$. Taking Λ^{-1} on both the sides we get $\overline{\Lambda^{-1}(B)} \subseteq \Lambda^{-1}(\overline{B})$.
$(5) \longrightarrow (1)$. Let $B \subseteq \mathfrak{X}$ be NSCS. Then $B = \overline{B}$. From (5), $\overline{\Lambda^{-1}(B)}$ is a subset of $\Lambda^{-1}(B)$ and by definition of closure $\overline{\Lambda^{-1}(B)}$ is a super set of $\Lambda^{-1}(B)$. We get $\overline{\Lambda^{-1}(B)} = \Lambda^{-1}(B)$. Hence $\Lambda^{-1}(B)$ is a $N_{\alpha\psi}SCS$.

Theorem 3.4. *Every neutrosophic supra continuous map is a* $N_{\alpha\psi}SCM$

Proof:
Let $\Lambda : (\mathfrak{W}, \tau) \longrightarrow (\mathfrak{X}, \kappa)$ be a continuous map from NSTS on $\mathfrak{W}$ to NSTS on $\mathfrak{X}$. Then $\Lambda^{-1}(U)$ is NSOS in $\mathfrak{W}$ for all NSOS U in $\mathfrak{X}$. We know that every NSOS is $N_{\alpha\psi}SOS$. Then $\Lambda^{-1}(U)$ is $N_{\alpha\psi}SOS$ in $\mathfrak{W}$ for all NSOS U in $\mathfrak{X}$. Hence $\Lambda : (\mathfrak{W}, \tau) \longrightarrow (\mathfrak{X}, \kappa)$ is a $N_{\alpha\psi}SCM$.

Theorem 3.5. *The map* $\Lambda : (\mathfrak{W}, \tau) \longrightarrow (\mathfrak{X}, \kappa)$ *is a* $N_{\alpha\psi}SCM$, *if one the following holds*

1. $\Lambda^{-1}(N_{\alpha\psi}Sint(A)) \subseteq NSint(\Lambda^{-1}(A))$ *for every NSOS A in* $\mathfrak{X}$.

2. $\Lambda^{-1}(N_{\alpha\psi}Scl(A)) \supseteq NScl(\Lambda^{-1}(A))$ *for every NSOS A in* $\mathfrak{X}$.

3. $\Lambda(NScl(B)) \subseteq N_{\alpha\psi}Scl(\Lambda(B))$ *for every* $N_{\alpha\psi}SOS$ *B in* $\mathfrak{W}$

Proof:
Let A be any NSOS of $\mathfrak{X}$ then $Nsint(A) = A$. If (i) hold $\Lambda^{-1}(N_{\alpha\psi}Sint(A)) \subseteq NSint(\Lambda^{-1}(A))$. Then $\Lambda^{-1}(A) \subseteq NSint(\Lambda^{-1}(A))$. Therefore, $\Lambda^{-1}(A)$ is a NSOS. Since every NSOS is a $N_{\alpha\psi}SOS$. Hence Λ is a $N_{\alpha\psi}SCM$. Similarly for condition (ii). If (iii) hold and A be any NSOS in $\mathfrak{X}$. Then $\Lambda^{-1}(A) = B$ is a NSOS in $\mathfrak{W}$ and $\Lambda(NScl(\Lambda^{-1}(A))) \subseteq N_{\alpha\psi}Scl(\Lambda(\Lambda^{-1}(A)))$. this implies $\Lambda(NScl(\Lambda^{-1}(A))) \subseteq N_{\alpha\psi}Scl(A)$. By condition (ii), Λ is a $N_{\alpha\psi}SCM$.

4 Conclusion

Neutrosophic $\alpha\psi$-supra topological space, Neutrosophic $\alpha\psi$-supra interior and Neutrosophic $\alpha\psi$-supra closure of a NS is introduced. Properties of $N_{\alpha\psi}SOS$ is investigated. Finally, we introduced the $N_{\alpha\psi}SCM$ and investigated its properties. In future work, one can look for an application of $N_{\alpha\psi}SOS$ in NSTS in data mining, medical diagnosis and in many decision making problem.

Abbreviation

1. Neutrosophic semi-supra open set - NSSOS
2. Neutrosophic semi-supra closed set - NSSCS
3. Neutrosophic α-supra open set - NαSOS
4. Neutrosophic supra open set - NSOS
5. Neutrosophic $\alpha\psi$-supra open set - N$\alpha\psi$SOS
6. Neutrosophic $\alpha\psi$-supra closed set - N$\alpha\psi$SCS
7. Neutrosophic $\alpha\psi$-supra continuous map - $N_{\alpha\psi}SCM$

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