Design and analysis of process capability indices cpm and cpmk by neutrosophic sets

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Abstract

Process capability indices (PCIs) have been widely used to analyze capability of the process that measures how the customer expectations have been conformed. Two of the well-known PCIs, named indices C_{pm} and C_{pmk} have been developed to consider customers' ideal value that called target value (T). Although, these indices have similar features of the well-known indices C_p and C_{pk}, one of the most important differences is to consider T. In real case problems, we need to add some uncertainties related with human's evaluations into process capability analysis (PCA). One of the uncertainty modelling methods called neutrosophic sets (NSs), have an important role in modeling uncertainty based on incomplete and inconsistent information. For this aim, the PCIs have been designed by using NSs to manage the uncertainties of systems and to increase sensitiveness, flexibility and to obtain more detailed results of PCA in this paper. For this aim, the indices C_{pm} and C_{pmk} have been performed and re-designed by using single valued neutrosophic numbers for the first time in the literature. Additionally, specification limits (SLs) have been re-considered by using NSs. The neutrosophic state of the SLs provide us to have more knowledge about the process and easily applied for engineering problems that includes uncertainty. Finally, the neutrosophic process capability indices (NPCIs) $\widetilde{\widetilde{C}}_{pm}$ and $\widetilde{\widetilde{C}}_{pmk}$ have been obtained and the main formulas of them have been produced. Additionally, the proposed $\widetilde{\widetilde{C}}_{pm}$ and $\widetilde{\widetilde{C}}_{pmk}$ have been applied on real case studies from manufacturing industry. The obtained results show that the indices $\widetilde{\widetilde{C}}_{pm}$ and $\widetilde{\widetilde{C}}_{pmk}$ include more informative and flexible results to evaluate capability of process.

Keywords: Process capability analysis, process capability index, neutrosophic set, single valued neutrosophic numbers, neutrosophic specification limits.

1 Introduction

Companies should regularly analyze the capability of the process and interpret the results correctly to produce the output with desired quality. Process capability analysis (PCA) is an important tool for measuring the ability of whether the product to be produced is according to specified specifications or not. The process must be under control before a process capability analysis is carried out based on specification limits (SLs). The high variability causes the process to fall outside of the SLs [31]. Specifications are a set of predefined rules for a product to occur. For example, the hardness or strength of the gear wheel is determined by specifications. In other words, the specifications define the limits of the product or service [22]. The customer wants to know if the product he buys meets the specifications. Process capability indices (PCIs) are one of the most used the techniques that enable to improve the quality of the process. For this reasons, many indices have been developed to measure the capability of the process. PCIs are one of the methods used to measure the adequacy of the process. PCIs are calculated based on SLs [41]. If the more accurately SLs are defined, the more accurate and realistic the PCA will be obtained. For this reason, defining the SLs are very critical for calculating the PCA. The results obtained from PCA may be incorrect if the parameters of these indices are

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not defined properly, more flexible and accurate definition of parameters such as SLs, mean and variance that used in PCIs would increase the effectiveness of PCIs.

We also know that uncertainty is very critical for the process. The expressions we use when describing SLs or explaining any event can include uncertainty. For these definitions, generally fuzzy expressions that do not contain certainty are used. The human brain frequently uses fuzzy expressions in the face of uncertainty and uncertain events [34]. For this reason, it is not possible to express the decisions made by people with crisp numerical values. Using linguistic terms while modeling human decisions makes the model more realistic and increases its accuracy. The most important feature that distinguishes fuzzy logic from other systems is that it allows the use of variables that provide linguistic terms [38]. Uncertain, complex and incomplete sources of information are called fuzzy sources. Zadeh [58] stated that the more realistic and deeply examined the problems in the real world, the more fuzzy solutions would become. Fuzzy logic gives successful results in models where it is difficult or judgmental to understand, in the processes where human decisions, human opinions and evaluations are too much [21]. One of the fuzzy extension methods named neutrosophic sets (NSs) that have an important to express incomplete, inconsistent and uncertain information can be successfully used to manage the uncertainty of the process. Defining parameters by using fuzzy values rather than precise values due to time, cost, and sampling difficulty would ensure that the higher information content and sensitivity of the process. NSs provide an effective approach on PCA since they have more advantages compared to other fuzzy set extensions in terms of both ease of application and flexibility. Defining PCIs with more than one membership function instead of an only one membership function would enable to evaluate the process more broadly. The being of NSs of SLs provide more information and more sensitiveness in PCA. The NPCIs show all possible values of PCIs with functions of truth, indeterminacy and falsity. This wide range of information makes it easy for process engineers to follow the process. The calculation of PCIs by neutrosophic numbers (NNs) provides more flexibility to the process engineer. For this aim, the PCA are re-analyzed by considering the indices C_{pm} and C_{pmk} by using NSs in this paper.

Kaya and olak [28] examined the studies on fuzzy process capability analysis and concluded that the majority of the studies were on traditional fuzzy sets named type-1 fuzzy sets. However, when studies on PCA had been examined, it was noticeable that studies with the extensions of fuzzy sets such as hesitant, intuitionistic and type-2 fuzzy sets were quite limited. Several authors contributed to analyzing the process capability using the extensions of fuzzy sets. Hesamian and Akbari [23] calculated the index $\hat{C}_{pm}$ when the SLs, mean, and target value were defined by using intuitionistic fuzzy sets (IFSs). Cao et al. [11] also calculated the multivariate process capability index (MPCI) S_{pk}^T using IFSs. They compared the index S_{pk}^T with the MPCI (MC_p). Aslam and Albassam [4] developed the index C_{pk} by taking into account neutrosophic mean and standard deviation for the sampling plan. Kahraman et al. [25] utilized intuitionistic fuzzy SLs to obtain the indices C_p and C_{pk}. Parchami et al. [40] introduced the indices C_p, C_{pk} and C_{pm} by using interval type-2 fuzzy SLs. Senvar and Kahraman [47] proposed the percentile-based indices C_p and C_{pk} by using interval type-2 fuzzy sets for non-normal processes.

It is observed that the PCA and SLs are not previously examined by using NSs. Differently from the existing papers into literature, the indices C_{pm} and C_{pmk} are analyzed by using NSs for the first time in this paper. The SLs are defined by neutrosophic sets, the single-valued neutrosophic states of the SLs, and the state of being flexible NSs have been investigated. The flexibility of the SLs will facilitate the expression of uncertainty and will allow easier integration into the real life. NSs are described by three values that are all independent from each other. These are truth-membership, indeterminacy-membership and falsity-membership functions. Independency of truth-membership and indeterminacy-membership functions from each other and expressing the fact that an individual does not have full control of the issue with the indeterminacy-membership function has an important place in modeling uncertainty problems. For example, Tomorrow it will be raining. This proposition may be 70 % true, indeterminate 38 % and 25 % false. Neutrosophic logic reflects human thinking a better compared to fuzzy logic. Since the NSs are included more information related to uncertainty, they are more suitable to model these uncertainties for engineering and scientific applications [44]. So, it is clear that these advantages can be used to obtain more realistic results about the PCA. Also quality inspector or quality engineer who defines the SLs may not always have full control of the subject. In order to eliminate this problem, the indeterminacy-membership function involved in defining NSs is very significant in expressing uncertainty. Additionally the proposed PCIs are also applied on real case applications.

The rest of this paper has been structured as follows: Section 2 includes a brief summary about the indices C_{pm} and C_{pmk}. Section 3 introduces the concepts of NSs. Section 4 shows details of the PCIs based on NSs. An application from manufacturing industry for the proposed neutrosophic PCIs are detailed into Section 5. The obtained results and suggestions for further research discussed into Section 6.

2 Process capability indices C_{pm} and C_{pmk}

Process performance can be analyzed by using some statistical PCIs that are summarized statistics which measure the actual or the potential performance of the process characteristics relative to the target and SLs by considering process location and dispersion [30]. PCIs, which provide numerical measures on whether a process meets the customer expectations or not, have been popularly applied for evaluating process performance. Several PCIs such as C_p, C_{pk}, C_{pm} and C_{pmk} are used to estimate the capability of a process [35],[32]. Since the designs of C_p and C_{pk} are independent of target value (T), they can fail to account for process loss incurred by the departure from the target. A well-known pioneer in the quality control, G. Taguchi, pays special attention on the loss in product's worth when one of product's characteristics deviates from the customers ideal value T. To take this factor into account, Hsiang and Taguchi introduced the index C_{pm} [24]. Chan et al. [14] developed the index C_{pm}, which provides indicators of both process variability and deviation of process mean from T, and also provides a quadratic loss interpretation, taking into account the process departure. As a result, the index C_{pm} incorporates with the variation of production items with respect to T and SLs, and emphasizes on measuring the ability of the process to cluster around the target. The index C_{pm} is defined as follow [53]:

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}} = \frac{d}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \text{ where } d = \frac{USL - LSL}{2}. \tag{1}$$

Pearn et al. [42] proposed the index C_{pmk}, which combines the features of the three earlier indices C_p, C_{pk}, and C_{pm}. The index C_{pmk} alerts the user whenever the process variance increases and/or the process mean deviates from its T. The index C_{pmk} is defined as follow [53]:

$$C_{pmk} = \min\left\{ \frac{USL - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - LSL}{3\sqrt{\sigma^2 + (\mu - T)^2}} \right\}. \tag{2}$$

If the process closer to T and the less variability, the more capable the process is. In Eq. 2, the numerator represents how close the process is to the T, and the denominator represents the variability of the process. If the process mean departs from the T, the reduced value of C_{pmk} is more significant than three indices C_p, C_{pk} and C_{pm}. Hence, the index C_{pmk} responds to the departure of the process mean from the T faster than the other three basic indices C_p, C_{pk} and C_{pm}, while it remains sensitive to the changes of process variation [15].

3 Neutrosophic sets

Zadeh was first introduced the fuzzy set theory (FST) as an extension of the crisp sets [58]. The fuzzy sets have only degrees of membership. Traditional fuzzy sets describe the uncertain information only with membership function. By added non-membership degree to fuzzy sets, Atanassov developed intuitionistic fuzzy set (IFSs) that handle fuzziness with both membership and non-membership functions as an extension of the fuzzy sets [5]. The sum of degrees of these functions cannot exceed 1. But the sum of degrees of these functions can exceed 1 in vague sets. Another type of FST is called Pythagorean fuzzy sets (PFSs) that have space to define with the sum of membership and non-membership functions greater than 1, but the sum of their square cannot be greater than 1 is developed. The sum of the total values received for each element in the universal set X for membership and non-membership values in IFS theory should be in the range of [0,1]. This restriction of membership and non-membership functions creates problems in modeling uncertain problems. [48]. Smarandache presented NSs, that independently include the truth-membership degree, indeterminacy-membership degree and falsity-membership degree to reflect the humanitarian style of thinking [48]. This theory is very important to solve the real problems since indeterminacy is quantified explicitly and to provide more flexibility compared to the other extension of fuzzy sets. NSs can deal with indeterminacy on the data [10]. Wang et al. [51] have developed single-valued NSs that is a special form of NSs. NSs are expressed independently from each other in the range of [0, 1] with defined truth-membership, $T_{\widetilde{Z}}(x)$, indeterminacy-membership, $I_{\widetilde{Z}}(x)$ and falsity-membership functions, $F_{\widetilde{Z}}(x)$, respectively. The fact that the $T_{\widetilde{Z}}(x)$ and the $I_{\widetilde{Z}}(x)$ functions are independent of each other are more flexible and realistic in the modeling of uncertain problems compared to modeling using other types of fuzzy sets. Since an individual may not always be fully informed about the problem, the $I_{\widetilde{Z}}(x)$ function that eliminates this problem has an important role in the modeling of uncertain problems [44]. Some basic notions of NSs can be briefly summarized as follows:

Definition 3.1. *Let X be a universe. A single valued neutrosophic set $\widetilde{\widetilde{Z}}$ in X is characterized by a truth-membership function $T_{\widetilde{\widetilde{Z}}}(x)$, an indeterminacy-membership function $I_{\widetilde{\widetilde{Z}}}(x)$ and a falsity-membership function $F_{\widetilde{\widetilde{Z}}}(x)$. A single valued neutrosophic set $\widetilde{\widetilde{Z}}$ is defined as follows [18]:*

$$\widetilde{\widetilde{Z}} = \left\{ < x, \left(T_{\widetilde{\widetilde{Z}}}(x), I_{\widetilde{\widetilde{Z}}}(x), F_{\widetilde{\widetilde{Z}}}(x) \right) >: x \in X, T_{\widetilde{\widetilde{Z}}}(x), I_{\widetilde{\widetilde{Z}}}(x), F_{\widetilde{\widetilde{Z}}}(x) \in [0,1] \right\}. \tag{3}$$

There is no restriction on the sum of $T_{\widetilde{\widetilde{Z}}}(x)$, $I_{\widetilde{\widetilde{Z}}}(x)$ and $F_{\widetilde{\widetilde{Z}}}(x)$. So that $0 \leqslant T_{\widetilde{\widetilde{Z}}}(x) + I_{\widetilde{\widetilde{Z}}}(x) + F_{\widetilde{\widetilde{Z}}}(x) \leqslant 3$.

Definition 3.2. *Let $w_{\widetilde{\widetilde{z}}}$, $u_{\widetilde{\widetilde{z}}}$ and $y_{\widetilde{\widetilde{z}}} \in [0,1]$ be any real numbers, $a_i, b_i, c_i, d_i \in \mathbb{R}$ and $a_i \leqslant b_i \leqslant c_i \leqslant d_i$ for $(i = 1,2,3)$. A single valued neutrosophic number (SVN-Number)*

$$\widetilde{\widetilde{z}} = \langle ((a_1, b_1, c_1, d_1), w_{\widetilde{\widetilde{z}}}), ((a_2, b_2, c_2, d_2), u_{\widetilde{\widetilde{z}}}), ((a_3, b_3, c_3, d_3), y_{\widetilde{\widetilde{z}}}) \rangle,$$

is a special neutrosophic set in $\mathbb{R}$ with truth-membership function $\mu_{\widetilde{\widetilde{z}}} : \mathbb{R} \to [0, w_{\widetilde{\widetilde{z}}}]$, indeterminacy-membership function $v_{\widetilde{\widetilde{z}}} : \mathbb{R} \to [0, u_{\widetilde{\widetilde{z}}}]$ and falsity-membership function $\lambda_{\widetilde{\widetilde{z}}} : \mathbb{R} \to [0, y_{\widetilde{\widetilde{z}}}]$ as follows [19],[57]:

$$\mu_{\widetilde{\widetilde{z}}}(x) = \begin{cases} f_{\mu}^l(x) & a_1 \leq x < b_1 \\ w_{\widetilde{\widetilde{z}}} & b_1 \leq x < c_1 \\ f_{\mu}^r(x) & c_1 \leq x \leq d_1 \\ 0 & Otherwise \end{cases}, v_{\widetilde{\widetilde{z}}}(x) = \begin{cases} f_v^l(x) & a_2 \leq x < b_2 \\ u_{\widetilde{\widetilde{z}}} & b_2 \leq x < c_2 \\ f_v^r(x) & c_2 \leq x \leq d_2 \\ 1 & Otherwise \end{cases}, \lambda_{\widetilde{\widetilde{z}}}(x) = \begin{cases} f_{\lambda}^l(x) & a_3 \leq x < b_3 \\ y_{\widetilde{\widetilde{z}}} & b_3 \leq x < c_3 \\ f_{\lambda}^r(x) & c_3 \leq x \leq d_3 \\ 1 & Otherwise \end{cases} \tag{4}$$

where the functions $f_{\mu}^l : [a_1, b_1] \to [0, w_{\widetilde{\widetilde{z}}}], f_v^r : [c_2, d_2] \to [u_{\widetilde{\widetilde{z}}}, 1], f_{\lambda}^r : [c_3, d_3] \to [y_{\widetilde{\widetilde{z}}}, 1]$ are increasing continuous functions. Then the functions $f_{\mu}^r : [c_1, d_1] \to [0, w_{\widetilde{\widetilde{z}}}], f_v^l : [a_2, b_2] \to [u_{\widetilde{\widetilde{z}}}, 1], f_{\lambda}^l : [a_3, b_3] \to [y_{\widetilde{\widetilde{z}}}, 1]$ are decreasing continuous functions.

Definition 3.3. *A single valued trapezoidal neutrosophic number (SVTNN) $\widetilde{\widetilde{z}} = \langle (a,b,c,d); w_{\widetilde{\widetilde{z}}}, u_{\widetilde{\widetilde{z}}}, y_{\widetilde{\widetilde{z}}} \rangle$ is a special neutrosophic set in $\mathbb{R}$ with truth-membership function, indeterminacy-membership function and falsity-membership function as follows [19]:*

$$\mu_{\widetilde{\widetilde{z}}}(x) = \begin{cases} (x-a)w_{\widetilde{\widetilde{z}}}/(b-a) & a \leq x < b \\ w_{\widetilde{\widetilde{z}}} & b \leq x \leq c \\ (d-x)w_{\widetilde{\widetilde{z}}}/(d-c) & c < x \leq d \\ 0 & Otherwise \end{cases} \tag{5}$$

$$v_{\widetilde{\widetilde{z}}}(x) = \begin{cases} ((b-x) + u_{\widetilde{\widetilde{z}}}(x-a))/(b-a) & a \leq x < b \\ u_{\widetilde{\widetilde{z}}} & b \leq x \leq c \\ ((x-c) + u_{\widetilde{\widetilde{z}}}(d-x))/(d-c) & c < x \leq d \\ 1 & Otherwise \end{cases} \tag{6}$$

$$\lambda_{\widetilde{\widetilde{z}}}(x) = \begin{cases} ((b-x) + y_{\widetilde{\widetilde{z}}}(x-a))/(b-a) & a \leq x < b \\ y_{\widetilde{\widetilde{z}}} & b \leq x \leq c \\ ((x-c) + y_{\widetilde{\widetilde{z}}}(d-x))/(d-c) & c < x \leq d \\ 0 & Otherwise \end{cases} \tag{7}$$

Definition 3.4. *A single valued triangular neutrosophic number (SVTrNN) $\widetilde{\widetilde{z}} = \langle (a,b,c); w_{\widetilde{\widetilde{z}}}, u_{\widetilde{\widetilde{z}}}, y_{\widetilde{\widetilde{z}}} \rangle$ is a special neutrosophic set in $\mathbb{R}$ with truth-membership function, indeterminacy-membership function and falsity-membership function as follows [19]:*

$$\mu_{\widetilde{\widetilde{z}}}(x) = \begin{cases} (x-a)w_{\widetilde{\widetilde{z}}}/(b-a) & a \leq x < b \\ w_{\widetilde{\widetilde{z}}} & x = b \\ (c-x)w_{\widetilde{\widetilde{z}}}/(c-b) & b \leq x \leq c \\ 0 & Otherwise \end{cases} \tag{8}$$

$$v_{\widetilde{\widetilde{z}}}(x) = \begin{cases} ((b-x) + u_{\widetilde{\widetilde{z}}}(x-a))/(b-a) & a \leq x < b \\ u_{\widetilde{\widetilde{z}}} & x = b \\ ((x-b) + u_{\widetilde{\widetilde{z}}}(c-x))/(c-b) & b \leq x \leq c \\ 1 & Otherwise \end{cases} \tag{9}$$

$$\lambda_{\widetilde{\widetilde{z}}}(x) = \begin{cases} ((b-x) + y_{\widetilde{\widetilde{z}}}(x-a))/(b-a) & a \leq x < b \\ y_{\widetilde{\widetilde{z}}} & x = b \\ ((x-b) + y_{\widetilde{\widetilde{z}}}(c-x))/(c-b) & b \leq x \leq c \\ 1 & Otherwise \end{cases} \tag{10}$$

If $a \geq 0$ at least $c > 0$ then SVTrNN is called a positive SVTrNN, denoted by $\widetilde{\widetilde{z}} > 0$. In the same way, if $c \leq 0$ at least $a < 0$ then SVTrNN is called a negative SVTrNN, denoted by $\widetilde{\widetilde{z}} < 0$.

Definition 3.5. $\widetilde{\widetilde{\Psi}} = \left\langle (a_1, b_1, c_1, d_1) ; w_{\widetilde{\widetilde{\Psi}}}, u_{\widetilde{\widetilde{\Psi}}}, y_{\widetilde{\widetilde{\Psi}}} \right\rangle$ and $\widetilde{\widetilde{\Omega}} = \left\langle (a_2, b_2, c_2, d_2) ; w_{\widetilde{\widetilde{\Omega}}}, u_{\widetilde{\widetilde{\Omega}}}, y_{\widetilde{\widetilde{\Omega}}} \right\rangle$ be two SVTNNs and $\gamma \neq 0$ any real number. Then the arithmetic operations of SVTNNs are as follows [18]:

$$\widetilde{\widetilde{\Psi}} \oplus \widetilde{\widetilde{\Omega}} = \left\langle (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2) ; w_{\widetilde{\widetilde{\Psi}}} \wedge w_{\widetilde{\widetilde{\Omega}}}, u_{\widetilde{\widetilde{\Psi}}} \vee u_{\widetilde{\widetilde{\Omega}}}, y_{\widetilde{\widetilde{\Psi}}} \vee y_{\widetilde{\widetilde{\Omega}}} \right\rangle \tag{11}$$

$$\widetilde{\widetilde{\Psi}} \ominus \widetilde{\widetilde{\Omega}} = \left\langle (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2) ; w_{\widetilde{\widetilde{\Psi}}} \wedge w_{\widetilde{\widetilde{\Omega}}}, u_{\widetilde{\widetilde{\Psi}}} \vee u_{\widetilde{\widetilde{\Omega}}}, y_{\widetilde{\widetilde{\Psi}}} \vee y_{\widetilde{\widetilde{\Omega}}} \right\rangle \tag{12}$$

$$\widetilde{\widetilde{\Psi}} \otimes \widetilde{\widetilde{\Omega}} = \begin{cases} \left\langle (a_1 a_2, b_1 b_2, c_1 c_2, d_1 d_2) ; w_{\widetilde{\widetilde{\Psi}}} \wedge w_{\widetilde{\widetilde{\Omega}}}, u_{\widetilde{\widetilde{\Psi}}} \vee u_{\widetilde{\widetilde{\Omega}}}, y_{\widetilde{\widetilde{\Psi}}} \vee y_{\widetilde{\widetilde{\Omega}}} \right\rangle & d_1 > 0, d_2 > 0 \\ \left\langle (a_1 d_2, b_1 c_2, c_1 b_2, d_1 a_2) ; w_{\widetilde{\widetilde{\Psi}}} \wedge w_{\widetilde{\widetilde{\Omega}}}, u_{\widetilde{\widetilde{\Psi}}} \vee u_{\widetilde{\widetilde{\Omega}}}, y_{\widetilde{\widetilde{\Psi}}} \vee y_{\widetilde{\widetilde{\Omega}}} \right\rangle & d_1 < 0, d_2 > 0 \\ \left\langle (d_1 d_2, c_1 c_2, b_1 b_2, a_1 a_2) ; w_{\widetilde{\widetilde{\Psi}}} \wedge w_{\widetilde{\widetilde{\Omega}}}, u_{\widetilde{\widetilde{\Psi}}} \vee u_{\widetilde{\widetilde{\Omega}}}, y_{\widetilde{\widetilde{\Psi}}} \vee y_{\widetilde{\widetilde{\Omega}}} \right\rangle & d_1 < 0, d_2 < 0 \end{cases} \tag{13}$$

$$\widetilde{\widetilde{\Psi}} \oslash \widetilde{\widetilde{\Omega}} = \begin{cases} \left\langle (a_1/d_2, b_1/c_2, c_1/b_2, d_1/a_2) ; w_{\widetilde{\widetilde{\Psi}}} \wedge w_{\widetilde{\widetilde{\Omega}}}, u_{\widetilde{\widetilde{\Psi}}} \vee u_{\widetilde{\widetilde{\Omega}}}, y_{\widetilde{\widetilde{\Psi}}} \vee y_{\widetilde{\widetilde{\Omega}}} \right\rangle & d_1 > 0, d_2 > 0 \\ \left\langle (d_1/d_2, c_1/c_2, b_1/b_2, a_1/a_2) ; w_{\widetilde{\widetilde{\Psi}}} \wedge w_{\widetilde{\widetilde{\Omega}}}, u_{\widetilde{\widetilde{\Psi}}} \vee u_{\widetilde{\widetilde{\Omega}}}, y_{\widetilde{\widetilde{\Psi}}} \vee y_{\widetilde{\widetilde{\Omega}}} \right\rangle & d_1 < 0, d_2 > 0 \\ \left\langle (d_1/a_2, c_1/b_2, b_1/c_2, a_1/d_2) ; w_{\widetilde{\widetilde{\Psi}}} \wedge w_{\widetilde{\widetilde{\Omega}}}, u_{\widetilde{\widetilde{\Psi}}} \vee u_{\widetilde{\widetilde{\Omega}}}, y_{\widetilde{\widetilde{\Psi}}} \vee y_{\widetilde{\widetilde{\Omega}}} \right\rangle & d_1 < 0, d_2 < 0 \end{cases} \tag{14}$$

$$\gamma \widetilde{\widetilde{\Psi}} = \begin{cases} \left\langle (\gamma a_1, \gamma b_1, \gamma c_1, \gamma d_1) ; w_{\widetilde{\widetilde{\Psi}}}, u_{\widetilde{\widetilde{\Psi}}}, y_{\widetilde{\widetilde{\Psi}}} \right\rangle & \gamma > 0 \\ \left\langle (\gamma d_1, \gamma c_1, \gamma b_1, \gamma a_1) ; w_{\widetilde{\widetilde{\Psi}}}, u_{\widetilde{\widetilde{\Psi}}}, y_{\widetilde{\widetilde{\Psi}}} \right\rangle & \gamma < 0 \end{cases} \tag{15}$$

$$\widetilde{\widetilde{\Psi}}^{\gamma} = \begin{cases} \left\langle (a_1^{\gamma}, b_1^{\gamma}, c_1^{\gamma}, d_1^{\gamma}) ; w_{\widetilde{\widetilde{\Psi}}}, u_{\widetilde{\widetilde{\Psi}}}, y_{\widetilde{\widetilde{\Psi}}} \right\rangle & \gamma > 0 \\ \left\langle (d_1^{\gamma}, c_1^{\gamma}, b_1^{\gamma}, a_1^{\gamma}) ; w_{\widetilde{\widetilde{\Psi}}}, u_{\widetilde{\widetilde{\Psi}}}, y_{\widetilde{\widetilde{\Psi}}} \right\rangle & \gamma < 0 \end{cases} \tag{16}$$

Definition 3.6. Let $\widetilde{\widetilde{\Psi}} = \left\langle (a_1, b_1, c_1) ; w_{\widetilde{\widetilde{\Psi}}}, u_{\widetilde{\widetilde{\Psi}}}, y_{\widetilde{\widetilde{\Psi}}} \right\rangle$ and $\widetilde{\widetilde{\Omega}} = \left\langle (a_2, b_2, c_2) ; w_{\widetilde{\widetilde{\Omega}}}, u_{\widetilde{\widetilde{\Omega}}}, y_{\widetilde{\widetilde{\Omega}}} \right\rangle$ be two SVTrNNs and $\gamma \neq 0$ any real number. Then the arithmetic operations of SVTrNNs are as follows [18]:

$$\widetilde{\widetilde{\Psi}} \oplus \widetilde{\widetilde{\Omega}} = \left\langle (a_1 + a_2, b_1 + b_2, c_1 + c_2) ; w_{\widetilde{\widetilde{\Psi}}} \wedge w_{\widetilde{\widetilde{\Omega}}}, u_{\widetilde{\widetilde{\Psi}}} \vee u_{\widetilde{\widetilde{\Omega}}}, y_{\widetilde{\widetilde{\Psi}}} \vee y_{\widetilde{\widetilde{\Omega}}} \right\rangle \tag{17}$$

$$\widetilde{\widetilde{\Psi}} \ominus \widetilde{\widetilde{\Omega}} = \left\langle (a_1 - c_2, b_1 - b_2, c_1 - a_2) ; w_{\widetilde{\widetilde{\Psi}}} \wedge w_{\widetilde{\widetilde{\Omega}}}, u_{\widetilde{\widetilde{\Psi}}} \vee u_{\widetilde{\widetilde{\Omega}}}, y_{\widetilde{\widetilde{\Psi}}} \vee y_{\widetilde{\widetilde{\Omega}}} \right\rangle \tag{18}$$

$$\widetilde{\widetilde{\Psi}} \otimes \widetilde{\widetilde{\Omega}} = \begin{cases} \left\langle (a_1 a_2, b_1 b_2, c_1 c_2) ; w_{\widetilde{\widetilde{\Psi}}} \wedge w_{\widetilde{\widetilde{\Omega}}}, u_{\widetilde{\widetilde{\Psi}}} \vee u_{\widetilde{\widetilde{\Omega}}}, y_{\widetilde{\widetilde{\Psi}}} \vee y_{\widetilde{\widetilde{\Omega}}} \right\rangle & c_1 > 0, c_2 > 0 \\ \left\langle (a_1 c_2, b_1 b_2, c_1 a_2) ; w_{\widetilde{\widetilde{\Psi}}} \wedge w_{\widetilde{\widetilde{\Omega}}}, u_{\widetilde{\widetilde{\Psi}}} \vee u_{\widetilde{\widetilde{\Omega}}}, y_{\widetilde{\widetilde{\Psi}}} \vee y_{\widetilde{\widetilde{\Omega}}} \right\rangle & c_1 < 0, c_2 > 0 \\ \left\langle (c_1 c_2, b_1 b_2, a_1 a_2) ; w_{\widetilde{\widetilde{\Psi}}} \wedge w_{\widetilde{\widetilde{\Omega}}}, u_{\widetilde{\widetilde{\Psi}}} \vee u_{\widetilde{\widetilde{\Omega}}}, y_{\widetilde{\widetilde{\Psi}}} \vee y_{\widetilde{\widetilde{\Omega}}} \right\rangle & c_1 < 0, c_2 < 0 \end{cases} \tag{19}$$

$$\widetilde{\widetilde{\Psi}} \oslash \widetilde{\widetilde{\Omega}} = \begin{cases} \left\langle (a_1/c_2, b_1/b_2, c_1/a_2) ; w_{\widetilde{\widetilde{\Psi}}} \wedge w_{\widetilde{\widetilde{\Omega}}}, u_{\widetilde{\widetilde{\Psi}}} \vee u_{\widetilde{\widetilde{\Omega}}}, y_{\widetilde{\widetilde{\Psi}}} \vee y_{\widetilde{\widetilde{\Omega}}} \right\rangle & c_1 > 0, c_2 > 0 \\ \left\langle (c_1/c_2, b_1/b_2, a_1/a_2) ; w_{\widetilde{\widetilde{\Psi}}} \wedge w_{\widetilde{\widetilde{\Omega}}}, u_{\widetilde{\widetilde{\Psi}}} \vee u_{\widetilde{\widetilde{\Omega}}}, y_{\widetilde{\widetilde{\Psi}}} \vee y_{\widetilde{\widetilde{\Omega}}} \right\rangle & c_1 < 0, c_2 > 0 \\ \left\langle (c_1/a_2, b_1/c_2, a_1/c_2) ; w_{\widetilde{\widetilde{\Psi}}} \wedge w_{\widetilde{\widetilde{\Omega}}}, u_{\widetilde{\widetilde{\Psi}}} \vee u_{\widetilde{\widetilde{\Omega}}}, y_{\widetilde{\widetilde{\Psi}}} \vee y_{\widetilde{\widetilde{\Omega}}} \right\rangle & c_1 < 0, c_2 < 0 \end{cases} \tag{20}$$

$$\gamma \widetilde{\widetilde{\Psi}} = \begin{cases} \left\langle (\gamma a_1, \gamma b_1, \gamma c_1) ; w_{\widetilde{\widetilde{\Psi}}}, u_{\widetilde{\widetilde{\Psi}}}, y_{\widetilde{\widetilde{\Psi}}} \right\rangle & \gamma > 0 \\ \left\langle (\gamma c_1, \gamma b_1, \gamma a_1) ; w_{\widetilde{\widetilde{\Psi}}}, u_{\widetilde{\widetilde{\Psi}}}, y_{\widetilde{\widetilde{\Psi}}} \right\rangle & \gamma < 0 \end{cases} \tag{21}$$

Score and accuracy functions are very important to compare between two SVTrNNs. Different methods are developed to compare NNs in the literature. A comparison has been made with the accuracy and score functions defined as follows:

Definition 3.7. *Let $\widetilde{\widetilde{z}} = \langle (a, b, c); w_{\widetilde{\widetilde{z}}}, u_{\widetilde{\widetilde{z}}}, y_{\widetilde{\widetilde{z}}} \rangle$ be a SVTrNN. Score function denoted by $S(\widetilde{\widetilde{z}})$ and accuracy function denoted by $A(\widetilde{\widetilde{z}})$, are defined as follow* [9]:

$$S(\widetilde{\widetilde{z}}) = \frac{1}{12}(a + 2b + c)\left(2 + \mu_{\widetilde{\widetilde{z}}} - v_{\widetilde{\widetilde{z}}} - \gamma_{\widetilde{\widetilde{z}}}\right). \tag{22}$$

$$A(\widetilde{\widetilde{z}}) = \frac{1}{12}(a + 2b + c)\left(2 + \mu_{\widetilde{\widetilde{z}}} - v_{\widetilde{\widetilde{z}}} + \gamma_{\widetilde{\widetilde{z}}}\right). \tag{23}$$

Definition 3.8. *Score and accuracy functions are used to compare between two SVTNNs. Let $\widetilde{\widetilde{z}} = \langle (a, b, c, d); w_{\widetilde{\widetilde{z}}}, u_{\widetilde{\widetilde{z}}}, y_{\widetilde{\widetilde{z}}} \rangle$ be a SVTNN. $S(\widetilde{\widetilde{z}})$ and $A(\widetilde{\widetilde{z}})$ are defined as follow* [57]:

$$S(\widetilde{\widetilde{z}}) = \frac{1}{12}(a + b + c + d)\left(2 + \mu_{\widetilde{\widetilde{z}}} - v_{\widetilde{\widetilde{z}}} - \gamma_{\widetilde{\widetilde{z}}}\right). \tag{24}$$

$$A(\widetilde{\widetilde{z}}) = \frac{1}{12}(a + b + c + d)\left(2 + \mu_{\widetilde{\widetilde{z}}} - v_{\widetilde{\widetilde{z}}} + \gamma_{\widetilde{\widetilde{z}}}\right). \tag{25}$$

Definition 3.9. *Let $\widetilde{\widetilde{z}}_1$ and $\widetilde{\widetilde{z}}_2$ be two SVTNNs. The ranking $\widetilde{\widetilde{z}}_1$ and $\widetilde{\widetilde{z}}_2$ by score and accuracy functions are defined as follows* [57]:

  - *If $S\left(\widetilde{\widetilde{z}}_1\right) < S\left(\widetilde{\widetilde{z}}_2\right)$, then $\widetilde{\widetilde{z}}_1 < \widetilde{\widetilde{z}}_2$.*
  - *If $S\left(\widetilde{\widetilde{z}}_1\right) > S\left(\widetilde{\widetilde{z}}_2\right)$, then $\widetilde{\widetilde{z}}_1 > \widetilde{\widetilde{z}}_2$.*
  - *If $S\left(\widetilde{\widetilde{z}}_1\right) = S\left(\widetilde{\widetilde{z}}_2\right)$ and*
      - *If $A\left(\widetilde{\widetilde{z}}_1\right) < A\left(\widetilde{\widetilde{z}}_2\right)$, then $\widetilde{\widetilde{z}}_1 < \widetilde{\widetilde{z}}_2$.*
      - *If $A\left(\widetilde{\widetilde{z}}_1\right) > A\left(\widetilde{\widetilde{z}}_2\right)$, then $\widetilde{\widetilde{z}}_1 > \widetilde{\widetilde{z}}_2$.*
      - *If $A\left(\widetilde{\widetilde{z}}_1\right) = A\left(\widetilde{\widetilde{z}}_2\right)$, then $\widetilde{\widetilde{z}}_1 = \widetilde{\widetilde{z}}_2$.*

Definition 3.10. *A SVNN $\widetilde{\widetilde{z}} = \langle ((a_1, b_1, c_1, d_1), w_{\widetilde{\widetilde{z}}}), ((a_2, b_2, c_2, d_2), u_{\widetilde{\widetilde{z}}}), ((a_3, b_3, c_3, d_3), y_{\widetilde{\widetilde{z}}}) \rangle$ for $w_{\widetilde{\widetilde{z}}} = 1$, $u_{\widetilde{\widetilde{z}}} = 0$, $y_{\widetilde{\widetilde{z}}} = 0$* [56] *is defined as follows* [17]:

$$\widetilde{\widetilde{z}} = \langle (a_1, b_1, c_1, d_1), ((a_2, b_2, c_2, d_2), (a_3, b_3, c_3, d_3) \rangle.$$

For SVNN, there exist twelve numbers $a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2, a_3, b_3, c_3, d_3 \in \mathrm{R}$ such that $a_3 \leq a_2 \leq a_1 \leq b_3 \leq b_2 \leq b_1 \leq c_1 \leq c_2 \leq c_3 \leq d_1 \leq d_2 \leq d_3$ and the six functions $T_{\widetilde{\widetilde{z}}}^L(x), T_{\widetilde{\widetilde{z}}}^R(x), I_{\widetilde{\widetilde{z}}}^L(x), I_{\widetilde{\widetilde{z}}}^R(x), F_{\widetilde{\widetilde{z}}}^L(x), F_{\widetilde{\widetilde{z}}}^R(x) : R \to [0, 1]$ to represent the truth, indeterminacy and falsity membership degrees of $\widetilde{\widetilde{z}}$ [8]:

Definition 3.11. *Let $\widetilde{\widetilde{z}} = \langle (a_1, b_1, c_1, d_1), ((a_2, b_2, c_2, d_2), (a_3, b_3, c_3, d_3) \rangle$ be a SVTNN. Score function denoted by $S\left(\widetilde{\widetilde{z}}\right)$ and accuracy function denoted by $H\left(\widetilde{\widetilde{z}}\right)$, are defined as* [57]:

$$S(\widetilde{\widetilde{z}}) = \frac{1}{3}\left(2 + \frac{a_1 + b_1 + c_1 + d_1}{4} - \frac{a_2 + b_2 + c_2 + d_2}{4} - \frac{a_3 + b_3 + c_3 + d_3}{4}\right), S(\widetilde{\widetilde{z}}) \in [0, 1]. \tag{26}$$

$$H(\widetilde{\widetilde{z}}) = \left(\frac{a_1 + b_1 + c_1 + d_1}{4} - \frac{a_3 + b_3 + c_3 + d_3}{4}\right), H(\widetilde{\widetilde{z}}) \in [-1, 1]. \tag{27}$$

Definition 3.12. *Let $\widetilde{\widetilde{z}} = \langle (a_1, b_1, c_1), ((a_2, b_2, c_2), (a_3, b_3, c_3) \rangle$ be a SVTrNN. $S(\widetilde{\widetilde{z}})$ and $H(\widetilde{\widetilde{z}})$ are defined as follow* [56]:

$$S(\widetilde{\widetilde{z}}) = \frac{1}{3}\left(2 + \frac{a_1 + 2b_1 + c_1}{4} - \frac{a_2 + 2b_2 + c_2}{4} - \frac{a_3 + 2b_3 + c_3}{4}\right), S(\widetilde{\widetilde{z}}) \in [0, 1]. \tag{28}$$

$$H(\widetilde{\widetilde{z}}) = \left(\frac{a_1 + 2b_1 + c_1}{4} - \frac{a_3 + 2b_3 + c_3}{4}\right), H(\widetilde{\widetilde{z}}) \in [-1, 1]. \tag{29}$$

Definition 3.13. *Let* $\widetilde{\widetilde{z}}_1 = \langle (a_{11}, b_{11}, c_{11}, d_{11}), (a_{21}, b_{21}, c_{21}, d_{21}), (a_{31}, b_{31}, c_{31}, d_{31}) \rangle$ *and*
$\widetilde{\widetilde{z}}_2 = \langle (a_{12}, b_{12}, c_{12}, d_{12}), (a_{22}, b_{22}, c_{22}, d_{22}), (a_{32}, b_{32}, c_{32}, d_{32}) \rangle$ *be two SVTNNs. The ranking* $\widetilde{\widetilde{z}}_1$ *and* $\widetilde{\widetilde{z}}_2$ *by score function and accuracy function are defined as follows* [17]:

- *If* $S\left(\widetilde{\widetilde{z}}_1\right) > S\left(\widetilde{\widetilde{z}}_2\right)$, *then* $\widetilde{\widetilde{z}}_1 > \widetilde{\widetilde{z}}_2$.
- *If* $S\left(\widetilde{\widetilde{z}}_1\right) = S\left(\widetilde{\widetilde{z}}_2\right)$ *and*
  - *If* $H\left(\widetilde{\widetilde{z}}_1\right) > H\left(\widetilde{\widetilde{z}}_2\right)$, *then* $\widetilde{\widetilde{z}}_1 > \widetilde{\widetilde{z}}_2$.
  - *If* $H\left(\widetilde{\widetilde{z}}_1\right) = H\left(\widetilde{\widetilde{z}}_2\right)$, *then* $\widetilde{\widetilde{z}}_1 = \widetilde{\widetilde{z}}_2$.

NSs have been used in many studies because of their advantages for modelling of uncertainty. Sun et al. [50] developed Choquet integral operator that based on NNs to handle multi criteria decision making (MCDM) problem. They solved third party logistics provider selection problem via developed methodology. Wang et al. [52] extended TODIM with NSs and presented a safety assessment methodology for a construction project. Kumar et al. [36] introduced the neutrosophic shortest path problem. Yang and Pang [55] proposed the multi valued neutrosophic based DEMATEL integrated TOPSIS methodology to solve truck selection problem. Yang et al. [54] proposed neutrosophic preference relations based MCDM methodology to prioritize medical tourism destinations. Aydin et al. [7] extended analytic hierarchy process (AHP) with NSs and evaluate criteria for safe cities. Otay and Kahraman [39] presented NSs based analytic network process (ANP) to solve supplier selection problem. Aslam [3] developed an acceptance sampling plan considering the NNs. Kumar et al. [37] focused on multi objective shortest path problem in the neutrosophic environment. Subasri and Selvakumari [49] handled neutrosophic travelling salesman problem with trapezoidal numbers. Ayber and Erginel [6] developed neutrosophic failure mode and effects analysis (FMEA) to use as a risk management tool. Sastri et al. [46] dealt with robot design and functions using NNs. Chakraborty et al. [13] applied de-neutrosophication method to the crash model in networking field and job-sequencing problem. Chakraborty et al. [12] introduced the de-neutrosophication method for triangular NNs and then applied it to project evaluation review technique and route selection problem. Radwan et al. [44] presented a neutrosophic expert system for learning management systems evaluation. Broumi et al. [10] compared the shortest path problem with various existing algorithms and concluded the best algorithm for certain environment. Kumar Das and Chakraborty [16] developed the pentagonal neutrosophic approach to solve linear programming problem. Ahmad and Adhami [1] formulated a multi objective nonlinear transportation problem based on neutrosophic environment. Rizk-Allah et al. [45] developed a neutrosophic mathematical model for multi objective transportation problem. Ahmad et al. [2] focused on manufacturing system under the neutrosophic hesitant fuzzy environment. Pramanik and Dey [43] presented a goal programming methodology to solve bi level linear programming problem for neutrosophic environments. In this paper, traditional PCIs have been analyzed and re-structured based on NSs. analyzed and re-structured based on NSs. For this aim, two of the well-known PCIs named C_{pm} and C_{pmk} have been re-formulated by using SVN-Numbers. Finally, the neutrosophic process capability indices (NPCIs) are obtained. As a result, the neutrosophic C_{pm} ($\widetilde{\widetilde{C}}_{pm}$) and neutrosophic C_{pmk} ($\widetilde{\widetilde{C}}_{pmk}$) indices have been obtained.

4   Process capability indices based on neutrosophic sets

It is clear that traditional PCA methods are unavailable when the process includes uncertainty. We also know that real case modelling problem includes many uncertainties. Because of the subjective style of thinking of quality engineer who plays a role in determining the SLs of a product or due to measurement errors caused by measuring system or inspector factor during quality control of parts that coming out of each process in production; the analysis and the monitoring of the process cannot be done effectively. Type-1 fuzzy sets cannot provide sufficient information about the process, in incomplete, inconsistent and imprecise information. In order to eliminate this situation, when the SLs are defined with NSs which are closest to human thinking and are more successful in modeling uncertainty according to other sets, they enable us to make more realistic information and more accurate decisions about the process of the product. By the way, they have an ability to clearer reflect for inspector hesitancy on PCA as a result of three parts structure. Since the quality engineer considers the truth as well as the indeterminacy and falsity degrees when defining the SLs, the uncertainty in defining the SLs is minimized and the efficiency of the PCA has been increased. Thus, the obtained results of PCIs will be more correct and usable. In this paper, design and analysis of PCIs based on NSs have been detailed. For this aim, we analyze the effect on process capability by categorizing SVN-Numbers that can be written in different forms in this section. In this way, it is aimed to give a different perspective to the PCA. Previously, the researcher handles maximum the degrees of truth, indeterminacy and falsity functions. But

the degrees of these MFs can be defined less than one. NNs are also used to provide these definitions. SVTNN is represented by four components as $(a, b, c$ and $d)$ and degree of three membership functions. But, the authors define sixteen different components $(a_1, b_1, c_1, d_1; a_2, b_2, c_2, d_2; a_3, b_3, c_3, d_3)$ [13]. According to this, analysis is performed in two categories. In Category 1, the effect of SVN-Numbers defined as $\widetilde{\widetilde{z}} = \langle (a, b, c, d); w_{\widetilde{\widetilde{z}}}, u_{\widetilde{\widetilde{z}}}, y_{\widetilde{\widetilde{z}}} \rangle$ on process capability is analyzed. In Category 2, the effect of SVN-Numbers defined as $\widetilde{\widetilde{z}} = \langle (a_1, b_1, c_1, d_1), ((a_2, b_2, c_2, d_2), (a_3, b_3, c_3, d_3)) \rangle$ on process capability is analyzed. This case considers in the asymmetric view of three different MFs individually. Let $a_1 = a_2 = a_3 = a; b_1 = b_2 = b_3 = b; c_1 = c_2 = c_3 = c; d_1 = d_2 = d_3 = d$ and take the maximum value of truth, indeterminacy and falsity MF as generalized value $w_{\widetilde{\widetilde{z}}}, u_{\widetilde{\widetilde{z}}}, y_{\widetilde{\widetilde{z}}}$ rather than 1, respectively then the SVTNN in Definition 3.3 comes [13]. Finally, the neutrosophic C_{pm} $(\widetilde{\widetilde{C}}_{pm})$ and neutrosophic C_{pmk} $(\widetilde{\widetilde{C}}_{pmk})$ indices have been obtained based on the following sub-sections:

4.1   Category 1: The case that specifications limits are defined as single-valued neutrosophic numbers

In this subsection, we consider the case that the degrees of truth, indeterminacy and falsity for NSs are not independent. The indeterminacy-membership function that involved in the definition of NSs is very important in expressing uncertainty. For this reason, SLs are defined as NNs. We firstly consider that the SLs are defined by using SVTrNNs. Now assume single-valued triangular neutrosophic SLs are defined as follow: $\widetilde{L\ddot{S}L} = \left\langle (lsl_1, lsl_2, lsl_3); w_{\widetilde{L\ddot{S}L}}, u_{\widetilde{L\ddot{S}L}}, y_{\widetilde{L\ddot{S}L}} \right\rangle$ and $\widetilde{U\ddot{S}L} = \left\langle (usl_1, usl_2, usl_3); w_{\widetilde{U\ddot{S}L}}, u_{\widetilde{U\ddot{S}L}}, y_{\widetilde{U\ddot{S}L}} \right\rangle$. The membership functions (MFs) of neutrosophic SLs $(\widetilde{S\ddot{L}}s)$ as SVTrNN are obtained as shown in Figure 1 .
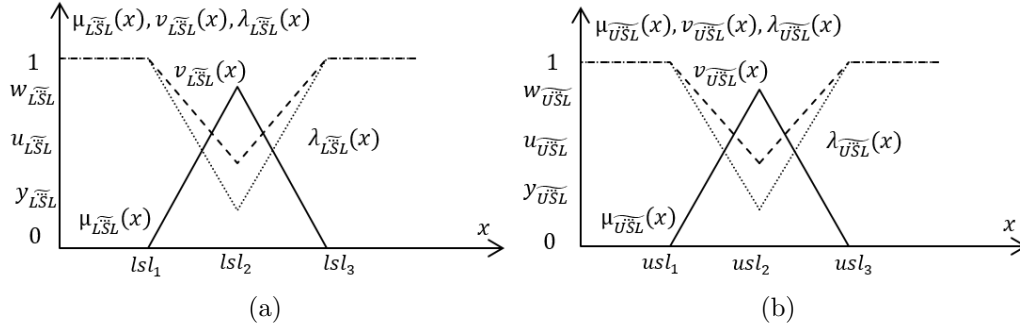
Figure 1: (a) The MFs of $\widetilde{L\ddot{S}L}$ by using SVTrNN (b) The MFs of $\widetilde{U\ddot{S}L}$ by using SVTrNN

Now assume that single-valued trapezoidal neutrosophic SLs are defined as follow: $\widetilde{L\ddot{S}L} = \left\langle (lsl_1, lsl_2, lsl_3, lsl_4); w_{\widetilde{L\ddot{S}L}}, u_{\widetilde{L\ddot{S}L}}, y_{\widetilde{L\ddot{S}L}} \right\rangle$ and $\widetilde{U\ddot{S}L} = \left\langle (usl_1, usl_2, usl_3, usl_4); w_{\widetilde{U\ddot{S}L}}, u_{\widetilde{U\ddot{S}L}}, y_{\widetilde{U\ddot{S}L}} \right\rangle$. The MFs of $\widetilde{S\ddot{L}}s$ as SVTNN are obtained as shown in Figure 2 .
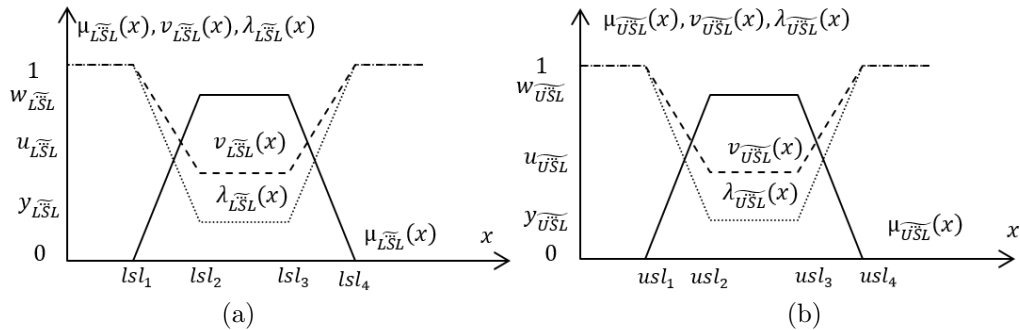
Figure 2: (a) The MFs of $\widetilde{L\ddot{S}L}$ by using SVTNN (b) The MFs of $\widetilde{U\ddot{S}L}$ by using SVTNN

4.1.1 The index $\widetilde{\ddot{C}}_{pm}$

The general definition of the index $\widetilde{\ddot{C}}_{pm}$ is as shown in Eq. (30) :

$$\widetilde{\ddot{C}}_{pm} = \frac{\widetilde{U\ddot{S}L} \ominus \widetilde{L\ddot{S}L}}{6\sqrt{\sigma^2 + (\mu - T)^2}}. \tag{30}$$

The index $\widetilde{\ddot{C}}_{pm}$ is calculated by using SVTrNNs as follows:

$$\widetilde{\ddot{C}}_{pm} = \frac{\left\langle (usl_1, usl_2, usl_3); w_{\widetilde{U\ddot{S}L}}, u_{\widetilde{U\ddot{S}L}}, y_{\widetilde{U\ddot{S}L}} \right\rangle \ominus \left\langle (lsl_1, lsl_2, lsl_3); w_{\widetilde{L\ddot{S}L}}, u_{\widetilde{L\ddot{S}L}}, y_{\widetilde{L\ddot{S}L}} \right\rangle}{6\sqrt{\sigma^2 + (\mu - T)^2}}. \tag{31}$$

$$\widetilde{\ddot{C}}_{pm} = \left\langle \left(\frac{usl_1 - lsl_3}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_2 - lsl_2}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_3 - lsl_1}{6\sqrt{\sigma^2 + (\mu - T)^2}} \right); w_{\widetilde{U\ddot{S}L}} \wedge w_{\widetilde{L\ddot{S}L}}, u_{\widetilde{U\ddot{S}L}} \vee u_{\widetilde{L\ddot{S}L}}, y_{\widetilde{U\ddot{S}L}} \vee y_{\widetilde{L\ddot{S}L}} \right\rangle. \tag{32}$$

Then the MFs of the index $\widetilde{\ddot{C}}_{pm}$ is shown in Figure 3 .
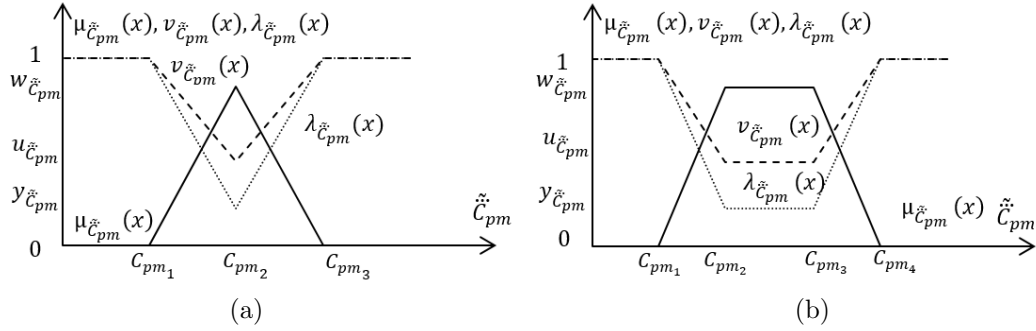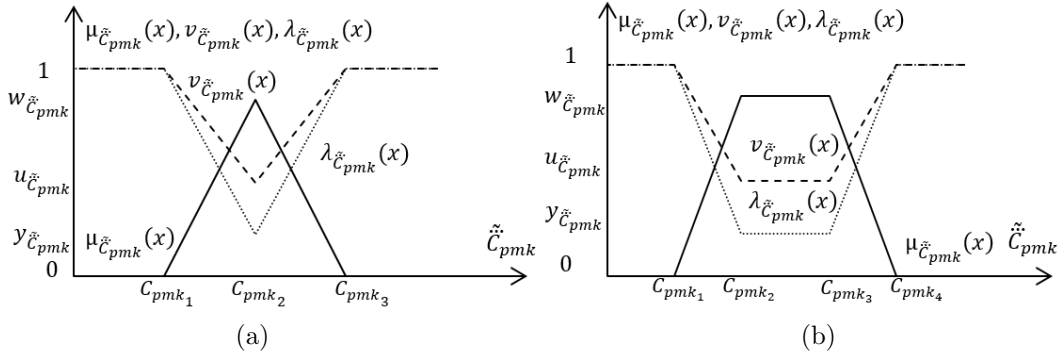

Figure 3: (a) The MFs of the index $\widetilde{\ddot{C}}_{pm}$ by using SVTrNN (b) The MFs of the index $\widetilde{\ddot{C}}_{pm}$ by using SVTNN

The index $\widetilde{\ddot{C}}_{pm}$ is calculated by using SVTNNs as follows:

$$\widetilde{\ddot{C}}_{pm} = \frac{\left\langle (usl_1, usl_2, usl_3, usl_4); w_{\widetilde{U\ddot{S}L}}, u_{\widetilde{U\ddot{S}L}}, y_{\widetilde{U\ddot{S}L}} \right\rangle \ominus \left\langle (lsl_1, lsl_2, lsl_3, lsl_4); w_{\widetilde{L\ddot{S}L}}, u_{\widetilde{L\ddot{S}L}}, y_{\widetilde{L\ddot{S}L}} \right\rangle}{6\sqrt{\sigma^2 + (\mu - T)^2}}. \tag{33}$$

$$\widetilde{\ddot{C}}_{pm} = \left\langle \left(\frac{usl_1 - lsl_4}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_2 - lsl_3}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_3 - lsl_2}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_4 - lsl_1}{6\sqrt{\sigma^2 + (\mu - T)^2}} \right); \atop w_{\widetilde{U\ddot{S}L}} \wedge w_{\widetilde{L\ddot{S}L}}, u_{\widetilde{U\ddot{S}L}} \vee u_{\widetilde{L\ddot{S}L}}, y_{\widetilde{U\ddot{S}L}} \vee y_{\widetilde{L\ddot{S}L}} \right\rangle. \tag{34}$$

Then the MFs of the index $\widetilde{\ddot{C}}_{pm}$ is shown in Figure 3 .

4.1.2 The index $\widetilde{\ddot{C}}_{pmk}$

The general definition of the index $\widetilde{\ddot{C}}_{pmk}$ can be obtained as detailed below:

$$\widetilde{\ddot{C}}_{pmk} = \min \left\{ \widetilde{\ddot{C}}_{pmku}, \widetilde{\ddot{C}}_{pmkl} \right\}. \tag{35}$$

$$\widetilde{\ddot{C}}_{pmk} = \min \left\{ \frac{\widetilde{U\ddot{S}L} \ominus \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu \ominus \widetilde{L\ddot{S}L}}{3\sqrt{\sigma^2 + (\mu - T)^2}} \right\}. \tag{36}$$

The index $\widetilde{\widetilde{C}}_{pmk}$ is calculated by using SVTrNNs as follows:

$$\widetilde{\widetilde{C}}_{pmk} = \min \left\{ \frac{\left\langle (usl_1, usl_2, usl_3); w_{\widetilde{U\widetilde{S}L}}, u_{\widetilde{U\widetilde{S}L}}, y_{\widetilde{U\widetilde{S}L}} \right\rangle \ominus \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu \ominus \left\langle (lsl_1, lsl_2, lsl_3); w_{\widetilde{L\widetilde{S}L}}, u_{\widetilde{L\widetilde{S}L}}, y_{\widetilde{L\widetilde{S}L}} \right\rangle}{3\sqrt{\sigma^2 + (\mu - T)^2}} \right\}. \tag{37}$$

$$\widetilde{\widetilde{C}}_{pmk} = \min \left\{ \begin{array}{l} \left\langle \left(\frac{usl_1 - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_2 - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_3 - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}} \right); w_{\widetilde{U\widetilde{S}L}}, u_{\widetilde{U\widetilde{S}L}}, y_{\widetilde{U\widetilde{S}L}} \right\rangle \\ \left\langle \left(\frac{\mu - lsl_3}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - lsl_2}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - lsl_1}{3\sqrt{\sigma^2 + (\mu - T)^2}} \right); w_{\widetilde{L\widetilde{S}L}}, u_{\widetilde{L\widetilde{S}L}}, y_{\widetilde{L\widetilde{S}L}} \right\rangle \end{array} \right\}. \tag{38}$$

These one-sided capability indices are calculated by using Eq. (2) . Additionally, the indices $\widetilde{\widetilde{C}}_{pmkl}$ and $\widetilde{\widetilde{C}}_{pmku}$ named as one-sided capability indices can be also obtained by using SVTRNNs as follows:

$$\widetilde{\widetilde{C}}_{pmkl} = \left\langle \left(\frac{\mu - lsl_3}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - lsl_2}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - lsl_1}{3\sqrt{\sigma^2 + (\mu - T)^2}} \right); w_{\widetilde{L\widetilde{S}L}}, u_{\widetilde{L\widetilde{S}L}}, y_{\widetilde{L\widetilde{S}L}} \right\rangle. \tag{39}$$

$$\widetilde{\widetilde{C}}_{pmku} = \left\langle \left(\frac{usl_1 - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_2 - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_3 - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}} \right); w_{\widetilde{U\widetilde{S}L}}, u_{\widetilde{U\widetilde{S}L}}, y_{\widetilde{U\widetilde{S}L}} \right\rangle. \tag{40}$$

Then the MFs of the index $\widetilde{\widetilde{C}}_{pmk}$ is shown in Figure 4 .

Figure 4: (a) The MFs of the index $\widetilde{\widetilde{C}}_{pmk}$ by using SVTrNN (b) The MFs of the index $\widetilde{\widetilde{C}}_{pmk}$ by using SVTNN

The index $\widetilde{\widetilde{C}}_{pmk}$ is also derivate by using SVTNNs as follows:

$$\widetilde{\widetilde{C}}_{pmk} = \min \left\{ \frac{\left\langle (usl_1, usl_2, usl_3, usl_4); w_{\widetilde{U\widetilde{S}L}}, u_{\widetilde{U\widetilde{S}L}}, y_{\widetilde{U\widetilde{S}L}} \right\rangle \ominus \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu \ominus \left\langle (lsl_1, lsl_2, lsl_3, lsl_4); w_{\widetilde{L\widetilde{S}L}}, u_{\widetilde{L\widetilde{S}L}}, y_{\widetilde{L\widetilde{S}L}} \right\rangle}{3\sqrt{\sigma^2 + (\mu - T)^2}} \right\}. \tag{41}$$

The Eq. (41) can be re-consider as follows:

$$\widetilde{\widetilde{C}}_{pmk} = \min \left\{ \begin{array}{l} \left\langle \left(\frac{usl_1 - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_2 - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_3 - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_4 - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}} \right); w_{\widetilde{U\widetilde{S}L}}, u_{\widetilde{U\widetilde{S}L}}, y_{\widetilde{U\widetilde{S}L}} \right\rangle, \\ \left\langle \left(\frac{\mu - lsl_4}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - lsl_3}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - lsl_2}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - lsl_1}{3\sqrt{\sigma^2 + (\mu - T)^2}} \right); w_{\widetilde{L\widetilde{S}L}}, u_{\widetilde{L\widetilde{S}L}}, y_{\widetilde{L\widetilde{S}L}} \right\rangle \end{array} \right\}. \tag{42}$$

Additionally, the indices $\widetilde{\widetilde{C}}_{pmkl}$ and $\widetilde{\widetilde{C}}_{pmku}$ can be obtained by using SVTNNs as follows:

$$\widetilde{\widetilde{C}}_{pmkl} = \left\langle \left(\frac{\mu - lsl_4}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - lsl_3}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - lsl_2}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - lsl_1}{3\sqrt{\sigma^2 + (\mu - T)^2}} \right); w_{\widetilde{L\widetilde{S}L}}, u_{\widetilde{L\widetilde{S}L}}, y_{\widetilde{L\widetilde{S}L}} \right\rangle. \tag{43}$$

$$\widetilde{\widetilde{C}}_{pmku} = \left\langle \left(\frac{usl_1 - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_2 - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_3 - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}} \frac{usl_4 - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}} \right); w_{\widetilde{U\widetilde{S}L}}, u_{\widetilde{U\widetilde{S}L}}, y_{\widetilde{U\widetilde{S}L}} \right\rangle. \tag{44}$$

The minimum value for the $\widetilde{\widetilde{C}}_{pmk}$ index is calculated with the score and accuracy functions in Definition 3.7 and Definition 3.8. Based on STVNNs, the MFs of the index $\widetilde{\widetilde{C}}_{pmk}$ is shown in Figure 4 .

4.2 Category 2: The Case That Specifications Limits are Flexible Structure of Single-Valued Neutrosophic numbers

In this subsection, we consider the case that degrees of truth, indeterminacy and falsity are independent. For this aim, a new case is related to a flexible structure that is obtained by using the SLs. Instead of expressed as Definition 3.4, the expanding border values of truth, indeterminacy and falsity membership functions provides flexibility to SLs. The flexibility of the SLs makes them easier to apply for real case problems. Now assume that the SLs are defined in a flexible structure of SVTrNN as $\widetilde{L\ddot{S}L} = \left\langle (lsl_1, lsl_2, lsl_3), (lsl_1', lsl_2, lsl_3'), (lsl_1'', lsl_2, lsl_3'') \right\rangle$ and $\widetilde{U\ddot{S}L} = \left\langle (usl_1, usl_2, usl_3), (usl_1', usl_2, usl_3'), (usl_1'', usl_2, usl_3'') \right\rangle$. The MFs of the neutrosophic $\widetilde{S\ddot{L}s}$ as SVTrNN are obtained in a flexible structure as shown in Figure 5 .
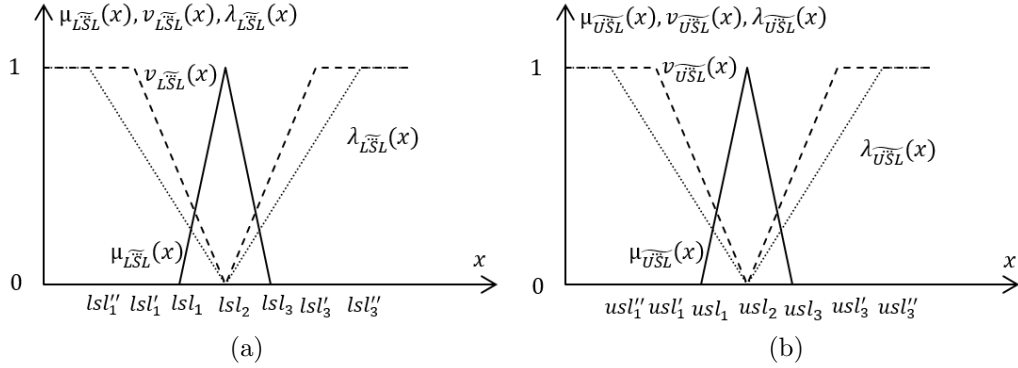
Figure 5: (a) The MFs of $\widetilde{L\ddot{S}L}$ by using SVTrNN for flexible structure (b) The MFs of $\widetilde{U\ddot{S}L}$ by using SVTrNN for flexible structure

SLs are defined in a flexible structure of SVTNN as $\widetilde{L\ddot{S}L} = \left\langle (lsl_1, lsl_2, lsl_3, lsl_4), (lsl_1', lsl_2, lsl_3, lsl_4'), (lsl_1'', lsl_2, lsl_3, lsl_4'') \right\rangle$ and $\widetilde{U\ddot{S}L} = \left\langle (usl_1, usl_2, usl_3, usl_4), (usl_1', usl_2, usl_3, usl_4'), (usl_1'', usl_2, usl_3, usl_4'') \right\rangle$. The MFs of the neutrosophic $\widetilde{S\ddot{L}s}$ as SVTNN are obtained in a flexible structure as shown in Figure 6 .
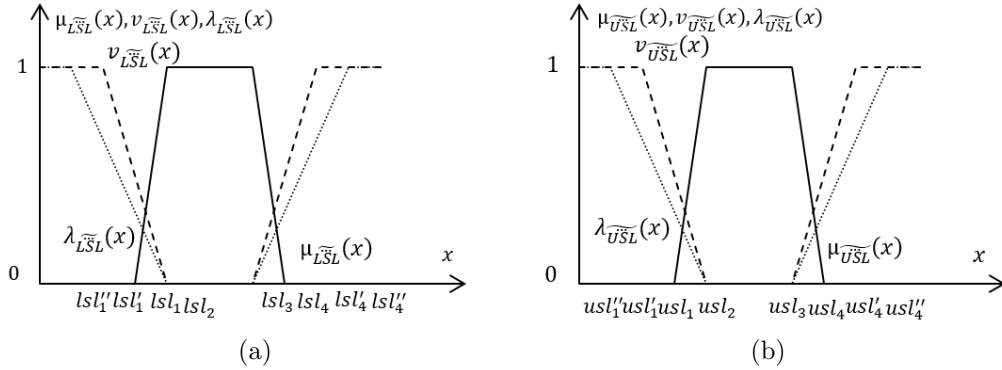
Figure 6: (a) The MFs of $\widetilde{L\ddot{S}L}$ by using SVTNN for flexible structure (b) The MFs of $\widetilde{U\ddot{S}L}$ by using SVTNN for flexible structure

4.2.1 The index $\widetilde{\widetilde{C}}_{pm}$

The index $\widetilde{\widetilde{C}}_{pm}$ for a flexible structure that is obtained by using the SLs which expressed by extending the limit values of the truth, indeterminacy and falsity membership functions can be obtained as follows:

$$\widetilde{\widetilde{C}}_{pm} = \frac{\left\langle (usl_1, usl_2, usl_3), (usl_1', usl_2, usl_3'), (usl_1'', usl_2, usl_3'') \right\rangle \ominus \left\langle (lsl_1, lsl_2, lsl_3), (lsl_1', lsl_2, lsl_3'), (lsl_1'', lsl_2, lsl_3'') \right\rangle}{6\sqrt{\sigma^2 + (\mu - T)^2}}.$$

(45)

$$\widetilde{\widetilde{C}}_{pm} = \left\langle \begin{array}{c} \left(\frac{usl_1 - lsl_3}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_2 - lsl_2}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_3 - lsl_1}{6\sqrt{\sigma^2 + (\mu - T)^2}} \right), \left(\frac{usl_1' - lsl_3'}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_2 - lsl_2}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_3' - lsl_1'}{6\sqrt{\sigma^2 + (\mu - T)^2}} \right), \\ \left(\frac{usl_1'' - lsl_3''}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_2 - lsl_2}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_3'' - lsl_1''}{6\sqrt{\sigma^2 + (\mu - T)^2}} \right) \end{array} \right\rangle.$$

(46)

The MFs of the index $\widetilde{\widetilde{C}}_{pm}$ based on SVTrNN in a flexible structure is shown in Figure 7 .
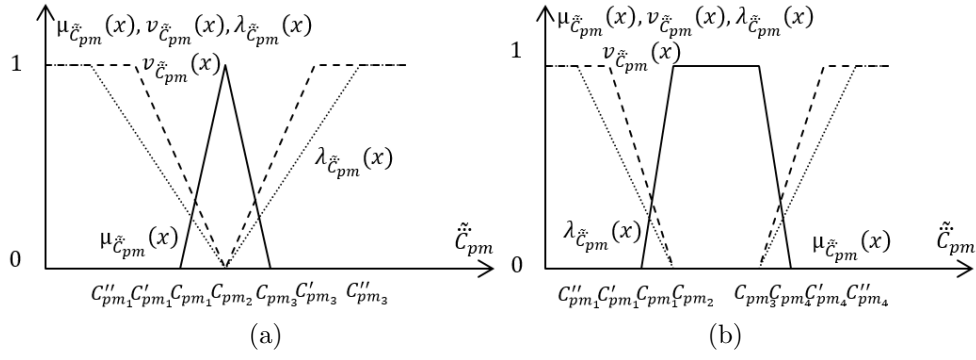
Figure 7: (a) The MFs of the index $\widetilde{\widetilde{C}}_{pm}$ by using SVTrNN for flexible structure (b) The MFs of the index $\widetilde{\widetilde{C}}_{pm}$ by using SVTNN for flexible structure

Based on SVTNN, the index $\widetilde{\widetilde{C}}_{pm}$ can be derivate as follows:

$$\widetilde{\widetilde{C}}_{pm} = \frac{\langle (usl_1, usl_2, usl_3, usl_4), (usl_1', usl_2, usl_3, usl_4'), (usl_1'', usl_2, usl_3, usl_4'') \rangle \ominus}{6\sqrt{\sigma^2 + (\mu - T)^2}} \langle (lsl_1, lsl_2, lsl_3, lsl_4), (lsl_1', lsl_2, lsl_3, lsl_4'), (lsl_1'', lsl_2, lsl_3, lsl_4'') \rangle}{6\sqrt{\sigma^2 + (\mu - T)^2}}.$$

(47)

$$\widetilde{\widetilde{C}}_{pm} = \left\langle \begin{array}{c} \left(\frac{usl_1 - lsl_4}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_2 - lsl_3}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_3 - lsl_2}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_4 - lsl_1}{6\sqrt{\sigma^2 + (\mu - T)^2}} \right), \\ \left(\frac{usl_1' - lsl_4'}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_2 - lsl_3}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_3 - lsl_2}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_4' - lsl_1'}{6\sqrt{\sigma^2 + (\mu - T)^2}} \right), \\ \left(\frac{usl_1'' - lsl_4''}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_2 - lsl_3}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_3 - lsl_2}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{usl_4'' - lsl_1''}{6\sqrt{\sigma^2 + (\mu - T)^2}} \right) \end{array} \right\rangle.$$

(48)

Then the MFs of the index $\widetilde{\widetilde{C}}_{pm}$ based on SVTNN in a flexible structure can be produced as shown in Figure 7 .

4.2.2 The index $\widetilde{\widetilde{C}}_{pmk}$

The index $\widetilde{\widetilde{C}}_{pmk}$ based on SVTrNN for a flexible structure can be obtained as detailed in below:

$$\widetilde{\widetilde{C}}_{pmk} = \min \left\{ \frac{\langle (usl_1, usl_2, usl_3), (usl_1', usl_2, usl_3'), (usl_1'', usl_2, usl_3'') \rangle \ominus \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu \ominus ((lsl_1, lsl_2, lsl_3), (lsl_1', lsl_2, lsl_3'), (lsl_1'', lsl_2, lsl_3''))}{3\sqrt{\sigma^2 + (\mu - T)^2}} \right\}.$$

(49)

$$\widetilde{\widetilde{C}}_{pmk} = \min \left\{ \begin{array}{l} \left\langle \left(\frac{usl_1-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl_2-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl_3-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}} \right) \left(\frac{usl'_1-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl_2-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl'_3-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}} \right), \right\rangle \\ \qquad\qquad \left(\frac{usl''_1-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl_2-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl''_3-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}} \right) \\ \left\langle \left(\frac{\mu-lsl_3}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl_2}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl_1}{3\sqrt{\sigma^2+(\mu-T)^2}} \right) \left(\frac{\mu-lsl'_3}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl_2}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl'_1}{3\sqrt{\sigma^2+(\mu-T)^2}} \right), \right\rangle \\ \qquad\qquad \left(\frac{\mu-lsl''_3}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl_2}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl''_1}{3\sqrt{\sigma^2+(\mu-T)^2}} \right) \end{array} \right\}. \tag{50}$$

Then the one-sided capability indices based on SVTrNN are defined as follows:

$$\widetilde{\widetilde{C}}_{pmkl} = \left\langle \left(\frac{\mu-lsl_3}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl_2}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl_1}{3\sqrt{\sigma^2+(\mu-T)^2}} \right), \left(\frac{\mu-lsl'_3}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl_2}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl'_1}{3\sqrt{\sigma^2+(\mu-T)^2}} \right) \right\rangle \tag{51}$$
$$\left(\frac{\mu-lsl''_3}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl_2}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl''_1}{3\sqrt{\sigma^2+(\mu-T)^2}} \right).$$

$$\widetilde{\widetilde{C}}_{pmku} = \left\langle \left(\frac{usl_1-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl_2-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl_3-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}} \right), \left(\frac{usl'_1-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl_2-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl'_3-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}} \right) \right\rangle \tag{52}$$
$$\left(\frac{usl''_1-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl_2-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl''_3-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}} \right).$$

The MFs of the index $\widetilde{\widetilde{C}}_{pmk}$ based on SVTrNN for flexible structure is drawn in Figure 8 .
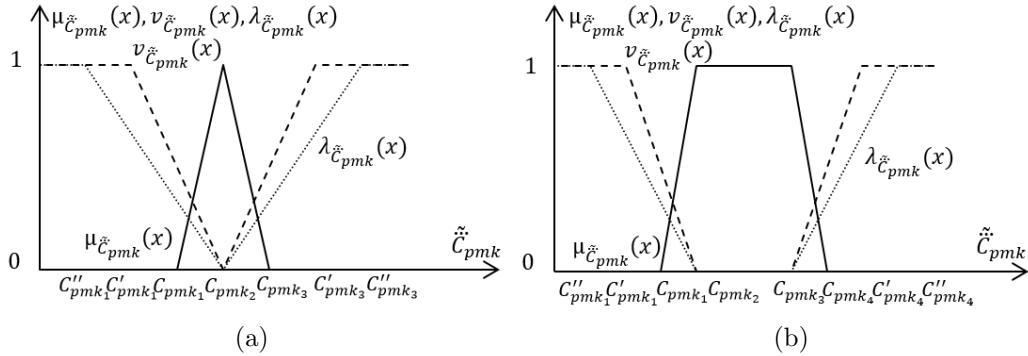

Figure 8: (a) The MFs of the index $\widetilde{\widetilde{C}}_{pmk}$ by using SVTrNN for flexible structure (b) The MFs of the index $\widetilde{\widetilde{C}}_{pmk}$ based on SVTNN for flexible structure

Similarly, the index $\widetilde{\widetilde{C}}_{pmk}$ based on SVTNN for a flexible structure is derivate as follows:

$$\widetilde{\widetilde{C}}_{pmk} = \min \left\{ \begin{array}{l} \frac{\left\langle (usl_1,usl_2,usl_3,usl_4),\left(usl'_1,usl_2,usl_3,usl'_4\right),\left(usl''_1,usl_2,usl_3,usl''_4\right)\right\rangle \ominus \mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \\ \frac{\mu \ominus \left\langle (lsl_1,lsl_2,lsl_3,lsl_4),\left(lsl'_1,lsl_2,lsl_3,lsl'_4\right),\left(lsl''_1,lsl_2,lsl_3,lsl''_4\right)\right\rangle}{3\sqrt{\sigma^2+(\mu-T)^2}}. \end{array} \right\} \tag{53}$$

$$\widetilde{\widetilde{C}}_{pmk} = \min \left\{ \begin{array}{l} \left\langle \begin{array}{l} \left(\frac{usl_1-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl_2-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl_3-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl_4-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}} \right), \\ \left(\frac{usl'_1-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl_2-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl_3-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl'_4-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}} \right), \\ \left(\frac{usl''_1-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl_2-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl_3-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl''_4-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}} \right) \end{array} \right\rangle, \\ \left\langle \begin{array}{l} \left(\frac{\mu-lsl_4}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl_3}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl_2}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl_1}{3\sqrt{\sigma^2+(\mu-T)^2}} \right), \\ \left(\frac{\mu-lsl'_4}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl_3}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl_2}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl'_1}{3\sqrt{\sigma^2+(\mu-T)^2}} \right), \\ \left(\frac{\mu-lsl''_4}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl_3}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl_2}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl''_1}{3\sqrt{\sigma^2+(\mu-T)^2}} \right) \end{array} \right\rangle \end{array} \right\}. \tag{54}$$

Then the one-sided capability indices named $\widetilde{\widetilde{C}}_{pmkl}$ and $\widetilde{\widetilde{C}}_{pmku}$ based on SVTNN are defined as follows:

$$
\widetilde{\widetilde{C}}_{pmkl} = \left\langle \begin{pmatrix} \left(\frac{\mu-lsl_4}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl_3}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl_2}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl_1}{3\sqrt{\sigma^2+(\mu-T)^2}} \right), \\ \left(\frac{\mu-lsl_4'}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl_3}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl_2}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl_1'}{3\sqrt{\sigma^2+(\mu-T)^2}} \right), \\ \left(\frac{\mu-lsl_4''}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl_3}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl_2}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{\mu-lsl_1''}{3\sqrt{\sigma^2+(\mu-T)^2}} \right) \end{pmatrix}, \right\rangle. \tag{55}
$$

$$
\widetilde{\widetilde{C}}_{pmku} = \left\langle \begin{pmatrix} \left(\frac{usl_1-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl_2-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl_3-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl_4-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}} \right), \\ \left(\frac{usl_1'-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl_2-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl_3-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl_4'-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}} \right), \\ \left(\frac{usl_1''-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl_2-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl_3-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}}, \frac{usl_4''-\mu}{3\sqrt{\sigma^2+(\mu-T)^2}} \right) \end{pmatrix}, \right\rangle. \tag{56}
$$

The minimum value of the $\widetilde{\widetilde{C}}_{pmk}$ index is calculated with the score and accuracy functions in Definition 3.11 and Definition 3.12. Consequently, the MFs of the index $\widetilde{\widetilde{C}}_{pmk}$ based on SVTNN for flexible structure is obtained as shown in Figure 8 .

5   An application for the proposed capability indices

In this section, some real case application have been analyzed for the proposed methodologies. For this aim, a manufacturing company in Konyas Industrial Area that produces piston, liner, and piston have been analyzed based on NSs [33],[20],[29],[26],[27]. The piston production process has been analyzed and total length of piston diameters are handled in this study. The process has been analyzed to improve its capability. The defective and non-defective products produced in the company have been examined. As the PCA of a product requires a sensitive measurement, the SLs of its may not only define as crisp values. The quality department define the SLs of the piston lengths by using NSs to improve the sensitivity and flexibility. The mean and variance values of ideal dimension values are estimated as 175.20 and 0.00009580, respectively. For two cases detailed above, the indices $\widetilde{\widetilde{C}}_{pm}$ and $\widetilde{\widetilde{C}}_{pmk}$ have been obtained based on real case as detailed below.

5.1   Category 1: An application for the case that specifications limits are single-valued neutrosophic numbers

A real case application for piston length has been provided in this subsection to illustrate the performance of the proposed approach for the NPCIs. Thus, the uncertainty and fuzziness in the opinions of the quality inspector are handled in this way. For this purpose, the SLs based SVTrNN and SVTNN for the length of the piston can be defined as follows: $\widetilde{L\ddot{S}L} = \langle (175.164, 175.165, 175.166); 0.950, 0.050, 0.075 \rangle$, $\widetilde{U\ddot{S}L} = \langle (175.234, 175.235, 175.236); 0.950, 0.050, 0.075 \rangle$ and $\widetilde{L\ddot{S}L} = \langle (175.234, 175.235.175.236, 175.237); 0.950, 0.055, 0.070 \rangle$, $\widetilde{U\ddot{S}L} = \langle (175.164, 175.165, 175.166, 175.167); 0.950, 0.055, 0.070 \rangle$, respectively. The indices $\widetilde{\widetilde{C}}_{pm}$ and $\widetilde{\widetilde{C}}_{pmk}$ based SVTrNN are calculated as follows:

$$
\widetilde{\widetilde{C}}_{pm} = \langle (1.16, 1.19, 1.22); 0.950, 0.050, 0.075 \rangle.
$$

$$
\widetilde{\widetilde{C}}_{pmk} = \min\{ \langle (1.16, 1.19, 1.22); 0.950, 0.050, 0.075 \rangle; \langle (1.16, 1.19, 1.22); 0.950, 0.050, 0.075 \rangle \}
$$

$$
\widetilde{\widetilde{C}}_{pmk} = \langle (1.16, 1.19, 1.22); 0.950, 0.050, 0.075 \rangle.
$$

The score values of $\widetilde{\widetilde{C}}_{pmkl}$ and $\widetilde{\widetilde{C}}_{pmku}$ indices are calculated by using Eq. (22). Score values are found as 1.12. Then the accuracy values of $\widetilde{\widetilde{C}}_{pmkl}$ and $\widetilde{\widetilde{C}}_{pmku}$ indices are calculated by using Eq. (23). Accuracy values are found as 1.18. Since the score and accuracy values of $\widetilde{\widetilde{C}}_{pmkl}$ and $\widetilde{\widetilde{C}}_{pmku}$ indices are the same, either one can be chosen as $\widetilde{\widetilde{C}}_{pmk}$ index. The process capability of this process within the based on crisp SLs is approximately 1.19. However, the quality inspector

may not be able to define these SLs precisely. Consequently, the indeterminacy and falsity degrees of the process are expressed as 0.05 and 0.075. Then the indices $\widetilde{\widetilde{C}}_{pm}$ and $\widetilde{\widetilde{C}}_{pmk}$ based on SVTNN indices are calculated as follows:

$$\widetilde{\widetilde{C}}_{pm} = \langle (1.14, 1.17, 1.21, 1.24); 0.950, 0.055, 0.070 \rangle .$$

$$\widetilde{\widetilde{C}}_{pmk} = \min\{\langle (1, 16, 1.19, 1.22, 1.26); 0.950, 0.055, 0.070 \rangle; \langle (1.12, 1.16, 1.19, 1.22); 0.950, 0.055, 0.070 \rangle\}$$

$$\widetilde{\widetilde{C}}_{pmk} = \langle (1.12, 1.16, 1.19, 1.22); 0.950, 0.055, 0.070 \rangle.$$

The score values of $\widetilde{\widetilde{C}}_{pmkl}$ and $\widetilde{\widetilde{C}}_{pmku}$ indices are calculated by using Eq.(24). Score values are found as 1.10 and 1.14, respectively. Since $\widetilde{\widetilde{C}}_{pmkl}$ index is smaller than $\widetilde{\widetilde{C}}_{pmku}$ index, it was chosen as $\widetilde{\widetilde{C}}_{pmk}$ index.

5.2 Category 2: An application for the case that specifications limits are flexible structure of single-valued neutrosophic numbers

A real case application is also analyzed based on similar piston length measurements in this subsection. The SLs for a flexible structure based on SVTrNN and SVTNN, respectively can be defined as follows:

$\widetilde{L\ddot{S}L} = \langle (175.164, 175.165, 175.166), (175.163, 175.165, 175.167), (175.162, 175.165, 175.168) \rangle,$

$\widetilde{U\ddot{S}L} = \langle (175.234, 175.235, 175.236), (175.233, 175.235, 175.237), (175.232, 175.235, 175.238) \rangle$ and

$\widetilde{L\ddot{S}L} = \langle (175.164, 175.165, 175.166, 175.167), (175.163, 175.165, 175.166, 175.168), (175.162, 175.165, 175.166, 175.169) \rangle,$

$\widetilde{U\ddot{S}L} = \langle (175.234, 175.235, 175.236, 175.237), (175.233, 175.235, 175.236, 175.238), (175.232, 175.235, 175.236, 175.239) \rangle,$

respectively. The indices $\widetilde{\widetilde{C}}_{pm}$ and $\widetilde{\widetilde{C}}_{pmk}$ based on SVTrNN for a flexible structure can be obtained as follows:

$$\widetilde{\widetilde{C}}_{pm} = \langle (1.16, 1.19, 1.22), (1.12, 1.19, 1.26), (1.09, 1.19, 1.29) \rangle.$$

$$\widetilde{\widetilde{C}}_{pmk} = \min\left\{ \begin{array}{l} \langle (1.16, 1.19, 1.22), (1.12, 1.19, 1.26), (1.09, 1.19, 1.29) \rangle, \\ \langle (1.16, 1.19, 1.22), (1.12, 1.19, 1.26), (1.09, 1.19, 1.29) \rangle \end{array} \right\}.$$

$$\widetilde{\widetilde{C}}_{pmk} = \langle (1.16, 1.19, 1.22), (1.12, 1.19, 1.26), (1.09, 1.19, 1.29) \rangle.$$

The score values of $\widetilde{\widetilde{C}}_{pmkl}$ and $\widetilde{\widetilde{C}}_{pmku}$ indices are calculated by using Eq. (28). The score values of $\widetilde{\widetilde{C}}_{pmkl}$ and $\widetilde{\widetilde{C}}_{pmku}$ indices are determined as 0.27. Then the accuracy values of $\widetilde{\widetilde{C}}_{pmkl}$ and $\widetilde{\widetilde{C}}_{pmku}$ indices are calculated by using Eq. (29). The accuracy values of $\widetilde{\widetilde{C}}_{pmkl}$ and $\widetilde{\widetilde{C}}_{pmku}$ indices are determined as 0. Since the score and accuracy values of $\widetilde{\widetilde{C}}_{pmkl}$ and $\widetilde{\widetilde{C}}_{pmku}$ indices are the same, either one can be chosen as $\widetilde{\widetilde{C}}_{pmk}$ index. The process capability of this process within the defined SLs is approximately 1.19. Neutrosophic process capability shows that considering the indeterminacy-membership function, the capability of the process cannot be less than 1.12 and more than 1.26, and considering the falsity-membership function, it cannot be less than 1.09 and more than 1.29. Similarly, the indices $\widetilde{\widetilde{C}}_{pm}$ and $\widetilde{\widetilde{C}}_{pmk}$ based on STVNN for flexible structure can be calculated as follows:

$$\widetilde{\widetilde{C}}_{pm} = \langle (1.14, 1.17, 1.21, 1.24), (1.11, 1.17, 1.21, 1.28), (1.07, 1.17, 1.21, 1.31) \rangle.$$

$$\widetilde{\widetilde{C}}_{pmk} = \min\left\{ \begin{array}{l} \langle (1.16, 1.19, 1.22, 1.26), (1.12, 1.19, 1.22, 1.29), (1.09, 1.19, 1.22, 1.33) \rangle, \\ \langle (1.12, 1.16, 1.19, 1.22), (1.09, 1.16, 1.19, 1.26), (1.05, 1.16, 1.19, 1.29) \rangle \end{array} \right\}$$

$$\widetilde{\widetilde{C}}_{pmk} = \langle (1.16, 1.19, 1.22, 1.26), (1.12, 1.19, 1.22, 1.29), (1.09, 1.19, 1.22, 1.33) \rangle.$$

Similar comments are also valid for the flexible structure based on SVTNN.

6 The obtained results and discussion

When the results of category 1 for SVN-Numbers are compared with the studies based on type-1 fuzzy sets, it should be taken into account that the membership degree of the result is 1.00. The truth, indeterminacy and falsity membership degrees of neutrosophic $\widetilde{\widetilde{C}}_{pm}$ are determined 0.95, 0.05 and 0.0.075, respectively. It is clear that the obtained results contain more information about process capability. Thus, the instability of human decisions is better integrated into the model with the proposed methodology. As a result, more precise and more informational process capability measurement

tools have been developed in this study. In addition, NSs provide an advantage over type-2 fuzzy sets in terms of ease of operation to the quality engineers. The obtained results of category 2 of SVN-Numbers are also compared with IFSs. It is observed that in addition to membership and non-membership functions of IFSs, indeterminacy function in NSs provides an opportunity to examine process capability in more detail. According to the results, it is seen that the crisp value of neutrosophic $\widetilde{\widetilde{C}}_{pm}$ is 1.19, but at the same time, it cannot be less than 1.09 and greater than 1.29. The result shows that the biggest advantage of NSs over other new fuzzy set extensions is the uncertainty function.

7   Conclusions

The indices C_{pm} and C_{pmk} are used to measure the capability of any process. In this study, NSs, one of the extensions of traditional fuzzy sets and have an important role in understanding uncertainty and incomplete information have been used to calculate PCIs in a mode detailed and flexible ways. NSs are described by independency truth, indeterminacy and falsity functions to increase the ability related to model uncertainty of the process. Sometimes, a quality engineer has not enough information or cannot express their ideas with perfect judgments, then for example he defines opinions with 0.7, 0.5, 0.3 as true, indeterminacy, false value, respectively. For this reason, NSs are successfully used to overcome the vague and incomplete information. When the literature on PCA is analyzed, it is seen that traditional fuzzy sets on PCA are studied too much. However, the studies with the extensions of fuzzy sets such as hesitant fuzzy sets, intuitionistic fuzzy sets and type-2 fuzzy sets are very limited. So, this paper is also aim to overcome the gap of literature. The neutrosophic state of the SLs provide us to more flexibility in order to apply to engineering problems. For this aim, traditional PCIs have been analyzed and re-structured based on NSs in this paper. For this aim, the SLs are considered as the SVN-Numbers PCA has been performed by examining the status of SLs to be neutrosophic. We also try to integrate the advantages of NSs on PCA. Finally, the neutrosophic C_{pm} ($\widetilde{\widetilde{C}}_{pm}$) and neutrosophic C_{pmk} ($\widetilde{\widetilde{C}}_{pmk}$) indices by using the NSs are developed in this study. The NPCIs provide wider knowledge about the capability of process than the analysis that used traditional fuzzy sets. The best of our knowledge, this study is the first study that considers the indices C_{pm} and C_{pmk} with NSs. Additionally, a real case application is added and this real case application shows that NPCIs are more effective than PCIs using traditional fuzzy sets under the uncertainty environment. In the future studies, the other extensions of fuzzy sets such as intuitionistic fuzzy sets or pythagorean fuzzy sets can be used to the PCA.

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