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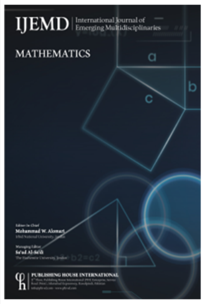

Neutrosophic Delta-Beta Connected Topological Spaces

Raja Mohammad Latif

Department of Mathematics and Natural Science, Prince Mohammad Bin Fahd University Al Khobar Kingdom of Saudi Arabia

Abstract

The real-life situations always include indeterminacy. The Mathematical tool which is well known in deal with indeterminacy is neutrosophic. The notion of neutrosophic set is generally referred to as the generalization of intuitionistic fuzzy set. In this paper, the notion of neutrosophic $\delta\beta$-connectedness and $\delta\beta$-disconnectedness in neutrosophic topological spaces is introduced. Also, we introduce neutrosophic $\delta\beta$-separated sets, neutrosophic super $\delta\beta$-connected spaces, neutrosophic extremely $\delta\beta$-disconnected spaces and neutrosophic strongly $\delta\beta$-connected spaces. We investigate and study several properties and characterizations concerning connectedness in these spaces.

Keywords: neutrosophic topology, neutrosophic $\delta\beta$-open set, neutrosophic $\delta\beta$-closed set, neutrosophic $\delta\beta$-interior, neutrosophic $\delta\beta$-closure, neutrosophic $\delta\beta$-connected space, neutrosophic $\delta\beta$-separated sets, neutrosophic super $\delta\beta$-connected space, neutrosophic extremely $\delta\beta$-disconnected space, neutrosophic strongly $\delta\beta$-connected space.

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1. Introduction

Many real-life problems in Business, Finance, Medical sciences, Engineering and Social sciences deal with uncertainties. There are difficulties in solving the uncertainties in these data by traditional mathematical models. Neutrosophic system plays important role in the fields of Information Systems, Computer Science, Artificial Intelligence, Applied Mathematics, Mathematics decision making, Medicine, Management Science, and Electrica & Electronic, etc. In 2021, Vadivel et al. introduced neutrosophic δ-pre open set, neutrosophic δ-semi open set, neutrosophic $\delta\alpha$-open set and $\delta\beta$-open set in neutrosophic topological spaces. In this paper, the concepts of neutrosophic $\delta\beta$-connected space, neutrosophic $\delta\beta$-disconnected space, neutrosophic $\delta\beta$-separated sets, neutrosophic super $\delta\beta$-connected space, neutrosophic extremely $\delta\beta$-disconnected space and

Email addresses: **rlatif@pmu.edu.sa (R. Latif)**

neutrosophic strongly $\delta\beta$-connected space are discussed in neutrosophic topological spaces. We investigate and study several properties and characterizations concerning connectedness in these spaces.

2. Preliminaries

We recall basic definitions and operations of neutrosophic sets and neutrosophic topological space.

Definition 2.1. Let X be a non-empty fixed set. A neutrosophic set P is an object having the form $P=\{\langle x,\mu_P(x),\sigma_P(x),\gamma_P(x)\rangle : x\in X\}$, *where* $\mu_P(x)$- represents the degree of membership, $\sigma_P(x)$- represents the degree of indeterminacy and $\gamma_P(x)$- represents the degree of non-membership. The class of all neutrosophic sets on X will be denoted by $\mathrm{N}\ (X)$.

Definition 2.2. Let X be a non-empty set and let $P=\{\langle x,\mu_P(x),\sigma_P(x),\gamma_P(x)\rangle : x\in X\}$ and $Q=\{\langle x,\mu_Q(x),\sigma_Q(x),\gamma_Q(x)\rangle : x\in X\}$ be two neutrosophic sets, *Then*

1. $(Empty\ set)$ $0_N=\langle x,0,0.1\rangle$ is called the neutrosophic empty se.
2. $(Universal\ set)$ $1_N=\langle x,1,1.0\rangle$ is called the neutrosophic universal set.
3. (Inclusion): $P\subseteq Q$ if and only if $\mu_P(x)\le\mu_Q(x)$, $\sigma_P(x)\le\sigma_Q(x)$ and $\gamma_P(x)\ge\gamma_Q(x)$: $\forall\ x\in X$.
4. (Equality): $P=Q$ if and only if $P\subseteq Q$ and $Q\subseteq P$.
5. $(Complement)$: $P^C=1_N-P=\{\langle x,\gamma_P(x),1-\sigma_P(x),\mu_P(x)\rangle : x\in X\}$.
6. $(Union)$: $P\bigcup Q=\{\langle x,\max(\mu_P(x),\mu_Q(x)),\max(\sigma_P(x),\sigma_Q(x)),\min(\gamma_P(x),\gamma_Q(x))\rangle : x\in X\}$.
7. $(Intersection)$: $P\bigcap Q=\{\langle x,\min(\mu_P(x),\mu_Q(x)),\min(\sigma_P(x),\sigma_Q(x)),\max(\gamma_P(x),\gamma_Q(x))\rangle : x\in X\}$.

Definition 2.3. A neutrosophic point $x_{(\alpha,\beta,\gamma)}$ is said to be in the neutrosophic set A- in symbols $x_{(\alpha,\beta,\gamma)}\in A$ if and only if $\alpha<\mu_A(x)$, $\beta<\sigma_A(x)$ and $\gamma>\gamma_A(x)$.

Definition 2.4. A neutrosophic topology on a non-empty set X is a family T_N of neutrosophic subsets of X satisfying:

(i) $0_N, 1_N\in\mathrm{T}_N$.

(ii) $G\bigcap H\in\mathrm{T}_N$ for every $G, H\in\mathrm{T}_N$.

(iii) $\bigcup_{j\in J} G_j\in\mathrm{T}_N$ for every $\{G_j : j\in J\}\subseteq\tau_N$.

Then the pair (X,T_N) is called a neutrosophic topological space. The elements of T_N are called neutrosophic open sets in X. A neutrosophic set A is called a neutrosophic closed set if and only if its complememnt A^C is a neutrosophic open set.

Definition 2.5. Let (X,T_N) be a neutrosophic topological space and A be a neutrosophic set. Then

(i) The neutrosophic interior of A, denoted by $\mathrm{N}\,Int(A)$ is the union of all neutrosophic open subsets of A. Clearly $\mathrm{N}\,Int(A)$ is the biggest neutrosophic open subset of X contained in A.

(ii) The neutrosophic closure of A denoted by $\mathrm{N}\,Cl(A)$ is the intersection of all neutrosophic closed sets containing A. Clearly $\mathrm{N}\,Cl(A)$ is the smallest neutrosophic closed set which contains A.

Definition 2.6. A neutrosophic subset A of a neutrosophic topological space (X,T_N) is said to be a neutrosophic regular open set if $A\subseteq N\ Int\left[N\ Cl(A)\right]$. The complement of a neutrosophic regular open set is called a neutrosophic regular closed set in X.

Definition 2.7. Let (X,T_N) be a neutrosophic topological space and A be a neutrosophic set. Then neutrosophic δ-interior of A, denoted by $(briefly\ N\ \delta Int(A))$ is defined as the union of all neutrosophic regular open subsets of A. Equivalently, it could be as given below: $N\ \delta Int(A)=\bigcup\{B: B\subseteq A\ \&\ B\ is\ a\ neutrosophic\ regular\ open\ set\ in\ X\}$.

Definition 2.8. Let (X,T_N) be a neutrosophic topological space and A be a neutrosophic set. Then neutrosophic δ-closure of A, denoted by $(briefly\ N\ \delta Cl(A))$ is defined as the intersection of all neutrosophic regular closed sets containing A. Equivalently, it could be as given below: $N\ \delta Cl(A)=\bigcap\{B: A\subseteq B\ \&\ B\ is\ a\ neutrosophic\ regular\ closed\ set\ in\ X\}$.

Definition 2.9. Let (X,T_N) be a neutrosophic topological space and A be a neutrosophic set on X. Then A is said to be a neutrosophic δ-open (resp. δ-closed) set if $A=N\ \delta Int(A)$ $(resp.\ A=N\ \delta Cl(A))$.

Definition 2.10. Let (X,T_N) be a neutrosophic topological space and A be a neutrosophic set on X. Then A is said to be a neutrosophic $\delta\beta$-open (briefly $N\ \delta\beta$-OS) set if $A\subseteq N\ Cl\left[N\ Int\left(N\ \delta Cl(A)\right)\right]$.

Definition 2.11. Let (X,T_N) be a neutrosophic topological space and A be a neutrosophic set on X. Then A is said to be a neutrosophic $\delta\beta$-closed (briefly $N\ \delta\beta$-CS) set if its complement A^C is a neutrosophic $\delta\beta$-open set in X.

The family of all neutrosophic $\delta\beta$-open (resp. $\delta\beta$-closed) in a neutrosophic topological space (X,T_N) is denoted by $N\ \delta\beta OS(X,T_N)$ or $N\ \delta\beta OS(X)$ $(resp.\ N\ \delta\beta CS(X,T_N))$ or $(resp.\ N\ \delta\beta CS(X))$.

Proposition 2.12. Let (X,T_N) be a neutrosophic topological space. Then the following are true:

(i) Every neutrosophic δ-open (resp. δ-closed) set is neutrosophic open (resp. closed) set in X.

(ii) Every neutrosophic open (resp. closed) set is neutrosophic $\delta\beta$-open (resp. $\delta\beta$-closed) set in X.

Proposition 2.13. Let (X,T_N) be a neutronsophic topological space. Then the union (resp. intersection) of any family of $N\ \delta\beta OS(X,T_N)$ $(resp.\ N\ \delta\beta CS(X,T_N))$ is an $N\ \delta\beta OS(X,T_N)$ $(resp.\ N\ \delta\beta CS(X,T_N))$.

Proposition 2.14. Let (X,T_N) be a neutrosophic topological space. Let A be an $N\ \delta$-OS and B be an $N\ \delta\beta$-OS. Then $A\cap B$ is an $N\ \delta\beta$-OS.

Definition 2.15. Let (X,T_N) be a neutrosophic topological space and A be a neutrosophic set on X. Then A is said to be a neutrosophic regular $\delta\beta$-open set if $A=N\ \delta\beta Int\left[N\ \delta\beta Cl(A)\right]$. The complement of a neutrosophic $\delta\beta$-regular open set is called a neutrosophic $\delta\beta$-regular closed set in X.

Lemma 2.16. Assume that A is a neutrosophic subset of a neutrosophic topological space (X,T_N). Then the following relations hold.

(a) $X-N\ \delta\beta Int(U)=N\ \delta\beta Cl(X-U)$.

(b) $X - \mathrm{N}\ \delta\beta Cl(U) = \mathrm{N}\ \delta\beta Int(X - U)$.

Definition 2.17. A function $f:(X,\mathrm{T}_N)\to(Y,\sigma_N)$ is called a neutrosophic $\delta\beta$-irresolute function if $f^{-1}(B)$ is a neutrosophic $\delta\beta$-open set in X, for every neutrosophic $\delta\beta$-open set B in Y.

Lemma 2.18. A function $f:(X,\mathrm{T}_N)\to(Y,\sigma_N)$ is a neutrosophic $\delta\beta$-irresolute function if and only if $f^{-1}(B)$ is a neutrosophic $\delta\beta$-closed set in X, for every neutrosophic $\delta\beta$-closed set B in Y.

3 Neutrosophic $\delta\beta$-Connected Spaces

In this section, we study the notion of neutrosophic $\delta\beta$-connected, and neutrosophic $\delta\beta$-disconnected spaces in neutrosophic topological spaces.

Definition 3.1. A neutrosophic topological space (X,T_N) is neutrosophic $\delta\beta$-disconnected if there exist neutrosophic $\delta\beta$-open sets A, B in X, $A\neq 0_N$, $B\neq 0_N$ such that $A\bigcup B=1_N$ and $A\bigcap B=0_N$. If (X,T_N) is not neutrosophic $\delta\beta$-disconnected then it is said to be neutrosophic $\delta\beta$-connected.

Theorem 3.2. A neutrosophic topological space (X,T_N) is neutrosophic $\delta\beta$-connected space if and only if there exists no non-zero neutrosophic $\delta\beta$-open sets U and V in (X,T_N) such that $U=V^C$.

Proof. Necessity: Let U and V be two neutrosophic $\delta\beta$-open sets in (X,T_N) such that $U\neq 0_N$, $V\neq 0_N$ and $U=V^C$. Therefore V^C is a neutrosophic $\delta\beta$-closed set. Since $U\neq 0_N$, $V\neq 1_N$. This implies V is a proper neutrosophic subset which is both neutrosophic $\delta\beta$-open set and neutrosophic $\delta\beta$-closed set in X. Hence X is not a neutrosophic $\delta\beta$-connected space. But this is a contradiction to our hypothesis. Thus, there exist no non-zero neutrosophic $\delta\beta$-open sets U and V in X, such that $U=V^C$.

Sufficiency: Let U be both neutrosophic $\delta\beta$-open and neutrosophic $\delta\beta$-closed set in X such that $U\neq 0_N$, $U\neq 1_N$. Now let $V=U^C$. Then V is a neutrosophic $\delta\beta$-open set and $V\neq 1_N$. This implies $U^C=V\neq 0_N$, which is a contradiction to our hypothesis. Therefore X is neutrosophic $\delta\beta$-connected space.

Theorem 3.3. A neutrosophic topological space (X,T_N) is neutrosophic $\delta\beta$-connected space if and only if there do not exist non-zero neutrosophic subsets U and V in X such that $U=V^C$, $V=\left[\mathrm{N}\ \delta\beta Cl(U)\right]^C$ and $U=\left[\mathrm{N}\ \delta\beta Cl(V)\right]^C$.

Proof. Necessity: Let U and V be two neutrosophic subsets of (X,T_N) such that $U\neq 0_N$, $V\neq 0_N$ and $U=V^C$, $V=\left[\mathrm{N}\ \delta\beta Cl(U)\right]^C$ and $U=\left[\mathrm{N}\ \delta\beta Cl(V)\right]^C$. Since $\left[\mathrm{N}\ \delta\beta Cl(U)\right]^C$ and $\left[\mathrm{N}\ \delta\beta Cl(V)\right]^C$ are neutrosophic $\delta\beta$-open sets in X, so U and V are neutrosophic $\delta\beta$-open sets in X. This implies X is not a neutrosophic $\delta\beta$-connected space, which is a contradiction. Therefore, there exist no non-zero neutrosophic $\delta\beta$-open sets U and V in X, such that $U=V^C$, $V=\left[\mathrm{N}\ \delta\beta Cl(U)\right]^C$ and $U=\left[\mathrm{N}\ \delta\beta Cl(V)\right]^C$.

Sufficiency: Let U be both neutrosophic $\delta\beta$-open and neutrosophic $\delta\beta$-closed set in X such that $U\neq 0_N$, $U\neq 1_N$. Now by taking $V=U^C$ we obtain a contradiction to our hypothesis. Hence X is neutrosophic $\delta\beta$-connected space.

Theorem 3.4. Let $f:(X,T_N)\longrightarrow(Y,\sigma_N)$ be a neutrosophic $\delta\beta$-irresolure surjection and X be neutrosophic $\delta\beta$-connected. Then Y is neutrosophic $\delta\beta$-connected.

Proof. Assume that Y is not neutrosophic $\delta\beta$-connected, then there exist nonempty neutrosophic $\delta\beta$-open sets U and V in Y such that $U\bigcup V=1_N$ and $U\bigcap V=0_N$. Since f is neutrosophic $\delta\beta$-irresolure mapping, $A=f^{-1}(U)\neq 0_N$, $B=f^{-1}(V)\neq 0_N$, which are neutrosophic $\delta\beta$-open sets in X and $f^{-1}(U)\bigcup f^{-1}(V)=f^{-1}(1_N)=1_N$, which implies $A\bigcup B=1_N$. Also $f^{-1}(U)\bigcap f^{-1}(V)=f^{-1}(0_N)=0_N$, which implies $A\bigcap B=0_N$. Thus, X is neutrosophic $\delta\beta$-disconnected, which is a contradiction to our hypothesis. Hence Y is neutrosophic $\delta\beta$-connected.

4 Neutrosophic $\delta\beta$-Separated Sets

In this section, we introduce the concept of neutrosophic $\delta\beta$-separated sets in neutrosophic topological spaces. Also, we study some of the main results depending on neutrosophic $\delta\beta$-separated sets.

Definition 4.1. Let A and B be non-zero neutrosophic subsets in a neutrosophic topological space (X,T_N). Then A and B are said to be neutrosophic $\delta\beta$-separated if $N\ \delta\beta Cl(A)\bigcap B=A\bigcap N\ \delta\beta Cl(B)=0_N$.

Remark 4.2. Any two disjoint non-empty neutrosophic $\delta\beta$-closed sets are neutrosophic $\delta\beta$-separated.

Proof. Suppose A and B are disjoint non-empty neutrosophic $\delta\beta$-closed sets. Then $N\ \delta\beta Cl(A)\bigcap B=A\bigcap N\ \delta\beta Cl(B)=A\bigcap B=0_N$. This shows that A and B are neutrosophic $\delta\beta$-separated.

Theorem 4.3. (i) Let A and B be two neutrosophic $\delta\beta$-separated subsets of a neutrosophic topological space (X,T_N) and $C\subseteq A$, $D\subseteq B$. Then C and D are also neutrosophic $\delta\beta$-separated.

(ii) Let A and B be both neutrosophic $\delta\beta$-separated subsets of a neutrosophic topological space (X,T_N) and let $H=A\bigcap B^C$ and $G=B\bigcap A^C$. Then H and G are also neutrosophic $\delta\beta$-separated.

Proof. (i) Let A and B be two neutrosophic $\delta\beta$-separated sets in neutrosophic topological space (X,T_N). Then $N\ \delta\beta Cl(A)\bigcap B=0_N=A\bigcap N\ \delta\beta Cl(B)$. Since $C\subseteq A$ and $D\subseteq B$, then $N\ \delta\beta Cl(C)\subseteq N\ \delta\beta Cl(A)$ and $N\ \delta\beta Cl(D)\subseteq N\ \delta\beta Cl(B)$. This implies that, $N\ \delta\beta Cl(C)\bigcap D\subseteq N\ \delta\beta Cl(A)\bigcap B=0_N$ and hence $N\ \delta\beta Cl(C)\bigcap D=0_N$. Similarly $N\ \delta\beta Cl(D)\bigcap C\subseteq N\ \delta\beta Cl(B)\bigcap A=0_N$ and hence $N\ \delta\beta Cl(D)\bigcap C=0_N$. Therefore C and D are neutrosophic $\delta\beta$-separated.

(ii) Let A and B be both neutrosophic $\delta\beta$-open subsets of X. Then A^C and B^C are neutrosophic $\delta\beta$-closed sets. Since $H\subseteq B^C$, then $N\ \delta\beta Cl(H)\subseteq N\ \delta\beta Cl(B^C)=B^C$ and so $N\ \delta\beta Cl(H)\bigcap B=0_N$. Since $G\subseteq H$, then $N\ \delta\beta Cl(H)\bigcap G\subseteq N\ \delta\beta Cl(H)\bigcap B=0_N$. Thus, $N\ \delta\beta Cl(H)\bigcap G=0_N$. Similarly, $N\ \delta\beta Cl(G)\bigcap H=0_N$. Hence H and G are neutrosophic $\delta\beta$-separated.

Theorem 4.4. The neutrosophic subsets A and B of a neutrosophic topological space (X,T_N) are neutrosophic $\delta\beta$-separated if and only if there exist neutrosophic $\delta\beta$-open sets U and V in X such that $A\subseteq U$, $B\subseteq V$ and $A\bigcap V=0_N$ and $B\bigcap U=0_N$.

Proof. Let A and B be neutrosophic $\delta\beta$-separated. Then $A \cap N\ \delta\beta Cl(B) = 0_N = B \cap N\ \delta\beta Cl(A)$. Take $V = (N\ \delta\beta Cl(A))^C$ and $U = (N\ \delta\beta Cl(B))^C$. Then U and V are neutrosophic $\delta\beta$-open sets in X such that $A \subseteq U$, $B \subseteq V$ and $A \cap V = 0_N$ and $B \cap U = 0_N$.

Conversely, let U and V be neutrosophic $\delta\beta$-open sets such that $A \subseteq U$, $B \subseteq V$ and $A \cap V = 0_N$, $B \cap U = 0_N$. Then $A \subseteq V^C$ and $B \subseteq U^C$ and V^C and U^C are neutrosophic $\delta\beta$-closed. This implies that $N\ \delta\beta Cl(A) \subseteq N\ \delta\beta Cl(V^C) = V^C \subseteq B^C$ and $N\ \delta\beta Cl(B) \subseteq N\ \delta\beta Cl(U^C) = U^C \subseteq A^C$. That is, $N\ \delta\beta Cl(A) \subseteq B^C$ and $N\ \delta\beta Cl(B) \subseteq A^C$. Therefore $A \cap N\ \delta\beta Cl(B) = 0_N = N\ \delta\beta Cl(A) \cap B$. Hence A and B are neutrosophic $\delta\beta$-separated.

Theorem 4.5. Each two neutrosophic $\delta\beta$-separated sets are always disjoint.

Proof. Let A and B be neutrosophic $\delta\beta$-separated. Then $A \cap N\ \delta\beta Cl(B) = 0_N = N\ \delta\beta Cl(A) \cap B$. Now, $A \cap B \subseteq A \cap N\ \delta\beta Cl(B) = 0_N$. Therefore $A \cap B = 0_N$ and hence A and B are disjoint.

Theorem 4.6. A neutrosophic topological space (X, T_N) is neutrosophic $\delta\beta$-connected if and only if $A \cup B \neq 1_N$, where A and B are neutrosophic $\delta\beta$-separated sets.

Proof. Assume that (X, T_N) is neutrosophic $\delta\beta$-connected space. Suppose $A \cup B = 1_N$, where A and B are neutrosophic $\delta\beta$-separated sets. Then $N\ \delta\beta Cl(A) \cap B = A \cap N\ \delta\beta Cl(B) = 0_N$. Since $A \subseteq N\ \delta\beta Cl(A)$, we have $A \cap B \subseteq N\ \delta\beta Cl(A) \cap B = 0_N$. Therefore $N\ \delta\beta Cl(A) \subseteq B^C = A$ and $N\ \delta\beta Cl(B) \subseteq A^C = B$. Hence $A = N\ \delta\beta Cl(A)$ and $B = N\ \delta\beta Cl(B)$. Therefore A and B are neutrosophic $\delta\beta$-closed sets and hence $A = B^C$ and $B = A^C$ are disjoint neutrosophic $\delta\beta$-open sets. Thus $A \neq 0_N$, $B \neq 0_N$ such that $A \cup B = 1_N$ and $A \cap B = 0_N$, and A, B are neutrosophic $\delta\beta$-open sets. That is X is not neutrosophic $\delta\beta$-connected, which is a contradiction to X is a neutrosophic $\delta\beta$-connected space. Hence 1_N is not the union of any two neutrosophic $\delta\beta$-separated sets.

Conversely, assume that 1_N is not the union of any two neutrosophic $\delta\beta$-separated sets. Suppose X is not neutrosophic $\delta\beta$-connected. Then $A \cup B = 1_N$, where $A \neq 0_N$, $B \neq 0_N$ such that $A \cap B = 1_N$, A and B are neutrosophic $\delta\beta$-open sets in X. Since $A \subseteq B^C$ and $B \subseteq A^C$, $N\ \delta\beta Cl(A) \cap B \subseteq B^C \cap B = 0_N$ and $A \cap N\ \delta\beta Cl(B) \subseteq A \cap A^C = 0_N$. That is A and B are neutrosophic $\delta\beta$-separated sets. This is a contradiction. Therefore X is neutrosophic $\delta\beta$-connected.

5 Neutrosophic Super $\delta\beta$-Connected Spaces

In this section, we introduce the concept of neutrosophic super $\delta\beta$-connected spaces in neutrosophic topological spaces and study some properties of this space.

Definition 5.1. A neutrosophic topological space (X, T_N) is neutrosophic super $\delta\beta$-disconnected if there exists a neutrosophic regular $\delta\beta$-open set A in X such that $A \neq 0_N$ and $A \neq 1_N$. A neutrosophic topological space (X, T_N) is called neutrosophic super $\delta\beta$-connected if X is not neutrosophic super $\delta\beta$-disconnected.

Theorem 5.2. Let (X, T_N) be a neutrosophic topological space. Then the following assertions are equivalent:

(i) X is neutrosophic super $\delta\beta$-connected

(ii) For each neutrosophic $\delta\beta$-open set $U \neq 0_N$ in X, we have $N\ \delta\beta Cl(U) = 1_N$.

(iii) For each neutrosophic $\delta\beta$-closed set $U \neq 1_N$ in X, we have $N\ \delta\beta Int(U) = 0_N$.

(iv) There do not exist neutrosophic $\delta\beta$-open subsets U and V in (X, T_N), such that $U \neq 0_N$, $V \neq 0_N$ and $U \subseteq V^C$.

(v) There do not exist neutrosophic $\delta\beta$-open subsets U and V in (X, T_N), such that $U \neq 0_N$, $V \neq 0_N$, $V = (N\ \delta\beta Cl(U))^C$ and $U = (N\ \delta\beta Cl(V))^C$.

(vi) There do not exist neutrosophic $\delta\beta$-closed subsets U and V in (X, T_N), such that $U \neq 1_N$, $V \neq 1_N$, $V = (N\ \delta\beta Int(U))^C$ and $U = (N\ \delta\beta Int(V))^C$.

Proof. (i)$\Rightarrow$(ii): Assume that there exists a neutrosophic $\delta\beta$-open set $A \neq 0_N$ such that $N\ \delta\beta Cl(A) \neq 1_N$. Now take $B = N\ \delta\beta Int[N\ \delta\beta Cl(A)]$. Then B is a proper neutrosophic regular $\delta\beta$-open set in X which contradicts that X is neutrosophic super $\delta\beta$-connected. Therefore $N\ \delta\beta Cl(A) = 1_N$.

(ii)$\Rightarrow$(iii): Let $A \neq 1_N$ be a neutrosophic $\delta\beta$-closed set in X. Then A^C is neutrosophic $\delta\beta$-open set in X and $A^C \neq 0_N$. Hence by hypothesis, $N\ \delta\beta Cl(A^C) = 1_N$, and so $N\ \delta\beta Cl(A^C) = (N\ \delta\beta Int(A))^C = 1_N$. This implies that $N\ \delta\beta Int(A) = 0_N$.

(iii)$\Rightarrow$(iv): Let A and B be neutrosophic $\delta\beta$-open sets in X such that $A \neq 0_N \neq B$ and $A \subseteq B^C$ Since B^C is neutrosophic $\delta\beta$-closed set in X and $B \neq 0_N$ implies $B^C \neq 1_N$, we obtain $N\ \delta\beta Int(B^C) = 0_N$. But, from $A \subseteq B^C$, $0_N \neq A = N\ \delta\beta Int(A) \subseteq N\ \delta\beta Int(B^C) = 0_N$, which is a contradiction.

(iv)$\Rightarrow$(i): Let $0_N \neq A \neq 1_N$ be neutrosophic regular $\delta\beta$-open set in X. Let $B = (N\ \delta\beta Cl(A))^C$. Since $N\ \delta\beta Int[N\ \delta\beta Cl(B)] = N\ \delta\beta Int\left[N\ \delta\beta Cl\left((N\ \delta\beta Cl(A))^C\right)\right] = N\ \delta\beta Int[N\ \delta\beta Int(N\ \delta\beta Cl(A))]^C = N\ \delta\beta Int(A^C) = [N\ \delta\beta Cl(A)]^C = B$. Also we get $B \neq 0_N$, since otherwise, we have $B = 0_N$ and this implies $(N\ \delta\beta Cl(A))^C = 0_N$. That implies $N\ \delta\beta Cl(A) = 1_N$. That shows that $A = N\ \delta\beta Int[N\ \delta\beta Cl(A)] = N\ \delta\beta Int(1_N) = 1_N$. That is $A = 1_N$, which is a contradiction. Therefore $B \neq 0_N$ and $A \subseteq B^C$. But this is a contradiction to (iv). Therefore (X, T_N) is neutrosophic super $\delta\beta$-connected space.

(i)$\Rightarrow$(v): Let A and B be neutrosophic $\delta\beta$-open sets in (X, T_N) such that $A \neq 0_N \neq B$, $B = [N\ \delta\beta Cl(A)]^C$, $A = [N\ \delta\beta Cl(B)]^C$. Now we have $N\ \delta\beta Int[N\ \delta\beta Cl(A)] = N\ \delta\beta Int(B^C) = [N\ \delta\beta Cl(B)]^C = A$, $A \neq 0_N$ and $A \neq 1_N$, since if $A = 1_N$, then $1_N = [N\ \delta\beta Cl(B)]^C$. This implies $N\ \delta\beta Cl(B) = 0_N$. But $B \neq 0_N$. Therefore $A \neq 1_N$ implies that A is proper neutrosophic regular $\delta\beta$-open set in (X, T_N), which is a contradiction to (i). Hence (v) is true.

(v)$\Rightarrow$(i): Let A be neutrosophic regular $\delta\beta$-open set in (X, T_N) such that $A = N\ \delta\beta Int[N\ \delta\beta Cl(A)]$ and $0_N \neq A \neq 1_N$. Now take $B = [N\ \delta\beta Cl(A)]^C$. In this case we get $B \neq 0_N$ and B is neutrosophic regular $\delta\beta$-open set in (X, T_N). $B = [N\ \delta\beta Cl(A)]^C$ and $[N\ \delta\beta Cl(B)]^C = \left[N\ \delta\beta Cl(N\ \delta\beta Cl(A))^C\right]^C = N\ \delta\beta Int\left[(N\ \delta\beta Cl(A))^C\right]^C =$

$N\ \delta\beta Int\left[\left(N\ \delta\beta Cl(A)\right)\right]=A$. But this is a contradiction. Therefore (X,T_N) is neutrosophic super $\delta\beta$-connected space.

$(\mathrm{v})\Rightarrow(\mathrm{vi})$: Let A and B be two neutrosophic regular $\delta\beta$-closed sets in (X,T_N) such that $A\neq 1_N\neq B$, $B=\left[N\ \delta\beta Int(A)\right]^C$, $A=\left[N\ \delta\beta Int(B)\right]^C$. Take $C=A^C$ and $D=B^C$, C and D become neutrosophic regular $\delta\beta$-open sets in (X,T_N) with $C\neq 0_N\neq D$, $D=\left[N\ \delta\beta Int(C)\right]^C$, $C=\left[N\ \delta\beta Int(D)\right]^C$, which is a contradiction to (v). Hence (vi) is true.

$(\mathrm{vi})\Rightarrow(\mathrm{v})$: It can be easily proved in the similar way as in $(\mathrm{v})\Rightarrow(\mathrm{vi})$.

6 Neutrosophic Extremely $\delta\beta$-Disconnected Spaces

In this section, we introduce the concept of neutrosophic extremely $\delta\beta$-disconnected spaces in neutrosophic topological spaces and study its properties.

Definition 6.1. A neutrosophic topological space (X,T_N) is said to be neutrosophic extremely $\delta\beta$-disconnected if the neutrosophic $\delta\beta$-closure of every neutrosophic $\delta\beta$-open set in (X,T_N) is neutrosophic $\delta\beta$-open set in X.

Theorem 6.2. Let (X,T_N) be a neutrosophic topological space. The following statements are equivalent.

(i) X is neutrosophic extremely $\delta\beta$-disconnected space.

(ii) For each neutrosophic $\delta\beta$-closed set A, $N\ \delta\beta Int(A)$ is neutrosophic $\delta\beta$-closed set.

(iii) For each neutrosophic $\delta\beta$-open set A, $N\ \delta\beta Cl(A)=\left[N\ \delta\beta Cl\left(N\ \delta\beta Cl(A)\right)^C\right]^C$.

(iv) For each neutrosophic $\delta\beta$-open sets A and B with $N\ \delta\beta Cl(A)=B^C$, $N\ \delta\beta Cl(A)=\left[N\ \delta\beta Cl(B)\right]^C$.

Proof. $(\mathrm{i})\Rightarrow(\mathrm{ii})$: Let A be any neutrosophic $\delta\beta$-closed set in (X,T_N). Then A^C is neutrosophic $\delta\beta$-open set. So (i) implies that $N\ \delta\beta Cl\left(A^C\right)=\left[N\ \delta\beta Int(A)\right]^C$ is neutrosophic $\delta\beta$-open set. Thus $N\ \delta\beta Int(A)$ is neutrosophic $\delta\beta$-closed set in (X,T_N).

$(\mathrm{ii})\Rightarrow(\mathrm{iii})$: Let A be neutrosophic $\delta\beta$-open set. Then we have $\left[N\ \delta\beta Cl\left(N\ \delta\beta Cl(A)\right)^C\right]^C=\left[N\ \delta\beta Cl\left(N\ \delta\beta Int\left(A^C\right)\right)\right]^C$. Since A is neutrosophic $\delta\beta$-open set. Then A^C is neutrosophic $\delta\beta$-closed set. So, by (ii) $N\ \delta\beta Int\left(A^C\right)$ is neutrosophic $\delta\beta$-closed set. That is $N\ \delta\beta Cl\left[N\ \delta\beta Int\left(A^C\right)\right]=N\ \delta\beta Int\left(A^C\right)$. Hence $\left[N\ \delta\beta Cl\left(N\ \delta\beta Int\left(A^C\right)\right)\right]^C=\left[N\ \delta\beta Int\left(A^C\right)\right]^C=N\ \delta\beta Cl(A)$.

$(\mathrm{iii})\Rightarrow(\mathrm{iv})$: Let A and B be any two neutrosophic $\delta\beta$-open sets in (X,T_N) such that $N\ \delta\beta Cl(A)=B^C$. $(\mathrm{iii})\Rightarrow$ $N\ \delta\beta Cl(A)=\left[N\ \delta\beta Cl\left(N\ \delta\beta Cl(A)\right)^C\right]^C=\left[N\ \delta\beta Cl\left(B^C\right)^C\right]^C=\left[N\ \delta\beta Cl(B)\right]^C$.

$(\mathrm{iv})\Rightarrow(\mathrm{i})$: Let A be any neutrosophic $\delta\beta$-open set in (X,T_N). Let $B=\left[N\ \delta\beta Cl(A)\right]^C$. Then $N\ \delta\beta Cl(A)=B^C$. Then (iv) implies $N\ \delta\beta Cl(A)=\left[N\ \delta\beta Cl(B)\right]^C$. Since $N\ \delta\beta Cl(B)$ is neutrosophic $\delta\beta$-closed set, this implies that $N\ \delta\beta Cl(A)$ is neutrosophic $\delta\beta$-open set. This implies that (X,T_N) is neutrosophic extremely $\delta\beta$-disconnected space.

7 Neutrosophic Strongly $\delta\beta$-Connected Spaces

In this section, we study neutrosophic strongly $\delta\beta$-connectedness.

Definition 7.1. A neutrosophic topological space (X, T_N) is neutrosophic strongly $\delta\beta$-connected, if there does not exist any nonempty neutrosophic $\delta\beta$-closed sets A and B in X such that $A \cap B = 0_N$.

Theorem 7.2. Let $f:(X, T_N) \longrightarrow (Y, \sigma_N)$ be a neutrosophic $\delta\beta$-irresolure surjection and X be a neutrosophic strongly $\delta\beta$-connected space. Then Y is neutrosophic strongly $\delta\beta$-connected.

Proof. Assume that Y is not neutrosophic strongly $\delta\beta$-connected, then there exist nonempty neutrosophic $\delta\beta$-closed sets U and V in Y such that $U \neq 0_N$, $V \neq 0_N$, and $U \cap V = 0_N$. Since f is neutrosophic $\delta\beta$-irresolure mapping, $A = f^{-1}(U) \neq 0_N$, $B = f^{-1}(V) \neq 0_N$, which are neutrosophic $\delta\beta$-closed sets in X and $f^{-1}(U) \cap f^{-1}(V) = f^{-1}(0_N) = 0_N$, which implies $A \cap B = 0_N$. Thus, X is not neutrosophic strongly $\delta\beta$-connected, which is a contradiction to our hypothesis. Hence Y is neutrosophic strongly $\delta\beta$-connected.

8 Conclusion

We have introduced the notion of neutrosophic $\delta\beta$-connectedness and $\delta\beta$-disconnectedness in neutrosophic topological spaces. We also have introduced neutrosophic $\delta\beta$-separated sets, neutrosophic super $\delta\beta$-connected spaces, neutrosophic extremely $\delta\beta$-disconnected spaces, and neutrosophic strongly $\delta\beta$-connected spaces. We have investigated and studied several properties and characterizations concerning connectedness in these spaces. In the future we plan to introduce neutrosophic $\delta\beta$-compact spaces, neutrosophic countably $\delta\beta$-compact spaces and neutrosophic $\delta\beta$-Lindelof spaces as well as neutrosophic $\delta\beta$-separation axioms in neutrosophic topological spaces and investigate their basic properties and characterizations in neutrosophic topological spaces.

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Competing Interests

The author declares no competing interests.

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