PENTAPARTITIONED ROUGH FERMATEAN NEUTROSOPHIC NORMED SPACES

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BEŞ BİLEŞENLİ KABA FERMATEAN NÖTROSOFİK NORMLU UZAYLAR

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ÖZET

Bu çalışma nötrosofik normlu uzayların kaba küme yaklaşımıyla birlikte değerlendirildiği bir çalışmadır. Klasik kaba küme yaklaşımı Pawlak tarafından 1982 yılında tanımlanmış olup fuzzy kaba kümeler ise 1990 yılında Dubois ve Prade tarafından çalışılmıştır. Thomas ve Nair, 2011 yılında kaba intuitionistic fuzzy kümeleri tanımlamıştır. Kaba nötrosofik kümeler ise Broumi ve diğerleri tarafından 2014 yılında tanımlamıştır. Bileşenlerin bağımlı ya da bağımsız olma durumuna göre Smarandache' ın verdiği tanım dikkate alınarak beş bileşenli nötrosofik kümelerin fermatean yapılara uygulanması ile çalışacağımız uzay kurulmuştur. Bu uzay üzerinde temel küme işlemleri ve örnekleri verilmiş daha sonra denklik bağıntıları kullanılarak beş parçalı kaba fermatean nötrosofik kümeler inşa edilmiştir. Yaklaşım uzayı üzerinde kümelerin kapsama ve birleşim gibi bazı önemli özellikleri tanımlanmıştır. Diğer taraftan Phu tarafından verilen kaba yakınsaklık kavramı Aytar tarafından istatiksel yakınsaklığa aktarılmıştır. Akçay ve Aytar tarafından fuzzy sayı dizilerine taşınan kaba yakınsaklık kavramı, bu sayılar üzerinde istatistiksel açıdan Debnath ve Rakshit tarafından incelenmiştir. Intuitionistic fuzzy normlu uzaylarda kaba istatiksel yakınsaklık ise Antal ve diğerleri tarafından 2021 yılında çalışılmıştır. Bu kavramın nötrosofik normlu uzaylara taşınması ise Bilgin tarafından 2022 yılında yapılmıştır. Bununla birlikte Senapati ve Yager fuzzy fermatean kümeleri tanımlamış ve önemli küme işlemlerini incelemiştir. Fermatean nötrosofik kümeler ilk olarak Sweety ve Jansi tarafından 2021 de tanımlanmış olup 2022 de Bilgin ve diğerleri tarafından modifiye edilmiştir. Bu çalışmalar dikkate alınarak beş parçalı fermatean kaba kümeler yardımıyla nötrosofik normlu uzaylar kurularak istatiksel yakınsaklığa ilişkin önemli teorik bilgiler verilecektir.

Anahtar Kelimeler: Fermatean Nötrosofik Küme, Kaba Nötrosofik Küme, Kaba Fermatean Nötrosofik Normlu Uzaylar

PENTAPARTITIONED ROUGH FERMATEAN NEUTROSOPHIC NORMED SPACES

ABSTRACT

This study is a study in which neutrosophic normed spaces are evaluated together with the rough set approach. The classical rough set approach was defined by Pawlak in 1982, and fuzzy rough sets were studied by Dubois and Prade in 1990. Thomas and Nair defined rough intuitionistic fuzzy sets in 2011. Rough neutrosophic sets were described by Broumi et. al. in 2014. Considering the definition given by Smarandache according to whether the components are dependent or independent, the space we will work with has been established by applying the five-component neutrosophic sets to fermatean structures. Basic set operations and examples are given on this space, then five-part rough fermatean neutrosophic sets are constructed using equivalence relations. Some important properties of sets such as coverage and union are defined on the approximation space. On the other hand, the concept of rough convergence given by Phu was transferred to statistical convergence by Aytar. The concept of rough convergence carried to fuzzy number sequences by Akçay and Aytar has been statistically examined by Debnath and Rakshit on these numbers. Rough statistical convergence in intuitionistic fuzzy normed spaces was studied by Antal et. al. in 2021. The transfer of this concept to spaces with neutrosophic norms was made by Bilgin in 2022. However, Senapati and Yager defined fuzzy fermatean sets and examined important set operations. Fermatean neutrosophic sets were first described by Sweety and Jansi in 2021 and modified by Bilgin et. al. in 2022. Considering these studies, important theoretical information about statistical convergence will be given by establishing neutrosophic normed spaces with the help of pentapartitioned fermatean rough sets.

Keywords: Fermatean Neutrosophic Set, Rough Neutrosophic Set, Rough Fermatean Neutrosophic Normed Spaces

1. INTRODUCTION AND PRELIMINARIES

Florentin Smarandache first introduced the concept of neutrophisophic logic and neutrophisofic set in 1995. In fact, it is a generalization of the neutrophic set, classical sets, fuzzy set, intuitive fuzzy set, and interval-valued fuzzy set. In the neutrophic approach, uncertainty is measured explicitly and truth can be free of uncertainty and falsehood. This is especially important in many applications where it is necessary for solving daily life problems.

The neutrosophic set approach, which provides a different perspective for the solution of many daily life problems, has become an important field in which mathematicians collaborate with researchers from different fields.After the classical rough set approach defined by Pawlak was transferred to fuzzy sets and intuitionistic fuzzy sets, with the rapid development of neutrosophic sets, it was also applied to neutrosophic structures by Broumi et. al. Fuzzy fermatean sets were described by Senapati and Yager, and Fermatean neutrosophic sets were first described by Sweety and Jansi in 2021. This definition was modified by Bilgin et. al. and an application was made on the decision-making process.

After Aytar’s study, in which the concept of rough convergence was evaluated together with statistical convergence, a new door was opened for researchers working in the field. The study in which rough statistical convergence was examined on fuzzy number sequences was followed by a study by Antal et. al. in 2021 on Intuitionistic fuzzy normed spaces.

In this study, with the help of Smarandache's definition based on whether the components are dependent or independent, the application of penta-component neutrosophic sets to rough fermatean structures using equivalence relations is examined. Basic set operations and examples are given on this space, and then some important properties of sets such as coverage and union are defined on the approximation space. Finally, with the help of penta-part fermatean rough sets, neutrosophic normed spaces will be established and statistical convergence definition and important theorems will be given on this space.

Definition 1.1 (Smarandache, 1995) A neutrosophic set V over a fixed set W is defined as follows:

$V = \{(r, H_V(r), E_V(r), S_V(r)): r \in W\}$, where $H_V, E_V, S_V: W \to [0,1]$ are the truth, indeterminacy and falsity membership functions respectively.

Definition 1.2. (Das & Pramanik, 2020) Let W be a fixed set. Then, a pentapartitioned neutrosophic set Z (in short P-NS) over W is defined as follows:

$Z = \{(r, H_Z(r), A_Z(r), O_Z(r), M_Z(r), S_Z(r)): r \in W\}$

where $H_Z(r), A_Z(r), O_Z(r), M_Z(r), S_Z(r) \in [0,1]$ are the truth, contradiction, ignorance, unknown, falsity membership values of each $r \in W$. So, for all $r \in W$, $0 \leq H_Z(r) + A_Z(r) + O_Z(r) + M_Z(r) + S_Z(r) \leq 5$ if truth, falsity, contradiction, ignorance, unknown are independent components.

But for all $r \in W$, truth, falsity are dependent components and contradiction, ignorance, unknown are independent components and so

$0 \leq H_Z(r) + A_Z(r) + O_Z(r) + M_Z(r) + S_Z(r) \leq 4.$

Now, we give (1_{PN}) and (0_{PN}) it should be noted that there are different definitions for this sets but we use as in Definition 1.3

Definition 1.3. (Das & Pramanik, 2020) Let W be a fixed set. Then, (1_{PN}) and (0_{PN}) on W are defined as follows:

(i) $1_{PN} = \{(r, 1,1,0,0,0): r \in W\}$;

(ii) $0_{PN} = \{(r, 0,0,1,1,1): r \in W\}$.

For pentapartitioned neutrosophic set general set operations is given in [Das and Pramanic, 2020].

Remark 1.1. (Das, et.al.2022) Let M be a P-NS. Clearly, $0_{PN} \subseteq M \subseteq 1_{PN}$.

Definition 1.4. ((Sweety & Jansi 2021) and (Bilgin et. al. 2022)) Let $H_{\text{฿}}(r)$, $E_{\text{฿}}(r)$, $S_{\text{฿}}(r): W \longrightarrow [0,1]$ demonstrate the degree of membership indeterminacy and non-membership of all $r \in W$ to ฿. A Fermatean neutrosophic set ฿ has the form ฿ $= \{(r, H_{\text{฿}}(r), E_{\text{฿}}(r), S_{\text{฿}}(r)): r \in W\}$ so that membership and non-membership are dependent components and indeterminacy is an independent component sc. $0 \leq H_{\text{฿}}(r) + E_{\text{฿}}(r) + S_{\text{฿}}(r) \leq 2$ for all $r \in W$, $0 \leq H_{\text{฿}}(r)^3(r) + {S_{\text{฿}}}^3(r) \leq 1$ such that $H_{\text{฿}}(r)^3 + {E_{\text{฿}}}^3(r) + {S_{\text{฿}}}^3(r) \leq 2$.

Definition 1.5 (Broumi, et al. 2014) Let W be any non-empty set. Suppose Ʀ is an equivalence relation over W. For any non-null subset Φ of W, the sets

$\mathfrak{Q}_1(r) = \{r: [r]_{\text{Ʀ}} \subseteq \Phi\}$ and $\mathfrak{Q}_2(r) = \{r: [r]_{\text{Ʀ}} \cap \text{Ʀ} \neq \emptyset\}$

are called the lower approximation and upper approximation, respectively of $\mathfrak{R}$, where the pair $\mathfrak{H} = (W, \mathfrak{R})$ is called an approximation space. This equivalent relation $\mathfrak{R}$ is called indiscernibility relation. The pair $\mathfrak{Q}(\Phi) = (\mathfrak{Q}_1(r), \mathfrak{Q}_2(r))$ is called the rough set of Φ in $\mathfrak{H}$. Here $[r]_{\mathfrak{R}}$ denotes the equivalence class of $\mathfrak{R}$ containing r.

2. Pentapartitioned Fermatean Neutrosophic Set

Definition 2.1 Let W be a fixed set. Then, a pentapartitioned fermatean neutrosophic set (in short P-FNS) Z on W is defined as follows:

$Z = \{(r, H_Z(r), A_Z(r), O_Z(r), M_Z(r), S_Z(r)) : r \in W\}$ where $H_Z(r), A_Z(r), O_Z(r), M_Z(r), S_Z(r) \in [0,1]$ are the truth, contradiction, ignorance, unknown, falsity membership values of each $r \in W$. Then, $0 \le H_Z(r) + A_Z(r) + O_Z(r) + M_Z(r) + S_Z(r) \le 4$, for all $r \in W$. Here $H_Z(r), S_Z(r)$ are dependent components and $A_Z(r), O_Z(r), M_Z(r)$ is an independent component for all $r \in W$ also, for all $r \in W$, $0 \le H_Z(r)^3(r) + {S_Z}^3(r) \le 1$ such that $0 \le A_Z(r)^3 + {O_Z}^3(r) + {M_Z}^3(r) \le 3$.

Definition 2.2 Let $\mathfrak{Q}_1 = \left\{\left(r, H_{\mathfrak{Q}_1}(r), A_{\mathfrak{Q}_1}(r), O_{\mathfrak{Q}_1}(r), M_{\mathfrak{Q}_1}(r), S_{\mathfrak{Q}_1}(r)\right) : r \in W\right\}$ and $\mathfrak{Q}_2 = \left\{\left(r, H_{\mathfrak{Q}_2}(r), A_{\mathfrak{Q}_2}(r), O_{\mathfrak{Q}_2}(r), M_{\mathfrak{Q}_2}(r), S_{\mathfrak{Q}_2}(r)\right) : r \in W\right\}$ be two P-FNSs on W and let $H_{\mathfrak{Q}_i}(r), S_{\mathfrak{Q}_i}(r)$ are dependent components and $A_{\mathfrak{Q}_i}(r), O_{\mathfrak{Q}_i}(r), M_{\mathfrak{Q}_i}(r)$ is an independent component for all $r \in W$. Then, $\mathfrak{Q}_1 \sqsubseteq \mathfrak{Q}_2$ iff $H_{\mathfrak{Q}_1}(r) \le H_{\mathfrak{Q}_2}(r), A_{\mathfrak{Q}_1}(r) \le A_{\mathfrak{Q}_2}(r), O_{\mathfrak{Q}_1}(r) \ge O_{\mathfrak{Q}_2}(r), M_{\mathfrak{Q}_1}(r) \ge M_{\mathfrak{Q}_2}(r), S_{\mathfrak{Q}_1}(r) \ge S_{\mathfrak{Q}_2}(r)$, for all $r \in W$.

Example 2.1. Consider two P-FNS $X = \{(r, 0.3,0.6,0.4,0.8,0.4), (m, 0.3,0.4,0.5,0.7,0.3)\}$ and $Y = \{(r, 0.8,0.9,0.2,0.1,0.2), (m, 0.4,0.7,0.1,0.5,0.2)\}$ over a fixed set $W = \{r, m\}$. Then, $X \sqsubseteq Y$.

Definition 2.3. Let $\mathfrak{Q}_1 = \left\{\left(r, H_{\mathfrak{Q}_1}(r), A_{\mathfrak{Q}_1}(r), O_{\mathfrak{Q}_1}(r), M_{\mathfrak{Q}_1}(r), S_{\mathfrak{Q}_1}(r)\right) : r \in W\right\}$ and $\mathfrak{Q}_2 = \left\{\left(r, H_{\mathfrak{Q}_2}(r), A_{\mathfrak{Q}_2}(r), O_{\mathfrak{Q}_2}(r), M_{\mathfrak{Q}_2}(r), S_{\mathfrak{Q}_2}(r)\right) : r \in W\right\}$ be two P-FNSs on W. Then, the intersection of $\mathfrak{Q}_1$ and $\mathfrak{Q}_2$ is

$\mathfrak{Q}_1 \cap \mathfrak{Q}_2 = \{(r, \min\{H_{\mathfrak{Q}_1}(r), H_{\mathfrak{Q}_2}(r)\}, \min\{A_{\mathfrak{Q}_1}(r), A_{\mathfrak{Q}_2}(r)\}, \max\{O_{\mathfrak{Q}_1}(r), O_{\mathfrak{Q}_2}(r)\},$
$\max\{M_{\mathfrak{Q}_1}(r), M_{\mathfrak{Q}_2}(r)\}, \max\{S_{\mathfrak{Q}_1}(r), S_{\mathfrak{Q}_2}(r)\}) : r \in W\}.$

Example 2.2. Consider two P-FNSs $X = \{(r, 0.5,0.6,0.7,0.7,0.5), (m, 0.4,0.7,0.4,0.2,0.9)\}$ and $Y = \{(r, 0.5,0.8,0.7,0.2,0.9), (m, 0.9,0.2,0.8,0.4,0.3)\}$ over $W = \{r, m\}$. Then, the intersection of X and Y is $X \cap Y = \{(r, 0.5,0.6,0.7,0.7,0.9), (m, 0.4,0.2,0.8,0.4,0.9)\}$.

Definition 2.4. Let $\mathfrak{Q}_1 = \left\{\left(r, H_{\mathfrak{Q}_1}(r), A_{\mathfrak{Q}_1}(r), O_{\mathfrak{Q}_1}(r), M_{\mathfrak{Q}_1}(r), S_{\mathfrak{Q}_1}(r)\right) : r \in W\right\}$ and $\mathfrak{Q}_2 = \left\{\left(r, H_{\mathfrak{Q}_2}(r), A_{\mathfrak{Q}_2}(r), O_{\mathfrak{Q}_2}(r), M_{\mathfrak{Q}_2}(r), S_{\mathfrak{Q}_2}(r)\right) : r \in W\right\}$ be two P-FNSs on W. Then, the union of $\mathfrak{Q}_1$ and $\mathfrak{Q}_2$ is

$\mathfrak{Q}_1 \cup \mathfrak{Q}_2 = \{(r, \max\{H_{\mathfrak{Q}_1}(r), H_{\mathfrak{Q}_2}(r)\}, \max\{A_{\mathfrak{Q}_1}(r), A_{\mathfrak{Q}_2}(r)\}, \min\{O_{\mathfrak{Q}_1}(r), O_{\mathfrak{Q}_2}(r)\},$
$\min\{M_{\mathfrak{Q}_1}(r), M_{\mathfrak{Q}_2}(r)\}, \min\{S_{\mathfrak{Q}_1}(r), S_{\mathfrak{Q}_2}(r)\}) : r \in W\}.$

Example 2.3. Consider two P-FNSs $X = \{(r, 0.8,0.5,0.9,0.3,0.5), (m, 0.5,0.4,0.7,0.7,0.5)\}$ and $Y = \{(r, 0.9,0.9,0.4,0,0.1), (m, 0.6,0.7,0.1,0.5,0.2)\}$ over $W = \{r, m\}$. Then, their union is $X \cup Y = \{(r, 0.9,0.9,0.4,0,0.1), (m, 0.6,0.7,0.1,0.5,0.2)\}$.

Definition 2.5. Let $\mathfrak{Q}_1 = \left\{\left(r, H_{\mathfrak{Q}_1}(r), A_{\mathfrak{Q}_1}(r), O_{\mathfrak{Q}_1}(r), M_{\mathfrak{Q}_1}(r), S_{\mathfrak{Q}_1}(r)\right) : r \in W\right\}$ be a P-FNS over a fixed set W. Then, ${\mathfrak{Q}_1}^c = \left\{\left(r, S_{\mathfrak{Q}_1}(r), M_{\mathfrak{Q}_1}(r), 1 - O_{\mathfrak{Q}_1}(r), A_{\mathfrak{Q}_1}(r), H_{\mathfrak{Q}_1}(r)\right) : r \in W\right\}$.

Example 2.4. Let $M = \{(r, 0.7,0.1,0.5,0.7,0.1), (m, 0.4,0.5,0.9,0.1,0.8)\}$ be a P-FNS over $W = \{r, m\}$. Then, $M^c = \{(r, 0.1,0.7,0.5,0.1,0.7), (m, 0.8,0.1,0.1,0.5,0.4) : r \in W\}$.

3. Pentapartitioned Rough Fermatean Neutrosophic Sets

Definition 3.1. Let $\mathfrak{Q} = \{(r, H_{\mathfrak{Q}}(r), A_{\mathfrak{Q}}(r), O_{\mathfrak{Q}}(r), M_{\mathfrak{Q}}(r), S_{\mathfrak{Q}}(r)) : r \in W\}$ be a P-FNS on W. and $H_{\mathfrak{Q}_i}(r), S_{\mathfrak{Q}_i}(r)$ are dependent components and $A_{\mathfrak{Q}_i}(r), O_{\mathfrak{Q}_i}(r), M_{\mathfrak{Q}_i}(r)$ is an independent component for all $r \in W$. Also let Ɽ be an equivalence relation on W. Then, the lower approximation $\left(\underline{\mathrm{N}}(\mathfrak{Q})\right)$ and the upper approximation $\left(\overline{\mathrm{N}}(\mathfrak{Q})\right)$ of $\mathfrak{Q}$ in the approximation space $(W, Ɽ)$ are defined as follows:

$$\underline{\mathrm{N}}(\mathfrak{Q}) = \left\{\left(r, H_{\underline{\mathrm{N}}(\mathfrak{Q})}(r), A_{\underline{\mathrm{N}}(\mathfrak{Q})}(r), O_{\underline{\mathrm{N}}(\mathfrak{Q})}(r), M_{\underline{\mathrm{N}}(\mathfrak{Q})}(r), S_{\underline{\mathrm{N}}(\mathfrak{Q})}(r)\right) : p \in [r]_{Ɽ}, r \in W\right\}$$

$$\overline{\mathrm{N}}(\mathfrak{Q}) = \left\{\left(r, H_{\overline{\mathrm{N}}(\mathfrak{Q})}(r), A_{\overline{\mathrm{N}}(\mathfrak{Q})}(r), O_{\overline{\mathrm{N}}(\mathfrak{Q})}(r), M_{\overline{\mathrm{N}}(\mathfrak{Q})}(r), S_{\overline{\mathrm{N}}(\mathfrak{Q})}(r)\right) : p \in [r]_{Ɽ}, r \in W\right\}$$

where

$$H_{\underline{\mathrm{N}}(\mathfrak{Q})} = \wedge_{p \in [r]_{Ɽ}} H_{(\mathfrak{Q})}(p), A_{\underline{\mathrm{N}}(\mathfrak{Q})} = \vee_{p \in [r]_{Ɽ}} A_{(\mathfrak{Q})}(p), \ O_{\underline{\mathrm{N}}(\mathfrak{Q})} = \vee_{p \in [r]_{Ɽ}} O_{(\mathfrak{Q})}(p),$$

$$M_{\underline{\mathrm{N}}(\mathfrak{Q})} = \vee_{p \in [r]_{Ɽ}} M_{(\mathfrak{Q})}(p), S_{\underline{\mathrm{N}}(\mathfrak{Q})} = \vee_{p \in [r]_{Ɽ}} S_{(\mathfrak{Q})}(p),$$

$$H_{\overline{\mathrm{N}}(\mathfrak{Q})} = \vee_{p \in [r]_{Ɽ}} H_{(\mathfrak{Q})}(p), \quad A_{\overline{\mathrm{N}}(\mathfrak{Q})} = \wedge_{p \in [r]_{Ɽ}} A_{(\mathfrak{Q})}(p), \quad O_{\overline{\mathrm{N}}(\mathfrak{Q})} = \wedge_{p \in [r]_{Ɽ}} O_{(\mathfrak{Q})}(p),$$

$$M_{\overline{\mathrm{N}}(\mathfrak{Q})} = \wedge_{p \in [r]_{Ɽ}} M_{(\mathfrak{Q})}(p), S_{\overline{\mathrm{N}}(\mathfrak{Q})} = \wedge_{p \in [r]_{Ɽ}} S_{(\mathfrak{Q})}(p).$$

So, $0 \le H_{\underline{\mathrm{N}}(\mathfrak{Q})}(r) + A_{\underline{\mathrm{N}}(\mathfrak{Q})}(r) + O_{\underline{\mathrm{N}}(\mathfrak{Q})}(r) + M_{\underline{\mathrm{N}}(\mathfrak{Q})}(r) + S_{\underline{\mathrm{N}}(\mathfrak{Q})}(r) \le 4$ and

$0 \le H_{\overline{\mathrm{N}}(\mathfrak{Q})}(r) + A_{\overline{\mathrm{N}}(\mathfrak{Q})}(r) + O_{\overline{\mathrm{N}}(\mathfrak{Q})}(r) + M_{\overline{\mathrm{N}}(\mathfrak{Q})}(r) + S_{\overline{\mathrm{N}}(\mathfrak{Q})}(r) \le 4$.

Also, for all $r \in W$, $0 \le {H_{\underline{\mathrm{N}}(\mathfrak{Q})}}^3(r) + {S_{\underline{\mathrm{N}}(\mathfrak{Q})}}^3(r) \le 1$ such that $0 \le {A_{\underline{\mathrm{N}}(\mathfrak{Q})}}^3(r) + {O_{\underline{\mathrm{N}}(\mathfrak{Q})}}^3(r) + {M_{\underline{\mathrm{N}}(\mathfrak{Q})}}^3(r) \le 3$ and $0 \le {H_{\overline{\mathrm{N}}(\mathfrak{Q})}}^3(r) + {S_{\overline{\mathrm{N}}(\mathfrak{Q})}}^3(r) \le 1$ such that $0 \le {A_{\overline{\mathrm{N}}(\mathfrak{Q})}}^3(r) + {O_{\overline{\mathrm{N}}(\mathfrak{Q})}}^3(r) + {M_{\overline{\mathrm{N}}(\mathfrak{Q})}}^3(r) \le 3$.

Clearly, lower approximation $\left(\underline{\mathrm{N}}(\mathfrak{Q})\right)$ and the upper approximation $\left(\overline{\mathrm{N}}(\mathfrak{Q})\right)$ are the P-FNSs over W. The pair $\left(\underline{\mathrm{N}}(\mathfrak{Q}), \overline{\mathrm{N}}(\mathfrak{Q})\right)$ is said to be a pentapartitioned rough fermatean neutrosophic set (in short PR-FNS) in the approximation space $(W, Ɽ)$.

Here, the operators "∨" and "∧" means "max" and "min" or operators respectively.

Example 3.1 Let $W = \{r_1, r_2, r_3, r_4, r_5, r_6, r_7\}$ be a fixed set. Let Ꞧ be an equivalence relation, where its partition of W is given by $W/\text{Ꞧ} = \{(r_1, r_4, r_7), (r_2, r_3, r_6), (r_5)\}$. Suppose that

$\mathfrak{Q} = \{\langle r_1, 0.2,0.3,0.5,0.2,0.6\rangle, \langle r_2, 0.2,0.2,0.4,0.2,0.7\rangle, \langle r_3, 0.2,0.4,0.6,0.8,0.9\rangle,$

$\langle r_4, 0.3,0.5,0.7,0.6,0.4\rangle, \langle r_5, 0.3,0.6,0.9,0.2,0.5\rangle, \langle r_6, 0.1,0.3,0.5,0.7,0.8\rangle, \langle r_7, 0.1,0.3,0.2,0.4,0.6\rangle\}$

be a R-P-FNS over W. Then, the lower approximation and upper approximation set of the R-P-FNS $\mathfrak{Q}$ is

$\underline{N}(\mathfrak{Q}) = \{\langle r_1, 0.1,0.5,0.7,0.6,0.6\rangle, \langle r_2, 0.1,0.4,0.6,0.8,0.9\rangle, \langle r_3, 0.1,0.4,0.6,0.8,0.9\rangle,$

$\langle r_4, 0.1,0.5,0.7,0.6,0.6\rangle, \langle r_5, 0.3,0.6,0.9,0.2,0.5\rangle, \langle r_6, 0.1,0.4,0.6,0.8,0.9\rangle, \langle r_7, 0.1,0.5,0.7,0.6,0.6\rangle\}$

and $\overline{N}(\mathfrak{Q}) = \{\langle r_1, 0.3,0.3,0.2,0.2,0.4\rangle, \langle r_2, 0.2,0.2,0.4,0.2,0.7\rangle, \langle r_3, 0.2,0.2,0.4,0.2,0.7\rangle,$

$\langle r_4, 0.3,0.3,0.2,0.2,0.4\rangle, \langle r_5, 0.3,0.6,0.9,0.2,0.5\rangle, \langle r_6, 0.2,0.2,0.4,0.2,0.7\rangle, \langle r_7, 0.3,0.3,0.2,0.2,0.4\rangle\}.$

Therefore,

$\left(\underline{N}(\mathfrak{Q}), \overline{N}(\mathfrak{Q})\right) = (\{\langle r_1, 0.1,0.5,0.7,0.6,0.6\rangle, \langle r_2, 0.1,0.4,0.6,0.8,0.9\rangle, \langle r_3, 0.1,0.4,0.6,0.8,0.9\rangle,$

$\langle r_4, 0.1,0.5,0.7,0.6,0.6\rangle, \langle r_5, 0.3,0.6,0.9,0.2,0.5\rangle, \langle r_6, 0.1,0.4,0.6,0.8,0.9\rangle, \langle r_7, 0.1,0.5,0.7,0.6,0.6\rangle\},$

$\{\langle r_1, 0.3,0.3,0.2,0.2,0.4\rangle, \langle r_2, 0.2,0.2,0.4,0.2,0.7\rangle, \langle r_3, 0.2,0.2,0.4,0.2,0.7\rangle, \langle r_4, 0.3,0.3,0.2,0.2,0.4\rangle,$

$\langle r_5, 0.3,0.6,0.9,0.2,0.5\rangle, \langle r_6, 0.2,0.2,0.4,0.2,0.7\rangle, \langle r_7, 0.3,0.3,0.2,0.2,0.4\rangle\}).$

is a PR-FNS in $(W, \text{Ꞧ})$.

Definition 3.2 Let $N(\mathfrak{Q}) = \left(\underline{N}(\mathfrak{Q}), \overline{N}(\mathfrak{Q})\right)$ be a PR-FNS in the approximation space $(W, \text{Ꞧ})$. Then, the complement of $N(\mathfrak{Q}) = \left(\underline{N}(\mathfrak{Q}), \overline{N}(\mathfrak{Q})\right)$ is defined as follows:

$N(\mathfrak{Q})^c = \left(\underline{N}(\mathfrak{Q})^c, \overline{N}(\mathfrak{Q})^c\right)$,where

$\underline{N}(\mathfrak{Q})^c = \left\{\left(r, S_{\underline{N}(\mathfrak{Q})}(r), M_{\underline{N}(\mathfrak{Q})}(r), 1 - O_{\underline{N}(\mathfrak{Q})}(r), A_{\underline{N}(\mathfrak{Q})}(r), H_{\underline{N}(\mathfrak{Q})}(r)\right) : p \in [r]_{\text{Ꞧ}}, r \in W\right\}$ and

$\overline{N}(\mathfrak{Q})^c = \left\{\left(r, S_{\overline{N}(\mathfrak{Q})}(r), M_{\overline{N}(\mathfrak{Q})}(r), 1 - O_{\overline{N}(\mathfrak{Q})}(r), A_{\overline{N}(\mathfrak{Q})}(r), H_{\overline{N}(\mathfrak{Q})}(r)\right) : p \in [r]_{\text{Ꞧ}}, r \in W\right\}.$

Example 3.2. Let $N(\mathfrak{Q}) = \left(\underline{N}(\mathfrak{Q}), \overline{N}(\mathfrak{Q})\right)$ be a PR-FNS in the approximation space $(W, \text{Ꞧ})$ as it is shown Example 3.1. Then the complement of $N(\mathfrak{Q})$ is $N(\mathfrak{Q})^c = \left(\underline{N}(\mathfrak{Q})^c, \overline{N}(\mathfrak{Q})^c\right)$, where,

$\underline{N}(\mathfrak{Q})^c = \{\langle r_1, 0.6,0.6,0.3,0.5,0.1\rangle, \langle r_2, 0.9,0.8,0.4,0.4,0.1\rangle, \langle r_3, 0.9,0.8,0.4,0.4,0.1\rangle,$

$\langle r_4, 0.6,0.6,0.3,0.5,0.1\rangle, \langle r_5, 0.5,0.2,0.1,0.6,0.3\rangle, \langle r_6, 0.9,0.8,0.4,0.4,0.1\rangle, \langle r_7, 0.6,0.6,0.3,0.5,0.1\rangle\}$

and $\overline{N}(\mathfrak{Q})^c = \{\langle r_1, 0.4,0.2,0.8,0.3,0.3\rangle, \langle r_2, 0.7,0.2,0.6,0.2,0.2\rangle, \langle r_3, 0.7,0.2,0.6,0.2,0.2\rangle,$

$\langle r_4, 0.4,0.2,0.8,0.3,0.3\rangle, \langle r_5, 0.5,0.2,0.1,0.6,0.3\rangle, \langle r_6, 0.7,0.2,0.6,0.2,0.2\rangle, \langle r_7, 0.4,0.2,0.8,0.3,0.3\rangle\}.$

Definition 3.3 Let $N(\mathfrak{Q}) = \left(\underline{N}(\mathfrak{Q}), \overline{N}(\mathfrak{Q})\right)$ and $N(\mathrm{U}) = \left(\underline{N}(\mathrm{U}), \overline{N}(\mathrm{U})\right)$ be two PR-FNSs in the approximation space $(W, \text{Ꞧ})$. Then, the intersection and union of the PR-FNSs $N(\mathfrak{Q})$ and $N(\mathrm{U})$ are defined as fallows:

$Ɲ(Ձ \cap Ʋ) = \left(\underline{Ɲ}(Ձ \cap Ʋ), \overline{Ɲ}(Ձ \cap Ʋ)\right)$ and $Ɲ(Ձ \cup Ʋ) = \left(\underline{Ɲ}(Ձ \cup Ʋ), \overline{Ɲ}(Ձ \cup Ʋ)\right)$,

where,

$\underline{Ɲ}(Ձ \cap Ʋ) = \{\langle r, H_{\underline{Ɲ}(Ձ)}(r) \wedge H_{\underline{Ɲ}(Ʋ)}(r), A_{\underline{Ɲ}(Ձ)}(r) \wedge A_{\underline{Ɲ}(Ʋ)}(r), O_{\underline{Ɲ}(Ձ)}(r) \vee O_{\underline{Ɲ}(Ʋ)}(r), M_{\underline{Ɲ}(Ձ)}(r) \vee M_{\underline{Ɲ}(Ʋ)}(r), S_{\underline{Ɲ}(Ձ)}(r) \vee$

SƝUr:p∈r Ʀ,r∈W;

$\overline{Ɲ}(Ձ \cap Ʋ) =$

$\{\langle r, H_{\overline{Ɲ}(Ձ)}(r) \wedge H_{\overline{Ɲ}(Ʋ)}(r), A_{\overline{Ɲ}(Ձ)}(r) \wedge A_{\overline{Ɲ}(Ʋ)}(r), O_{\overline{Ɲ}(Ձ)}(r) \vee O_{\overline{Ɲ}(Ʋ)}(r), M_{\overline{Ɲ}(Ձ)}(r) \vee M_{\overline{Ɲ}(Ʋ)}(r), S_{\overline{Ɲ}(Ձ)}(r) \vee$

SƝUr:p∈r Ʀ,r∈W;

$\underline{Ɲ}(Ձ \cup Ʋ) = \{\langle r, H_{\underline{Ɲ}(Ձ)}(r) \vee H_{\underline{Ɲ}(Ʋ)}(r), A_{\underline{Ɲ}(Ձ)}(r) \vee A_{\underline{Ɲ}(Ʋ)}(r), O_{\underline{Ɲ}(Ձ)}(r) \wedge O_{\underline{Ɲ}(Ʋ)}(r), M_{\underline{Ɲ}(Ձ)}(r) \wedge M_{\underline{Ɲ}(Ʋ)}(r), S_{\underline{Ɲ}(Ձ)}(r) \wedge$

SƝUr:p∈r Ʀ,r∈W;

and

$\overline{Ɲ}(Ձ \cup Ʋ) =$

$\{\langle r, H_{\overline{Ɲ}(Ձ)}(r) \vee H_{\overline{Ɲ}(Ʋ)}(r), A_{\overline{Ɲ}(Ձ)}(r) \vee A_{\overline{Ɲ}(Ʋ)}(r), O_{\overline{Ɲ}(Ձ)}(r) \wedge O_{\overline{Ɲ}(Ʋ)}(r), M_{\overline{Ɲ}(Ձ)}(r) \wedge M_{\overline{Ɲ}(Ʋ)}(r), S_{\overline{Ɲ}(Ձ)}(r) \wedge$

SƝUr:p∈r Ʀ,r∈W.

Definition 3.4 Let $Ɲ(Ձ) = \left(\underline{Ɲ}(Ձ), \overline{Ɲ}(Ձ)\right)$ and $Ɲ(Ʋ) = \left(\underline{Ɲ}(Ʋ), \overline{Ɲ}(Ʋ)\right)$ be two PR-FNSs in the approximation space $(W, Ʀ)$ and $\overline{Ɲ}(Ձ), \overline{Ɲ}(Ʋ), \underline{Ɲ}(Ձ)$ and $\underline{Ɲ}(Ʋ)$ are P-FNSs Then we define the following

i. $Ɲ(Ձ) = Ɲ(Ʋ)$ iff $\underline{Ɲ}(Ձ) = \underline{Ɲ}(Ʋ)$ and $\overline{Ɲ}(Ձ) = \overline{Ɲ}(Ʋ)$

ii. $Ɲ(Ձ) \subseteq Ɲ(Ʋ)$ iff $\underline{Ɲ}(Ձ) \subseteq \underline{Ɲ}(Ʋ)$ and $\overline{Ɲ}(Ձ) \subseteq \overline{Ɲ}(Ʋ)$

iii. $Ɲ(Ձ) \cup Ɲ(Ʋ) = \left(\underline{Ɲ}(Ձ) \cup \underline{Ɲ}(Ʋ), \overline{Ɲ}(Ձ) \cup \overline{Ɲ}(Ʋ)\right)$

iv. $Ɲ(Ձ) \cap Ɲ(Ʋ) = \left(\underline{Ɲ}(Ձ) \cap \underline{Ɲ}(Ʋ), \overline{Ɲ}(Ձ) \cap \overline{Ɲ}(Ʋ)\right)$

Proposition 3.7 If $Ɲ(Ձ) = \left(\underline{Ɲ}(Ձ), \overline{Ɲ}(Ձ)\right)$ and $Ɲ(Ʋ) = \left(\underline{Ɲ}(Ʋ), \overline{Ɲ}(Ʋ)\right)$ be two PR-NSs in the approximation space $(W, Ʀ)$ such that $Ձ \subseteq Ʋ$, then $Ɲ(Ձ) \subseteq Ɲ(Ʋ)$

i. $Ɲ(Ձ \cup Ʋ) \supseteq Ɲ(Ձ) \cup Ɲ(Ʋ)$

ii. $Ɲ(Ձ \cap Ʋ) \subseteq Ɲ(Ձ) \cup Ɲ(Ʋ)$

Proof: (i) $H_{\underline{Ɲ}(Ձ \cup Ʋ)}(r) = \inf\{H_{(Ձ \cup Ʋ)}(r) : r \in \Phi_i\}$

$= \inf(\max\{H_Ձ(r), H_Ʋ(r) : r \in \Phi_i\})$

$\geq \max\{\inf\{H_Ձ(r) : r \in \Phi_i\}, \inf\{H_Ʋ(r) : r \in \Phi_i\}\}$

$= \max\left\{H_{\underline{\mathrm{N}}(\mathfrak{Q})}(r_i), H_{\underline{\mathrm{N}}(\mho)}(r_i)\right\}$

$= \left(H_{\underline{\mathrm{N}}(\mathfrak{Q})} \cup H_{\underline{\mathrm{N}}(\mho)}\right)(r_i)$

Similarly,

$A_{\underline{\mathrm{N}}(\mathfrak{Q}\cup\mho)}(r_i) \leq \left(A_{\underline{\mathrm{N}}(\mathfrak{Q})} \cup A_{\underline{\mathrm{N}}(\mho)}\right)(r_i)$

$O_{\underline{\mathrm{N}}(\mathfrak{Q}\cup\mho)}(r_i) \leq \left(O_{\underline{\mathrm{N}}(\mathfrak{Q})} \cup O_{\underline{\mathrm{N}}(\mho)}\right)(r_i)$

$M_{\underline{\mathrm{N}}(\mathfrak{Q}\cup\mho)}(r_i) \leq \left(M_{\underline{\mathrm{N}}(\mathfrak{Q})} \cup M_{\underline{\mathrm{N}}(\mho)}\right)(r_i)$

$S_{\underline{\mathrm{N}}(\mathfrak{Q}\cup\mho)}(r_i) \leq \left(S_{\underline{\mathrm{N}}(\mathfrak{Q})} \cup S_{\underline{\mathrm{N}}(\mho)}\right)(r_i)$

Thus,

$\underline{\mathrm{N}}(\mathfrak{Q} \cup \mho) \supseteq \underline{\mathrm{N}}(\mathfrak{Q}) \cup \underline{\mathrm{N}}(\mho)$

We can also see that

$\overline{\mathrm{N}}(\mathfrak{Q} \cup \mho) = \overline{\mathrm{N}}(\mathfrak{Q}) \cup \overline{\mathrm{N}}(\mho)$

Hence,

$\mathrm{N}(\mathfrak{Q} \cup \mho) \supseteq \mathrm{N}(\mathfrak{Q}) \cup \mathrm{N}(\mho)$

(ii) proof is similar to the proof of (i).

Remark 3.1 Let $\mathrm{N}(\mathfrak{Q}) = \left(\underline{\mathrm{N}}(\mathfrak{Q}), \overline{\mathrm{N}}(\mathfrak{Q})\right)$ be a PR-FNS in the approximation space $(W, Ɽ)$ and $\overline{\mathrm{N}}(\mathfrak{Q}), \underline{\mathrm{N}}(\mathfrak{Q})$ are P-FNSs. Then,

$\underline{\mathrm{N}}(\mathfrak{Q}) = \left(\overline{\mathrm{N}}(\mathfrak{Q}^c)\right)^c, \overline{\mathrm{N}}(\mathfrak{Q}) = \left(\underline{\mathrm{N}}(\mathfrak{Q}^c)\right)^c$ and $\underline{\mathrm{N}}(\mathfrak{Q}) \subseteq \overline{\mathrm{N}}(\mathfrak{Q})$ doesn't necessarily have to be.

Example 3.3 Let $\mathrm{N}(\mathfrak{Q}) = \left(\underline{\mathrm{N}}(\mathfrak{Q}), \overline{\mathrm{N}}(\mathfrak{Q})\right)$ be a PR-FNS in the approximation space $(W, Ɽ)$ as it is shown Example 3.1.

$\mathfrak{Q} = \{\langle r_1, 0.2,0.3,0.5,0.2,0.6\rangle, \langle r_2, 0.2,0.2,0.4,0.2,0.7\rangle, \langle r_3, 0.2,0.4,0.6,0.8,0.9\rangle,$
$\langle r_4, 0.3,0.5,0.7,0.6,0.4\rangle, \langle r_5, 0.3,0.6,0.9,0.2,0.5\rangle, \langle r_6, 0.1,0.3,0.5,0.7,0.8\rangle, \langle r_7, 0.1,0.3,0.2,0.4,0.6\rangle\}$

be a PR-FNS over W. Then, the lower approximation and upper approximation set of the PR-FNS $\mathfrak{Q}$ is

$\underline{\mathrm{N}}(\mathfrak{Q}) = \{\langle r_1, 0.1,0.5,0.7,0.6,0.6\rangle, \langle r_2, 0.1,0.4,0.6,0.8,0.9\rangle, \langle r_3, 0.1,0.4,0.6,0.8,0.9\rangle,$

$\langle r_4, 0.1,0.5,0.7,0.6,0.6\rangle, \langle r_5, 0.3,0.6,0.9,0.2,0.5\rangle, \langle r_6, 0.1,0.4,0.6,0.8,0.9\rangle, \langle r_7, 0.1,0.5,0.7,0.6,0.6\rangle\}$

and $\overline{\mathrm{N}}(\mathfrak{Q}) = \{\langle r_1, 0.3,0.3,0.2,0.2,0.4\rangle, \langle r_2, 0.2,0.2,0.4,0.2,0.7\rangle, \langle r_3, 0.2,0.2,0.4,0.2,0.7\rangle,$

$\langle r_4, 0.3,0.3,0.2,0.2,0.4\rangle, \langle r_5, 0.3,0.6,0.9,0.2,0.5\rangle, \langle r_6, 0.2,0.2,0.4,0.2,0.7\rangle, \langle r_7, 0.3,0.3,0.2,0.2,0.4\rangle\}.$

$\mathfrak{Q}^c = \{\langle r_1, 0.6,0.2,0.5,0.3,0.2\rangle, \langle r_2, 0.7,0.2,0.6,0.2,0.2\rangle, \langle r_3, 0.9,0.8,0.4,0.4,0.2\rangle,$
$\langle r_4, 0.4,0.6,0.3,0.5,0.3\rangle, \langle r_5, 0.5,0.2,0.1,0.6,0.3\rangle, \langle r_6, 0.8,0.7,0.5,0.3,0.1\rangle, \langle r_7, 0.6,0.4,0.8,0.3,0.1\rangle\}$

$\underline{N}(\mathfrak{Q}^c) = \{\langle r_1, 0.4,0.6,0.8,0.5,0.3\rangle, \langle r_2, 0.7,0.8,0.6,0.4,0.2\rangle, \langle r_3, 0.7,0.8,0.6,0.4,0.2\rangle,$

$\langle r_4, 0.4,0.6,0.8,0.5,0.3\rangle, \langle r_5, 0.5,0.2,0.1,0.6,0.3\rangle, \langle r_6, 0.7,0.8,0.6,0.4,0.2\rangle, \langle r_7, 0.4,0.6,0.8,0.5,0.3\rangle\}$

and $\overline{N}(\mathfrak{Q}^c) = \{\langle r_1, 0.6,0.2,0.3,0.3,0.3\rangle, \langle r_2, 0.9,0.2,0.4,0.2,0.1\rangle, \langle r_3, 0.9,0.2,0.4,0.2,0.1\rangle,$

$\langle r_4, 0.6,0.2,0.3,0.3,0.3\rangle, \langle r_5, 0.5,0.2,0.1,0.6,0.3\rangle, \langle r_6, 0.9,0.2,0.4,0.2,0.1\rangle, \langle r_7, 0.6,0.2,0.3,0.3,0.3\rangle\}$

$\left(\underline{N}(\mathfrak{Q}^c)\right)^c = \{\langle r_1, 0.3,0.5,0.2,0.6,0.4\rangle, \langle r_2, 0.2,0.4,0.4,0.8,0.7\rangle, \langle r_3, 0.2,0.4,0.4,0.8,0.7\rangle,$

$\langle r_4, 0.3,0.5,0.2,0.6,0.4\rangle, \langle r_5, 0.3,0.6,0.9,0.2,0.5\rangle, \langle r_6, 0.2,0.4,0.4,0.8,0.7\rangle, \langle r_7, 0.3,0.5,0.2,0.6,0.4\rangle\}$

and $\left(\overline{N}(\mathfrak{Q}^c)\right)^c = \{\langle r_1, 0.3,0.3,0.7,0.2,0.6\rangle, \langle r_2, 0.1,0.2,0.6,0.2,0.9\rangle, \langle r_3, 0.1,0.2,0.6,0.2,0.9\rangle,$

$\langle r_4, 0.3,0.3,0.7,0.2,0.6\rangle, \langle r_5, 0.3,0.6,0.9,0.2,0.5\rangle, \langle r_6, 0.1,0.2,0.6,0.2,0.9\rangle, \langle r_7, 0.3,0.3,0.7,0.2,0.6\rangle\}$

Example 3.4:

Let $W = \{r_1, r_2, r_3\}$ be a fixed set. Let Ꞧ be an equivalence relation, where its partition of W is given by $W/$Ꞧ $= \{(r_1, r_2), (r_3)\}$. Suppose that

$$\mathfrak{Q} = \{\langle r_1, 0.2,0.2,0.5,0.2,0.6\rangle, \langle r_2, 0.2,0.3,0.4,0.3,0.7\rangle, \langle r_3, 0.2,0.4,0.6,0.4,0.9\rangle\}$$

be a R-P-FNS over W. Then, the lower approximation and upper approximation set of the R-P-FNS $\mathfrak{Q}$ is

$\underline{N}(\mathfrak{Q}) = \{\{\langle r_1, 0.2,0.3,0.5,0.3,0.7\rangle, \langle r_2, 0.2,0.3,0.5,0.3,0.7\rangle, \langle r_3, 0.2,0.4,0.6,0.4,0.9\rangle\}$

and $\overline{N}(\mathfrak{Q}) = \{\{\langle r_1, 0.2,0.2,0.4,0.2,0.6\rangle, \langle r_2, 0.2,0.2,0.4,0.2,0.6\rangle, \langle r_3, 0.2,0.4,0.6,0.4,0.9\rangle\}$

$$\mathfrak{Q}^c = \{\langle r_1, 0.6,0.2,0.5,0.2,0.2\rangle, \langle r_2, 0.7,0.3,0.6,0.3,0.2\rangle, \langle r_3, 0.9,0.4,0.4,0.4,0.2\rangle\}$$

$\underline{N}(\mathfrak{Q}^c) = \{\{\langle r_1, 0.6,0.3,0.6,0.3,0.2\rangle, \langle r_2, 0.6,0.3,0.6,0.3,0.2\rangle, \langle r_3, 0.9,0.4,0.4,0.4,0.2\rangle\}$

and $\overline{N}(\mathfrak{Q}^c) = \{\{\langle r_1, 0.7,0.2,0.5,0.2,0.2\rangle, \langle r_2, 0.7,0.2,0.5,0.2,0.2\rangle, \langle r_3, 0.9,0.4,0.4,0.4,0.2\rangle\}$

$\left(\underline{N}(\mathfrak{Q}^c)\right)^c = \{\{\langle r_1, 0.2,0.3,0.4,0.2,0.6\rangle, \langle r_2, 0.2,0.3,0.4,0.2,0.6\rangle, \langle r_3, 0.2,0.4,0.6,0.4,0.9\rangle\}$

and $\left(\overline{N}(\mathfrak{Q}^c)\right)^c = \{\{\langle r_1, 0.2,0.2,0.5,0.2,0.7\rangle, \langle r_2, 0.2,0.2,0.5,0.2,0.7\rangle, \langle r_3, 0.2,0.4,0.6,0.4,0.9\rangle\}$

4. $S_{\text{B}}(r)$ Statistical Convergence in PR-FNS

Definition 4.1 Let $\odot$ and $\otimes$ show the continuous tnorm and continuous tconorm, respectively. W be a linear spaces on $\mathbb{R}$. A Pentapartitioned Fermatean Neutrosophic Normed (shortly P-FNN) is a notation of the form $Z = \{(r, H_Z(r), A_Z(r), O_Z(r), M_Z(r), S_Z(r)) : r \in W\}$ where $H_Z(r)$ denote the truth degree, $A_Z(r)$ denote the contradiction degree, $O_Z(r)$ denote the ignorance degree, $M_Z(r)$ denote the unknown degree and $S_Z(r)$ denote the falsity degree of r on W satisfies following conditions:

For all $r, m \in W$,

i. For every $s \in \mathbb{R}^+$ $H_Z(r,s) + A_Z(r,s) + O_Z(r,s) + M_Z(r,s) + S_Z(r,s) \leq 4$,

$0 \leq H_Z(r)^3(r) + {S_Z}^3(r) \leq 1$ and $0 \leq A_Z(r)^3 + {O_Z}^3(r) + {M_Z}^3(r) \leq 3$.

ii. For every $s \in \mathbb{R}^+$ $H_Z(r,s), A_Z(r,s), O_Z(r,s), M_Z(r,s), S_Z(r,s) \in [0,1]$.

iii. For every $s \in \mathbb{R}^+$,

$H_Z(r,s) = 1 \Leftrightarrow r = 0$, $A_Z(r,s) = 1 \Leftrightarrow r = 0$, $O_Z(r,s) = 0 \Leftrightarrow r = 0$, $M_Z(r,s) = 0 \Leftrightarrow r = 0$, $S_Z(r,s) = 0 \Leftrightarrow r = 0$.

iv. For each $\beta \neq 0$,

$H_Z(\beta r,s) = H_Z\left(r, \frac{s}{|\beta|}\right)$, $A_Z(\beta r,s) = A_Z\left(r, \frac{s}{|\beta|}\right)$, $O_Z(\beta r,s) = O_Z\left(r, \frac{s}{|\beta|}\right)$, $M_Z(\beta r,s) = M_Z\left(r, \frac{s}{|\beta|}\right)$, $S_Z(\beta r,s) = S_Z\left(r, \frac{s}{|\beta|}\right)$.

v. For every $s_1, s_2 \in \mathbb{R}^+$,

$H_Z(r,s_2) \odot H_Z(m,s_1) \leq H_Z(r,s_1)(r+m, s_1+s_2)$,

$A_Z(r,s_2) \odot A_Z(m,s_1) \leq A_Z(r,s_1)(r+m, s_1+s_2)$,

$O_Z(r,s_2) \otimes O_Z(m,s_1) \geq O_Z(r+m, s_1+s_2)$,

$M_Z(r,s_2) \otimes M_Z(m,s_1) \geq M_Z(r+m, s_1+s_2)$

$S_Z(r,s_2) \otimes S_Z(m,s_1) \geq S_Z(r+m, s_1+s_2)$.

vi. $H_Z(r,\cdot), A_Z(r,\cdot)$ is continuous non-decreasing function, $O_Z(r,\cdot), M_Z(r,\cdot)$ and $S_Z(r,\cdot)$ are continuous non-increasing function.

vii. $\lim_{s\to\infty} H_Z(r,s) = 1, \lim_{s\to\infty} A_Z(r,s) = 1$,

$\lim_{s\to\infty} O_Z(r,s) = 0, \lim_{s\to\infty} M_Z(r,s) = 0, \lim_{s\to\infty} S_Z(r,s) = 0$.

viii. If $s \leq 0$, then $H_Z(r,s) = 0$, $A_Z(r,s) = 0$, $O_Z(r,s) = 1$, $M_Z(r,s) = 1$ and $S_Z(r,s) = 1$.

In this case, $(W, H_Z, A_Z, O_Z, M_Z, S_Z, \odot, \otimes)$ is called Pentapartitioned Fermatean Neutrosophic Normed Spaces (P-FNNS).

Definition 4.2 Let $(W, H_Z, A_Z, O_Z, M_Z, S_Z, \odot, \otimes)$ is a P-FNNS. A sequence (x_k) is called statistical convergence with respect to P-FNN, if there exist $\ell \in \mathbb{F}$ such that the set

$\{k \leq n: H_Z(x_k - \ell, s) \leq 1 - \varepsilon, A_Z(x_k - \ell, s) \leq 1 - \varepsilon$

$or\ O_Z(x_k - \ell, s) \geq \varepsilon, M_Z(x_k - \ell, s) \geq \varepsilon, S_Z(x_k - \ell, s) \geq \varepsilon\}$

or equivalently

$\{k \leq n: H_Z(x_k - \ell, s) > 1 - \varepsilon, A_Z(x_k - \ell, s) > 1 - \varepsilon$

$or\ O_Z(x_k - \ell, s) < \varepsilon, M_Z(x_k - \ell, s) < \varepsilon, S_Z(x_k - \ell, s) < \varepsilon\}$

has natural density zero, for every $\varepsilon > 0$ and $s > 0$. Namely,

$\lim_n \frac{1}{n} |\{k \leq n: H_Z(x_k - \ell, s) \leq 1 - \varepsilon, A_Z(x_k - \ell, s) \leq 1 - \varepsilon$

$or\ O_Z(x_k - \ell, s) \geq \varepsilon, M_Z(x_k - \ell, s) \geq \varepsilon, S_Z(x_k - \ell, s) \geq \varepsilon\}| = 0$.

Therefore, we write $st_{P_{FN}}-\lim x_k=\ell$ or $x_k\to\ell(st_{P_{FN}})$. The set of statistical convergence sequences with respect to P-FNN, will be denoted by $st_{P_{FN}}$.

Lemma 4.1 Let W be a P-FNNS. The following statements are equivalent, for every $\varepsilon>0,\ s>0$,

i) $st_{P_{FN}}-\lim x_k=\ell$.

ii) $\lim_{n\to\infty}\frac{1}{n}\left|\left\{k\le n: H_Z\left((x_k)-\ell,s\right)\le 1-\varepsilon\right\}\right|=\lim_{n\to\infty}\frac{1}{n}\left|\left\{k\le n: A_Z\left((x_k)-\ell,s\right)\le 1-\varepsilon\right\}\right|$

$=\lim\limits_{n\to\infty}\dfrac{1}{n}\left|\left\{k\le n: O_Z\left((x_k)-\ell,s\right)\ge\varepsilon\right\}\right|=\lim\limits_{n\to\infty}\dfrac{1}{n}\left|\left\{k\le n: M_Z\left((x_k)-\ell,s\right)\ge\varepsilon\right\}\right|$

$=\lim_{n\to\infty}\frac{1}{n}\left|\left\{k\le n: S_Z\left((x_k)-\ell,s\right)\ge\varepsilon\right\}\right|=0.$

iii) $\lim_{n\to\infty}\frac{1}{n}\left|\left\{k\le n: H_Z\left((x_k)-\ell,s\right)>1-\varepsilon, A_Z\left((x_k)-\ell,s\right)>1-\varepsilon\right.\right.$

$\left.\left. and\ \ O_Z\left((x_k)-\ell,s\right)<\varepsilon, M_Z\left((x_k)-\ell,s\right)<\varepsilon, S_Z\left((x_k)-\ell,s\right)<\varepsilon\right\}\right|=1.$

iv) $\lim_{n\to\infty}\frac{1}{n}\left|\left\{k\le n: H_Z\left((x_k)-\ell,s\right)>1-\varepsilon\right\}\right|=\lim_{n\to\infty}\frac{1}{n}\left|\left\{k\le n: A_Z\left((x_k)-\ell,s\right)>1-\varepsilon\right\}\right|$

$=\lim\limits_{n\to\infty}\dfrac{1}{n}\left|\left\{k\le n: O_Z\left((x_k)-\ell,s\right)<\varepsilon\right\}\right|=\lim\limits_{n\to\infty}\dfrac{1}{n}\left|\left\{k\le n: M_Z\left((x_k)-\ell,s\right)<\varepsilon\right\}\right|$

$=\lim\limits_{n\to\infty}\dfrac{1}{n}\left|\left\{k\le n: S_Z\left((x_k)-\ell,s\right)<\varepsilon\right\}\right|=1.$

v) $st_{P_{FN}}-\lim H_Z(x_k-\ell,s)=1, st_{P_{FN}}-\lim A_Z(x_k-\ell,s)=1$

$and\ st_{P_{FN}}-\lim O_Z(x_k-\ell,s)=0, st_{P_{FN}}-\lim M_Z(x_k-\ell,s)=0,$

$st_{P_N}-\lim S_Z(x_k-\ell,s)=0.$

Theorem 4.1 Let W be a P-FNNS. If (x_k) is PFNN, then $st_{P_{FN}}-\lim x_k=\ell$ is unique.

Proof Suppose that $st_{P_{FN}}-\lim x_k=\ell_1$ and $st_{P_{FN}}-\lim x_k=\ell_2$ for $\ell_1\ne\ell_2$. Choose $\varepsilon>0$. Then, for a given $m>0, (1-\varepsilon)\odot(1-\varepsilon)>1-m$ and $\varepsilon\otimes\varepsilon<m$. For any $s>0$, let's write the following set:

$$K_{H_1}(\varepsilon,m):=\left\{k\le n: H_Z\left((x_k)-\ell_1,\frac{s}{2}\right)\le 1-\varepsilon\right\},$$

$$K_{H_2}(\varepsilon,m):=\left\{k\le n: H_Z\left((x_k)-\ell_2,\frac{s}{2}\right)\le 1-\varepsilon\right\}$$

$$K_{A_1}(\varepsilon,m):=\left\{k\le n: A_Z\left((x_k)-\ell_1,\frac{s}{2}\right)\le 1-\varepsilon\right\}$$

$$K_{A_2}(\varepsilon,m):=\left\{k\le n: A_Z\left((x_k)-\ell_2,\frac{s}{2}\right)\le 1-\varepsilon\right\}$$

$$K_{O_1}(\varepsilon,m):=\left\{k\le n: O_Z\left((x_k)-\ell_1,\frac{s}{2}\right)\ge\varepsilon\right\}$$

$$K_{O_2}(\varepsilon,m):=\left\{k\le n: O_Z\left((x_k)-\ell_2,\frac{s}{2}\right)\ge\varepsilon\right\}$$

$$K_{M_1}(\varepsilon,m):=\left\{k\leq n: M_Z\left((x_k)-\ell_1,\frac{s}{2}\right)\geq\varepsilon\right\}$$

$$K_{M_2}(\varepsilon,m):=\left\{k\leq n: M_Z\left((x_k)-\ell_2,\frac{s}{2}\right)\geq\varepsilon\right\}$$

$$K_{S_1}(\varepsilon,m):=\left\{k\leq n: S_Z\left((x_k)-\ell_1,\frac{s}{2}\right)\geq\varepsilon\right\}$$

$$K_{S_2}(\varepsilon,m):=\left\{k\leq n: S_Z\left((x_k)-\ell_2,\frac{s}{2}\right)\geq\varepsilon\right\}.$$

We know that $st_{P_{FN}}-\lim x_k=\ell_1$. Then using the Lemma 4.1, for all $s>0$, density of these sets

$$d\left(K_{H_1}(\varepsilon,m)\right)=d\left(K_{A_1}(\varepsilon,m)\right)=d\left(K_{O_1}(\varepsilon,m)\right)=d\left(K_{M_1}(\varepsilon,m)\right)=d\left(K_{S_1}(\varepsilon,m)\right)=0.$$

Further, since we get $st_{P_{FN}}-\lim x_k=\ell_2$, using the Lemma 4.1, for $s>0$, density of these sets

$$d\left(K_{H_2}(\varepsilon,m)\right)=d\left(K_{A_2}(\varepsilon,m)\right)=d\left(K_{O_2}(\varepsilon,m)\right)=d\left(K_{M_2}(\varepsilon,m)\right)=d\left(K_{S_2}(\varepsilon,m)\right)=0.$$

Let

$$K_{P_N}(\varepsilon,m):=\left\{K_{H_1}(\varepsilon,m)\cup K_{H_2}(\varepsilon,m)\right\}\cup\left\{K_{A_1}(\varepsilon,m)\cup K_{A_2}(\varepsilon,m)\right\}\cap\left\{K_{O_1}(\varepsilon,m)\cup K_{O_2}(\varepsilon,m)\right\}$$
$$\cap\left\{K_{M_1}(\varepsilon,m)\cup K_{M_2}(\varepsilon,m)\right\}\cap\left\{K_{S_1}(\varepsilon,m)\cup K_{S_2}(\varepsilon,m)\right\}.$$

Then observe that $d\left(K_{P_{FN}}(\varepsilon,m)\right)=0$ which implies $d\left(\mathbb{N}/K_{P_{FN}}(\varepsilon,m)\right)=1$. Then, we have five possible situations, when take $k\in\mathbb{N}/K_{P_{FN}}(\varepsilon,m)$:

i. $k\in\mathbb{N}/\left(K_{H_1}(\varepsilon,m)\cup K_{H_2}(\varepsilon,m)\right)$,

ii. $k\in\mathbb{N}/\left(K_{A_1}(\varepsilon,m)\cup K_{A_2}(\varepsilon,m)\right)$,

iii. $k\in\mathbb{N}/\left(K_{O_1}(\varepsilon,m)\cup K_{O_2}(\varepsilon,m)\right)$,

iv. $k\in\mathbb{N}/\left(K_{M_1}(\varepsilon,m)\cup K_{M_2}(\varepsilon,m)\right)$,

v. $k\in\mathbb{N}/\left(K_{S_1}(\varepsilon,m)\cup K_{S_2}(\varepsilon,m)\right)$.

Firstly, consider (i). Then we have

$$H_Z(\ell_1-\ell_2,s)\geq H_Z\left((x_k)-\ell_1,\frac{s}{2}\right)\odot H_Z\left((x_k)-\ell_2,\frac{s}{2}\right)>(1-\varepsilon)\odot(1-\varepsilon).$$

and so since $(1-\varepsilon)\odot(1-\varepsilon)>1-m$,

$$H_Z(\ell_1-\ell_2,s)>1-m. \qquad (1)$$

Using the (1), for all $s>0$, we obtain $H_Z(\ell_1-\ell_2,s)=1$, where $m>0$ is arbitrary. That is, $\ell_1=\ell_2$ is obtained. (ii) can be shown in the same way.

For the situation (iii), if we take $k\in\mathbb{N}/\left(K_{O_1}(\varepsilon,m)\cup K_{O_2}(\varepsilon,m)\right)$, then we can write

$$A_Z(\ell_1-\ell_2,s)\leq A_Z\left((x_k)-\ell_1,\frac{s}{2}\right)\otimes A_Z\left((x_k)-\ell_2,\frac{s}{2}\right)<\varepsilon\otimes\varepsilon.$$

Using $\varepsilon\otimes\varepsilon<m$, we can see that $A_Z(\ell_1-\ell_2,s)<m$. For all $s>0$, we obtain $A_Z(\ell_1-\ell_2,s)=0$, where $m>0$ is arbitrary. Thus $\ell_1=\ell_2$. The proof can be completed by showing the others in the same way.

Theorem 4.2 Let W be a P-FNNS. If $N-\lim x_k=\ell$ in P-FNNS, then $st_{P_N}-\lim x_k=\ell$.

Proof If $N-\lim x_k=\ell$, then, for every $\varepsilon>0,\ s>0$ there exist a number $N\in\mathbb{N}$ such that $H_Z\big((x_k)-\ell,s\big)>1-\varepsilon, A_Z\big((x_k)-\ell,s\big)>1-\varepsilon$ *and* $O_Z\big((x_k)-\ell,s\big)<\varepsilon, M_Z\big((x_k)-\ell,s\big)<\varepsilon, S_Z\big((x_k)-\ell,s\big)<\varepsilon$ for all $k\geq N$. Therefore, the set

$$\{k\leq n: H_Z\big((x_k)-\ell,s\big)\leq 1-\varepsilon, A_Z\big((x_k)-\ell,s\big)\leq 1-\varepsilon$$
$$or\ O_Z\big((x_k)-\ell,s\big)\geq\varepsilon, M_Z\big((x_k)-\ell,s\big)\geq\varepsilon, S_Z\big((x_k)-\ell,s\big)\geq\varepsilon\}$$

has at most finitely many terms. Hence, since every finite subset of N has density zero,

$$\lim_{n\to\infty}\frac{1}{n}\big|\{k\leq n: H_Z\big((x_k)-\ell,s\big)\leq 1-\varepsilon, A_Z\big((x_k)-\ell,s\big)\leq 1-\varepsilon$$
$$or\ O_Z\big((x_k)-\ell,s\big)\geq\varepsilon, M_Z\big((x_k)-\ell,s\big)\geq\varepsilon, S_Z\big((x_k)-\ell,s\big)\geq\varepsilon\}\big|=0.$$

This completes the proof.

Definition 4.3. The sequence (x_k) is called statistically Cauchy with respect to P-FNN. If there exists $N=N(\varepsilon)$, for every $\varepsilon>0,\ s>0$ and $k>m$ such that

$$\big|\{k\leq n: H_Z\big((x_k)-(x_m),s\big)\leq 1-\varepsilon, A_Z\big((x_k)-(x_m),s\big)\leq 1-\varepsilon$$
$$or\ O_Z\big((x_k)-(x_m),s\big)\geq\varepsilon, M_Z\big((x_k)-(x_m),s\big)\geq\varepsilon, S_Z\big((x_k)-(x_m),s\big)\geq\varepsilon\}\big|$$

has ND zero.

Theorem 4.3 If a sequence (x_k) is statistical convergence sequences in P-FNNS, then it is statistical Cauchy in P-FNNS.

Definition 4.4 W is called statistically complete P-FNNS , if every statistical Cauchy P-FNN sequences is statistical converges P-FNN sequences.

Theorem 4.4 Let W be a PFNNS. Then W is statistical complete P-FNN.

Definition 4.5 Let $(W,H_Z,A_Z,O_Z,M_Z,S_Z,\odot,\otimes)$ be a P-FNNS. A sequence $x=x_k$ in W is said to be rough convergent to $\ell\in W$ for some non-negative number r if there exists $k_0\in\mathbb{N}$ for every $\varepsilon>0$ and $\lambda\in(0,1)$ such that

$$H_Z(x_k-\ell,r+\varepsilon)>1-\lambda, A_Z(x_k-\ell,r+\varepsilon)>1-\lambda$$
$$and\ O_Z(x_k-\ell,r+\varepsilon)<\lambda, M_Z(x_k-\ell,r+\varepsilon)<\lambda, S_Z(x_k-\ell,r+\varepsilon)<\lambda$$

for all $k\geq k_0$.

Definition 4.6 Let $(W, H_Z, A_Z, O_Z, M_Z, S_Z, \odot, \otimes)$ be a P-FNNS. A sequence $x = x_k$ in W is said to be rough statistically convergent to $\ell \in W$ for some non-negative number r if for every $\varepsilon > 0$ and $\lambda \in (0,1)$ such that

$\delta(\{k \in \mathbb{N}: H_Z(x_k - \ell, r + \varepsilon) \leq 1 - \lambda, A_Z(x_k - \ell, r + \varepsilon) \leq 1 - \lambda$

$or\ O_Z(x_k - \ell, r + \varepsilon) \geq \lambda, M_Z(x_k - \ell, r + \varepsilon) \geq \lambda, S_Z(x_k - \ell, r + \varepsilon) \geq \lambda\}) = 0.$

It is denoted by $st_{P_{FRN}} - \lim x_k = \ell$ or $x_k \to \ell(st_{P_{FRN}})$. The set of rough statistical convergence sequences with respect to PR-FNN, will be denoted by $st_{P_{FRN}}$.

Definition 4.7 Let $(W, H_Z, A_Z, O_Z, M_Z, S_Z, \odot, \otimes)$ be a P-FNNS. A sequence $x = x_k$ in W is said to be rough statistically bounded for some non-negative number r if for every $\varepsilon > 0$ and $\lambda \in (0,1)$ there exists a real number $\mathfrak{M}$ such that

$\delta(\{k \in \mathbb{N}: H_Z(x_k, \mathfrak{M}) \leq 1 - \lambda, A_Z(x_k, \mathfrak{M}) \leq 1 - \lambda$

$or\ O_Z(x_k, \mathfrak{M}) \geq \lambda, M_Z(x_k, \mathfrak{M}) \geq \lambda, S_Z(x_k, \mathfrak{M}) \geq \lambda\}) = 0.$

5. RESULTS

In this study, as a result of combining the rough set approach with the pentapartitioned neutrophic sets, sets were constructed to carry the fermatean structure. The dependency or independence of the components has shaped the definitions we will give, and with the help of equivalence classes, the basic properties of the sets are shown and the normed space construction that will carry the properties of all these structures has been made. This study can be evaluated from different perspectives using different types of convergence as (Olmez and Aytar, 2021). Our work has been completed with the definition of Rough statistical convergence and its properties. (Riaz et. al., 2022 a, b, c, d; Pamucar et. al., 2019, 2020, 2021; Naeem et. al. 2019; Wei et. al. 2022) and (Bilgin et. al. 2022) resources can be used for the applications of this study to daily life problems.

REFERENCES

Alharbi, N., Aydi, H., & Özel, C., (2019). Rough spaces on covering based roughsets. Eur. J. Pure Appl. Math., 12 (2), 533-543.

Akçay, F. G., & Aytar S. (2015). Rough convergence of a sequence of fuzzy numbers. Bull. Math. Anal. Appl., 7 (4), 17–23.

Antal, R., Chawla, M. & Kumar, V. (2021). Rough statistical convergence in intuitionistic fuzzy normed spaces. Filomat, 35 (13), 4405-4416.

Aytar, S. (2008). Rough statistical convergence. Numerical functional analysis and optimization, 29 (3-4), 291-303.

Bilgin, N. G., Pamučar, D., & Riaz, M. (2022). Fermatean Neutrosophic Topological Spaces and an Application of Neutrosophic Kano Method. Symmetry, 14 (11), 2442.

Bilgin, N. G. (2022). Rough Statistical Convergence in Neutrophic Norm Spaces. Euroasia Journal of Mathematics, Engineering, Natural & Medical Sciences, 9 (21), 47-55.

Broumi, S., Smarandache, F., & Dhar, M. (2014). Rough neutrosophic set. Neutrosophic Sets Syst., 3, 60–65.

Das, S., Das, R., & Tripathy, B. C. (2022). Topology on Rough Pentapartitioned Neutrosophic Set. Iraqi Journal of Science, 2630-2640.

Das, S., & Pramanik, S. (2020). Neutrosophic simply soft open set in neutrosophic soft topological space. Neutrosophic Sets and Systems, 38, 235-243.

Debnath, S., & Rakshit, D. (2019). Rough Statistical Convergence of a Sequence of Fuzzy Numbers. Mathematica, 61, 33–39.

Dubois, D., & Prade, H. (1990). Rough fuzzy sets and fuzzy rough sets. International Journal of Gerneral Systems, 17, 191–208.

Naeem, K., Riaz, M., Peng, X., & Afzal, D. (2019). Pythagorean fuzzy soft MCGDM methods based on TOPSIS, VIKOR and aggregation operators. Journal of Intelligent & Fuzzy Systems, 37(5), 6937-6957.

Olmez, O., & Aytar, S. (2021). On The Rough Hausdorff Convergence.Sigma: Journal of Engineering & Natural Sciences,39.

Pamučar, D., & Božanić, D. (2019). Selection of a location for the development of multimodal logistics center: Application of single-valued neutrosophic MABAC model. Operational Research in Engineering Sciences: Theory and Applications, 2 (2), 55-71.

Pamučar, D., Yazdani, M., Obradovic, R., Kumar, A., & Torres-Jiménez, M. (2020). A novel fuzzy hybrid neutrosophic decision-making approach for the resilient supplier selection problem. International Journal of Intelligent Systems, 35 (12), 1934-1986.

Pamučar, D., Ecer, F., & Deveci, M. (2021). Assessment of alternative fuel vehicles for sustainable road transportation of United States using integrated fuzzy FUCOM and neutrosophic fuzzy MARCOS methodology. Science of The Total Environment, 788, 147763.

Pawlak, Z. (1982). Rough sets. International Journal of Computer Science, 11, 341–356.

Phu, H. X. (2001). Rough convergence in normed linear spaces. Numer. Funct. Anal. Optim., 22, 201-224.

Riaz, M., Zeb, A., Ali, F., Naeem, M., & Arjika, S. (2022). Fermatean Cubic Fuzzy Aggregation Operators and Their Application in Multiattribute Decision-Making Problems. Journal of Function Spaces, 2022.

Riaz, M., Zeb, A., Ali, F., Naeem, M., & Arjika, S. (2022). Fermatean Cubic Fuzzy Aggregation Operators and Their Application in Multiattribute Decision-Making Problems. Journal of Function Spaces, 2022.

Riaz, M., Almalki, Y., Batool, S., & Tanveer, S. (2022). Topological Structure of Single-Valued Neutrosophic Hesitant Fuzzy Sets and Data Analysis for Uncertain Supply Chains. Symmetry, 14, 1382.

Riaz, M., Athar Farid, H. M., Shakeel, H. M., & Arif, D. (2022). Cost Effective Indoor HVAC Energy Efficiency Monitoring based on Intelligent Decision Support System under Fermatean Fuzzy Framework. Scientia Iranica.

Riaz, M., Naeem, K., Zareef, I., & Afzal, D. (2020). Neutrosophic N-soft sets with TOPSIS method for multiple attribute decision making. Neutrosophic Sets and Systems, 32, 145.

Smarandache, F. (1995). Neutrosophy, neutrosophic theories and applications.

Senapati, T., & Yager, R.R. (2019). Fermatean fuzzy sets. J Ambient Intell Hum Computi.

Sweety, C. A. C., & Jansi R. (2021). Fermatean Neutrosophic Sets. International Journal of Advanced Research in Computer and Communication Engineering, 10 (6), 24-27.

Thomas, K. V., & Nair, L. S. (2011). Rough intutionistic fuzzy sets in a lattice. Int. Math. Forum, 6 (27), 1327–1335.

Wei, D., Meng, D., Rong, Y., Liu, Y., Garg, H., & Pamucar, D. (2022). Fermatean Fuzzy Schweizer–Sklar Operators and BWM-Entropy-Based Combined Compromise Solution Approach: An Application to Green Supplier Selection. Entropy, 24(6), 776.