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Abstract

In this paper, we study the algebraic relationships between n- refined neutrosophic modules by using semi-module homomorphisms.

On the other hand, this work shows the relationship between neutrosophic geometrical AH-isometry and semi-module isomorphisms.

Keywords: AH-isometry, Semi homomorphism, semi isomorphism, n-refined neutrosophic module

1.Introduction

Neutrosophy is a new kind of philosophy founded by Smarandache [1] to study the uncertainty and the lack of information in many areas os science and life.

The indeterminacy element I and its refinements, were usefule in algebra where we find some generalizations of classical algebraic structures such as neutrosophic modules , n-refined neutrosophic matrices [2-5].

The concept of semi-modules homomorphism /isomorphism was presented in [6] as a tool to study the relations between modules which are defined over different rings.

In this work, we show a new application of semi homomorphisms in the study of n-refined neutrosophic modules. Also, we present a novel application in neutrosophic Euclidean geometry [9] based on semi- isomorphisms, where we show that algebraic isometries used in the study of neutrosophic Euclidean geometrical shapes can be considered as semi-module isomorphisms.

Main discussion

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Firs of all, we show that the concept of semi-homomorphism (semi-isomorphism) is essential in the study of neutrosophic Euclidean geometry.

Neutrosophic Euclidean geometry wae built over the idea of AH-isometry, where the AH-isometry is a function that preserves distances between neutrosophic points in $R(I) \times R(I)$.

Definition 1.

(a). Let $R(I) \times R(I)$ be the neutrosophic plane with two N-dimensions, then.

$$g: R(I) \times R(I) \to R^2 \times R^2$$

$$g(a + bI, c + dI) = \big((a, a + b), (c, c + d)\big)$$

is called the two dimensional AH-isometry.

(b). the function $T: R(I) \to R \times R; T(a + bI) = (a, a + b)$is called the one dimensional AH-isometry.

Theorem 2

g, T preserve operations and distances.

In the following, we prove that the two dimensional AH-isometry is a semi-module isomorphisim.

Definition 3:

Let (M,+,.) be a module over the ring R, then (M(I),+,.) is called a weak neutrosophic module over the ring R, and it is called a strong neutrosophic module if it is a module over the neutrosophic ring R(I).

Elements of M(I) have the form $\boldsymbol{x + yI; x, y \in M}$, i.e M(I) can be written as $\boldsymbol{M(I) = M + MI}$.

Definition 4:

(a) Let (M,+,.) be a module over the ring R, we say that $M_n(I) = M + MI_1 + \cdots + MI_n = \{x_0 + x_1I_1 + \cdots + x_nI_n;\ x_i \in M\}$ is a weak n-refined neutrosophic module over the ring R. Elements of $M_n(I)$ are called n-refined neutrosophic vectors, elements of R are called scalars.

(b) If we take scalars from the n-refined neutrosophic ring $R_n(I)$, we say that $M_n(I)$ is a strong n-refined neutrosophic module over the n-refined neutrosophic ring $M_n(I)$. Elements of $M_n(I)$ are called n-refined neutrosophic scalars.

Definition 5:

Let M be a module over a ring R, N be a module over a ring T, $\varphi: M \to N$ be a well defined map, we say that φ is an f-semi module homomorphism if and only if the following conditions are true:

(a) $\varphi(x + y) = \varphi(x) + \varphi(y)$ for all $x, y \in M$.

(b) There is a ring homomorphism $f: R \to T$ such $\varphi(r.x) = f(r).\varphi(x)$ for all $r \in R, x \in$M.

Example 6:

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Let M be a module over a ring R, M(I) be the corresponding strong neutrosophic module over R(I), $M(I_1, I_2)$ be the corresponding strong refined neutrosophic module over $R(I_1, I_2)$. Then

$\varphi: M(I_1, I_2) \to M(I);\ \varphi(a, bI_1, cI_2) = a + (b + c)I$ is a semi homomorphism.

Theorem 7.

Let $g: R(I) \times R(I) \to R^2 \times R^2$ be the two dimensional AH-isometry, and $T: R(I) \to R \times R$ be the one dimensional AH-isometry, then, g is a semi-module isomorphism.

Proof.

we have $R(I) \times R(I)$ is a module over the ring $R(I)$, and $R^2 \times R^2$ is a module over the ring $R \times R$.

According to [14], $T$is a ring isomorphism.

On the other hand, g preserves addition between $R(I) \times R(I)$ and $R^2 \times R^2$, and g is a bijection.

Now, we prove that g has the following property.

$$g[(m + nI).(a + bI, c + dI)] = T(m + nI)g(a + bI, c + dI)$$

$l_1 = [(m + nI).(a + bI, c + dI)] = g[(m + nI)(a + bI), (m + nI)(c + dI)]$

$= g\big[\big(ma + I(ab + na + nb)\big), \big(mc + I(amd + nc + nd)\big)\big]$

$= \big((m, m + n).(a, a + b), (m, m + n).(c, c + d)\big) = (m, m + n)\big((a, a + b), (c, c + d)\big)$

$= T(m + nI)g(a + bI, c + dI).$

Thus g is a semi isomorphism.

Result 8.

According to the previous theorem, the neutrosophic Euclidean geometry can be undersood as a semi isomorphic image of a neutrosophic module.

Now, we study semi homomorphisms between n-refined neutrosophic modules.

Theorem 9

Let R be a ring, $R_n(I)$ be the corresponding n-refined neutrosophic ring.

Let M be a module over R, $M_n(I)$ be the corresponding n-refined neutrosophic module over $R_n(I)$, then.

(a). $M_{n-1}(I)$ is a semi homomorphic image of $M_n(I)$.

(b). M is a semi homomorphic image of $M_n(I)$.

Proof.

(a). $g: R_n(I) \to R_{n-1}(I); g(a_0 + a_1I_1 + \cdots + a_nI_n) = a_0 + a_1I_1 + \cdots + (a_{n-1} + a_n)I_{n-1}$

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is a ring homomorphism.

We define $f: M_n(I) \to M_{n-1}(I); f(x_0 + x_1 I_1 + \cdots + x_n I_n) = x_0 + x_1 I_1 + \cdots + (x_{n-1} + x_n) I_{n-1}$.

It is clear that f is well defined and preserves addition.

Now, Let's compute:

$$f[(a_0 + a_1 I_1 + \cdots + a_n I_n).(x_0 + x_1 I_1 + \cdots + x_n I_n)] = f\left[\sum_{i,j=0}^{n} a_i x_j I_i I_j\right]$$

The coefficient of I_n is $a_0 x_n + a_n x_0 + a_n x_n$.

The coefficient of I_{n-1} is $a_0 x_{n-1} + a_{n-1} x_0 + a_n x_{n-1} + a_{n-1} x_n + a_{n-1} x_{n-1}$.

This implies that

$$f\left[\sum_{i,j=0}^{n} a_i x_j I_i I_j\right] = f[A + I_{n-1}(a_0 x_{n-1} + a_{n-1} x_0 + a_n x_{n-1} + a_{n-1} x_n + a_{n-1} x_{n-1}) + I_n(a_0 x_n + a_n x_0 + a_n x_n)]$$

$$= A + I_{n-1}(a_0 x_{n-1} + a_{n-1} x_0 + a_n x_{n-1} + a_{n-1} x_n + a_{n-1} x_{n-1}) + I_n(a_0 x_n + a_n x_0 + a_n x_n)$$

Now, we compute.

$g(a_0 + a_1 I_1 + \cdots + a_n I_n). f(x_0 + x_1 I_1 + \cdots + x_n I_n)$

$= \big(a_0 + \cdots + a_{n-2} I_{n-2} + I_{n-1}(a_n + a_{n-1})\big).\big(x_0 + \cdots + x_{n-2} I_{n-2} + I_{n-1}(x_n + x_{n-1})\big)$

Where $B = a_0 x_{n-1} + a_0 x_n + a_n x_0 + a_{n-1} x_0 + a_n x_{n-1} + a_n x_n + a_{n-1} x_{n-1} + a_{n-1} x_n$

Which is exactly equal to the coefficient I_{n-1} in $f[(\sum_{i=0}^{n} a_i I_i . \sum_{i=0}^{n} x_i I_i)]$.

This means that $f[(\sum_{i=0}^{n} a_i I_i . \sum_{i=0}^{n} x_i I_i)] = g(\sum_{i=0}^{n} a_i I_i). f(\sum_{i=0}^{n} x_i I_i)$, hence f is a semi module homomorphism.

(b). since $M_{n-i}(I)$ is a semi homomorphic image of $M_{n-i+1}(I)$, we get the following sequence:

$$M_n(I) \to M_{n-1}(I) \to \cdots \to M_2(I) \to M_1(I) \to M(I)$$

Thus M is a semi homomorphic image of $M_n(I)$.

Example 10

Let $R = Z, M = Z_3$. M is a module over R .

Let $M_3(I) = \{a_0 + a_1 I_1 + a_2 I_2 + a_3 I_3;\ a_i \in Z_3\}$be the corresponding 3-refined neutrosophic module over $R_3(I)$

$Z_3(I) = \{b_0 + b_1 I_1 + b_2 I_2 + b_3 I_3;\ b_i \in Z\}$.

(a). $g: Z_3(I) \to Z_2(I); g(a_0 + a_1 I_1 + a_2 I_2 + a_3 I_3) = a_0 + a_1 I_1 + (a_2 + a_3) I_2$

Is a ring homomorphism.

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(b). $f: M_3(I) \to M_2(I); f(x_0 + x_1 I_1 + x_2 I_2 + x_3 I_3) = x_0 + x_1 I_1 + (x_2 + x_3) I_2$

is a semi homomorphism, that is because:

f preserves addition clearly.

$f[(a_0 + a_1 I_1 + a_2 I_2 + a_3 I_3).(x_0 + x_1 I_1 + x_2 I_2 + x_3 I_3)]$

$$= f[a_0x_0 + I_1(a_0x_1 + a_1x_0 + a_1x_1 + a_1x_2 + a_1x_3 + a_2x_1 + a_3x_1) + I_2(a_0x_2 + a_2x_2 + a_2x_3 + a_3x_2) + I_3(a_0x_3 + a_3x_0 + a_3x_3)] + a_2x_0$$

$$= a_0x_0 + I_1(a_0x_1 + a_1x_0 + a_1x_1 + a_1x_2 + a_1x_3 + a_2x_1 + a_3x_1) + I_2(a_0x_2 + a_2x_2 + a_2x_3 + a_3x_2 + a_2x_0 + a_0x_2 + a_3x_0 + a_3x_3)$$

On the other hand.

$$g(a_0 + a_1 I_1 + a_2 I_2 + a_3 I_3).f(x_0 + x_1 I_1 + x_2 I_2 + x_3 I_3) = \big(a_0 + a_1 I_1 + I_2(a_3 + a_2)\big)\big(x_0 + x_1 I_1 + I_2(x_3 + x_2)\big)$$

$$= a_0x_0 + I_1(a_0x_1 + a_1x_0 + a_1x_1 + a_1x_2 + a_1x_3 + a_2x_1 + a_3x_1) + I_2(a_2x_0 + a_3x_0 + a_2x_2 + a_2x_3 + a_3x_2 + a_0x_2 + a_3x_3 + a_0x_3) = f[(a_0 + a_1 I_1 + a_2 I_2 + a_3 I_3).(x_0 + x_1 I_1 + x_2 I_2 + x_3 I_3)]$$

(c).$kerf = \{a_2 I_2 - a_2 I_3 \in M_3(I)\} = \{a_2(I_2 - I_3); a_2 \in M\}$

$M_3(I)/kerf \cong_s f\big(M_3(I)\big) = M_2(I)$.

The symbol $\cong_s$means semi isomorphic property.

Conclusion

In this paper, we have presented some novel applications of semi module homomorphisms/isomorphisms, where we proved that every module is a semi homomorphic image of it corresponding n-refined neutrosophic module.

Also, we have shown that the AH-isometry used in the theory of neutrosophic Euclidean geometry is a semi-module isomorphism.

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