Neutrosophic Bipolar Vague Incidence Graph

P. Anitha [1] and P. Chitra Devi [2]

[1] *PG & Research Department of Mathematics, H.K.R.H. College, Uthamapalayam, Theni-625 533, Tamil Nadu,.*

[2] *PG & Research Department of Mathematics, Mannar Thirumali Naicker College , Madurai-625 004, Tamil Nadu,.*

Abstract: Neutrosophic Bipolar Vague sets give more intuitive graphical notation of vague data, that devotes better analysis in information relationships, incompleteness and similarity measures. Neutrosophic bipolar vague graphs are used as a mathematical tool to kept an imprecise and unspecified information. In this paper, the neutrosophic bipolar vague incidence graphs are introduced. The edge-connectivity, vertex-connectivity and pair-connectivity in neutrosophic bipolar vague incidence graphs are established. The given results are illustrated with suitable example.

Keywords: Neutrosophic bipolar vague incidence graph, Edge-connectivity, Vertex-connectivity and Pair-connectivity. Neutrosophic bipolar vague graph

2020 Mathematics Subject Classification: 05C72; 05C76; 03E72; 86A08.

1 Introduction

The single-valued neutrosophic set is the generalisation of intuitionistic fuzzy sets and is used expediently to deal with real-world problems, especially in decision support [16, 17]. The computation of believe in that element (truth), the disbelieve in that element (falsehood) and the indeterminacy part of that element with the sum of these three components are strictly less than 1. Neutrosophic sets are the base of neutrosophic logic, a multiple value logic that generalizes the fuzzy logic which deals with paradoxes, contradictions, antitheses, antinomies is proposed by Smarandache [32, 34] and references therein.

The neutrosophic set is introduced by the author Smarandache in order to use the inconsistent and indeterminate information, and has been studied extensively (see [32, 33, 34, 37, 35, 36, 38]). In the definition of neutrosophic set, the indeterminacy value is quantified explicitly and truth-membership, indeterminacy membership, and false-membership are defined completely independent with the sum of these values lies between 0 and 3. Neutrosophic set and related notions paid attention by the researchers in many weird domains [5, 6]. The combination of neutrosophic set and vague set are introduced by Alkhazaleh in 2015 [7]. Single valued neutrosophic graph are established in the papers [12, 13]. Some types of neutrosophic graphs and co-neutrosophic graphs are discussed in [19]. Intuitionistic bipolar neutrosophic set and its application to graphs are established in [27]. SS Hussain et al. in [25] introduced a combination of neutrosophic bipolar vague set to improving the reason-ability of decision making in real life application. Neutrosophic vague graphs and Neutrosophic bipolar vague graphs are investigated by [26, 30]. Authors in[3] presented some properties of single-valued neutrosophic incidence graphs, neutrosophic vague incidence graphs, neutrosophic vague line graphs and discussed the edge-connectivity, vertex-connectivity and pair-connectivity in neutrosophic incidence graphs. Motivated by papers [7, 3, 26, 24, 25, 31], we introduce the concept of neutrosophic bipolar vague incidence graphs. The main contributions of this paper are to introduce the neutrosophic bipolar vague incidence graphs, and the edge-connectivity, vertex-connectivity and pair-connectivity in neutrosophic vague incidence graphs are discussed in Section 3.

2 Preliminaries

In this section, basic definitions and example are given.

Definition 2.1 *[39] A vague set* $\mathbb{A}$ *on a non empty set* $\mathbb{X}$ *is a pair* $(T_{\mathbb{A}}, F_{\mathbb{A}})$*, where* $T_{\mathbb{A}}: \mathbb{X} \to [0,1]$ *and* $F_{\mathbb{A}}: \mathbb{X} \to [0,1]$ *are true membership and false membership functions, respectively, such that*

$$0 \leq T_{\mathbb{A}}(x) + F_{\mathbb{A}}(y) \leq 1 \text{ for any } x \in \mathbb{X}.$$

Let $\mathbb{X}$ and $\mathbb{Y}$ be two non-empty sets. A vague relation $\mathbb{R}$ of $\mathbb{X}$ to $\mathbb{Y}$ is a vague set $\mathbb{R}$ on $\mathbb{X} \times \mathbb{Y}$ that is $\mathbb{R} = (T_{\mathbb{R}}, F_{\mathbb{R}})$, where $T_{\mathbb{R}}: \mathbb{X} \times \mathbb{Y} \to [0,1], F_{\mathbb{R}}: \mathbb{X} \times \mathbb{Y} \to [0,1]$ and satisfy the condition:

$$0 \leq T_{\mathbb{R}}(x,y) + F_{\mathbb{R}}(x,y) \leq 1 \text{ for any } x \in \mathbb{X}.$$

Definition 2.2 *[8] Let $\mathbb{G}^* = (\mathbb{V}, \mathbb{E})$ be a graph. A pair $\mathbb{G} = (\mathbb{J}, \mathbb{K})$ is called a vague graph on $\mathbb{G}^*$, where $\mathbb{J} = (T_{\mathbb{J}}, F_{\mathbb{J}})$ is a vague set on $\mathbb{V}$ and $\mathbb{K} = (T_{\mathbb{K}}, F_{\mathbb{K}})$ is a vague set on $\mathbb{E} \subseteq \mathbb{V} \times \mathbb{V}$ such that for each $xy \in \mathbb{E}$,*

$$T_{\mathbb{K}}(xy) \leq \min(T_{\mathbb{J}}(x), T_{\mathbb{J}}(y)) \text{ and } F_{\mathbb{K}}(xy) \geq \max(F_{\mathbb{J}}(x), F_{\mathbb{J}}(y)).$$

Definition 2.3 *[15, 32] Let $\mathbb{X}$ be a space of points (objects), with a generic elements in $\mathbb{X}$ denoted by x. A single valued neutrosophic set $\mathbb{A}$ in $\mathbb{X}$ is characterised by truth-membership function $T_{\mathbb{A}}(x)$, indeterminacy-membership function $I_{\mathbb{A}}(x)$ and falsity-membership-function $F_{\mathbb{A}}(x)$, For each point x in $\mathbb{X}$, $T_{\mathbb{A}}(x), F_{\mathbb{A}}(x), I_{\mathbb{A}}(x) \in [0,1]$. Also*

$$A = \{x, T_{\mathbb{A}}(x), F_{\mathbb{A}}(x), I_{\mathbb{A}}(x)\} \text{ and } 0 \leq T_{\mathbb{A}}(x) + I_{\mathbb{A}}(x) + F_{\mathbb{A}}(x) \leq 3.$$

Definition 2.4 *[32] A Neutrosophic set is contained in another neutrosophic set $\mathbb{B}$, (i.e) $\mathbb{A} \subseteq \mathbb{B}$ if $\forall x \in \mathbb{X}, T_{\mathbb{A}}(x) \leq T_{\mathbb{B}}(x), I_{\mathbb{A}}(x) \geq I_{\mathbb{B}}(x)$and $F_{\mathbb{A}}(x) \geq F_{\mathbb{B}}(x)$.*

Definition 2.5 *[2, 13] A neutrosophic graph is defined as a pair $\mathbb{G}^* = (\mathbb{V}, \mathbb{E})$ where*

(i) $\mathbb{V} = \{v_1, v_2, \ldots, v_n\}$ such that $T_1: \mathbb{V} \to [0,1]$, $I_1: \mathbb{V} \to [0,1]$ and $F_1: \mathbb{V} \to [0,1]$ denote the degree of truth-membership function, indeterminacy function and falsity-membership function, respectively and

$$0 \leq T_1(v) + I_1(v) + F_1(v) \leq 3,$$

(ii) $\mathbb{E} \subseteq \mathbb{V} \times \mathbb{V}$ where $T_2: \mathbb{E} \to [0,1]$, $I_2: \mathbb{E} \to [0,1]$ and $F_2: \mathbb{E} \to [0,1]$ are such that

$$T_2(uv) \leq \min\{T_1(u), T_1(v)\},$$
$$I_2(uv) \leq \min\{I_1(u), I_1(v)\},$$
$$F_2(uv) \leq \max\{F_1(u), F_1(v)\},$$
$$\text{and } 0 \leq T_2(uv) + I_2(uv) + F_2(uv) \leq 3, \ \forall uv \in \mathbb{E}.$$

Definition 2.6 *[7] A neutrosophic vague set $\mathbb{A}_{NV}$ (NVS in short) on the universe of discourse $\mathbb{X}$ written as*

$$\mathbb{A}_{NV} = \{\langle x, \hat{T}_{\mathbb{A}_{NV}}(x), \hat{I}_{\mathbb{A}_{NV}}(x), \hat{F}_{\mathbb{A}_{NV}}(x)\rangle, x \in \mathbb{X}\},$$

whose truth-membership, indeterminacy membership and falsity-membership function is defined as

$$\hat{T}_{\mathbb{A}_{NV}}(x) = [T^-(x), T^+(x)], \hat{I}_{\mathbb{A}_{NV}}(x) = [I^-(x), I^+(x)] \text{and} \hat{F}_{\mathbb{A}_{NV}}(x) = [F^-(x), F^+(x)],$$

where $T^+(x) = 1 - F^-(x), F^+(x) = 1 - T^-(x)$, and $0 \leq T^-(x) + I^-(x) + F^-(x) \leq 2$.

Definition 2.7 *[7] The complement of NVS $\mathbb{A}_{NV}$ is denoted by $\mathbb{A}^c_{NV}$ and it is defined by*

$$\hat{T}^c_{\mathbb{A}_{NV}}(x) = [1 - T^+(x), 1 - T^-(x)],$$
$$\hat{I}^c_{\mathbb{A}_{NV}}(x) = [1 - I^+(x), 1 - I^-(x)],$$
$$\hat{F}^c_{\mathbb{A}_{NV}}(x) = [1 - F^+(x), 1 - F^-(x)].$$

Definition 2.8 *[7] Let $\mathbb{A}_{NV}$ and $\mathbb{B}_{NV}$ be two NVSs of the universe $\mathbb{U}$. If for all $u_i \in \mathbb{U}$,*

$$\hat{T}_{\mathbb{A}_{NV}}(u_i) \leq \hat{T}_{\mathbb{B}_{NV}}(u_i), \hat{I}_{\mathbb{A}_{NV}}(u_i) \geq \hat{I}_{\mathbb{B}_{NV}}(u_i), \hat{F}_{\mathbb{A}_{NV}}(u_i) \geq \hat{F}_{\mathbb{B}_{NV}}(u_i),$$

then the NVS, $\mathbb{A}_{NV}$ are included in $\mathbb{B}_{NV}$, denoted by $\mathbb{A}_{NV} \subseteq \mathbb{B}_{NV}$ where $1 \leq i \leq n$.

Definition 2.9 *[7] The union of two NVSs $\mathbb{A}_{NV}$ and $\mathbb{B}_{NV}$ is a NVSs, $\mathbb{C}_{NV}$, written as $\mathbb{C}_{NV} = \mathbb{A}_{NV} \cup \mathbb{B}_{NV}$, whose truth membership function, indeterminacy-membership function and false-membership function are related to those of $\mathbb{A}_{NV}$ and $\mathbb{B}_{NV}$ by*

$$\hat{T}_{\mathbb{C}_{NV}}(x) = [\max(T^-_{\mathbb{A}_{NV}}(x), T^-_{\mathbb{B}_{NV}}(x)), \max(T^+_{\mathbb{A}_{NV}}(x), T^+_{\mathbb{B}_{NV}}(x))]$$
$$\hat{I}_{\mathbb{C}_{NV}}(x) = [\min(I^-_{\mathbb{A}_{NV}}(x), I^-_{\mathbb{B}_{NV}}(x)), \min(I^+_{\mathbb{A}_{NV}}(x), I^+_{\mathbb{B}_{NV}}(x))]$$
$$\hat{F}_{\mathbb{C}_{NV}}(x) = [\min(F^-_{\mathbb{A}_{NV}}(x), F^-_{\mathbb{B}_{NV}}(x)), \min(F^+_{\mathbb{A}_{NV}}(x), F^+_{\mathbb{B}_{NV}}(x))].$$

Definition 2.10 *[7] The intersection of two NVSs,* A_{NV} *and* B_{NV} *is a NVSs* C_{NV}*, written as* $C_{NV} = A_{NV} \cap B_{NV}$*, whose truth-membership function, indeterminacy-membership function and false-membership function are related to those of* A_{NV} *and* B_{NV} *by*

$$\hat{T}_{\mathbb{C}_{NV}}(x) = [\min(T^-_{\mathbb{A}_{NV}}(x), T^-_{\mathbb{B}_{NV}}(x)), \min(T^+_{\mathbb{A}_{NV}}(x), T^+_{\mathbb{B}_{NV}}(x))]$$
$$\hat{I}_{\mathbb{C}_{NV}}(x) = [\max(I^-_{\mathbb{A}_{NV}}(x), I^-_{\mathbb{B}_{NV}}(x)), \max(I^+_{\mathbb{A}_{NV}}(x), I^+_{\mathbb{B}_{NV}}(x))]$$
$$\hat{F}_{\mathbb{C}_{NV}}(x) = [\max(F^-_{\mathbb{A}_{NV}}(x), F^-_{\mathbb{B}_{NV}}(x)), \max(F^+_{\mathbb{A}_{NV}}(x), F^+_{\mathbb{B}_{NV}}(x))].$$

Definition 2.11 *[26] Let* $\mathbb{G}^* = (\mathbb{R}, \mathbb{S})$ *be a graph. A pair* $\mathbb{G} = (\mathbb{A}, \mathbb{B})$ *is called a neutrosophic vague graph (NVG) on* $\mathbb{G}^*$ *or a neutrosophic vague graph where* $\mathbb{A} = (\hat{T}_{\mathbb{A}}, \hat{I}_{\mathbb{A}}, \hat{F}_{\mathbb{A}})$ *is a neutrosophic vague set on* $\mathbb{R}$ *and* $\mathbb{B} = (\hat{T}_{\mathbb{B}}, \hat{I}_{\mathbb{B}}, \hat{F}_{\mathbb{B}})$ *is a neutrosophic vague set* $\mathbb{S} \subseteq \mathbb{R} \times \mathbb{R}$ *where*

(1)$\mathbb{R} = \{v_1, v_2, \ldots, v_n\}$ such that $(T^-)_{\mathbb{A}}: \mathbb{R} \to [0,1], (I^-)_{\mathbb{A}}: \mathbb{R} \to [0,1], (F^-)_{\mathbb{A}}: \mathbb{R} \to [0,1]$ which satisfies the condition $F^-_{\mathbb{A}} = [1 - T^+_{\mathbb{A}}]$

$T^+_{\mathbb{A}}: \mathbb{R} \to [0,1], I^+_{\mathbb{A}}: \mathbb{R} \to [0,1], F^+_{\mathbb{A}}: \mathbb{R} \to [0,1]$ which satisfies the condition $F^+_{\mathbb{A}} = [1 - T^-_{\mathbb{A}}]$

denotes the degree of truth membership function, indeterminacy membership and falsity membership of the element $v_i \in \mathbb{R}$, and

$$0 \le T^-_{\mathbb{A}}(v_i) + I^-_{\mathbb{A}}(v_i) + F^-_{\mathbb{A}}(v_i) \le 2$$
$$0 \le T^+_{\mathbb{A}}(v_i) + I^+_{\mathbb{A}}(v_i) + F^+_{\mathbb{A}}(v_i) \le 2.$$

(2) $\mathbb{S} \subseteq \mathbb{R} \times \mathbb{R}$ where

$$T^-_{\mathbb{B}}: \mathbb{R} \times \mathbb{R} \to [0,1], I^-_{\mathbb{B}}: \mathbb{R} \times \mathbb{R} \to [0,1], F^-_{\mathbb{B}}: \mathbb{R} \times \mathbb{R} \to [0,1]$$
$$T^+_{\mathbb{B}}: \mathbb{R} \times \mathbb{R} \to [0,1], I^+_{\mathbb{B}}: \mathbb{R} \times \mathbb{R} \to [0,1], F^+_{\mathbb{B}}: \mathbb{R} \times \mathbb{R} \to [0,1]$$

denotes the degree of truth membership function, indeterminacy membership and falsity membership of the element $v_i, v_j \in \mathbb{S}$, respectively and such that,

$$0 \le T^-_{\mathbb{B}}(v_iv_j) + I^-_{\mathbb{B}}(v_iv_j) + F^-_{\mathbb{B}}(v_iv_j) \le 2$$
$$0 \le T^+_{\mathbb{B}}(v_iv_j) + I^+_{\mathbb{B}}(v_iv_j) + F^+_{\mathbb{B}}(v_iv_j) \le 2,$$

such that

$$T^-_{\mathbb{B}}(v_iv_j) \le \min\{T^-_{\mathbb{A}}(v_i), T^-_{\mathbb{A}}(v_j)\}$$
$$I^-_{\mathbb{B}}(v_iv_j) \le \min\{I^-_{\mathbb{A}}(v_i), I^-_{\mathbb{A}}(v_j)\}$$
$$F^-_{\mathbb{B}}(v_iv_j) \le \max\{F^-_{\mathbb{A}}(v_i), F^-_{\mathbb{A}}(v_j)\},$$

and similarly

$$T^+_{\mathbb{B}}(v_iv_j) \le \min\{T^+_{\mathbb{A}}(v_i), T^+_{\mathbb{A}}(v_j)\}$$
$$I^+_{\mathbb{B}}(v_iv_j) \le \min\{I^+_{\mathbb{A}}(v_i), I^+_{\mathbb{A}}(v_j)\}$$
$$F^+_{\mathbb{B}}(v_iv_j) \le \max\{F^+_{\mathbb{A}}(v_i), F^+_{\mathbb{A}}(v_j)\}.$$

Definition 2.12 *[3] A neutrosophic incidence graph of an incidence graph,* $G^* = (V, E, I)$*, is an ordered triplet,* $\tilde{G} = (A, B, C)$*, such that*

1. A is a neutrosophic set on V,
2. B is a neutrosophic relation on V and
3. C is a neutrosophic subset of $V \times E$ such that

$$T_C(x, xy) \le \min\{T_A(x), T_B(xy)\},$$
$$I_C(x, xy) \le \min\{I_A(x), I_B(xy)\},$$
$$F_C(x, xy) \le \max\{F_A(x), F_B(xy)\}, \text{forall} xy \in E$$

Definition 2.13 *Let* $G^* = (V, E)$ *be a crisp graph. A pair* $G = (J, K)$ *is called a neutrosophic bipolar vague graph (NBVG) on* G^* *or a neutrosophic bipolar vague graph where* $J^P = ((\hat{T}_J)^P, (\hat{I}_J)^P, (\hat{F}_J)^P), J^N = ((\hat{T}_J)^N, (\hat{I}_J)^N, (\hat{F}_J)^N)$ *is a neutrosophic bipolar vague set on* V *and* $K^P = ((\hat{T}_K)^P, (\hat{I}_K)^P, (\hat{F}_K)^P), K^N = ((\hat{T})_K)^N, (\hat{I})_K)^N, (\hat{F})_K)^N)$ *is a neutrosophic Bipolar vague* *set* $E \subseteq V \times V$ *where*

(1)$V = \{v_1, v_2, \dots, v_n\}$ such that $(T_J^-)^P: V \to [0,1], (I_J^-)^P: V \to [0,1], (F_J^-)^P: V \to [0,1]$ which satisfies the condition $(F_J^-)^P = [1 - (T_J^+)^P]$

$(T_J^+)^P: V \to [0,1], (I_J^+)^P: V \to [0,1], (F_J^+)^P: V \to [0,1]$ which satisfies the condition $(F_J^+)^P = [1 - (T_J^-)^P]$

And

$(T_J^-)^N: V \to [-1,0], (I_J^-)^N: V \to [-1,0], (F_J^-)^N: V \to [-1,0]$ which satisfies the condition $(F_J^-)^N = [-1 - (T_J^+)^N]$

$(T_J^+)^N: V \to [-1,0], (I_J^+)^N: V \to [-1,0], (F_J^+)^N: V \to [-1,0]$ which satisfies the condition $(F_J^+)^N = [-1 - (T_J^-)^N]$

denotes the degree of truth membership function, indeterminacy membership and falsity membership of the element $v_i \in V$., and

$$0 \le (T_J^-)^P(v_i) + (I_J^-)^P(v_i) + (F_J^-)^P(v_i) \le 2.$$
$$0 \le (T_J^+)^P(v_i) + (I_J^+)^P(v_i) + (F_J^+)^P(v_i) \le 2.$$
$$0 \ge (T_J^-)^N(v_i) + (I_J^-)^N(v_i) + (F_J^-)^N(v_i) \ge -2.$$
$$0 \le (T_J^+)^N(v_i) + (I_J^+)^N(v_i) + (F_J^+)^N(v_i) \ge -2.$$

(2) $E \subseteq V \times V$ where

$$(T_K^-)^P: V \times V \to [0,1], (I_K^-)^P: V \times V \to [0,1], (F_K^-)^P: V \times V \to [0,1]$$
$$(T_K^+)^P: V \times V \to [0,1], (I_K^+)^P: V \times V \to [0,1], (F_K^+)P: V \times V \to [0,1]$$

And

$$(T_K^-)^N: V \times V \to [-1,0], (I_K^-)^N: V \times V \to [-1,0], (F_K^-)^N: V \times V \to [-1,0]$$
$$(T_K^+)^N: V \times V \to [-1,0], (I_K^+)^N: V \times V \to [-1,0], (F_K^+)N: V \times V \to [-1,0]$$

denotes the degree of truth membership function, indeterminacy membership and falsity membership of the element $v_i, v_j \in E$. respectively and such that

$$0 \le (T_K^-)^P(v_i, v_j) + (I_K^-)^P(v_i, v_j) + (F_K^-)^P(v_i, v_j) \le 2.$$
$$0 \le (T_K^+)^P(v_i, v_j) + (I_K^+)^P(v_i, v_j) + (F_K^+)^P(v_i, v_j) \le 2.$$
$$0 \ge (T_K^-)^N(v_i, v_j) + (I_K^-)^N(v_i, v_j) + (F_K^-)^N(v_i, v_j) \ge -2.$$
$$0 \ge (T_K^+)^N(v_i, v_j) + (I_K^+)^N(v_i, v_j) + (F_K^+)^N(v_i, v_j) \ge -2.$$

such that

$$(T_K^-)^P(xy) \le \min\{(T_J^-)^P(x), (T_J^-)^P(y)\}$$
$$(I_K^-)^P(xy) \le \max\{(I_J^-)^P(x), (I_J^-)^P(y)\}$$
$$(F_K^-)^P(xy) \le \max\{(F_J^-)^P(x), (F_J^-)^P(y)\}$$
$$(T_K^+)^P(xy) \le \min\{(T_J^+)^P(x), (T_J^+)^P(y)\}$$
$$(I_K^+)^P(xy) \le \max\{(I_J^+)^P(x), (I_J^+)^P(y)\}$$
$$(F_K^+)^P(xy) \le \max\{(F_J^+)^P(x), (F_J^+)^P(y)\},$$

And

$$(T_K^-)^N(xy) \ge \max\{(T_J^-)^N(x), (T_J^-)^N(y)\}$$
$$(I_K^-)^N(xy) \ge \min\{(I_J^-)^N(x), (I_K^-)^N(y)\}$$
$$(F_K^-)^N(xy) \ge \min\{(F_J^-)^N(x), (F_J^-)^N(y)\},$$
$$(T_K^+)^N(xy) \ge \max\{(T_J^+)^N(x), (T_J^+)^N(y)\}$$
$$(I_K^+)^N(xy) \ge \min\{(I_J^+)^N(x), (I_J^+)^N(y)\}$$
$$(F_K^+)^N(xy) \ge \min\{(F_J^+)^N(x), (F_J^+)^N(y)\},$$

3 Neutrosophic Bipolar Vague incidence Graph

In this section, the definition of NBVIGs are introduced. Some properties on edge-connectivity, vertex-connectivity and pair-connectivity in neutrosophic bipolar vague incidence graphs are established.

Definition 3.1 *A neutrosophic bipolar vague incidence graph of an incidence graph* $G = (\mathbb{V}, \mathbb{E}, \mathbb{I})$*, is an ordered triplet,* $G^* = (Q, R, S)$*, such that*

1. Q is a neutrosophic bipolar vague set on $\mathbb{V}$,
2. R is a neutrosophic bipolar vague relation on $\mathbb{V}$ and
3. S is a neutrosophic bipolar vague subset of $\mathbb{V} \times \mathbb{E}$ such that

$$(T_S^-)^P(a, ab) \leq \min\{(T_Q^-)^P(a), (T_R^-)^P(ab)\},$$
$$(I_S^-)^P(a, ab) \leq \min\{(I_Q^-)^P(a), (I_R^-)^P(ab)\},$$
$$(F_S^-)^P(a, ab) \leq \max\{(F_Q^-)^P(a), (F_R^-)^P(ab)\},$$

similarly

$$(T_S^+)^P(a, ab) \leq \min\{(T_Q^+)^P(a), (T_R^+)^P(ab)\},$$
$$(I_S^+)^P(a, ab) \leq \min\{(I_Q^+)^P(a), (I_R^+)^P(ab)\},$$
$$(F_S^+)^P(a, ab) \leq \max\{(F_Q^+)^P(a), (F_R^+)^P(ab)\}, \forall a \in \mathbb{V}, ab \in \mathbb{E}.$$

similarly negative part

$$(T_S^-)^N(a, ab) \geq \max\{(T_Q^-)^N(a), (T_R^-)^N(ab)\},$$
$$(I_S^-)^N(a, ab) \geq \max\{(I_Q^-)^N(a), (I_R^-)^N(ab)\},$$
$$(F_S^-)^N(a, ab) \geq \min\{(F_Q^-)^N(a), (F_R^-)^N(ab)\},$$

similarly

$$(T_S^+)^N(a, ab) \geq \max\{(T_Q^+)^N(a), (T_R^+)^N(ab)\},$$
$$(I_S^+)^N(a, ab) \geq \max\{(I_Q^+)^N(a), (I_R^+)^N(ab)\},$$
$$(F_S^+)^N(a, ab) \geq \min\{(F_Q^+)^N(a), (F_R^+)^N(ab)\}, \forall a \in \mathbb{V}, ab \in \mathbb{E}.$$

Example 3.1 *Consider an incidence graph* $G = (\mathbb{V}, \mathbb{E}, \mathbb{I})$ *such that* $\mathbb{V} = \{q, r, s, t\}$, $\mathbb{E} = \{qr, rs, rt, st, qt\}$ *and* $\mathbb{I} = \{(q, qr), (r, qr), (r, rs), (s, rs), (r, rt), (t, rt), (s, st), (t, st), (q, qt), (t, qt)\}$, *as shown in figure 1*

Let $G^* = (Q, R, S)$ be a neutrosophic bipolar vague incidence graph associated with G, as shown in figure 3, where $q^P = [0.5,0.5], [0.4,0.4], [0.5,0.5]$, $r^P = [0.3,0.4], [0.5,0.6], [0.6,0.7]$, $s^P = [0.7,0.5], [0.3,0.4], [0.5,0.3]$, $t^P = [0.4,0.7], [0.4,0.6], [0.3,0.6]$

$(q^-)^P = (0.5,0.4,0.5), (q^+)^P = (0.5,0.4,0.5)$, $(r^-)^P = (0.3,0.5,0.6), (r^+)^P = (0.4,0.6,0.7)$

$(s^-)^P = (0.7,0.3,0.5), (s^+)^P = (0.5,0.4,0.3)$ $(t^-)^P = (0.3,0.4,0.3), (t^+)^P = (0.7,0.6,0.6)$

$q^N = [-0.5, -0.4], [-0.4, -0.3], [-0.6, -0.5]$, $r^N = [-0.3, -0.4], [-0.4, -0.5], [-0.6, -0.7]$, $s^N = [-0.7, -0.5], [-0.2, -0.2], [-0.5, -0.3]$, $t^N = [-0.4, -0.7], [-0.4, -0.6], [-0.3, -0.6]$

$(q^-)^N = (-0.5, -0.4, -0.6), (q^+)^N = (-0.5, -0.4, -0.6)$

$(r^-)^N = (-0.3, -0.4, -0.6), (r^+)^N = (-0.4, -0.5, -0.7)$

$(s^-)^N = (-0.7, -0.2, -0.5), (s^+)^N = (-0.5, -0.4, -0.3)$

$$(t^-)^N = (-0.4, -0.4, -0.3), (t^+)^N = (-0.7, -0.6, -0.6)$$

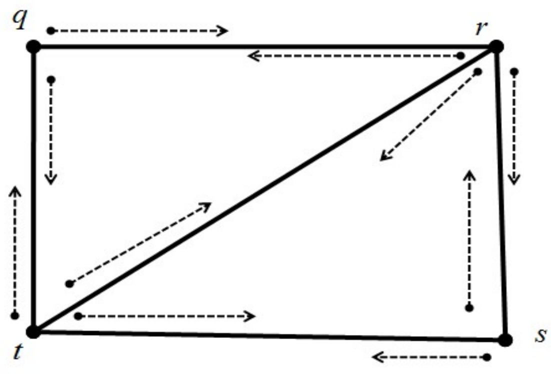

Figure 1
Incidence graph

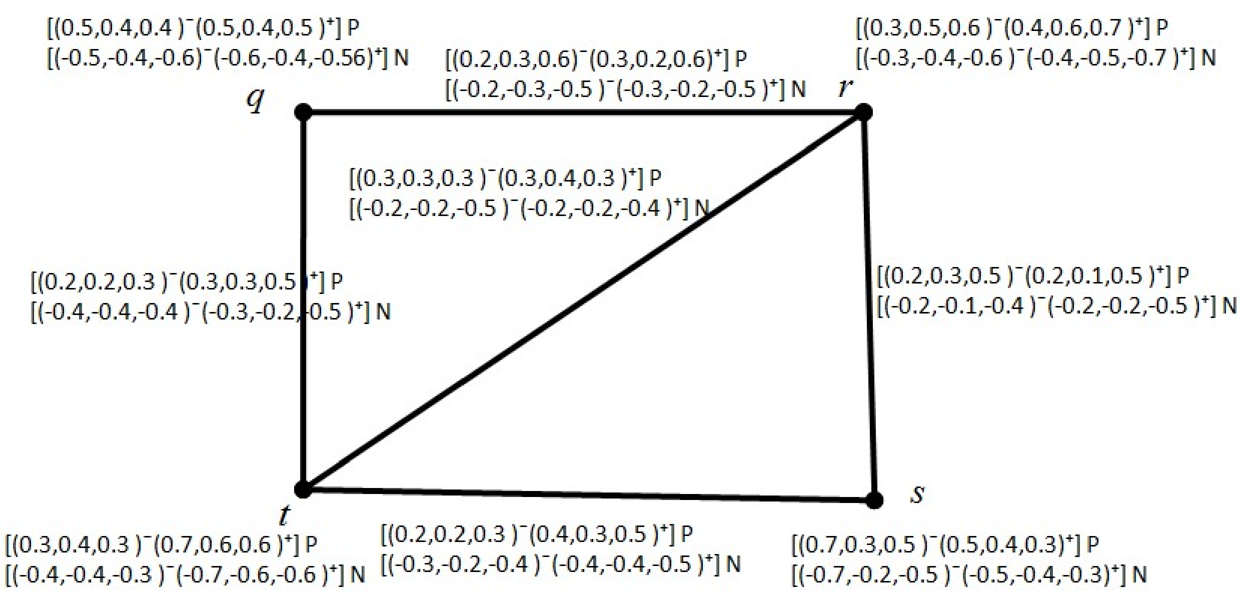

Figure 2
Neutrosophic Vague Graph

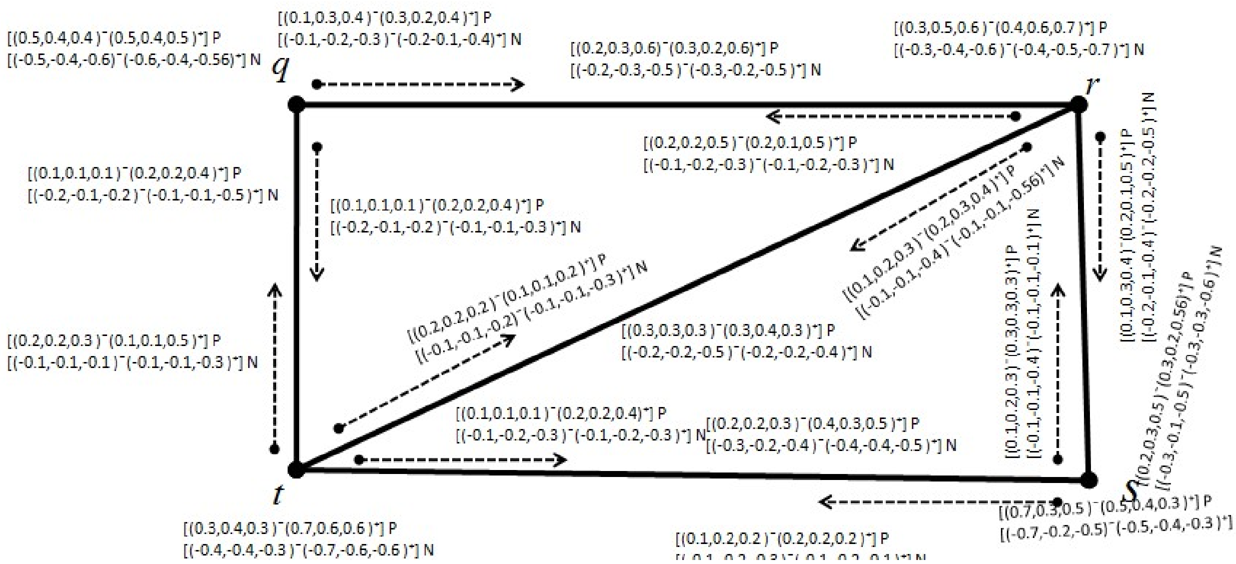

Figure 3
Neutrosophic Bipolar Vague Incidence Graph

Definition 3.2 *The support of an NBVIG* $G^* = (Q, R, S)$ *is denoted by* $G^{**} = (Q^*, R^*, S^*)$ *where*

$Q^* =$ support of $Q^P = \{a \in \mathbb{V}: \hat{T}_Q^P(a) > 0, \hat{I}_Q^P(a) > 0, \hat{F}_Q^P(a) > 0\}$

$R^* =$ support of $R^P = \{ab \in \mathbb{E}: \hat{T}_R^P(ab) > 0, \hat{I}_R^P(ab) > 0, \hat{F}_R^P(ab) > 0\}$

$S^* =$ support of $S^P = \{(a, ab) \in \mathbb{I}: \hat{T}_S^P(a, ab) > 0, \hat{I}_S^P(a, ab) > 0, \hat{F}_S^P(a, ab) > 0\}$.

negative part $Q^* =$ support of $Q^N = \{a \in \mathbb{V}: \hat{T}_Q^N(a) < 0, \hat{I}_Q^N(a) < 0, \hat{F}_Q^N(a) < 0\}$

$R^* =$ support of $R^N = \{ab \in \mathbb{E}: \hat{T}_R^N(ab) < 0, \hat{I}_R^N(ab) < 0, \hat{F}_R^N(ab) < 0\}$

$S^* =$ support of $S^N = \{(a, ab) \in \mathbb{I}: \hat{T}_S^N(a, ab) < 0, \hat{I}_S^N(a, ab) < 0, \hat{F}_S^N(a, ab) < 0\}$.

Definition 3.3 *If* $ab \in R^*$, *then* ab *is the edges of the NBVIG* $G^* = (Q, R, S)$ *and if* $(a, ab), (b, ab) \in S^*$ *then* (a, ab) *and* (b, ab) *are called pair of* G^*.

Definition 3.4 *Suppose*
$P = a_0, (a_0, a_0a_1), a_0a_1, (a_1, a_0, a_1), a_1, (a_1, a_1a_2), a_1a_2, (a_2, a_1, a_2), \ldots a_{n-1}$, $(a_{n-1}, a_{n-1}a_n), a_{n-1}a_n, (a_n, a_{n-1}, a_n)$ *of vertices, edges and pairs in* G^* *is a walk. It is a closed walk if* $a_0 = a_n$. *In the above sequence, if all edges are distinct, then it is trail, and if the pairs are distinct, then it is an incidence trail.* P *is called a path, if the vertices are distinct. A path is called a cycle if the initial and end vertices of the path are same. Any two vertices of* G^* *are said to be* connected, if they are joined by a path.

Example 3.2 *In the above example, one can see that*

$P_1 = q, (q, qr), qr, (r, qr), r, (r, rs), rs, (s, rs), t, (t, tq), tq, (q, tq), q$ is a walk. It is a closed walk since the initial and final vertices are same. (i.e) it is not a path, but it is a trail and incidence trail $P_2 = q, (q, qr), qr, (r, qr), r, (r, rs), rt, (t, rt), t$. Then, P_2 is a walk, path trail and incidence trail.

Definition 3.5 *Let $G^* = (Q, R, S)$ be an NBVIG, then $H = (L, M, N)$ is a neutrosophic bipolar vague incidence subgraph of G^*, if $L \subseteq Q, M \subseteq$ and $N \subseteq S$. Also, H is a neutrsophic incidence spanning subgraph of G^*, if $L = Q$.*

Definition 3.6 *In an NBVIG, the strength of a path, P is an ordered triplet denoted by $\mathbb{S}(P) = (s_1, s_2, s_3)$, where*

$$s_1 = \min\{\hat{T}_R^P(ab): ab \in P\}, s_2 = \min\{\hat{I}_R^P(ab): ab \in P\}, s_3 = \max\{\hat{F}_R^P(ab): ab \in P\}.$$

$$s_1 = \max\{\hat{T}_R^N(ab): ab \in P\}, s_2 = \max\{\hat{I}_R^N(ab): ab \in P\}, s_3 = \min\{\hat{F}_R^N(ab): ab \in P\}.$$

Similarly, the incidence strength of a path, P, in an NBVG is denoted by $I\mathbb{S}(P) = (is_1, is_2, is_3)$, where

$$is_1 = \min\{\hat{T}_S^P(ab): (a, ab) \in P\}, is_2 = \min\{\hat{I}_S^P(ab): (a, ab) \in P\}, is_3 = \max\{\hat{F}_S^P(ab): (a, ab) \in P\}.$$

$$is_1 = \max\{\hat{T}_S^N(ab): (a, ab) \in P\}, is_2 = \max\{\hat{I}_S^N(ab): (a, ab) \in P\}, is_3 = \min\{\hat{F}_S^N(ab): (a, ab) \in P\}.$$

Definition 3.7 *In an NBVG, $G^* = (Q, R, S)$ the greatest strength of the path from l to m, where $l, m \in Q^* \cup R^*$ is the maximum of strength of all paths from l to m.*

$$(\mathbb{S}^P)^\infty(l, m) = \max\{\mathbb{S}(P_1), \mathbb{S}(P_2), \mathbb{S}(P_3), \dots\}$$
$$= (s_1^\infty, s_2^\infty, s_3^\infty)$$
$$= (\max(s_{11}, s_{12}, s_{13}, \dots), \max(s_{21}, s_{22}, s_{23}, \dots), \min(s_{31}, s_{32}, s_{33}, \dots)),$$

and

$$(\mathbb{S}^N)^\infty(l, m) = \min\{\mathbb{S}(P_1), \mathbb{S}(P_2), \mathbb{S}(P_3), \dots\}$$
$$= (s_1^\infty, s_2^\infty, s_3^\infty)$$
$$= (\min(s_{11}, s_{12}, s_{13}, \dots), \min(s_{21}, s_{22}, s_{23}, \dots), \max(s_{31}, s_{32}, s_{33}, \dots)),$$

$(\mathbb{S}^N)^\infty(l, m)$ is sometimes called the connectedness between l and m.

Similarly, the greatest incidence strength of the path from l to m, where $l, m \in Q^* \cup R^*$ is the maximum of incidence strength of all paths from l to m.

$$(I\mathbb{S}^P)^\infty(l, m) = \max\{I\mathbb{S}(P_1), I\mathbb{S}(P_2), I\mathbb{S}(P_3), \dots\}$$
$$= (is_1^\infty, is_2^\infty, is_3^\infty)$$
$$= (\max(is_{11}, is_{12}, is_{13}, \dots), \max(is_{21}, is_{22}, is_{23}, \dots), \min(is_{31}, is_{32}, is_{33}, \dots)),$$

and

$$(I\mathbb{S}^N)^\infty(l, m) = \min\{I\mathbb{S}(P_1), I\mathbb{S}(P_2), I\mathbb{S}(P_3), \dots\}$$
$$= (is_1^\infty, is_2^\infty, is_3^\infty)$$
$$= (\min(is_{11}, is_{12}, is_{13}, \dots), \min(is_{21}, is_{22}, is_{23}, \dots), \max(is_{31}, is_{32}, is_{33}, \dots)),$$

where $P_j, j = 1,2,3, \dots$ are different paths from l to m.

$(I\mathbb{S}^P)^\infty(l, m)$ and $(I\mathbb{S}^N)^\infty(l, m)$ is sometimes represented as the incidence connectedness between l to m.

Definition 3.8 *An NBVG, $G^* = (Q, R, S)$ is a cycle if and only if, the underlying graph, $G^{**} = (Q^*, R^*, S^*)$ is a cycle.*

Definition 3.9 *An NBVG, $G^* = (Q, R, S)$ is a is a neutrosophic bipolar vague cycle if and only if, $G^{**} = (Q^*, R^*, S^*)$ is a cycle and there exist no unique edge, $ab \in R^*$ such that*

$$(\hat{T})_R^P(xy) = \min\{(\hat{T})_R^P(ab): ab \in R^*\},$$
$$(\hat{I})_R^P(xy) = \min\{(\hat{I})_R^P(ab): ab \in R^*\},$$
$$(\hat{F})_R^P(xy) = \max\{(\hat{F})_R^P(ab): ab \in R^*\}. \textit{ and}$$

$(\hat{T})_R^N(xy) = \max\{(\hat{T})_R^N(ab): ab \in R^*\},$
$(\hat{I})_R^N(xy) = \max\{(\hat{I})_R^N(ab): ab \in R^*\},$
$(\hat{F})_R^N(xy) = \min\{(\hat{F})_R^N(ab): ab \in R^*\}.$

Definition 3.10 *An NBVG,* $G^* = (Q, R, S)$ *is a neutrosophic bipolar vague incidence cycle if and only if,* $G^{**} = (Q^*, R^*, S^*)$ *is a cycle and there exist no unique edge,* $ab \in S^*$ *such that*

$(\hat{T})_S^P(x, xy) = \min\{(\hat{T})_S^P(a, ab): ab \in S^*\},$
$(\hat{I})_S^P(x, xy) = \min\{(\hat{I})_S^P(a, ab): ab \in S^*\},$
$(\hat{F})_S^P(x, xy) = \max\{(\hat{F})_S^P(a, ab): ab \in S^*\}. and$
$(\hat{T})_S^N(x, xy) = \max\{(\hat{T})_S^N(a, ab): ab \in S^*\},$
$(\hat{I})_S^N(x, xy) = \max\{(\hat{I})_S^N(a, ab): ab \in S^*\},$
$(\hat{F})_S^N(x, xy) = \min\{(\hat{F})_S^N(a, ab): ab \in S^*\}.$

Definition 3.11 *Let* $G^* = (Q, R, S)$ *be an NBVIG. An edge* ab *in* G *is called a bridge if and only if,* ab *is a bridge in* $G^{**} = (Q^*, R^*, S^*)$ *that is, the removal of* ab *disconnectes* G^{**}*. An edge,* ab *is called a neutrosophic vague bridge if*

$(\mathbb{S}^P)'^{\infty}(x, y) < (\mathbb{S}^P)^{\infty}(x, y), forsomex, y \in Q^*$
$(s_1'^{\infty}, s_2'^{\infty}, s_3'^{\infty},) < (s_1^{\infty}, s_2^{\infty}, s_3^{\infty},)$
$\Rightarrow s_1'^{\infty} < s_1^{\infty}, s_2'^{\infty} < s_2^{\infty}, s_3'^{\infty} > s_3^{\infty},$

and

$(\mathbb{S}^N)'^{\infty}(x, y) > (\mathbb{S}^N)^{\infty}(x, y), forsomex, y \in Q^*$
$(s_1'^{\infty}, s_2'^{\infty}, s_3'^{\infty},) < (s_1^{\infty}, s_2^{\infty}, s_3^{\infty},)$
$\Rightarrow s_1'^{\infty} < s_1^{\infty}, s_2'^{\infty} < s_2^{\infty}, s_3'^{\infty} > s_3^{\infty},$

where $(\mathbb{S}^P)'^{\infty}, (\mathbb{S}^N)'^{\infty}$ and $(\mathbb{S}^P)^{\infty}, (\mathbb{S}^N)^{\infty}$ denote the connectedness between x and y in $G' = G^* - ab$ respectively.

An edge ab is called a neutrosophic bipolar vague incidence bridge if

$(I\mathbb{S}^P)'^{\infty}(x, y) < (I\mathbb{S}^P)^{\infty}(x, y), forsomex, y \in Q^*$
$(is_1'^{\infty}, is_2'^{\infty}, is_3'^{\infty},) < (is_1^{\infty}, is_2^{\infty}, is_3^{\infty},)$
$\Rightarrow is_1'^{\infty} < is_1^{\infty}, is_2'^{\infty} < is_2^{\infty}, is_3'^{\infty} > is_3^{\infty},$

and

$(I\mathbb{S}^N)'^{\infty}(x, y) > (I\mathbb{S}^N)^{\infty}(x, y), forsomex, y \in Q^*$
$(is_1'^{\infty}, is_2'^{\infty}, is_3'^{\infty},) < (is_1^{\infty}, is_2^{\infty}, is_3^{\infty},)$
$\Rightarrow is_1'^{\infty} < is_1^{\infty}, is_2'^{\infty} < is_2^{\infty}, is_3'^{\infty} > is_3^{\infty},$

where $(I\mathbb{S}^P)'^{\infty}$, $(I\mathbb{S}^N)'^{\infty}$ and $(I\mathbb{S}^P)^{\infty}$ denote the connectedness between x and y in $G' = G^* - ab$ respectively.

Definition 3.12 *Let* $G^* = (Q, R, S)$ *be an NBVG. A vertex* v *in* G^* *is a cutvertex if and only if it is a cutvertex in* $G^{**} = (Q^*, R^*, S^*)$ *that is* $G^* - v$ *is a disconnected graph.*

A vertex v in an NBVIG is called a neutrosophic bipolar vague cutvertex if the connectedness between any two vertices in $G' = G^* - v$ is less than the conncetedness between the same vertices in G^* that is,

$$(\mathbb{S}^P)'^{\infty}(x, y) < (\mathbb{S}^P)^{\infty}(x, y), forsomex, y \in Q^*$$

$$(\mathbb{S}^N)'^{\infty}(x, y) > (\mathbb{S}^N)^{\infty}(x, y), forsomex, y \in Q^*$$

A vertex v in NBVIG G^* is a neutrosophic bipolar vague incidence cutvertex if for any pair of vertices, x, y other than v the following condition holds:

$$(I\mathbb{S}^P)^{'\infty}(x,y) < (I\mathbb{S}^P)^{\infty}(x,y), for some x, y \in Q^*$$

$$(I\mathbb{S}^N)^{'\infty}(x,y) > (I\mathbb{S}^N)^{\infty}(x,y), for some x, y \in Q^*$$

where $(I\mathbb{S}^P)^{'\infty}$,$(I\mathbb{S}^N)^{'\infty}$ and $(I\mathbb{S}^P)^{\infty}$ $(I\mathbb{S}^N)^{\infty}$ denote the connectedness between x and y in $G' = G^* - ab$ respectively.

Definition 3.13 *Let $G^* = (Q, R, S)$ be an NBVIG. A pair (a, ab) is called a cutpair if and only if, (a, ab) is a cutpair in $G^{**} = (Q^*, R^*, S^*)$ that is after removing the pair (a, ab) there is no path between a and ab. Let $G^* = (Q, R, S)$ be an NBVIG. A pair (a,ab) is called a neutrosophic bipolar vague cutpair if deleting the pair (a, ab) reduces the connectedness between $a, ab \in Q^* \cup R^*$ that is*

$$(\mathbb{S}^P)^{'\infty}(a,ab) < (\mathbb{S}^P)^{\infty}(a,ab),$$

and

$$(\mathbb{S}^N)^{'\infty}(a,ab) > (\mathbb{S}^N)^{\infty}(a,ab),$$

where $(\mathbb{S}^P)^{'\infty}(a,ab)$, $(\mathbb{S}^N)^{'\infty}(a,ab)$ and $(\mathbb{S}^P)^{\infty}(a,ab)$, $(\mathbb{S}^N)^{\infty}(a,ab)$ denote the connectedness between a and ab in $G' = G^* - \{(a,ab)\}$ and G^* respectively.

A pair (a, ab) is called neutrosophic bipolar vague incidence cutpair if

$$(I\mathbb{S}^P)^{'\infty}(a,ab) < (I\mathbb{S}^P)^{\infty}(a,ab),$$

for $a, ab \in Q^* \cup R^*$

and

$$(I\mathbb{S}^N)^{'\infty}(a,ab) > (I\mathbb{S}^N)^{\infty}(a,ab),$$

for $a, ab \in Q^* \cup R^*$

where $(I\mathbb{S}^P)^{'\infty}(a,ab)$, $(I\mathbb{S}^N)^{'\infty}(a,ab)$ and $(I\mathbb{S}^P)^{\infty}(a,ab)$, $(I\mathbb{S}^N)^{\infty}(a,ab)$ denotes the connectedness between a and ab in $G' = G^* - \{(a,ab)\}$ and G^* respectively.

Theorem 3.14 *Let $G^* = (Q, R, S)$ be a NBVIG. If ab is a neutrosophic vague bridge, then ab is not a weakest edge in any cycle.*

Proof. Let ab be a neutrosophic bipolar vague bridge and suppose, on the contrary that ab is the weakest edge of a cycle. Then, in this cycle, we can find an alternative path, P_1 from a to b that does not contain the edge ab and $\mathbb{S}P_1$ is greater than of equal to $\mathbb{S}P_2$, where P_2 is the path involving the edge ab. Thus, removal of the edge ab from G^* does not affect the connectedness between a and $v - a$ contradiction to our assumption. Hence, ab is not the weakest edge in any cycle.

Theorem 3.15 *If (a, ab) is a neutrosophic bipolar vague incidence cutpair, then (a, ab) is not the weakest pair any cycle.*

Proof. Let (a, ab) be a neutrosophic bipolar vague incidence cutpair in G^*. On contrary, suppose that (a, ab) is a weakest pair of a cycle. Then we can find an alternative path from a and ab having incidence strength greater than or equal to that of the path involving the pair (a, ab). Thus, removal of the pair (a, ab) does not affect the incidence connectedness between a and

ab, but this is a contradiction to our assumption that (a, ab) is a neutrosophic vague incidence cutpair. Hence (a, ab) is not a weakest pair in any cycle.

Theorem 3.16 *Let $G^* = (Q, R, S)$ be a NBVIG. If ab is a neutrosophic bipolar vague bridge in G^*, then*

$$(\mathbb{S}^P)^\infty(a,b) = (s_1^\infty, s_2^\infty, s_3^\infty) = ((\hat{T}_R^P)(ab), (\hat{I}_R^P)(ab), (\hat{F}^P)_R)(ab))$$

$$(\mathbb{S}^N)^\infty(a,b) = (s_1^\infty, s_2^\infty, s_3^\infty) = ((\hat{T}_R^N)(ab), (\hat{I}_R^N)(ab), (\hat{F}^N)_R)(ab))$$

Proof. Let G^* be an NBVIG and ab is a neutrosophic bipolar vague bridge in G^*. On the contrary, suppose that

$$(\mathbb{S}^P)^\infty(a,b) > ((\hat{T}_R^P)(ab), (\hat{I}_R^P)(ab), (\hat{F}^P)_R)(ab))$$

Then, there exists a $a - b$ path, P with

$$(\mathbb{S}^P)(P) > ((\hat{T}_R^P)(ab), (\hat{I}_R^P)(ab), (\hat{F}^P)_R)(ab))$$

and

$$(((\hat{T}_R^P)(xy), (\hat{I}_R^P)(xy), (\hat{F}^P)_R)(xy)))) > (((\hat{T}_R^P)(ab), (\hat{I}_R^P)(ab), (\hat{F}^P)_R)(ab)))$$

for all edges on path P. Now, P together with the edge, ab forms a cycle in which ab is the weakest edge, but it is a contradiction to the fact that ab is a neutrosophic bipolar vague bridge. Hence

$$(\mathbb{S}^P)^\infty(a,b) = (s_1^\infty, s_2^\infty, s_3^\infty) = ((\hat{T}_R^P)(ab), (\hat{I}_R^P)(ab), (\hat{F}^P)_R)(ab))$$

$$(\mathbb{S}^N)^\infty(a,b) = (s_1^\infty, s_2^\infty, s_3^\infty) = ((\hat{T}_R^N)(ab), (\hat{I}_R^N)(ab), (\hat{F}^N)_R)(ab))$$

Theorem 3.17 *If (a, ab) is a neutrosophic bipolar vague incidence cutpair in an NBVIG $G^* = (Q, R, S)$ then*

$$(I\mathbb{S}^P)^\infty(a,ab) = (is_1^\infty, is_2^\infty, is_3^\infty) = ((\hat{T}^P)_S(a,ab), (\hat{I}^P)_S(a,ab), (\hat{F}^P)_S(a,ab))$$

. and

$$(I\mathbb{S}^N)^\infty(a,ab) = (is_1^\infty, is_2^\infty, is_3^\infty) = ((\hat{T}^N)_S(a,ab), (\hat{I}^P)_S(a,ab), (\hat{F}^N)_S(a,ab))$$

.

Proof. The proof is on the same line as the above theorem.

Theorem 3.18 *Let $G^* = (Q, R, S)$ be an NBVIG and G^{**} is a cycle. then an edge ab is a neutrosophic bipolar vague bridge of G^* if and only if it is an edge common to two neutrosophic vague incidence cutpairs.*

Proof. Suppose that ab is a neutrosophic bipolar vague bridge of G^*. Then there exist vertices a and b with the ab edge lying on every path with the greatest incidence strength between a and b. Consequently, there exists only one path, $P(say)$ between a and b which contains a ab edge and has the greatest incidence strength. Any pair on P will be a neutrosophic bipolar vague incidence cutpair, since the removal of any one of them will disconnect P and reduce the incidence strength. Conversely, let ab be an edge common to two neutrosophic bipolar vague incidence cutpairs (a, ab) and (b, ab). Thus both (a, ab) and (b, ab) are not the weakest cutpair of G^*. Now, G^{**} being a cycle, there exists only two paths between any two vertices. Also the path P_1 from the vertex a and b not containing the pairs (a, ab) and (b, ab)

has less incidence strength than the path containing them. Thus, the path with the greatest incidence strength from a to b is

$$P_2: a, (a, ab), ab, (b, ab), b.$$

Also,

$$(\mathbb{S}^P)^\infty(a,b) = \mathbb{S}(P_2) = ((\hat{T}^P)_R(ab), (\hat{I}^P)_R(ab), (\hat{F}^P)_R(ab)).$$

and

$$(\mathbb{S}^N)^\infty(a,b) = \mathbb{S}(P_2) = ((\hat{T}^N)_R(ab), (\hat{I}^P)_R(ab), (\hat{F}^N)_R(ab)).$$

Therefore, ab is a neutrosophic bipolar vague bridge.

4 Conclusion

In this work, the neutrosophic bipolar vague incidence graphs have been introduced. The edge-connectivity, vertex-connectivity and pair-connectivity in neutrosophic bipolar vague incidence graphs have been established. The given results are illustrated with suitable example. In future, interval neutrosophic incidence graphs and neutrosophic soft incidence graphs with their properties will be developed.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflicts of interest.

Acknowledgement: The authors thank both the anonymous reviewers for their valuable comments to improve the quality of this manuscript.

References

[1] Ajay, D. and Chellamani, P., Fuzzy magic labelling of Neutrosophic path and star graph, Advances in Mathematics: Scientific Journal, Vol 9, No. 8, pp. 6059?6070, 2020.

[2] Akram, M. and Shahzadi, G., Operations on single-valued neutrosophic graphs, Journal of Uncertain Systems, 11(1) (2017), 1-26.

[3] Akram, M., Sayed, S. and Smarandache, F., Neutrosophic incidence graphs with application, Axioms, Vol 7, No. 3, pp. 1-47, 2018

[4] Al-Quran, A. and Hassan, N., "Neutrosophic vague soft expert set theory, Journal of Intelligent & Fuzzy Systems, Vol 30, No. 6, pp. 3691-3702, 2016

[5] Al-Tahan, M. and Davvaz, B., "On Single Valued Neutrosophic Sets and Neutrosophic $\mathcal{N}$-Structures: Applications on Algebraic Atructures (Hyperstructures)?, International Journal of Neutrosophic Science (IJNS), Vol 3, No. 2, pp. 108-117, 2020.

[6] Al- Tahan, M., "Some Results on Single Valued Neutrosophic (Weak) Polygroups?, International Journal of Neutrosophic Science (IJNS), Vol 2, No. 1, pp. 38-46, 2020

[7] Alkhazaleh, S., "Neutrosophic vague set theory?, Critical Review, Vol 10, pp. 29-39, 2015.

[8] Borzooei, A. R. and Rashmanlou, H., "Degree of vertices in vague graphs?, Journal of Applied Mathematics and Informatics, Vol 33, pp. 545-557, 2015

[9] Borzooei, A. R. and Rashmanlou, H., "Domination in vague graphs and its applications, Journal of Intelligent & Fuzzy Systems, Vol 29, pp. 1933-1940, 2015.

[10] Borzooei, A. R., Rashmanlou, H., Samanta, S. and Pal, M., "Regularity of vague graphs, Journal of Intelligent & Fuzzy Systems, Vol 30, pp. 3681-3689, 2016.

[11] Broumi, S. and Smarandache, F., "Intuitionistic neutrosophic soft set, Journal of Information and Computer Science, Vol 8, No. 2, pp. 130-140, 2013.

[12] Broumi, S., Smarandache, F., Talea, M. and Bakali., "Single-valued neutrosophic graphs: Degree, Order and Size, IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), 2016.

[13] Broumi, S., Mohamed, T., Assia, B. and Smarandache, F., "Single-valued neutrosophic graphs, The Journal of New Theory, Vol 2016, No. 10, pp. 861-101, 2016.

[14] Darabian, E., Borzooei, R. A., Rashmanlou, H. and Azadi, M., "New concepts of regular and (highly) irregular vague graphs with applications, Fuzzy Information and Engineering, Vol 9, No. 2, pp. 161-179, 2017.

[15] Deli, I. and Broumi, S., "Neutrosophic soft relations and some properties, Annals of Fuzzy Mathematics and Informatics, Vol 9, No. 1, pp. 169-182, 2014.

[16] Deli, I. and jun, Y., "Some weighted geometric operators with SVTrN-numbers and their application to multi-criteria decision-making problems, Journal of Intelligent & Fuzzy Systems, Vol 32, No. 1, pp. 291-301, 2017.

[17] Deli, I. and , E. K., "Two Centroid Point for SVTN-Numbers and SVTrN-Numbers: SVN-MADM Method?, In Neutrosophic Graph Theory and Algorithms, IGI Global, pp. 279-307, 2020.

[18] Dhavaseelan, R., Vikramaprasad, R. and Krishnaraj, V., Certain types of neutrosophic graphs, International Journal of Mathematical Sciences and Applications, Vol 5, No. 2, pp. 333-339, (2015).

[19] Dhavaseelan, R., Jafari, S., Farahani M. R and Broumi S, "On single-valued co-neutrosophic graphs, Neutrosophic Sets and Systems, An International Book Series in Information Science and Engineering, Vol 22, 2018.

[20] Molodtsov D, "Soft set theory-first results, Computers and Mathematics with Applications, Vol 37, No. 2, pp. 19-31, 1999.

[21] Muhiuddin, G. and Al-roqi, A. M., "Cubic soft sets with applications in BCK/BCI-algebras, Annals of Fuzzy Mathematics and Informatics, Vol 8, No. 2, pp. 291-304, 2014.

[22] Muhiuddin, G., "Neutrosophic Sub semi-groups, Annals of Communications in Mathematics, 1(1), (2018).

[23] Mordeson, J.N., "Fuzzy line graphs", Pattern Recognition Letters, Vol 14, pp. 381-384, 1993.

[24] Hussain, S. S., Hussain, R. J. and Muhiuddin, G., "Neutrosophic Vague Line Graphs, Neutrosophic Sets and Systems, Vol 36, pp. 121-130, 2020.

[25] Hussain, S. S., Hussain, R. J., Jun, Y. B. and Smarandache, F., "Neutrosophic bipolar vague set and its application to neutrosophic bipolar vague graphs?, Neutrosophic Sets and Systems, Vol 28, pp. 69-86, 2020.

[26] Hussain, S. S., Hussain, R. J. and Smarandache, F., "On neutrosophic vague graphs, Neutrosophic Sets and Systems, Vol 28, pp. 245-258, 2019.

[27] Hussain, S. S., Broumi, S., Jun, Y. B. and Durga, N., "Intuitionistic bipolar neutrosophic set and its application to intuitionistic bipolar neutrosophic graphs, Annals of Communication in Mathematics, Vol 2, No. 2, pp. 121-140, 2019.

[28] Hussain, S. S., Hussain, R. J. and Smarandache, F., "Domination Number in Neutrosophic Soft Graphs, Neutrosophic Sets and Systems, Vol 28, No. 1, pp. 228-244, 2019.

[29] Hussain, R. J., Hussain, S. S., Sahoo, S., Pal, M. and Pal, A., "Domination and Product Domination in Intuitionistic Fuzzy Soft Graphs, International Journal of Advanced Intelligence Paradigms, 2020 (In Press), DOI:10.1504/IJAIP.2019.10022975.

[30] Hussain S S and , Hussain R J and Babu M V, Neutrosophic Vague Incidence Graph, International Journal of Neutrosophic Sciences, 12(1), 29-38

[31] Hussain S. S., Rosyida I., Rashmanlou H and Mofidnakhaei F, Interval intuitionistic neutrosophic sets with its applications to interval intuitionistic neutrosophic graphs and climatic analysis. Computational and Applied Mathematics, 40(4), 1-20.

[32] Smarandache, F., "A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic, Rehoboth: American Research Press, 1999.

[33] Smarandache, F., "Neutrosophy, Neutrosophic Probability, Set, and Logic?, Amer../ Res. Press, Rehoboth, USA, 105 pages, 1998 http://fs.gallup.unm.edu/eBookneutrosophics4.pdf(4th edition).

[34] Smarandache, F., "Neutrosophic Graphs, in his book Symbolic Neutrosophic Theory, Europa, Nova, 2015.

[35] Smarandache. F., "Neutro-Algebra is a Generalization of Partial Algebra, International Journal of Neutrosophic Science (IJNS), Vol 2, No. 1, pp. 8-17, 2020.

[36] Smarandache, F., and Abobala, M, "n-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science (IJNS), Vol 7, No. 1, pp. 47-54, 2020
[37] Smarandache, F., "Neutrosophic set, a generalisation of the intuitionistic fuzzy sets, International Journal of Pure and Applied Mathematics, Vol 24, pp. 289-297, 2010.
[38] Wang, H., Smarandache, F., Zhang, Y. and Sunderraman, R., "Single-valued neutrosophic sets, Multispace and Multistructure, Vol 4, pp. 410-413, 2010.
[39] Gau, W. L. and Buehrer, D. J.,"Vague sets, IEEE Transactions on Systems. Man and Cybernetics, Vol 23, No. 2, pp. 610-614, 1993.