Fuzzy Sets and Systems

The MADM problems with interval valued neutrosophic numbers: A nonlinear programming model

--Manuscript Draft--

Manuscript Number:	FSS-D-22-00658
Article Type:	VSI:Uncertainty aggregation
Section/Category:	
Keywords:	Multiple attribute group decision making (MAGDM); Interval valued neutrosophic number (IVNN); Nonlinear programming; Variable transformation; Aggregation operator

The MADM problems with interval valued neutrosophic numbers: A nonlinear programming model

M. Arshi

Department of Mathematics, Khorramabad Branch, Islamic Azad University, Khorramabad, Iran
maryamarshi1@gmail.com

A. Hadi-Vencheh*

Department of Mathematics, Isfahan (Khorasgan) Branch, Islamic Azad University, Isfahan, Iran
ahadi@khuisf.ac.ir

M. Nazari

Department of Mathematics, Khorramabad Branch, Islamic Azad University, Khorramabad, Iran
m.nazari@khoiau.ac.ir

A. Jamshidi

Department of Mathematics, Isfahan (Khorasgan) Branch, Islamic Azad University, Isfahan, Iran
ali.jamshidi@khuisf.ac.ir

*Corresponding author

Abstract

The purpose of this paper is to present a new technique to solve multiple attribute group decision making (MAGDM) problems with interval valued neutrosophic numbers (IVNN). The proposed approach solves the MAGDM problem using mathematical programming methodology. To reinforce the mathematical programming approach in IVNN environment, an IVNN multiple attribute group decision making problem is formulated as a non-linear programming model which can be solved easily. The proposed approach helps decision maker (DM) to find an alternative which has best performance. The main advantage of the proposed approach is that decision making process does not rely on subjective weights of attributes. Moreover, the proposed methodology is easy and simple to understand.

Keywords**:** Multiple attribute group decision making (MAGDM); Interval valued neutrosophic number (IVNN); Nonlinear programming; Variable transformation; Aggregation operator

1. Introduction

In today's highly competitive market, real-world decision-making has indeed become indispensable often possessing multiple conflicting and non-commensurable evaluation standards. Consequently, most of the real-world decision-making problems can be viewed as multiple attribute decision-making (MADM) [1] problems that are used in cases of discrete and limited number of alternatives characterized by multiple and conflicting attributes. The attributes are classified in two ways; firstly, as subjective (qualitative/intangible), objective (quantitative/tangible) and critical (that need to be satisfied before further processing) attributes, and secondly, as benefit type (the more the better) and cost type (the less the better) attributes. The main advantage of MADM is that it can provide many dimensions to the decision-maker for considering the related elements, and evaluate all possible alternatives under variable degrees. Furthermore, to reinforce democracy and rationality of the decision-making, many real-world processes take place in group settings. Multiple attribute group decision-making (MAGDM) [2] is a key component of group decision-making, and is among the most important and frequently encountered process in a variety of important application fields including engineering, economy, management, medicine, military affairs, etc. Moreover, in practical group decision-making, because of the complexity and subjectivity of the decision systems and the indefinite source of human judgment, the evaluation results given by the decision experts are not necessarily crisp numbers, but may be linguistic terms or labels of fuzzy sets [3] as for the qualitative attributes, quantification of the uncertain information is a difficult task. This has led to the emergence of fuzzy multiple attribute group decision-making (FMAGDM) [4] methods that are needed to treat imprecise, vague and uncertain information, both qualitative and quantitative.

In many practical FMAGDM problems, there may exist hesitation in either evaluation process or in the preferences of the attributes. The intuitionistic fuzzy set (IFS) [5] is more useful and flexible in dealing with fuzziness and uncertainty originating from vague knowledge or information involving hesitation. The IFS theory is considered to solve the imprecision of cognitive thinking of humans due to its prominent characteristic that both the attached and non-attached information can be taken into consideration in the decision-process. The IFS theory has been successfully integrated with MADM and MAGDM approaches [6-9]. Gençet al. [10] studied the issue of consistency, missing value(s) and derivation of the priority vector of interval fuzzy preference relations. Boran and Akay [11] proposed a biparametric similarity measure for IFSs with applications to pattern recognition. Shen et al. [12] proposed a new outranking sorting method for group decision making using IFSs. Wan et al. [13] developed a new method for solving MAGDM problems with Atanassov's interval-valued intuitionistic fuzzy values and incomplete attribute weight information**.**

Beyond the extension of previous models, some recent developed methods are proposed for IVIF MADM problems. Li [14] developed a nonlinear programming methodology based on TOPSIS to solve MADM problems using ratings of alternatives on attributes and weights of attributes expressed with IVIFSs. Li [15] used the concept of relative closeness coefficients and constructed a pair of non-linear fractional programming models being transformed into two simpler auxiliary linear programming models in order to calculated the relative closeness coefficient of alternatives to the IVIF positive ideal solution, being employed to generate ranking order of alternatives.

IFSs were extended by Smarandache [17] to neutrosophic sets (NSs) which deals with imprecise, incomplete and uncertain information from philosophical point of view. NSs have been introduced as a generalization of crisp sets, FSs and IFSs. NS is a component of neutrosophy which deals with the study of origin, nature, and scope of neutralites, as well as their connections with different ideational spectra Smarandache [18]. It has three independent components: " truth membership function" (TMF), " indeterminacy membership function" (IMF) and " Falsity membership function" (FMF) (Smarandache [18-19]). Later on, single valued neutrosophic set (SVNS) has been introduced (Smarandache [17-19], Deli and Subas [20], Abdel-Basset and Mohamed [21] and Edalatpanah [22]). Moreover, some generalization of neutrosophic sets, including interval neutrosophic set (Garg [23], Liu and Shi [24]), bipolar neutrosophic set (Deli et.al [25], Ulucay et.al [26]), multi-valued neutrosophic set (Ji, Zhang and Wang [27], Peng, Wang and Yang [28]), simplified neutrosophic sets (Edalatpanah and Smarandache [29]; Peng et al. [30]) have been presented. Edalatpanah [31] presented a concept in neutrosphic sets (NSs) called neutrosophic structured element (NSE). He proposed a decision-making method for a multi-attribute decision making (MADM) problem under NSE information.

Biswas et al. [32] presented a value and ambiguity-based ranking approach for trapezoidal neutrosophic number (TrNNs) and presented an MCDM strategy. Liang et al. [33] defined score, accuracy and certainty function in the context of single-valued trapezoidal neutrosophic number (SVTrNN) by using the concept of center of gravity (COG). Moreover, Edalatpanah [34] proposed an algorithm for solving the single-valued neutrosophic linear programming problem.

The purpose of this paper is to present a formulation of multi attribute group decision making (MAGDM) problem in the presence of interval valued neutrosophic numbers. In the proposed methodology, the MAGDM problem is formulated in the form of a nonlinear programming problem and then, a novel approach is proposed to solve this problem. The rest of this paper is organized as follows. A brief overview of neutrosophic sets and required concepts are given in section 2. The considered problem and its formulations are expressed in section 3. Section 4 explains the proposed approach for solving the problem. In section 5, applicability of the proposed method is presented by a numerical example. Finally, section 6 makes some conclusions.

2. Neutrosophic Sets

In this section, we present some basic definitions and arithmetic operations neutrosophic numbers.

Definition 1. Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth- membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. If the function $T_A(x)$, $I_A(x)$ and $F_A(x)$ are singleton subintervals/subsets in the real standard [0,1], that is $T_A(x): X \rightarrow [0,1], I_A(x): X \rightarrow [0,1]$ and $F_A(x): X \rightarrow [0,1]$. Then, a single valued neutrosophic set A is denoted by $A = \{(x, T_A(x), I_A(x), F_A(x) \mid x \in X\}$ which is called a SVNS. Also, SVNS satisfies the condition $0 \leq T_A(x) + \ I_A(x) + F_A(x) \leq 3$. [29]

Definition 2. For SVNSs A and B, $A \subseteq B$ if and only if $T_A(x) \leq T_B(x), I_A(x) \geq I_B(x)$, and $F_A(x) \geq F_B(x)$ for every x in X. [35]

Definition 3. An interval-valued neutrosophic set (IVNS) $\tilde{A}$ on universal set X is defined as:

$$\tilde{A} = \{\langle \varepsilon, ([T_{\tilde{A}^L}(\varepsilon), T_{\tilde{A}^U}(\varepsilon)], [\ I_{\tilde{A}^L}(\varepsilon), I_{\tilde{A}^U}(\varepsilon)], [\ F_{\tilde{A}^L}(\varepsilon), F_{\tilde{A}^U}(\varepsilon)]\ \rangle : \varepsilon \in X\} \tag{1}$$

With the condition $0 \leq T_{\tilde{A}}(\varepsilon) + \ I_{\tilde{A}}(\varepsilon) + F_{\tilde{A}}(\varepsilon) \leq 3$ [36].

Definition 4. Following are some of the arithmetic operations defined on IVNNs:

Here we consider two IVNS of $\tilde{A} = \langle [T_{\tilde{A}^L}(\varepsilon), T_{\tilde{A}^U}(\varepsilon)], [\ I_{\tilde{A}^L}(\varepsilon), I_{\tilde{A}^U}(\varepsilon)], [\ F_{\tilde{A}^L}(\varepsilon), F_{\tilde{A}^U}(\varepsilon)] \rangle$ and $\tilde{B} = \langle [T_{\tilde{B}^L}(\varepsilon), T_{\tilde{B}^U}(\varepsilon)], [\ I_{\tilde{B}^L}(\varepsilon), I_{\tilde{B}^U}(\varepsilon)], [\ F_{\tilde{B}^L}(\varepsilon), F_{\tilde{B}^U}(\varepsilon)] \rangle$ can be defined as follow [29]:

(i) $\tilde{A} \oplus \tilde{B} = \langle [T_{\tilde{A}^L}(\varepsilon) + T_{\tilde{B}^L}(\varepsilon) - T_{\tilde{A}^L}(\varepsilon).T_{\tilde{B}^L}(\varepsilon), T_{\tilde{A}^U}(\varepsilon) + T_{\tilde{B}^U}(\varepsilon) - T_{\tilde{A}^U}(\varepsilon).T_{\tilde{B}^U}(\varepsilon)][I_{\tilde{A}^L}(\varepsilon).I_{\tilde{B}^L}(\varepsilon), I_{\tilde{A}^U}(\varepsilon).I_{\tilde{B}^U}(\varepsilon)], [F_{\tilde{A}^L}(\varepsilon).F_{\tilde{B}^L}(\varepsilon), F_{\tilde{A}^U}(\varepsilon).F_{\tilde{B}^U}(\varepsilon)\ ]\ \rangle$ (2)

(ii) $\tilde{A} \otimes \tilde{B} = \langle [T_{\tilde{A}^L}(\varepsilon).T_{\tilde{B}^L}(\varepsilon), T_{\tilde{A}^U}(\varepsilon).T_{\tilde{B}^U}(\varepsilon)\ ], [I_{\tilde{A}^L}(\varepsilon) + I_{\tilde{B}^L}(\varepsilon) - I_{\tilde{A}^L}(\varepsilon).I_{\tilde{B}^L}(\varepsilon), I_{\tilde{A}^U}(\varepsilon) + I_{\tilde{B}^U}(\varepsilon) - I_{\tilde{A}^U}(\varepsilon).I_{\tilde{B}^U}(\varepsilon)\ ], [\ F_{\tilde{A}^L}(\varepsilon) + F_{\tilde{B}^L}(\varepsilon) - F_{\tilde{A}^L}(\varepsilon).F_{\tilde{B}^L}(\varepsilon), F_{\tilde{A}^U}(\varepsilon) + F_{\tilde{B}^U}(\varepsilon) - F_{\tilde{A}^U}(\varepsilon).F_{\tilde{B}^U}(\varepsilon)]\rangle$ (3)

(iii) $\lambda\tilde{A} = \langle \left[1 - \left(1 - T_{\tilde{A}^L}(\varepsilon)\right)^{\lambda}, 1 - \left(1 - T_{\tilde{A}^U}(\varepsilon)\right)^{\lambda}\right], \left[\left(I_{\tilde{A}^L}(\varepsilon)\right)^{\lambda}, \left(I_{\tilde{A}^U}(\varepsilon)\right)^{\lambda}\right], [(F_{\tilde{A}^L}(\varepsilon)^{\lambda}, \left(F_{\tilde{A}^U}(\varepsilon)\right)^{\lambda}]\rangle\ \ \lambda > 0$ (4)

(iv) $\tilde{A}^{\lambda} = \langle \left[(T_{\tilde{A}^L}(\varepsilon)\ )^{\lambda}, (T_{\tilde{A}^U}(\varepsilon)\ )^{\lambda}\right], \left[1 - (1 - I_{\tilde{A}^L}(\varepsilon)\ )^{\lambda}, 1 - (1 - I_{\tilde{A}^U}(\varepsilon)\ )^{\lambda}\right], [1 - (1 - F_{\tilde{A}^L}(\varepsilon)\ )^{\lambda}, 1 - (1 - F_{\tilde{A}^U}(\varepsilon)\ )^{\lambda}]\ \rangle\ \lambda > 0$ (5)

Definition 5. Let $\tilde{A} = \langle \varepsilon, [T_{\tilde{A}^L}(\varepsilon), T_{\tilde{A}^U}(\varepsilon)], [\ I_{\tilde{A}^L}(\varepsilon), I_{\tilde{A}^U}(\varepsilon)], [\ F_{\tilde{A}^L}(\varepsilon), F_{\tilde{A}^U}(\varepsilon)]\rangle$ be an IVNN, then we define score function, and accuracy function as follows:

$S(\tilde{A}) = \frac{1}{2}([T_{\tilde{A}^L}(\varepsilon), T_{\tilde{A}^U}(\varepsilon)] + (1 - [\ I_{\tilde{A}^L}(\varepsilon), I_{\tilde{A}^U}(\varepsilon)]) + (1 - [\ F_{\tilde{A}^L}(\varepsilon), F_{\tilde{A}^U}(\varepsilon)])$ (6)

accuracy $(\tilde{A})$= $\frac{1}{2}([T_{\tilde{A}^L}(\varepsilon), T_{\tilde{A}^U}(\varepsilon)] - (1 - [\ I_{\tilde{A}^L}(\varepsilon), I_{\tilde{A}^U}(\varepsilon)]) - (1 - [\ F_{\tilde{A}^L}(\varepsilon), F_{\tilde{A}^U}(\varepsilon)]$ (7)

Let $\tilde{A} = \langle [T_{\tilde{A}^L}(\varepsilon), T_{\tilde{A}^U}(\varepsilon)], [\ I_{\tilde{A}^L}(\varepsilon), I_{\tilde{A}^U}(\varepsilon)], [\ F_{\tilde{A}^L}(\varepsilon), F_{\tilde{A}^U}(\varepsilon)]\rangle$ and $\tilde{B} = \langle [T_{\tilde{B}^L}(\varepsilon), T_{\tilde{B}^U}(\varepsilon)], [\ I_{\tilde{B}^L}(\varepsilon), I_{\tilde{B}^U}(\varepsilon)], [\ F_{\tilde{B}^L}(\varepsilon), F_{\tilde{B}^U}(\varepsilon)]\rangle$ be two arbitrary IVNN, the ranking of $\tilde{A}$ and $\tilde{B}$ by score function is defined as follows:

- If $S(\tilde{A}) < S(\tilde{B})$ then $\tilde{A} < \tilde{B}$

- If $S(\tilde{A})= S(\tilde{B})$ and if
- accuracy $(\tilde{A})<$ accuracy $(\tilde{B})$ then $\tilde{A}< \tilde{B}$
- accuracy $(\tilde{A})>$ accuracy $(\tilde{B})$ then $\tilde{A}> \tilde{B}$
- accuracy $(\tilde{A})=$ accuracy $(\tilde{B})$ then $\tilde{A}= \tilde{B}$

Definition 6. Let $\widetilde{A_J} = \langle \left[T_{\widetilde{A_J}^L}(\varepsilon), T_{\widetilde{A_J}^U}(\varepsilon)\right], \left[I_{\widetilde{A_J}^L}(\varepsilon), I_{\widetilde{A_J}^U}(\varepsilon)\right], \left[F_{\widetilde{A_J}^L}(\varepsilon), F_{\widetilde{A_J}^U}(\varepsilon)\right]\rangle$ $(j = 1, \dots n)$ be an IVNN. The arithmetic average operator is as follows:

$$\text{IVNAA}= \sum_{j=1}^{n} w_j A_j \tag{8}$$

Where $W = (w_1, \dots, w_n)$ is the weight vector of $\widetilde{A_J}$, $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$.

Definition 7. For the IVNN weighted arithmetic average operator (IVNNWAA), the aggregated result is as follows:

$$\text{IVNNWA}\left(\tilde{A}_1, \dots, \tilde{A}_n\right) = \tag{9}$$

$$\langle [1-\prod_{j=1}^{n}\left(1-T_{\widetilde{A_J}^L}(\varepsilon)\right)^{w_j}, 1-\prod_{j=1}^{n}\left(1-T_{\widetilde{A_J}^U}(\varepsilon)\right)^{w_i}], [\prod_{j=1}^{n}(I_{\widetilde{A_J}^L}(\varepsilon))^{w_j}, \prod_{j=1}^{n}(I_{\widetilde{A_J}^U}(\varepsilon))^{w_j}], [\prod_{j=1}^{n}(F_{\widetilde{A_J}^L}(\varepsilon))^{w_j}, \prod_{j=1}^{n}(F_{\widetilde{A_J}^U}(\varepsilon))^{w_j}]\rangle$$

3. MAGDM problem formulation

Suppose that there is a group of K decision maker who apprise the alternative set A= {A_1, A_2, …, A_n} regard to criteria set C= {C_1, C_2, …, C_n}. Each decision maker, makes his/her evaluations individually and constructs an individual decision matrix $D^k=[\tilde{x}_{ij}^k]$. The $\tilde{x}_{ij}^k$ elements of D^k are expressed in form of an IVNN $\tilde{x}_{ij}^k = [\left(\underline{T}_{ij,}^k \overline{T}_{ij}^k\right), \left(\underline{I}_{ij}^k, \overline{I}_{ij}^k\right), \left(\underline{F}_{ij}^k, \overline{F}_{ij}^k\right)]$, $i = 1,2, \dots, m$; $j = 1,2, \dots, n$. The aim of the problem is to determine a ranking of alternatives or rating them, to enable decision makers for choosing their final alternative(s) or ranking them.

At the first step, an aggregated decision matrix D is constituted using IVNNWAA (when a predetermined weight is assigned to different experts). The aggregated decision matrix will be obtained as D=[$\tilde{x}_{ij}$] where,

$$\tilde{x}_{ij} = \text{IVNNWA}(\tilde{x}_{ij}^{1}, \tilde{x}_{ij}^{2}, \dots, \tilde{x}_{ij}^{k}) \qquad (10)$$

The extended form of aggregated matrix D can be illustrated as:

$$D = \begin{bmatrix} \tilde{x}_{11} & \cdots & \tilde{x}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \cdots & \tilde{x}_{mn} \end{bmatrix} \qquad (11)$$

If the problem seeks to rank alternatives of the set A, the MAGDM problem can be formulated as below:

$$\begin{aligned} &Max \;\; \textstyle\sum_{j=1}^{n} w_j \tilde{x}_{ij} \\ &\text{s.t.} \quad \textstyle\sum_{j=1}^{n} {w_j}^2 = \tilde{1} \quad i = 1, \dots, m \quad \text{(i)} \\ &\qquad\; w_j \geq 0 \qquad\qquad j = 1, \dots, n \qquad \text{(ii)} \end{aligned} \qquad (12)$$

Where $\tilde{1}$ is an IVNN, e.g. [(0.9, 0.95), (0.01, 0.05), (0.02,0.06)], and w_j , $j = 1,2, \dots, n$ is the importance weight of criterion j. This non-linear model was initially proposed by Hadi-Vencheh [16] as a weighted optimization model for multi criteria inventory classification problem. In this model, objective function maximizes the total score of each alternative i as a linear function of criteria. Canstraint (i) restricts all alternatives square weights to be equal 1 with similar weight of alternative i. Canstraint (ii) means that all weights of criteria must be positive. This model repeatedly is solved for each alternative and ranked according to the descending order of their scores.

4. Solving approach

The MAGDM problem formulated in Eq. (12) can be considered as a nonlinear programming problem under IVNN environment.

Mathematically, an optimization problem is to find the infimum or supremum of a given real-valued function f over a specified set G of a universal set X, i.e.

$$\alpha = \inf\{ f(x) : x \in G \}, \quad G \subseteq X \tag{13}$$

The optimization problem includes finding the value of α or equivalently, an $x_0 \in G$ that $f(x_0) = \alpha$ [28]. A matrix presentation of non linear programming are revealed in Eq. (13).

$$\begin{aligned} & Max\ CX \\ & \text{s.t.} \quad (AX)^2 \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b, \quad X \geq 0 \end{aligned} \tag{14}$$

Where X is the $(n \times 1)$ column vector of decision variables, C is the $(1 \times m)$ row vector of profit (cost) coefficients, A denotes the $(m \times n)$ technological matrix, and b represents the $(m \times 1)$ resources or right-hand side vector. Regarding the certainty axiom, all the elements of matrix A and also vectors C and b are defined as deterministic numbers.

In this paper, a new approach is developed to solve the non-linear programming problem which their parameters are defined as IVN numbers. Consider a non-linear programming with IVN information whose parameters C, A, and b are defined as IVNNs.

$$\begin{aligned} & Max\ \tilde{C}X \\ & \text{s.t.} \quad (\tilde{A}X)^2 = \tilde{b} \\ & \qquad X \geq 0 \end{aligned} \tag{15}$$

In Eq. (15), X is a $(n \times 1)$ column vector of the decision variables, $\tilde{C}$ is a $(1 \times m)$ IVNN row vector of profit (cost) coefficients, $\tilde{A}$ denotes a $(m \times n)$ IVNN technological matrix, and $\tilde{b}$ gives $(m \times 1)$ IVNN resources or right hand side vector. In an extended form, IVNN-NLP in Eq. (15), can be written as follow:

$$\max \sum_{j=1}^{n} \tilde{c}_j x_j$$

$$\text{s.t.} \quad \sum_{j=1}^{n} (\tilde{a}_{ij} x_j)^2 = \tilde{b}_i, \quad i = 1,2,\dots,m, \tag{16}$$

$$x_j \geq 0 \qquad j = 1,2,\dots,n$$

The considered parameters in Eq. (16), are a set of IVNNs as follows:

$\tilde{c}_j = \left[\left(c_{1j}, c_{2j}\right), \left(c_{3j}, c_{4j}\right), \left(c_{5j}, c_{6j}\right)\right]$, $j = 1,2,\dots,n$ where (c_{1j}, c_{2j}) is truth membership and (c_{3j}, c_{4j}) is indeterminacy membership and $\left(c_{5j}, c_{6j}\right)$ is falsity membership intervals.

$\tilde{a}_{ij} = \left[\left(a_{1ij}, a_{2ij}\right), \left(a_{3ij}, a_{4ij}\right), \left(a_{5ij}, a_{6ij}\right)\right]$, $i = 1,\dots,m; j = 1,\dots,n$ where (a_{1ij}, a_{2ij}) is truth membership and (a_{3ij}, a_{4ij}) is indeterminacy membership and $\left(a_{5ij}, a_{6ij}\right)$ is falsity membership intervals.

$\tilde{b}_i = [(b_{1i}, b_{2i}), (b_{3i}, b_{4i}), (b_{5i}, b_{6i})]$, $i = 1,2,\dots,m$ where (b_{1i}, b_{2i}) is truth membership and (b_{3i}, b_{4i}) is indeterminacy membership and (b_{5i}, b_{6i}) is falsity membership intervals.

Now, consider the objective function $\sum_{j=1}^{n} \tilde{c}\, x_j$. Since the objective function coefficients $\tilde{c}_j, j = 1,\dots,n$ are IVNNs; thus, the objective function can be interpreted as the linear combination of these IVNNs by the non-negative coefficients $x_j \geq 0, j = 1,\dots,n$. The result of this linear combination can be achieved by interactively applying the summation and multiplication operators, as defined in Eqs. (2) and (4), respectively. This induction process involves n scalar multiplication plus $(n-1)$ IVN summation operation, totally include $(2n-1)$ operations. To avoid the amount of operations, a simple variable transformation can be used. Define the variable t as below:

$$t = \frac{1}{x_1 + x_2 + \cdots + x_n} \tag{17}$$

Now, the objective function $\sum_{j=1}^{n} \tilde{c}_j x_j$ is multiplied by t. Defining the variable $tx_j = y_j, j = 1,\dots,n$, the objective function is transformed into:

$$\sum_{j=1}^{n} \tilde{c}_j y_j \quad (18)$$

Since $\sum_{j=1}^{n} y_j = 1$ and $y_j \geq 0, j = 1, \ldots, n$, Eq. (18) can be interpreted as IVNWA of a set of IVNNs $\tilde{c}_j, j = 1, \ldots, n$. According to Eq. (9), Eq. (18) is transformed into the following one:

$$\left(\left[\left(1-\prod_{j=1}^{n}\left(1-c_{1j}\right)^{y_j}\right),\left(1-\prod_{j=1}^{n}\left(1-c_{2j}\right)^{y_j}\right)\right],\left[\prod_{j=1}^{n} c_{3j}^{y_j},\prod_{j=1}^{n} c_{4j}^{y_j}\right],\left[\prod_{j=1}^{n} c_{5j}^{y_j},\prod_{j=1}^{n} c_{6j}^{y_j}\right]\right) \quad (19)$$

In fact, application of the variable transformation in Eq. (17) simplicities obtaining a closed form for the objective functions. Based on score function definition, Eq. (6), an IVNN will be maximized when its truth membership degree is increased, while its indeterminacy membership degree and falsity membership degree are decreased. Also, an IVNN will be minimized when its truth membership degree is decreased, while its indeterminacy membership degree and falsity membership degree are increased. Suppose two interval numbers $A = [\underline{a}, \overline{a}]$ and $B = [\underline{b}, \overline{b}]$. Then, $A \geq B$ if $\underline{a} \geq \underline{b}$ and $\overline{a} \geq \overline{b}$ [29], thus, Eq. (19) will be maximized if $\left(1-\prod_{j=1}^{n}\left(1-c_{1j}\right)^{y_j}\right)$ and $\left(1-\prod_{j=1}^{n}\left(1-c_{2j}\right)^{y_j}\right)$ are maximized, while $\prod_{j=1}^{n} c_{3j}^{y_j}$,$\prod_{j=1}^{n} c_{4j}^{y_j}$,$\prod_{j=1}^{n} c_{5j}^{y_j}$ and $\prod_{j=1}^{n} c_{6j}^{y_j}]$ are minimized. These conditions are satisfied when:

$\prod_{j=1}^{n}(1-c_{1j})^{y_j}$ and $\prod_{j=1}^{n}(1-c_{2j})^{y_j}$ are minimized, and simultaneously;

$\prod_{j=1}^{n} c_{3j}^{y_j}$, $\prod_{j=1}^{n} c_{4j}^{y_j}$ and $\prod_{j=1}^{n} c_{5j}^{y_j}$, $\prod_{j=1}^{n} c_{6j}^{y_j}]$ are minimized.

Hence, the single objective function of IVNN-NLP problem is transformed into the following multi objective problem:

$$Min\left(\prod_{j=1}^{n}(1-c_{1j})^{y_j},\prod_{j=1}^{n}(1-c_{2j})^{y_j},\prod_{j=1}^{n} c_{3j}^{y_j},\prod_{j=1}^{n} c_{4j}^{y_j},\prod_{j=1}^{n} c_{5j}^{y_j},\prod_{j=1}^{n} c_{6j}^{y_j}\right) \quad (20)$$

Considering logarithm Neperien (Ln) as an increasing function, minimization of the elements of the above IVNN is equivalent to "minimizing" the Ln of its elements, as below:

$$Min\ (\sum_{j=1}^{n} y_j . Ln\left(1-c_{1j}\right), \sum_{j=1}^{n} y_j . Ln\left(1-c_{2j}\right), \sum_{j=1}^{n} y_j . Ln\left(c_{3j}\right), \sum_{j=1}^{n} y_j . Ln\left(c_{4j}\right), \sum_{j=1}^{n} y_j . Ln\left(c_{5j}\right), \sum_{j=1}^{n} y_j . Ln\left(c_{6j}\right)) \tag{21}$$

Since the all elements of the above elements are normalized number in [0,1] interval, its minimization is equivalent to "minimizing" the summation of elements,

$$Min\ \sum_{j=1}^{n} y_j . Ln\ ((1-c_{1j})(1-c_{2j})c_{3j}c_{4j}\, c_{5j}c_{6j} \tag{22}$$

Now, consider ith constraint $\sum_{j=1}^{n}(\tilde{a}_{ij}x_j)^2 = \tilde{b}_i$, for a given value $i, i = 1, \dots, m$, in Eq. (16). To handle this constraint, both sides of they are multiplied by t^2, Eq. (17). This operation will transform the initial constraint into $\sum_{j=1}^{n}(\tilde{a}_{ij}y_j)^2 = t^2\tilde{b}_i$. Considering the left hand side of constraint, it is an IVNNWA operator of a set of IVNNs $\tilde{a}_{ij}, j = 1,2, \dots, n$. This IVNNWA can be shown as follows:

$$([(1-\prod_{j=1}^{n}(1-a_{1ij})^{{y_j}^2}), (1-\prod_{j=1}^{n}\left(1-a_{2ij}\right)^{{y_j}^2})], [\prod_{j=1}^{n} a_{3ij}^{{y_j}^2}, \prod_{j=1}^{n} a_{4ij}^{{y_j}^2}], [\prod_{j=1}^{n} a_{5ij}^{{y_j}^2}, \prod_{j=1}^{n} a_{6ij}^{{y_j}^2}]) \tag{23}$$

On the right hand side, the product $t^2\tilde{b}$ can be handled based on scalar multiplication in Eq. (4). The result will be obtained as:

$$(\left[1-(1-b_1)^{t^2}, 1-(1-b_2)^{t^2}\right], \left[b_3^{t^2}, b_4^{t^2}\right], \left[b_5^{t^2}, b_6^{t^2}\right]) \tag{24}$$

For equality type constraints, the right hand side must be equal to the left hand side of constraints.

$$\begin{cases} 1-\prod_{j=1}^{n}(1-a_{1ij})^{{y_j}^2} = 1-(1-b_1)^{t^2} \\ 1-\prod_{j=1}^{n}(1-a_{2ij})^{{y_j}^2} = 1-(1-b_2)^{t^2} \\ \prod_{j=1}^{n}(a_{3ij})^{{y_j}^2} = (b_3)^{t^2} \\ \prod_{j=1}^{n}(a_{4ij})^{{y_j}^2} = (b_4)^{t^2} \\ \prod_{j=1}^{n}(a_{5ij})^{{y_j}^2} = (b_5)^{t^2} \\ \prod_{j=1}^{n}(a_{6ij})^{{y_j}^2} = (b_6)^{t^2} \end{cases} \tag{25}$$

The set of constraints in Eq. (25) is transformed into the non-linear form applying logarithm Neperien function:

$$\begin{cases} \sum_{j=1}^{n} {y_j}^2 Ln\left(1-a_{1ij}\right) = t^2 Ln(1-b_1) \\ \sum_{j=1}^{n} {y_j}^2 Ln\left(1-a_{2ij}\right) = t^2 Ln(1-b_2) \\ \sum_{j=1}^{n} {y_j}^2 Ln\left(a_{3ij}\right) = t^2 Ln(b_3) \\ \sum_{j=1}^{n} {y_j}^2 Ln\left(a_{4ij}\right) = t^2 Ln(b_4) \\ \sum_{j=1}^{n} {y_j}^2 Ln\left(a_{5ij}\right) = t^2 Ln(b_5) \\ \sum_{j=1}^{n} {y_j}^2 Ln\left(a_{6ij}\right) = t^2 Ln(b_6) \end{cases} \tag{26}$$

Finally, the IVNN-NLP problem in Eq. (16) is transformed into equivalent non-linear programming problem, as illustrated in Eq. (27). Solving this problem, the optimal values of $t*$ and y_j^* , $j = 1,2,\ldots,n$ are determined. Applying a reverse transformation, on the basis of Eq. (17), the optimal values of original variables $x_j^*, j = 1,\ldots,n$ are determined.

$$Min\ \sum_{j=1}^{n} y_j.\ Ln(\left(1-c_{1j}\right)\left(1-c_{2j}\right)c_{3j}c_{4j}c_{5j}c_{6j})$$

s.t.

$$\begin{cases} \sum_{j=1}^{n} {y_j}^2 Ln\left(1-a_{1ij}\right) = t^2 Ln(1-b_1) \\ \sum_{j=1}^{n} {y_j}^2 Ln\left(1-a_{2ij}\right) = t^2 Ln(1-b_2) \\ \sum_{j=1}^{n} {y_j}^2 Ln\left(a_{3ij}\right) = t^2 Ln(b_3) \\ \sum_{j=1}^{n} {y_j}^2 Ln\left(a_{4ij}\right) = t^2 Ln(b_4) \\ \sum_{j=1}^{n} {y_j}^2 Ln\left(a_{5ij}\right) = t^2 Ln(b_5) \\ \sum_{j=1}^{n} {y_j}^2 Ln\left(a_{6ij}\right) = t^2 Ln(b_6) \end{cases} \tag{27}$$

$$y_j \geq 0,\ \ t \geq 0$$

The problem in Eq. (27) is a nonlinear programming problem which can be solved by ordinal methods. This procedure can be applied to solve the MAGDM problem in Eq. (12).

5. Numerical Example

In this section, a numerical example problem has been solved using the proposed method to illustrate the applicability and efficiency of it.

Assume that there are four students $x1, x2, x3$ and $x4$ as the finalists after preliminary screening. These candidates evaluate to four attributes, academic records (A_1), college English test Band score (A_2), teamwork skills (A_3) and research potentials (A_4).

The above or any other scale can be used for subjective attributes. In the case of objective (quantitative) attributes, a method will be required to transform the exact quantitative values into IVNNs. Suppose that an objective attribute is evaluated as (100,150,120,90,80,70) for a set consist of four alternatives. This attribute vector is normalized by dividing its values to the maximum value of 150. The normalized vector is obtained as (0.7,1,0.8,0.6,0.5,0.4). Later, the DM can determine the desirability of these values according to the linguistic scale.

The IVNN assessments of DM on alternative performances regard to different criteria are shown in Table 1.

Table 1: Aggregated decision matrix

	A_1	A_2	A_3	A_4
$x1$	([0.1,0.2],[0.2,0.3],[0.4,0.5])	([0.2,0.4],[0.3,0.5],[0.1,0.2])	([0.3,0.4],[0.1,0.2],[0.3,0.5])	([0.1,0.3],[0. 3,0.4],[0.2,0.3])
$x2$	([0. 2,0.3],[0.2,0.5],[0.4,0.5])	([0.2,0.3],[0.2,0.6],[0.4,0.7])	([0.3,0.6],[0.1,0.2],[0.1,0.4])	([0.5,0.6],[0.4,0. 50],[0.1,0.3])
$x3$	([0.50.7],[0.2,0.3],[0.1,0.2])	([0.6,0.7],[0.3,0.4],[0.1,0.3])	([0.5,0.6],[0.3,0.4],[0.1,0.3])	([0.2,0. 5],[0.10.2],[0.4,0.5])
$x4$	([0.2,0.3],[0.1,0.5],[0.4,0.6])	([0,0.1],[0.4,0.6],[0.5,0.7])	([0.8,0.9],[0.3,0.4],[0.1,0,2])	([0.4,0.5],[0.3,0.7],[0.2,0.6])

Considering the alternative $x1$, based on Eq. (12), its MAGDM model is formulated as below:

$$Max\ ([0.1,0.2],[0.2,0.3],[0.4,0.5])w_1 + ([0.2,0.4],[0.3,0.5],[0.1,0.2])w_2 + ([0.3,0.4],[0.1,0.2],[0.3,0.5])w_3 + ([0.1,0.3],[0.3,0.4],[0.2,0.3])w_4$$

s.t.

$$([0.1,0.2],[0.2,0.3],[0.4,0.5]w_1 + ([0.2,0.4],[0.3,0.5],[0.1,0.2])w_2$$
$$+ ([0.3,0.4],[0.1,0.2],[0.3,0.5])w_3 + ([0.1,0.3],[0.3,0.4],[0.2,0.3])w_4)^2$$
$$= ([0.9,0.95],[0.01,0.05],[0.02,0.06])$$

$$(([0.2,0.3],[0.2,0.5],[0.4,0.5])w_1 + ([0.2,0.3],[0.2,0.6],[0.4,0.7])w_2$$
$$+ ([0.3,0.6],[0.1,0.2],[0.1,0.4])w_3 + ([0.5,0.6],[0.4,0.5],[0.1,0.3])w_4)^2$$
$$= ([0.9,0.95],[0.01,0.05],[0.02,0.06])$$

$$(([0.5,0.7],[0.2,0.3],[0.1,0.2])w_1 + ([0.6,0.7],[0.3,0.4],[0.1,0.3])w_2$$
$$+ ([0.5,0.6],[0.3,0.4],[0.1,0.3])w_3 + ([0.2,0.5],[0.1,0.2],[0.4,0.5])w_4)^2$$
$$= ([0.9,0.95],[0.01,0.05],[0.02,0.06])$$

$$(([0.2,0.3],[0.1,0.5],[0.4,0.6])w_1 + ([0,0.1],[0.4,0.6],[0.5,0.7])w_2$$
$$+ ([0.8,0.9],[0.3,0.4],[0.1,0.2])w_3 + ([0.4,0.5],[0.3,0.7],[0.2,0.6])w_4)^2$$
$$= ([0.9,0.95],[0.01,0.05],[0.02,0.06])$$

$$w_j \geq 0,\ j = 1,2,3,4$$

The above problem is an IVNN nonlinear programming problem solvable easily by transforming it into equivalent model of Eq. (27).

$$Min\ \ -4.75y_1 - 6.54y_2 - 6.67y_3 - 5.39y_4$$

s.t.

$$-0.1{y_1}^2 - 0.22{y_2}^2 - 0.35{y_3}^2 - 0.14{y_4}^2 = -2.30t^2$$

$$-0.22{y_1}^2 - 0.51{y_2}^2 - 0.51{y_3}^2 - 0.35{y_4}^2 = -2.99t^2$$

$$-1.6{y_1}^2 - 1.2{y_2}^2 - 2.3{y_3}^2 - 1.2{y_4}^2 = -4.6t^2$$

$$-1.2{y_1}^2 - 0.69{y_2}^2 - 1.6{y_3}^2 - 0.91{y_4}^2 = -2.99t^2$$

$$-0.91{y_1}^2 - 2.3{y_2}^2 - 1.2{y_3}^2 - 1.6{y_4}^2 = -3.91t^2$$

$$-0.69{y_1}^2 - 1.6{y_2}^2 - 0.69{y_3}^2 - 1.2{y_4}^2 = -2.81t^2$$

$$-0.22{y_1}^2 - 0.22{y_2}^2 - 0.35{y_3}^2 - 0.69{y_4}^2 = -2.3t^2$$

$$-0.35{y_1}^2 - 0.35y_2^2 - 0.91{y_3}^2 - 0.91{y_4}^2 = -2.99t^2$$

$$-1.6{y_1}^2 - 1.6{y_2}^2 - 2.3{y_3}^2 - 0.91{y_4}^2 = -4.6t^2$$

$$-0.69{y_1}^2 - 0.35{y_2}^2 - 1.6{y_3}^2 - 0.69{y_4}^2 = -2.99t^2$$

$$-0.91{y_1}^2 - 0.91{y_2}^2 - 2.3{y_3}^2 - 2.3{y_4}^2 = -3.91t^2$$

$$-0.69{y_1}^2 - 0.51{y_2}^2 - 0.91{y_3}^2 - 1.2{y_4}^2 = -2.81t^2$$

$$-0.69{y_1}^2 - 0.91{y_2}^2 - 0.69{y_3}^2 - 0.22{y_4}^2 = -2.30t^2$$

$$-1.2{y_1}^2 - 1.2{y_2}^2 - 0.91{y_3}^2 - 0.69{y_4}^2 = -2.99t^2$$

$$-0.95{y_1}^2 - 0.88{y_2}^2 - 1.15{y_3}^2 - 0.92{y_4}^2 = -4.61t^2$$

$$-1.2{y_1}^2 - 0.91{y_2}^2 - 0.91{y_3}^2 - 1.6{y_4}^2 = -2.99t^2$$

$$-2.3{y_1}^2 - 2.3{y_2}^2 - 2.3{y_3}^2 - 0.91{y_4}^2 = -3.91t^2$$

$$-1.6{y_1}^2 - 1.2{y_2}^2 - 1.2{y_3}^2 - 0.69{y_4}^2 = -2.81t^2$$

$$-1.2{y_1}^2 - 0.91{y_2}^2 - 0.91{y_3}^2 - 1.6{y_4}^2 = -2.99t^2$$

$$-3.5{y_1}^2 - 0.1{y_2}^2 - 2.3{y_3}^2 - 0.69{y_4}^2 = -2.99t^2$$

$$-2.3{y_1}^2 - 0.91{y_2}^2 - 1.2{y_3}^2 - 1.2{y_4}^2 = -4.6t^2$$

$$-0.69{y_1}^2 - 0.51{y_2}^2 - 0.91{y_3}^2 - 0.35{y_4}^2 = -2.99t^2$$

$$-0.91{y_1}^2 - 0.69{y_2}^2 - 2.3{y_3}^2 - 1.6{y_4}^2 = -3.91t^2$$

$$-0.51{y_1}^2 - 3.5{y_2}^2 - 1.6{y_3}^2 - 0.51{y_4}^2 = -2.81t^2$$

$$y_1, y_2, y_3, y_4 \geq 0, t \geq 0$$

By solving this model for each alternative, their ranking is determined as $x_3 > x_4 > x_1 > x_2$.

Table 2 presents the results obtained from solving model (27).

Table 2: objective values for alternative

$x1$	-0.821
$x2$	-0.837
$x3$	-0.106
$x4$	-0.795

6. Conclusion

In this paper we proposed a novel nonlinear programming-based model to solve MAGDM problems with IVNN information. The proposed formulation follows a strong logical context. This model solved a series of models iteratively and a final score is computed for each alternative. These scores are used to rank and to compare the set of alternatives. If there are a set of m alternatives to be compared, the proposed method includes solving m models, one for each alternative. Nevertheless, after formulating the problem for the first alternative, its objective function will be changed regarding to different alternatives and the feasible space remains unchangeably. Accordingly, it is only required to formulate m different objective functions with a similar set of constraints. Since the proposed model is a nonlinear programming problem with IVNN parameters, an approach is designed to solve this problem. The main advantage of the proposed method can be summarized as follows: there are increasing intends to apply mathematical optimization models in the context of MAGDM problems. With this fact in mind, the proposed method provides such a framework of decision making upon solving a series of optimization models. Second, the logic behind development of the proposed method is straightforward and acceptable. Maximizing the weighted average of each alternative while the scores restricted to be equal to one is very similar to the concepts of DEA as a well-known method. Third, the information requirements of the model is lower than other methods, since there isn't any necessity to specify attributes weights and the model itself determines them. Ultimately, while the applications of IVNN in MAGDM problems are wide, there is a narrow attention paid to mathematical programming with IVNN information. The proposed method can be generalized for solving nonlinear programming problems with IVNN parameters.

References

[1] C.L. Hwang, K.L. Yoon, Multiple Attribute Decision Making: Methods and Applications, Springer Verlag, New York, 1981.

[2] C.L. Hwang, M.J. Lin, Group Decision Making Under Multiple Criteria: Methods and Applications, Springer Verlag, New York, 1987.

[3] L.A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning-i, Inf. Sci. 8(1975) 199-249.

[4] S.J. Chen, C.L. Hwang, Fuzzy Multiple Attribute Decision Making, Springer Verlag Berlin Heidelberg, 1992.

[5] K.T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets Syst. 2. (1986) 87-96.

[6] F.E. Boran, Genc, M. Kurt, D. Akay, A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method, Expert Syst, Appl. 36(2009) 11363-11368.

[7] F.E. Boran, S. Genc, D. Akay, Personnel selection based on intuitionistic fuzzy sets, Hum. Factors Ergon. Manuf. Serv. Ind. 21 (2011) 493-503.

[8] F.E. Boran, An integrated intuitionistic fuzzy multi criteria decision making method for facility location selection, Math. Comput. Appl. 16(2011) 487-496.

[9] F.E. Boran, K. Boran, T. Menlik, The evaluation of renewable energy technologies for

electricity generation in turkey using intuitionistic fuzzy TOPSIS, Energy Sources Part B 7 (2012) 81-90.

[10] S. Genc, F.E. Boran, D. Akay, Z. Xu, Interval multiplicative transitivity for consistency, missing values and priority weights of interval fuzzy preference relations, Inf. Sci. 180(2010) 4877-4891.

[11] F.E. Boran, D. Akay, A biparametric similarity measure on intuitionistic fuzzy sets with applications to pattern recognition, Inf. Sci. 255(2014) 45-57.

[12] F. Shen, J. Xu, Z. Xu, An outranking sorting method for multi-criteria group decision making using intuitionistic fuzzy sets, Inf. Sci. 334-335(2016) 338-353.

[13] S.P. Wan, G.L. Xu, F. Wang, J. Y. Dong, A new method for Attanassov's interval-valued intuitionistic fuzzy MAGDM with incomplete attribute weight information, Inf. Sci. 316(2015) 329-347.

[14] D. F. Li, TOPSIS-based nonlinear-programming methodology for multiattribute decision making with interval-valued intuitionistic fuzzy sets, IEEE Transactions on Fuzzy Systems. 18(2) (2010a)299-311.

[15] D. F. Li, Linear programming method for MADM with interval-valued intuitionistic fuzzy sets, Expert Systems with Applications. 37(8) (2010b) 5939-5945.

[16] A. Hadi-Vencheh, An improvement to multiple criteria ABC inventory classification, European Journal of Operational Research. 201(2010) 962-965.

[17] F. Smarandache, A unifying field in logics. Neutrosophy; neutrosophic probability, set and logic. American Research Press, (1998) 01-157.

[18] F. Smarandache, Neutrosophy and neutrosophic logic. First international conference on neutrosophy, neutrosophic logic, set, probability and statistics, University of New Mexico, Gallup, USA, (2002) 01-147.

[19] F. Smarandache, Neutrosophic set, a generalization of the intuitionistic fuzzy sets. J New Theory,29. (2019),01-35.

[20] I. Deli, Y Subas, A ranking method of single valued neutrosophic numbers and its applications to multi-attribute decision making problems. Int J Mach Learn Cyb, (2017), 8:1309-1322.

[21] M. Abdel-Basset, M. Mohamed, The role of single valued neutrosophic sets and rough sets in smart city: Imperfect and incomplete information systems. Measurement, (2018), 124,47-55.

[22] S.A. Edalatpanah, Neutrosophic perspective on DEA. Journal of Applied Research on Industrial Engineering, (2018). 5(4), 339-345.

[23] H. Garg, Non-linear programming method for multi-criteria decision making problems under interval neutrosophic set environment. Applied Intelligence, (2018). 48(8), 2199-2213.

[24] P. Liu, L. Shi, The generalized hybrid weighted average operator based on interval neutrosophic hesitant set and its application to multiple attribute decision making. Neural Computing and Applications, (2015). 26(2),457-471.

[25] I. Deli, S. Yusuf, F. Smarandache, & M. Ali, Interval valued bipolar neutrosophic sets and their application in pattern recognition. In IEEE World Congress on Computational Intelligence. Vancouver, Canada: IEEE (2016).

[26] V. Ulucay, I. Deli, M. Sahin, Similarity measures of bipolar neutrosophic sets and their application to multiple criteria decision making. Neural Computing and Applications, (2018). 29(3), 739-748.

[27] P. Ji, H. Y. Zhang, J.Q. Wang, A projection-based TODIM method under multi-valued neutrosophic environments and its application in personal selection. Neural Computing and Applications, (2018). 29(1), 221-234.

[28] J.J Peng, J.Q. Wang, W.E. Yang, A multi-valued neutrosophic qualitative flexible approach based on are likelihood for multi criteria decision making problems. International Journal of Systems Science, (2017). 48(2), 425-435.

[29] S.A. Edalatpanah, F. Smarandache. Data envelopment analysis for simplified neutrosophic sets. Neutrosophic Sets and Systems. (2019) 29:215-226.

[30] J.J. Peng, J.Q. Wang, J. wang, H.Y. Zhang and X.H. Chen. Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems. International Journal of Systems Science. (2016). 47(10),2342-2358.

[31] S.A. Edalatpanah. Neutrosophic structured element. Expert Systems, (2020).37(5), 1-17.

[32] P. Biswas, S. Pramanik and B.C. Giri, Value and ambiguity index based ranking method of single-valued trapezoidal neutrosophic numbers and its application to multi-attribute decision making. Neutrosophic Sets Syst. (2016) 12:127-138

[33] R.X. Liang, J.Q. Wang and H.Y. Zhang, A multi criteria decision making method based on single valued trapezoidal neutrosophic preference relation with complete weight information. Neural Comput. Appl. (2018) 30:3383-3398.

[34] S.A. Edalatpanah. A Nonlinear Approach for Neutrosophic Linear Programming, Journal of Applied Research on Industrial Engineering, (2019). 6: 367-373.

[35] J. Ye, Similarity measures between interval neutrosophic sets and their applications in multicriteria decision- making. J Intell Fuzzy Syst, (2014). 26:165-172.

[36] S. Pramanik, K. Mondal, Interval neutrosophic multiattribute decision-making based on grey relational analysis. Neutr Sets Syst, (2015).9:13-22.

Hiighlights

A mathematical programming formulation of a multi attribute group decision making problem is proposed.

The model is developed for decision making with interval valued neutrosophic fuzzy information.

An approach is extended to solve the proposed mathematical programming model.

The results shown an acceptable consistency with previously presented methods.

Abstract

The purpose of this paper is to present a new technique to solve multiple attribute group decision making (MAGDM) problems with interval valued neutrosophic numbers (IVNN). The proposed approach solves the MAGDM problem using mathematical programming methodology. To reinforce the mathematical programming approach in IVNN environment, an IVNN multiple attribute group decision making problem is formulated as a non-linear programming model which can be solved easily. The proposed approach helps decision maker (DM) to find an alternative which has best performance. The main advantage of the proposed approach is that decision making process does not rely on subjective weights of attributes. Moreover, the proposed methodology is easy and simple to understand.

Declaration of Interest Statement

Conflict of interest: None