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Interval-Valued Bipolar Trapezoidal Neutrosophic Number Approach in Distribution Planning Problem

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Abstract: An interval valued bipolar trapezoidal neutrosophic set [IVBTrNS] is a special neutrosophic set on the set of real numbers R, is a new generalization of bipolar fuzzy sets, neutrosophic sets, interval valued neutrosophic set, and bipolar neutrosophic sets, so that it can handle uncertain information more flexibly in the optimization. Distribution planning is a process in which we study the way to get materials and distribute the product from delivery point to the consuming point after production planning in supply chain. In the present research article, we propose concept of interval valued bipolar trapezoidal neutrosophic number [IVBTrNN] and its operations in the fully neutrosophic transportation problem [FNTP], where neutrosophic variable are required to be equal either 0 or 1. During the covid-19 pandemic, to maintain physical distance among, human, used & unused equipments and researchers, in place of crisp numbers, the interval-valued fuzzy numbers [IVFNs] are much effective to address the uncertainty & hesitation in real world situations. To save the human lives in a covid-19 pandemic, the crisp cost, demand, and crisp supply in transportation problem are not so effective in compression of neutrosophic numbers. The use of IVBTrNN in place of crisp number, are more suitable to distribute the necessary equipments, medicines, food products, and other relevant items from one place to another. To understand the practical applications of interval-valued neutrosophic numbers [IVNNs], a numerical of FNTP and conclusion also the part of this paper for better execution in support of our proposed result & methodology with IVBTrNNs.

Keywords: *Interval Valued Bipolar Trapezoidal Neutrosophic Number, Fully Neutrosophic Transportation Problem, Interval Valued Fuzzy Numbers*

1. Introduction

Due to some vague information, inexact perception and environmental factors, the parameters of integer linear programming problem are essentials. To handle such type of uncertainty, Zadeh in 1965 introduced the concept of fuzzy set, by which the researchers can check the uncertainty in engineering, industrial, distribution and management problems [1, 2, 3, 4]. Since the fuzzy set is not absolute suitable to observe the uncertainty and hesitation, so to reduce the such problem, an extension of fuzzy set introduced by Atanassov in 1986 in the form of intuitionistic fuzzy set IFS [5] with membership and non-membership degrees. Detailed applications of IFS are in [6-11]. To reinforce and holding the uncertainty and hesitation in IFS Atanassov and Gargov in 1989 generalized the IFS by introducing the interval-valued IFS [12]. In the real world complication of inconsistent and uncertainty, the FS and IFS are unable to handle the situation, so to overcome such type of problem, Smarandache in 1988 introduced the concept of neutrosophic set [NS] [13, 14, 15]. A NS has the inconsistent data in the form of truth, indeterminacy and falsity membership degrees respectively. Due to the uncertain data of truth, indeterminacy and falsity in NS, the practical applications and some applied recommendation in real-world problems have some hurdles. To maintain the problem, Wang et. al. in 2010 introduced the notion of a single-valued neutrosophic set [SVNS][16].

To represent specify and non-specify issues in real-world problems, Samrandache in 2015 proposed the interval function [17]. For more uncertain linear programming problems (see [18-25] and references therein).

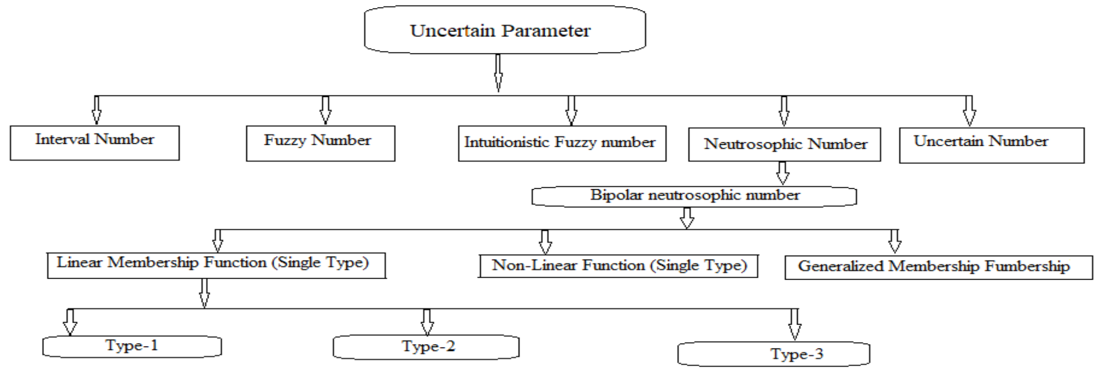
Lee in 2000 introduced the concept of bipolar fuzzy set, which was the extension of FS [26]. According Bosc and Pivert, bipolarity represents the tendency of human mind to reason and make decision on the basis of pessimistic and optimistic outcomes [27]. Optimistic information reflects, what is permitted, desirable, satisfactory or acceptable, while a pessimistic statement reflects, what is impossible, non-reachable, revertible or forbidden. The values or objects that are to be rejected or not satisfied the constraints are corresponds to negative preference, while positive preference corresponds user wishes, which are more acceptable than others. For more rapidly developments in bipolar fuzzy and neutrosophic set and its operation see ([28-35] and references therein).

In this FNTP, all parameters such as capital budgeting, fixed cost, distribution system and product market share are in the form of IVBTrNNs by using 0-1 variables to maintain the indeterminacy. For penetrating the quality solution of FNTP, there exists truth, indeterminacy and falsity membership function. The aim of this study is to design a fully neutrosophic mathematical model of distribution system that determines only economical and best site selection that provide minimum transportation cost for shipping the products to the issuing nodes in unsettled domain of supply chain. A layout uncertainty plan shown in figure 1.

$$\tilde{A}_{NS}^{IV}(x)=\left\{\left\langle x,\left\{\begin{matrix}\left[\inf T_{\tilde{A}_{NS}^{IV}}(x),\sup T_{\tilde{A}_{NS}^{IV}}(x)\right],\\ \left[\inf I_{\tilde{A}_{NS}^{IV}}(x),\sup I_{\tilde{A}_{NS}^{IV}}(x)\right],\\ \left[\inf F_{\tilde{A}_{NS}^{IV}}(x),\sup F_{\tilde{A}_{NS}^{IV}}(x)\right]\end{matrix}\right\}\right\rangle : x\in X\right\}$$

$$\tilde{B}_{NS}^{IV}(x)=\left\{\left\langle x,\left\{\begin{matrix}\left[\inf T_{\tilde{B}_{NS}^{IV}}(x),\sup T_{\tilde{B}_{NS}^{IV}}(x)\right],\\ \left[\inf I_{\tilde{B}_{NS}^{IV}}(x),\sup I_{\tilde{B}_{NS}^{IV}}(x)\right],\\ \left[\inf F_{\tilde{B}_{NS}^{IV}}(x),\sup F_{\tilde{B}_{NS}^{IV}}(x)\right]\end{matrix}\right\}\right\rangle : x\in X\right\}$$

Figure 1: Uncertainty plan



2. Preliminaries

In this study, to handling some uncertainties in fuzzy sets and neutrosophic sets, the extensions of fuzzy sets [1], bipolar fuzzy sets [30], neutrosophic sets [14] and bipolar neutrosophic sets [35], interval valued bipolar fuzzy weighted neutrosophic sets with application are introduced.

Definition 2.1. [16]: Let X be a universe of discourse. Then a single valued neutrosophic set is defined as:

$$\tilde{A}_{NS}=\left\{\left\langle x,T_{\tilde{A}_{NS}}(x),I_{\tilde{A}_{NS}}(x),F_{\tilde{A}_{NS}}(x)\right\rangle : x\in X\right\}$$

which is characterized by a truth-membership function $T_{\tilde{A}_{NS}}(x):X\to[0,1]$ an indeterminacy-membership function $I_{\tilde{A}_{NS}}(x):X\to[0,1]$ and a falsity-membership function $F_{\tilde{A}_{NS}}(x):X\to[0,1]$ there is not restriction on the sum of $T_{\tilde{A}_{NS}}(x)$, $I_{\tilde{A}_{NS}}(x)$ and $F_{\tilde{A}_{NS}}(x)$ so $0\le T_{\tilde{A}_{NS}}(x)+I_{\tilde{A}_{NS}}(x)+F_{\tilde{A}_{NS}}(x)\le 3$.

Definition 2.2. [17]: Let X be a space of points with generic elements x in X. An interval-valued neutrosophic set [IVNS] $\tilde{A}_{NS}^{IV}$ in X, is characterized by truth-membership function $T_{\tilde{A}_{NS}^{IV}}(x)$ indeteminacy-membership function $I_{\tilde{A}_{NS}^{IV}}(x)$ and falsity-membership function $F_{\tilde{A}_{NS}^{IV}}(x)$. For each point x in X, we have that $T_{\tilde{A}_{NS}^{IV}}(x),I_{\tilde{A}_{NS}^{IV}}(x),F_{\tilde{A}_{NS}^{IV}}(x)\subseteq[0,1]$. For two IVNS

$$\text{(i)}\quad \tilde{A}_{NS}^{IV}(x)\subseteq\tilde{B}_{NS}^{IV}(x)\Leftrightarrow\left\{\begin{matrix} x,\left\langle\begin{matrix}\left[\begin{matrix}\inf T_{\tilde{A}_{NS}^{IV}}(x)\le\inf T_{\tilde{B}_{NS}^{IV}}(x),\\ \sup T_{\tilde{A}_{NS}^{IV}}(x)\le\sup T_{\tilde{B}_{NS}^{IV}}(x)\end{matrix}\right],\\ \left[\begin{matrix}\inf I_{\tilde{A}_{NS}^{IV}}(x)\ge\inf I_{\tilde{B}_{NS}^{IV}}(x),\\ \sup I_{\tilde{A}_{NS}^{IV}}(x)\ge\sup I_{\tilde{B}_{NS}^{IV}}(x)\end{matrix}\right],\\ \left[\begin{matrix}\inf F_{\tilde{A}_{NS}^{IV}}(x)\ge\inf F_{\tilde{B}_{NS}^{IV}}(x),\\ \sup F_{\tilde{A}_{NS}^{IV}}(x)\ge\sup F_{\tilde{B}_{NS}^{IV}}(x)\end{matrix}\right]\end{matrix}\right\rangle\\ : x\in X\end{matrix}\right)$$

$$\text{(ii)}\quad \tilde{A}_{NS}^{IV}(x)=\tilde{B}_{NS}^{IV}(x)\Leftrightarrow\left\{\begin{matrix} x,\left\langle\begin{matrix}\left[\begin{matrix}\inf T_{\tilde{A}_{NS}^{IV}}(x)=\inf T_{\tilde{B}_{NS}^{IV}}(x),\\ \sup T_{\tilde{A}_{NS}^{IV}}(x)=\sup T_{\tilde{B}_{NS}^{IV}}(x)\end{matrix}\right]\\ \left[\begin{matrix}\inf I_{\tilde{A}_{NS}^{IV}}(x)=\inf I_{\tilde{B}_{NS}^{IV}}(x),\\ \sup I_{\tilde{A}_{NS}^{IV}}(x)=\sup I_{\tilde{B}_{NS}^{IV}}(x)\end{matrix}\right],\\ \left[\begin{matrix}\inf F_{\tilde{A}_{NS}^{IV}}(x)=\inf F_{\tilde{B}_{NS}^{IV}}(x),\\ \sup F_{\tilde{A}_{NS}^{IV}}(x)=\sup F_{\tilde{B}_{NS}^{IV}}(x)\end{matrix}\right]\end{matrix}\right\rangle\\ \forall x\in X\end{matrix}\right\}$$

$$\text{(iii)}\quad \tilde{A}_{NS}^{IV^c}(x)=\left\{x,\left\langle\begin{matrix}\left[\inf T_{\tilde{A}_{NS}^{IV^c}}(x),\ \sup T_{\tilde{A}_{NS}^{IV^c}}(x)\right],\\ \left[\inf I_{\tilde{A}_{NS}^{IV^c}}(x),\ \sup I_{\tilde{A}_{NS}^{IV^c}}(x)\right],\\ \left[\inf F_{\tilde{A}_{NS}^{IV^c}}(x),\ \sup F_{\tilde{A}_{NS}^{IV^c}}(x)\right]\end{matrix}\right\rangle : x\in X\right\}$$

$$\text{(iv)}\quad \tilde{A}_{NS}^{IV}(x)\cap\tilde{B}_{NS}^{IV}(x)\Leftrightarrow\left\{\begin{array}{l} x,\left\langle\begin{array}{l}\left[\begin{array}{l}\inf T_{\tilde{A}_{NS}^{IV}}(x)\wedge\inf T_{\tilde{B}_{NS}^{IV}}(x),\\ \sup T_{\tilde{A}_{NS}^{IV}}(x)\wedge\sup T_{\tilde{B}_{NS}^{IV}}(x)\end{array}\right],\\ \left[\begin{array}{l}\inf I_{\tilde{A}_{NS}^{IV}}(x)\vee\inf I_{\tilde{B}_{NS}^{IV}}(x),\\ \sup I_{\tilde{A}_{NS}^{IV}}(x)\vee\sup I_{\tilde{B}_{NS}^{IV}}(x)\end{array}\right],\\ \left[\begin{array}{l}\inf F_{\tilde{A}_{NS}^{IV}}(x)\vee\inf F_{\tilde{B}_{NS}^{IV}}(x),\\ \sup F_{\tilde{A}_{NS}^{IV}}(x)\vee\sup F_{\tilde{B}_{NS}^{IV}}(x)\end{array}\right]\end{array}\right\rangle\\ :x\in X\end{array}\right\}$$

$$\text{(v)}\quad \tilde{A}_{NS}^{IV}(x)\cup\tilde{B}_{NS}^{IV}(x)\Leftrightarrow\left\{\begin{array}{l} x,\left\langle\begin{array}{l}\left[\begin{array}{l}\inf T_{\tilde{A}_{NS}^{IV}}(x)\vee\inf T_{\tilde{B}_{NS}^{IV}}(x),\\ \sup T_{\tilde{A}_{NS}^{IV}}(x)\vee\sup T_{\tilde{B}_{NS}^{IV}}(x)\end{array}\right],\\ \left[\begin{array}{l}\inf I_{\tilde{A}_{NS}^{IV}}(x)\wedge\inf I_{\tilde{B}_{NS}^{IV}}(x),\\ \sup I_{\tilde{A}_{NS}^{IV}}(x)\wedge\sup I_{\tilde{B}_{NS}^{IV}}(x)\end{array}\right],\\ \left[\begin{array}{l}\inf F_{\tilde{A}_{NS}^{IV}}(x)\wedge\inf F_{\tilde{B}_{NS}^{IV}}(x),\\ \sup F_{\tilde{A}_{NS}^{IV}}(x)\wedge\sup F_{\tilde{B}_{NS}^{IV}}(x)\end{array}\right]\end{array}\right\rangle\\ :x\in X\end{array}\right\}$$

Definition 2.3. [31]: Let X be a non-empty set. Then, a bipolar valued fuzzy set, denoted by $\tilde{A}_{bi}$ is defined as;

$$\tilde{A}_{bi}(x)=\left\{\left\langle x,\mu_{bi}^{+}(x),\mu_{bi}^{-}(x)\right\rangle:x\in X\right\}$$

Where: $\mu_{bi}^{+}(x):X\to[0,1]$ and $\mu_{bi}^{-}(x):X\to[0,1]$. The positive membership degree $\mu_{bi}^{+}(x)$ denotes the satisfaction degree of an element x to the property corresponding to $\tilde{A}_{bi}$ and the negative membership degree $\mu_{bi}^{-}(x)$ denotes the satisfaction degree of x to some implicit counter property of $\tilde{A}_{bi}$.

Definition 2.4. [36]: Let X be a fixed set. A bipolar neutrosophic set $\tilde{A}_{bi}^{N}$ in X is defined as

$$\tilde{A}_{bi}^{N}(x)=\left\{\left\langle x,T_{\tilde{A}_{bi}^{N}}^{+}(x),I_{\tilde{A}_{bi}^{N}}^{+}(x),F_{\tilde{A}_{bi}^{N}}^{+}(x),T_{\tilde{A}_{bi}^{N}}^{-}(x),I_{\tilde{A}_{bi}^{N}}^{-}(x),F_{\tilde{A}_{bi}^{N}}^{-}(x)\right\rangle:x\in X\right\}$$

where $T_{\tilde{A}_{bi}^{N}}^{+},I_{\tilde{A}_{bi}^{N}}^{+},F_{\tilde{A}_{bi}^{N}}^{+}:X\to[0,1]$ and $T_{\tilde{A}_{bi}^{N}}^{-},I_{\tilde{A}_{bi}^{N}}^{-},F_{\tilde{A}_{bi}^{N}}^{-}:X\to[0,1]$. The positive membership degree $T_{\tilde{A}_{bi}^{N}}^{+},I_{\tilde{A}_{bi}^{N}}^{+},F_{\tilde{A}_{bi}^{N}}^{+}$ denotes the truth membership, indeterminate membership and false membership of an element $x\in X$ corresponding to a bipolar neutrosophic set $\tilde{A}_{bi}^{N}$ and the negative membership degree $T_{\tilde{A}_{bi}^{N}}^{-},I_{\tilde{A}_{bi}^{N}}^{-},F_{\tilde{A}_{bi}^{N}}^{-}$ denotes the truth membership, indeterminate membership and false membership of an element $x\in X$ to some implicit counter-property corresponding to a bipolar neutrosophic set $\tilde{A}_{bi}^{N}$. If X has only one element then bipolar neutrosophic set [BNS] becomes bipolar neutrosophic number [BNN] and denoted as:

$\tilde{A}_{bi}^{N}(x)=\left\{x,\left\langle T_{\tilde{A}_{bi}^{N}}^{+}(x),I_{\tilde{A}_{bi}^{N}}^{+}(x),F_{\tilde{A}_{bi}^{N}}^{+}(x),T_{\tilde{A}_{bi}^{N}}^{-}(x),I_{\tilde{A}_{bi}^{N}}^{-}(x),F_{\tilde{A}_{bi}^{N}}^{-}(x)\right\rangle\right\}$. The algebraic operation on BNN as follows:

Definition 2.5. [36]: Let

$$\tilde{A}_{bi}^{N}(x)=\left\{\left\langle x,T_{\tilde{A}_{bi}^{N}}^{+}(x),I_{\tilde{A}_{bi}^{N}}^{+}(x),F_{\tilde{A}_{bi}^{N}}^{+}(x),T_{\tilde{A}_{bi}^{N}}^{-}(x),I_{\tilde{A}_{bi}^{N}}^{-}(x),F_{\tilde{A}_{bi}^{N}}^{-}(x)\right\rangle:x\in X\right\}$$

and

$$\tilde{B}_{bi}^{N}(x)=\left\{\left\langle x,T_{\tilde{B}_{bi}^{N}}^{+}(x),I_{\tilde{B}_{bi}^{N}}^{+}(x),F_{\tilde{B}_{bi}^{N}}^{+}(x),T_{\tilde{B}_{bi}^{N}}^{-}(x),I_{\tilde{B}_{bi}^{N}}^{-}(x),F_{\tilde{B}_{bi}^{N}}^{-}(x)\right\rangle:x\in X\right\}$$

be two BNNs. If

(a) $\tilde{A}_{bi}^{N}\subseteq\tilde{B}_{bi}^{N}$ then $T_{\tilde{A}_{bi}^{N}}^{+}(x)\le T_{\tilde{B}_{bi}^{N}}^{+}(x)$, $I_{\tilde{A}_{bi}^{N}}^{+}(x)\le I_{\tilde{B}_{bi}^{N}}^{+}(x)$, $F_{\tilde{A}_{bi}^{N}}^{+}(x)\ge F_{\tilde{B}_{bi}^{N}}^{+}(x)$ and $T_{\tilde{A}_{bi}^{N}}^{-}(x)\ge T_{\tilde{B}_{bi}^{N}}^{-}(x)$, $I_{\tilde{A}_{bi}^{N}}^{-}(x)\ge I_{\tilde{B}_{bi}^{N}}^{-}(x)$, $F_{\tilde{A}_{bi}^{N}}^{-}(x)\le F_{\tilde{B}_{bi}^{N}}^{-}(x)$ for all $x\in X$.

(b) If $\tilde{A}_{bi}^{N}(x)=\tilde{B}_{bi}^{N}(x)$ then $T_{\tilde{A}_{bi}^{N}}^{+}(x)=T_{\tilde{B}_{bi}^{N}}^{+}(x)$, $I_{\tilde{A}_{bi}^{N}}^{+}(x)=I_{\tilde{B}_{bi}^{N}}^{+}(x)$ $F_{\tilde{A}_{bi}^{N}}^{+}(x)=F_{\tilde{B}_{bi}^{N}}^{+}(x)$ and $T_{\tilde{A}_{bi}^{N}}^{-}(x)=T_{\tilde{B}_{bi}^{N}}^{-}(x)$, $I_{\tilde{A}_{bi}^{N}}^{-}(x)=I_{\tilde{B}_{bi}^{N}}^{-}(x)$, $F_{\tilde{A}_{bi}^{N}}^{-}(x)=F_{\tilde{B}_{bi}^{N}}^{-}(x)$ for all $x\in X$.

$$\text{(c)}\quad \tilde{A}_{bi}^{N}(x)\cup\tilde{B}_{bi}^{N}(x)=\left\{\begin{array}{l} x,\left\langle\begin{array}{l}\left[\begin{array}{l}\max\left\{T_{\tilde{A}_{bi}^{N}}^{+}(x),T_{\tilde{B}_{bi}^{N}}^{+}(x)\right\},\\ \dfrac{I_{\tilde{A}_{bi}^{N}}^{+}(x),I_{\tilde{B}_{bi}^{N}}^{+}(x)}{2},\\ \min\left\{F_{\tilde{A}_{bi}^{N}}^{+}(x),F_{\tilde{B}_{bi}^{N}}^{+}(x)\right\}\end{array}\right],\\ \left[\begin{array}{l}\min\left\{T_{\tilde{A}_{bi}^{N}}^{-}(x),T_{\tilde{B}_{bi}^{N}}^{-}(x)\right\},\\ \dfrac{I_{\tilde{A}_{bi}^{N}}^{-}(x),I_{\tilde{B}_{bi}^{N}}^{-}(x)}{2},\\ \max\left\{F_{\tilde{A}_{bi}^{N}}^{-}(x),F_{\tilde{B}_{bi}^{N}}^{-}(x)\right\}\end{array}\right]\end{array}\right\rangle\\ \text{for all }x\in X.\end{array}\right\}$$

$$\text{(d)}\quad \tilde{A}_{bi}^{N}(x)\cap\tilde{B}_{bi}^{N}(x)=\left\{\begin{array}{l} x,\left\langle\begin{array}{l}\left[\begin{array}{l}\min\left\{T_{\tilde{A}_{bi}^{N}}^{+}(x),T_{\tilde{B}_{bi}^{N}}^{+}(x)\right\},\\ \dfrac{I_{\tilde{A}_{bi}^{N}}^{+}(x),I_{\tilde{B}_{bi}^{N}}^{+}(x)}{2},\\ \max\left\{F_{\tilde{A}_{bi}^{N}}^{+}(x),F_{\tilde{B}_{bi}^{N}}^{+}(x)\right\}\end{array}\right],\\ \left[\begin{array}{l}\max\left\{T_{\tilde{A}_{bi}^{N}}^{-}(x),T_{\tilde{B}_{bi}^{N}}^{-}(x)\right\},\\ \dfrac{I_{\tilde{A}_{bi}^{N}}^{-}(x),I_{\tilde{B}_{bi}^{N}}^{-}(x)}{2},\\ \min\left\{F_{\tilde{A}_{bi}^{N}}^{-}(x),F_{\tilde{B}_{bi}^{N}}^{-}(x)\right\}\end{array}\right]\end{array}\right\rangle\\ \text{for all }x\in X.\end{array}\right\}$$

$$\text{(e)}\quad \tilde{A}_{bi}^{N^{c}}(x)=\left\{x,\left\langle\begin{array}{l}T_{\tilde{A}_{bi}^{N^{c}}}^{+}(x),I_{\tilde{A}_{bi}^{N^{c}}}^{+}(x),F_{\tilde{A}_{bi}^{N^{c}}}^{+}(x),\\ T_{\tilde{A}_{bi}^{N^{c}}}^{-}(x),I_{\tilde{A}_{bi}^{N^{c}}}^{-}(x),F_{\tilde{A}_{bi}^{N^{c}}}^{-}(x)\end{array}\right\rangle:x\in X\right\}$$

where $T_{\tilde{A}_{bi}^{N^{c}}}^{+}(x)=\{1^{+}\}-T_{\tilde{A}_{bi}^{N}}^{+}(x)$, $I_{\tilde{A}_{bi}^{N^{c}}}^{+}(x)=\{1^{+}\}-I_{\tilde{A}_{bi}^{N}}^{+}(x)$ and $F_{\tilde{A}_{bi}^{N^{c}}}^{+}(x)=\{1^{+}\}-F_{\tilde{A}_{bi}^{N}}^{+}(x)$ for all $x\in X$.

$$\text{(f)}\quad \lambda\tilde{A}_{bi}^{N}(x)=\left\{x,\left\langle\begin{array}{l}1-(1-T_{\tilde{A}_{bi}^{N}}^{+})^{\lambda},(I_{\tilde{A}_{bi}^{N}}^{+})^{\lambda},(F_{\tilde{A}_{bi}^{N}}^{+})^{\lambda},\\ -(-T_{\tilde{A}_{bi}^{N}}^{-})^{\lambda},-(-I_{\tilde{A}_{bi}^{N}}^{-})^{\lambda},-(1-(1-(-F_{\tilde{A}_{bi}^{N}}^{-})))^{\lambda}\end{array}\right\rangle\right\}$$

$$\text{(g)}\quad (\tilde{A}_{bi}^{N}(x))^{\lambda}=\left\{x,\left\langle\begin{array}{l}(T_{\tilde{A}_{bi}^{N}}^{+})^{\lambda},1-(1-I_{\tilde{A}_{bi}^{N}}^{+})^{\lambda},1-(1-F_{\tilde{A}_{bi}^{N}}^{+})^{\lambda},\\ -(1-(1-(-T_{\tilde{A}_{bi}^{N}}^{-})))^{\lambda},-(-I_{\tilde{A}_{bi}^{N}}^{-})^{\lambda},-(-F_{\tilde{A}_{bi}^{N}}^{-})^{\lambda}\end{array}\right\rangle\right\}$$

$$\text{(h)}\quad \tilde{A}_{bi}^{N}(x)+\tilde{B}_{bi}^{N}(x)=\left\{x,\left\langle\begin{array}{l}\left[\begin{array}{l}\left(T_{\tilde{A}_{bi}^{N}}^{+}+T_{\tilde{B}_{bi}^{N}}^{+}-T_{\tilde{A}_{bi}^{N}}^{+}.T_{\tilde{B}_{bi}^{N}}^{+}\right),\\ I_{\tilde{A}_{bi}^{N}}^{+}.I_{\tilde{B}_{bi}^{N}}^{+},F_{\tilde{A}_{bi}^{N}}^{+}.F_{\tilde{B}_{bi}^{N}}^{+},-T_{\tilde{A}_{bi}^{N}}^{-}.T_{\tilde{B}_{bi}^{N}}^{-}\end{array}\right],\\ \left[\begin{array}{l}-\left(-I_{\tilde{A}_{bi}^{N}}^{-}-I_{\tilde{B}_{bi}^{N}}^{-}-I_{\tilde{A}_{bi}^{N}}^{-}.I_{\tilde{B}_{bi}^{N}}^{-}\right),\\ -\left(-F_{\tilde{A}_{bi}^{N}}^{-}-F_{\tilde{B}_{bi}^{N}}^{-}-F_{\tilde{A}_{bi}^{N}}^{-}.F_{\tilde{B}_{bi}^{N}}^{-}\right)\end{array}\right]\end{array}\right\rangle\right\}$$

$$(i)\quad \tilde{A}^N_{bi}(x).\tilde{B}^N_{bi}(x)=\left\{x,\left\langle \begin{matrix} \left[\begin{matrix} T^+_{\tilde{A}^N_{bi}}.T^+_{\tilde{B}^N_{bi}},\left(I^+_{\tilde{A}^N_{bi}}+I^+_{\tilde{B}^N_{bi}}-I^+_{\tilde{A}^N_{bi}}.I^+_{\tilde{B}^N_{bi}}\right),\\ \left(F^+_{\tilde{A}^N_{bi}}+F^+_{\tilde{B}^N_{bi}}-F^+_{\tilde{A}^N_{bi}}.F^+_{\tilde{B}^N_{bi}}\right)\end{matrix}\right],\\ \left[\begin{matrix} -\left(-T^-_{\tilde{A}^N_{bi}}-T^-_{\tilde{B}^N_{bi}}-T^-_{\tilde{A}^N_{bi}}.T^-_{\tilde{B}^N_{bi}}\right),\\ I^-_{\tilde{A}^N_{bi}}.I^-_{\tilde{B}^N_{bi}},F^-_{\tilde{A}^N_{bi}}.F^-_{\tilde{B}^N_{bi}}\end{matrix}\right]\end{matrix}\right\rangle\right\}$$

Definition 2.7. [36]: Let $\tilde{A}^N_{bi}=\left\{x,\left\langle\begin{matrix}T^+_{\tilde{A}^N_{bi}}(x),I^+_{\tilde{A}^N_{bi}}(x),F^+_{\tilde{A}^N_{bi}}(x),\\ T^-_{\tilde{A}^N_{bi}}(x),I^-_{\tilde{A}^N_{bi}}(x),F^-_{\tilde{A}^N_{bi}}(x)\end{matrix}\right\rangle:x\in X\right\}$ be a BNN. Then, the score function $s\left(\tilde{A}^N_{bi}\right)$, accuracy function $a\left(\tilde{A}^N_{bi}\right)$ and certainty function $c\left(\tilde{A}^N_{bi}\right)$ of an bipolar neutrosophic number are defined as follows:

$$s\left(\tilde{A}^N_{bi}\right)=\frac{T^+_{\tilde{A}^N_{bi}}+(1-I^+_{\tilde{A}^N_{bi}})+(1-F^+_{\tilde{A}^N_{bi}})+(1+T^-_{\tilde{A}^N_{bi}})-I^-_{\tilde{A}^N_{bi}}-F^-_{\tilde{A}^N_{bi}}}{6},$$

$$a\left(\tilde{A}^N_{bi}\right)=T^+_{\tilde{A}^N_{bi}}-F^+_{\tilde{A}^N_{bi}}+T^-_{\tilde{A}^N_{bi}}-F^-_{\tilde{A}^N_{bi}} \text{ and } c\left(\tilde{A}^N_{bi}\right)=T^+_{\tilde{A}^N_{bi}}-F^-_{\tilde{A}^N_{bi}}.$$

3. Interval Valued Bipolar Fuzzy Neutrosophic Set

Interval valued bipolar fuzzy neutrosophic set [IVBFNS] and its operations are as follows:

Definition 3.1. An IVBFNS is denoted as $\tilde{A}^{IVN}_{bi}$ in X and defined as

$$\tilde{A}^{IVN}_{bi}=\left[\left\{x,\left\langle\begin{matrix}T^+_{\tilde{A}^{IVN}_{bi}}(x),I^+_{\tilde{A}^{IVN}_{bi}}(x),F^+_{\tilde{A}^{IVN}_{bi}}(x),\\ T^-_{\tilde{A}^{IVN}_{bi}}(x),I^-_{\tilde{A}^{IVN}_{bi}}(x),F^-_{\tilde{A}^{IVN}_{bi}}(x)\end{matrix}\right\rangle\right\}:x\in X\right].$$

Here $T^+_{\tilde{A}^{IVN}_{bi}}(x)=\left[T^{L^+}_{\tilde{A}^{IVN}_{bi}}(x),T^{R^+}_{\tilde{A}^{IVN}_{bi}}(x)\right]$, $I^+_{\tilde{A}^{IVN}_{bi}}(x)=\left[I^{L^+}_{\tilde{A}^{IVN}_{bi}}(x),I^{R^+}_{\tilde{A}^{IVN}_{bi}}(x)\right]$ $F^+_{\tilde{A}^{IVN}_{bi}}(x)=\left[F^{L^+}_{\tilde{A}^{IVN}_{bi}}(x),F^{R^+}_{\tilde{A}^{IVN}_{bi}}(x)\right]$, where $T^{L^+}_{\tilde{A}^{IVN}_{bi}},T^{R^+}_{\tilde{A}^{IVN}_{bi}},I^{L^+}_{\tilde{A}^{IVN}_{bi}},I^{R^+}_{\tilde{A}^{IVN}_{bi}},F^{L^+}_{\tilde{A}^{IVN}_{bi}},F^{R^+}_{\tilde{A}^{IVN}_{bi}}:X\to[0,1]$ and $T^{L^-}_{\tilde{A}^{IVN}_{bi}},T^{R^-}_{\tilde{A}^{IVN}_{bi}},I^{L^-}_{\tilde{A}^{IVN}_{bi}},I^{R^-}_{\tilde{A}^{IVN}_{bi}},F^{L^-}_{\tilde{A}^{IVN}_{bi}},F^{R^-}_{\tilde{A}^{IVN}_{bi}}:X\to[0,1]$. If X has only one element then IVBFNS becomes interval valued bipolar fuzzy neutrosophic number [IVBFNN] and denoted as

$$\tilde{A}^{IVN}_{bi}=\left\{x,\left\langle\begin{matrix}\left[T^{L^+}_{\tilde{A}^{IVN}_{bi}}(x),T^{R^+}_{\tilde{A}^{IVN}_{bi}}(x)\right],\left[I^{L^+}_{\tilde{A}^{IVN}_{bi}}(x),I^{R^+}_{\tilde{A}^{IVN}_{bi}}(x)\right],\\ \left[F^{L^+}_{\tilde{A}^{IVN}_{bi}}(x),F^{R^+}_{\tilde{A}^{IVN}_{bi}}(x)\right],\\ \left[T^{L^-}_{\tilde{A}^{IVN}_{bi}}(x),T^{R^-}_{\tilde{A}^{IVN}_{bi}}(x)\right],\left[I^{L^-}_{\tilde{A}^{IVN}_{bi}}(x),I^{R^-}_{\tilde{A}^{IVN}_{bi}}(x)\right],\\ \left[F^{L^-}_{\tilde{A}^{IVN}_{bi}}(x),F^{R^-}_{\tilde{A}^{IVN}_{bi}}(x)\right]\end{matrix}\right\rangle\right\}$$

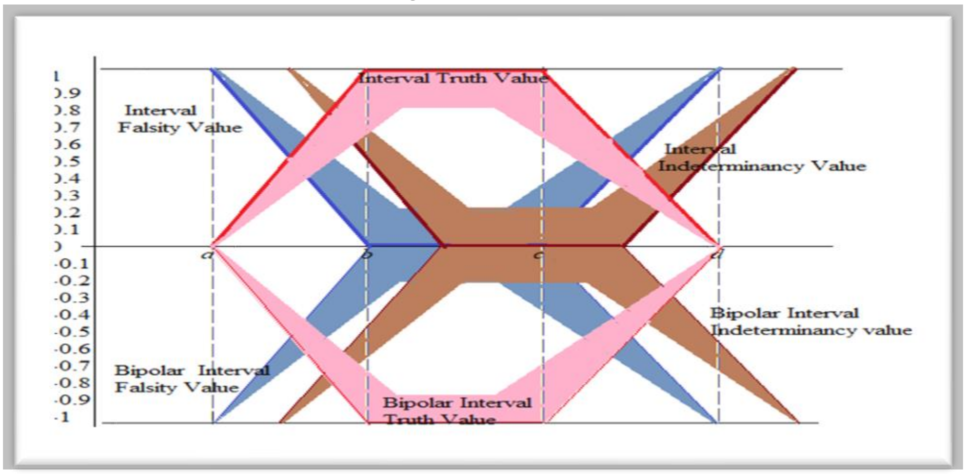
Definition 3.2. An interval valued bipolar trapezoidal neutrosophic number [IVBTrNN] is a special neutrosophic set on the set of real numbers R defined as:

$$\tilde{A}^{IVN}_{bi}=\left\{(a,b,c,d),\left\langle\begin{matrix}\left[T^{L^+}_{\tilde{A}^{IVN}_{bi}}(x),T^{R^+}_{\tilde{A}^{IVN}_{bi}}(x)\right],\left[I^{L^+}_{\tilde{A}^{IVN}_{bi}}(x),I^{R^+}_{\tilde{A}^{IVN}_{bi}}(x)\right],\\ \left[F^{L^+}_{\tilde{A}^{IVN}_{bi}}(x),F^{R^+}_{\tilde{A}^{IVN}_{bi}}(x)\right],\\ \left[T^{L^-}_{\tilde{A}^{IVN}_{bi}}(x),T^{R^-}_{\tilde{A}^{IVN}_{bi}}(x)\right],\left[I^{L^-}_{\tilde{A}^{IVN}_{bi}}(x),I^{R^-}_{\tilde{A}^{IVN}_{bi}}(x)\right],\\ \left[F^{L^-}_{\tilde{A}^{IVN}_{bi}}(x),F^{R^-}_{\tilde{A}^{IVN}_{bi}}(x)\right]\end{matrix}\right\rangle\right\}$$

or

$$\tilde{A}^{IVN}_{bi}=\left\{(a,b,c,d);\left\langle\begin{matrix}\left[T^-_{\tilde{A}^{IVN}_{bi}}(x),T^+_{\tilde{A}^{IVN}_{bi}}(x)\right],\left[I^-_{\tilde{A}^{IVN}_{bi}}(x),I^+_{\tilde{A}^{IVN}_{bi}}(x)\right],\\ \left[F^-_{\tilde{A}^{IVN}_{bi}}(x),F^+_{\tilde{A}^{IVN}_{bi}}(x)\right]\end{matrix}\right\rangle\right\}$$

Figure 2 IVBTrNN



where the left truth-membership $T^-_{\tilde{A}^{IVN}_{bi}}(x)=\left[T^{L^-}_{\tilde{A}^{IVN}_{bi}}(x),T^{R^-}_{\tilde{A}^{IVN}_{bi}}(x)\right]$, the right truth-membership $T^+_{\tilde{A}^{IVN}_{bi}}(x)=\left[T^{L^+}_{\tilde{A}^{IVN}_{bi}}(x),T^{R^+}_{\tilde{A}^{IVN}_{bi}}(x)\right]$, the left indeterminacy-membership $I^-_{\tilde{A}^{IVN}_{bi}}(x)=\left[I^{L^-}_{\tilde{A}^{IVN}_{bi}}(x),I^{R^-}_{\tilde{A}^{IVN}_{bi}}(x)\right]$, the right indeterminacy-membership $I_{\tilde{A}_{bi}^{IVN}}^{+}(x)=\left[I_{\tilde{A}_{bi}^{IVN}}^{L^{+}}(x), I_{\tilde{A}_{bi}^{IVN}}^{R^{+}}(x)\right]$ and the left falsity-membership $F_{\tilde{A}_{bi}^{IVN}}^{-}(x)=\left[F_{\tilde{A}_{bi}^{IVN}}^{L^{-}}(x), F_{\tilde{A}_{bi}^{IVN}}^{R^{-}}(x)\right]$, the right falsity-membership $F_{\tilde{A}_{bi}^{IVN}}^{+}(x)=\left[F_{\tilde{A}_{bi}^{IVN}}^{L^{+}}(x), F_{\tilde{A}_{bi}^{IVN}}^{R^{+}}(x)\right]$ are respectively defined as follows:

$$T_{\tilde{A}_{bi}^{IVN}}^{-}(x)=\begin{cases}(x-a)T_{\tilde{A}_{bi}^{IVN}}^{-}\big/(b-a); & a\le x\le b\\ T_{\tilde{A}_{bi}^{IVN}}^{-}; & b\le x\le c\\ (d-x)T_{\tilde{A}_{bi}^{IVN}}^{-}\big/(d-c); & c\le x\le d\\ 0; & \text{otherwise}\end{cases}$$

$$T_{\tilde{A}_{bi}^{IVN}}^{+}(x)=\begin{cases}(x-a)T_{\tilde{A}_{bi}^{IVN}}^{+}\big/(b-a); & a\le x\le b\\ T_{\tilde{A}_{bi}^{IVN}}^{+}; & b\le x\le c\\ (d-x)T_{\tilde{A}_{bi}^{IVN}}^{+}\big/(d-c); & c\le x\le d\\ 0; & \text{otherwise}\end{cases}$$

$$I_{\tilde{A}_{bi}^{IVN}}^{-}(x)=\begin{cases}\left((b-x+I_{\tilde{A}_{bi}^{IVN}}^{-}(x-a)\right)\Big/(b-a); & a\le x\le b\\ I_{\tilde{A}_{bi}^{IVN}}^{-}; & b\le x\le c\\ \left((x-c+I_{\tilde{A}_{bi}^{IVN}}^{-}(d-x)\right)\Big/(d-c); & c\le x\le d\\ 1; & \text{otherwise}\end{cases}$$

$$I_{\tilde{A}_{bi}^{IVN}}^{+}(x)=\begin{cases}\left((b-x+I_{\tilde{A}_{bi}^{IVN}}^{+}(x-a)\right)\Big/(b-a); & a\le x\le b\\ I_{\tilde{A}_{bi}^{IVN}}^{+}; & b\le x\le c\\ \left((x-c+I_{\tilde{A}_{bi}^{IVN}}^{+}(d-x)\right)\Big/(d-c); & c\le x\le d\\ 1; & \text{otherwise}\end{cases}$$

$$F_{\tilde{A}_{bi}^{IVN}}^{-}(x)=\begin{cases}\left((b-x+F_{\tilde{A}_{bi}^{IVN}}^{-}(x-a)\right)\Big/(b-a); & a\le x\le b\\ F_{\tilde{A}_{bi}^{IVN}}^{-}; & b\le x\le c\\ \left((x-c+F_{\tilde{A}_{bi}^{IVN}}^{-}(d-x)\right)\Big/(d-c); & c\le x\le d\\ 1; & \text{otherwise}\end{cases}$$

$$F_{\tilde{A}_{bi}^{IVN}}^{+}(x)=\begin{cases}\left((b-x+F_{\tilde{A}_{bi}^{IVN}}^{+}(x-a)\right)\Big/(b-a); & a\le x\le b\\ F_{\tilde{A}_{bi}^{IVN}}^{+}; & b\le x\le c\\ \left((x-c+F_{\tilde{A}_{bi}^{IVN}}^{+}(d-x)\right)\Big/(d-c); & c\le x\le d\\ 1; & \text{otherwise}\end{cases}$$

If $a\ge 0$ and at least one $d>0$, then $\tilde{A}_{bi}^{IVN}$ called positive IVBTrNN and denoted as $\tilde{A}_{bi}^{IVN}>0$. similarly if $d\le 0$, and atleast $a<0$, then IVBTrNN called negative i.e. $\tilde{A}_{bi}^{IVN}<0$.

Some important max-min norm operations on IVBTrNN. Let

$$\tilde{A}_{bi}^{IVN}=\left\{(a_1,b_1,c_1,d_1),\left\langle\begin{matrix}\left[T_{\tilde{A}_{bi}^{IVN}}^{L^{+}},T_{\tilde{A}_{bi}^{IVN}}^{R^{+}}\right],\left[I_{\tilde{A}_{bi}^{IVN}}^{L^{+}},I_{\tilde{A}_{bi}^{IVN}}^{R^{+}}\right],\left[F_{\tilde{A}_{bi}^{IVN}}^{L^{+}},F_{\tilde{A}_{bi}^{IVN}}^{R^{+}}\right],\\ \left[T_{\tilde{A}_{bi}^{IVN}}^{L^{-}},T_{\tilde{A}_{bi}^{IVN}}^{R^{-}}\right],\left[I_{\tilde{A}_{bi}^{IVN}}^{L^{-}},I_{\tilde{A}_{bi}^{IVN}}^{R^{-}}\right],\left[F_{\tilde{A}_{bi}^{IVN}}^{L^{-}},F_{\tilde{A}_{bi}^{IVN}}^{R^{-}}\right]\end{matrix}\right\rangle\right\}$$

$$\tilde{B}_{bi}^{IVN}=\left\{(a_2,b_2,c_2,d_2),\left\langle\begin{matrix}\left[T_{\tilde{B}_{bi}^{IVN}}^{L^{+}},T_{\tilde{B}_{bi}^{IVN}}^{R^{+}}\right],\left[I_{\tilde{B}_{bi}^{IVN}}^{L^{+}},I_{\tilde{B}_{bi}^{IVN}}^{R^{+}}\right],\left[F_{\tilde{B}_{bi}^{IVN}}^{L^{+}},F_{\tilde{B}_{bi}^{IVN}}^{R^{+}}\right],\\ \left[T_{\tilde{B}_{bi}^{IVN}}^{L^{-}},T_{\tilde{B}_{bi}^{IVN}}^{R^{-}}\right],\left[I_{\tilde{B}_{bi}^{IVN}}^{L^{-}},I_{\tilde{B}_{bi}^{IVN}}^{R^{-}}\right],\left[F_{\tilde{B}_{bi}^{IVN}}^{L^{-}},F_{\tilde{B}_{bi}^{IVN}}^{R^{-}}\right]\end{matrix}\right\rangle\right\}$$

then

a.

$$\tilde{A}_{bi}^{IVN}+\tilde{B}_{bi}^{IVN}=\left\{\begin{matrix}(a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2),\\ \left\langle\begin{matrix}\left(\begin{matrix}\left[\max\left(T_{\tilde{A}_{bi}^{IVN}}^{L^{+}},T_{\tilde{B}_{bi}^{IVN}}^{L^{+}}\right),\max\left(T_{\tilde{A}_{bi}^{IVN}}^{R^{+}},T_{\tilde{B}_{bi}^{IVN}}^{R^{+}}\right)\right],\\ \left[\dfrac{I_{\tilde{A}_{bi}^{IVN}}^{L^{+}}+I_{\tilde{B}_{bi}^{IVN}}^{L^{+}}}{2},\dfrac{I_{\tilde{A}_{bi}^{IVN}}^{R^{+}}+I_{\tilde{B}_{bi}^{IVN}}^{R^{+}}}{2}\right],\\ \left[\min\left(F_{\tilde{A}_{bi}^{IVN}}^{L^{+}},F_{\tilde{B}_{bi}^{IVN}}^{L^{+}}\right),\min\left(F_{\tilde{A}_{bi}^{IVN}}^{R^{+}},F_{\tilde{B}_{bi}^{IVN}}^{R^{+}}\right)\right]\end{matrix}\right),\\ \left(\begin{matrix}\left[\min\left(T_{\tilde{A}_{bi}^{IVN}}^{L^{-}},T_{\tilde{B}_{bi}^{IVN}}^{L^{-}}\right),\min\left(T_{\tilde{A}_{bi}^{IVN}}^{R^{-}},T_{\tilde{B}_{bi}^{IVN}}^{R^{-}}\right)\right],\\ \left[\dfrac{I_{\tilde{A}_{bi}^{IVN}}^{L^{-}}+I_{\tilde{B}_{bi}^{IVN}}^{L^{-}}}{2},\dfrac{I_{\tilde{A}_{bi}^{IVN}}^{R^{-}}+I_{\tilde{B}_{bi}^{IVN}}^{R^{-}}}{2}\right],\\ \left[\max\left(F_{\tilde{A}_{bi}^{IVN}}^{L^{-}}.F_{\tilde{B}_{bi}^{IVN}}^{L^{-}}\right),\max\left(F_{\tilde{A}_{bi}^{IVN}}^{R^{-}}.F_{\tilde{B}_{bi}^{IVN}}^{R^{-}}\right)\right]\end{matrix}\right)\end{matrix}\right\rangle\end{matrix}\right\}$$

b. $$\tilde{A}_{bi}^{IVN}-\tilde{B}_{bi}^{IVN}=\left\{\begin{matrix}(a_1-d_2,b_1-c_2,c_1-b_2,d_1-a_2),\\ \left\langle\begin{matrix}\left(\begin{matrix}\left[\min\left(T_{\tilde{A}_{bi}^{IVN}}^{L^{+}},T_{\tilde{B}_{bi}^{IVN}}^{L^{+}}\right),\min\left(T_{\tilde{A}_{bi}^{IVN}}^{R^{+}},T_{\tilde{B}_{bi}^{IVN}}^{R^{+}}\right)\right],\\ \left[\dfrac{I_{\tilde{A}_{bi}^{IVN}}^{L^{+}}+I_{\tilde{B}_{bi}^{IVN}}^{L^{+}}}{2},\dfrac{I_{\tilde{A}_{bi}^{IVN}}^{R^{+}}+I_{\tilde{B}_{bi}^{IVN}}^{R^{+}}}{2}\right],\\ \left[\max\left(F_{\tilde{A}_{bi}^{IVN}}^{L^{+}},F_{\tilde{B}_{bi}^{IVN}}^{L^{+}}\right),\max\left(F_{\tilde{A}_{bi}^{IVN}}^{R^{+}},F_{\tilde{B}_{bi}^{IVN}}^{R^{+}}\right)\right]\end{matrix}\right),\\ \left(\begin{matrix}\left[\max\left(T_{\tilde{A}_{bi}^{IVN}}^{L^{-}},T_{\tilde{B}_{bi}^{IVN}}^{L^{-}}\right),\max\left(T_{\tilde{A}_{bi}^{IVN}}^{R^{-}},T_{\tilde{B}_{bi}^{IVN}}^{R^{-}}\right)\right],\\ \left[\dfrac{I_{\tilde{A}_{bi}^{IVN}}^{L^{-}}+I_{\tilde{B}_{bi}^{IVN}}^{L^{-}}}{2},\dfrac{I_{\tilde{A}_{bi}^{IVN}}^{R^{-}}+I_{\tilde{B}_{bi}^{IVN}}^{R^{-}}}{2}\right],\\ \left[\min\left(F_{\tilde{A}_{bi}^{IVN}}^{L^{-}}.F_{\tilde{B}_{bi}^{IVN}}^{L^{-}}\right),\min\left(F_{\tilde{A}_{bi}^{IVN}}^{R^{-}}.F_{\tilde{B}_{bi}^{IVN}}^{R^{-}}\right)\right]\end{matrix}\right)\end{matrix}\right\rangle\end{matrix}\right\}$$

c. $$\lambda\tilde{A}_{bi}^{IVN}=\begin{cases}(\lambda a_1,\lambda b_1,\lambda c_1,\lambda d_1);\\ \left(\left\langle\begin{matrix}\left[T_{\tilde{A}_{bi}^{IVN}}^{L^{+}},T_{\tilde{A}_{bi}^{IVN}}^{R^{+}}\right],\left[I_{\tilde{A}_{bi}^{IVN}}^{L^{+}},I_{\tilde{A}_{bi}^{IVN}}^{R^{+}}\right],\left[F_{\tilde{A}_{bi}^{IVN}}^{L^{+}},F_{\tilde{A}_{bi}^{IVN}}^{R^{+}}\right],\\ \left[T_{\tilde{A}_{bi}^{IVN}}^{L^{-}},T_{\tilde{A}_{bi}^{IVN}}^{R^{-}}\right],\left[I_{\tilde{A}_{bi}^{IVN}}^{L^{-}},I_{\tilde{A}_{bi}^{IVN}}^{R^{-}}\right],\left[F_{\tilde{A}_{bi}^{IVN}}^{L^{-}},F_{\tilde{A}_{bi}^{IVN}}^{R^{-}}\right]\end{matrix}\right\rangle\right); \lambda>0\\ (\lambda d_1,\lambda c_1,\lambda b_1,\lambda a_1);\\ \left(\left\langle\begin{matrix}\left[T_{\tilde{A}_{bi}^{IVN}}^{L^{+}},T_{\tilde{A}_{bi}^{IVN}}^{R^{+}}\right],\left[I_{\tilde{A}_{bi}^{IVN}}^{L^{+}},I_{\tilde{A}_{bi}^{IVN}}^{R^{+}}\right],\left[F_{\tilde{A}_{bi}^{IVN}}^{L^{+}},F_{\tilde{A}_{bi}^{IVN}}^{R^{+}}\right],\\ \left[T_{\tilde{A}_{bi}^{IVN}}^{L^{-}},T_{\tilde{A}_{bi}^{IVN}}^{R^{-}}\right],\left[I_{\tilde{A}_{bi}^{IVN}}^{L^{-}},I_{\tilde{A}_{bi}^{IVN}}^{R^{-}}\right],\left[F_{\tilde{A}_{bi}^{IVN}}^{L^{-}},F_{\tilde{A}_{bi}^{IVN}}^{R^{-}}\right]\end{matrix}\right\rangle\right); \lambda<0\end{cases}$$

d. $$\tilde{A}_{bi}^{IVN}.\tilde{B}_{bi}^{IVN}=\left\{\begin{array}{l}(a_1.a_2,b_1.b_2,c_1.c_2,d_1.d_2),\\ \left\langle\begin{array}{l}\left(\begin{array}{r}\left[T^{L^+}_{\tilde{A}_{bi}^{IVN}}.T^{L^+}_{\tilde{B}_{bi}^{IVN}},T^{R+}_{\tilde{A}_{bi}^{IVN}}.T^{R^+}_{\tilde{B}_{bi}^{IVN}}\right],\\ \left[\begin{array}{l}\left(I^{L^+}_{\tilde{A}_{bi}^{IVN}}+I^{L^+}_{\tilde{B}_{bi}^{IVN}}-I^{L^+}_{\tilde{A}_{bi}^{IVN}}.I^{L^+}_{\tilde{B}_{bi}^{IVN}}\right),\\ \left(I^{R^+}_{\tilde{A}_{bi}^{IVN}}+I^{R^+}_{\tilde{B}_{bi}^{IVN}}-I^{R+}_{\tilde{A}_{bi}^{IVN}}.I^{R^+}_{\tilde{B}_{bi}^{IVN}}\right)\end{array}\right],\\ \left[\begin{array}{l}\left(F^{L^+}_{\tilde{A}_{bi}^{IVN}}+F^{L^+}_{\tilde{B}_{bi}^{IVN}}-F^{L^+}_{\tilde{A}_{bi}^{IVN}}.F^{L^+}_{\tilde{B}_{bi}^{IVN}}\right),\\ \left(F^{R^+}_{\tilde{A}_{bi}^{IVN}}+F^{R^+}_{\tilde{B}_{bi}^{IVN}}-F^{R+}_{\tilde{A}_{bi}^{IVN}}.F^{R^+}_{\tilde{B}_{bi}^{IVN}}\right)\end{array}\right]\end{array}\right),\\ \left(\begin{array}{r}\left[\begin{array}{l}-\left(-T^{L^-}_{\tilde{A}_{bi}^{IVN}}-T^{L^-}_{\tilde{B}_{bi}^{IVN}}-T^{L^-}_{\tilde{A}_{bi}^{IVN}}.T^{L^-}_{\tilde{B}_{bi}^{IVN}}\right),\\ -\left(-T^{R^-}_{\tilde{A}_{bi}^{IVN}}-T^{R^-}_{\tilde{B}_{bi}^{IVN}}-T^{R^-}_{\tilde{A}_{bi}^{IVN}}.T^{R^-}_{\tilde{B}_{bi}^{IVN}}\right)\end{array}\right],\\ \left[\begin{array}{l}-\left(-I^{L^-}_{\tilde{A}_{bi}^{IVN}}\right).\left(-I^{L^-}_{\tilde{B}_{bi}^{IVN}}\right),\\ -\left(-I^{R^-}_{\tilde{A}_{bi}^{IVN}}\right).\left(-I^{R^-}_{\tilde{B}_{bi}^{IVN}}\right)\end{array}\right],\\ \left[\begin{array}{l}-\left(-F^{L^-}_{\tilde{A}_{bi}^{IVN}}\right).\left(-F^{L^-}_{\tilde{B}_{bi}^{IVN}}\right),\\ -\left(-F^{R^-}_{\tilde{A}_{bi}^{IVN}}\right).\left(-F^{R^-}_{\tilde{B}_{bi}^{IVN}}\right)\end{array}\right]\end{array}\right)\end{array}\right\rangle\end{array}\right\}$$

e. $$\tilde{A}_{bi}^{IVN}.\tilde{B}_{bi}^{IVN}=\left\{\begin{array}{l}(a_1/a_2,b_1/b_2,c_1/c_2,d_1/d_2);\\ \left\langle\begin{array}{l}\left(\begin{array}{r}\left[T^{L^+}_{\tilde{A}_{bi}^{IVN}}.T^{L^+}_{\tilde{B}_{bi}^{IVN}},T^{R+}_{\tilde{A}_{bi}^{IVN}}.T^{R^+}_{\tilde{B}_{bi}^{IVN}}\right],\\ \left[\begin{array}{l}\left(I^{L^+}_{\tilde{A}_{bi}^{IVN}}+I^{L^+}_{\tilde{B}_{bi}^{IVN}}-I^{L^+}_{\tilde{A}_{bi}^{IVN}}.I^{L^+}_{\tilde{B}_{bi}^{IVN}}\right),\\ \left(I^{R^+}_{\tilde{A}_{bi}^{IVN}}+I^{R^+}_{\tilde{B}_{bi}^{IVN}}-I^{R+}_{\tilde{A}_{bi}^{IVN}}.I^{R^+}_{\tilde{B}_{bi}^{IVN}}\right)\end{array}\right],\\ \left[\begin{array}{l}\left(F^{L^+}_{\tilde{A}_{bi}^{IVN}}+F^{L^+}_{\tilde{B}_{bi}^{IVN}}-F^{L^+}_{\tilde{A}_{bi}^{IVN}}.F^{L^+}_{\tilde{B}_{bi}^{IVN}}\right),\\ \left(F^{R^+}_{\tilde{A}_{bi}^{IVN}}+F^{R^+}_{\tilde{B}_{bi}^{IVN}}-F^{R+}_{\tilde{A}_{bi}^{IVN}}.F^{R^+}_{\tilde{B}_{bi}^{IVN}}\right)\end{array}\right]\end{array}\right),\\ \left(\begin{array}{r}\left[\begin{array}{l}-\left(-T^{L^-}_{\tilde{A}_{bi}^{IVN}}-T^{L^-}_{\tilde{B}_{bi}^{IVN}}-T^{L^-}_{\tilde{A}_{bi}^{IVN}}.T^{L^-}_{\tilde{B}_{bi}^{IVN}}\right),\\ -\left(-T^{R^-}_{\tilde{A}_{bi}^{IVN}}-T^{R^-}_{\tilde{B}_{bi}^{IVN}}-T^{R^-}_{\tilde{A}_{bi}^{IVN}}.T^{R^-}_{\tilde{B}_{bi}^{IVN}}\right)\end{array}\right],\\ \left[\begin{array}{l}-\left(-I^{L^-}_{\tilde{A}_{bi}^{IVN}}\right).\left(-I^{L^-}_{\tilde{B}_{bi}^{IVN}}\right),\\ -\left(-I^{R^-}_{\tilde{A}_{bi}^{IVN}}\right).\left(-I^{R^-}_{\tilde{B}_{bi}^{IVN}}\right)\end{array}\right],\\ \left[\begin{array}{l}-\left(-F^{L^-}_{\tilde{A}_{bi}^{IVN}}\right).\left(-F^{L^-}_{\tilde{B}_{bi}^{IVN}}\right),\\ -\left(-F^{R^-}_{\tilde{A}_{bi}^{IVN}}\right).\left(-F^{R^-}_{\tilde{B}_{bi}^{IVN}}\right)\end{array}\right]\end{array}\right)\end{array}\right\rangle\\ (d_1>0,d_2>0)\end{array}\right\}$$

f. $$\left(\tilde{A}_{bi}^{IVN}\right)^{\lambda}=\left\{\begin{array}{l}(a_1^{\lambda},b_1^{\lambda},c_1^{\lambda},d_1^{\lambda});\\ \left\langle\begin{array}{l}\left[T^{L^+}_{\tilde{A}_{bi}^{IVN}},T^{R^+}_{\tilde{A}_{bi}^{IVN}}\right],\left[I^{L^+}_{\tilde{A}_{bi}^{IVN}},I^{R^+}_{\tilde{A}_{bi}^{IVN}}\right],\left[F^{L^+}_{\tilde{A}_{bi}^{IVN}},F^{R^+}_{\tilde{A}_{bi}^{IVN}}\right],\\ \left[T^{L^-}_{\tilde{A}_{bi}^{IVN}},T^{R^-}_{\tilde{A}_{bi}^{IVN}}\right],\left[I^{L^-}_{\tilde{A}_{bi}^{IVN}},I^{R^-}_{\tilde{A}_{bi}^{IVN}}\right],\left[F^{L^-}_{\tilde{A}_{bi}^{IVN}},F^{R^-}_{\tilde{A}_{bi}^{IVN}}\right]\end{array}\right\rangle:\lambda>0\\ ((d_1^{\lambda},c_1^{\lambda},b_1^{\lambda},a_1^{\lambda});\\ \left\langle\begin{array}{l}\left[T^{L^+}_{\tilde{A}_{bi}^{IVN}},T^{R^+}_{\tilde{A}_{bi}^{IVN}}\right],\left[I^{L^+}_{\tilde{A}_{bi}^{IVN}},I^{R^+}_{\tilde{A}_{bi}^{IVN}}\right],\left[F^{L^+}_{\tilde{A}_{bi}^{IVN}},F^{R^+}_{\tilde{A}_{bi}^{IVN}}\right],\\ \left[T^{L^-}_{\tilde{A}_{bi}^{IVN}},T^{R^-}_{\tilde{A}_{bi}^{IVN}}\right],\left[I^{L^-}_{\tilde{A}_{bi}^{IVN}},I^{R^-}_{\tilde{A}_{bi}^{IVN}}\right],\left[F^{L^-}_{\tilde{A}_{bi}^{IVN}},F^{R^-}_{\tilde{A}_{bi}^{IVN}}\right]\end{array}\right\rangle:\lambda<0\end{array}\right.$$

g. $$\left(\tilde{A}_{bi}^{IVN}\right)^{-1}=\left\{\begin{array}{l}(a_1^{-1},b_1^{-1},c_1^{-1},d_1^{-1});\\ \left\langle\begin{array}{l}\left[T^{L^+}_{\tilde{A}_{bi}^{IVN}},T^{R^+}_{\tilde{A}_{bi}^{IVN}}\right],\left[I^{L^+}_{\tilde{A}_{bi}^{IVN}},I^{R^+}_{\tilde{A}_{bi}^{IVN}}\right],\left[F^{L^+}_{\tilde{A}_{bi}^{IVN}},F^{R^+}_{\tilde{A}_{bi}^{IVN}}\right],\\ \left[T^{L^-}_{\tilde{A}_{bi}^{IVN}},T^{R^-}_{\tilde{A}_{bi}^{IVN}}\right],\left[I^{L^-}_{\tilde{A}_{bi}^{IVN}},I^{R^-}_{\tilde{A}_{bi}^{IVN}}\right],\left[F^{L^-}_{\tilde{A}_{bi}^{IVN}},F^{R^-}_{\tilde{A}_{bi}^{IVN}}\right]\end{array}\right\rangle\end{array}\right\}$$

Definition 2.7. Let

$$\tilde{A}_{bi}^{IVN}=\left\{\begin{array}{l}(a_1,b_1,c_1,d_1),\\ \left\langle\begin{array}{l}\left[T^{L^+}_{\tilde{A}_{bi}^{IVN}},T^{R^+}_{\tilde{A}_{bi}^{IVN}}\right],\left[I^{L^+}_{\tilde{A}_{bi}^{IVN}},I^{R^+}_{\tilde{A}_{bi}^{IVN}}\right],\left[F^{L^+}_{\tilde{A}_{bi}^{IVN}},F^{R^+}_{\tilde{A}_{bi}^{IVN}}\right],\\ \left[T^{L^-}_{\tilde{A}_{bi}^{IVN}},T^{R^-}_{\tilde{A}_{bi}^{IVN}}\right],\left[I^{L^-}_{\tilde{A}_{bi}^{IVN}},I^{R^-}_{\tilde{A}_{bi}^{IVN}}\right],\left[F^{L^-}_{\tilde{A}_{bi}^{IVN}},F^{R^-}_{\tilde{A}_{bi}^{IVN}}\right]\end{array}\right\rangle\end{array}\right\}$$

be a IVBTrNN. The primary application of score function is to drag the judgment of conversion of IVBTrNN into crisp number. The mean of IVBTrNN components is $\left(\dfrac{a_1+b_1+c_1+d_1}{4}\right)$ and the score value of the membership portion is

$$\left\{8+\left(\begin{array}{l}\left(T^{L^-}_{\tilde{A}_{bi}^{IVN}}-I^{L^-}_{\tilde{A}_{bi}^{IVN}}-F^{L^-}_{\tilde{A}_{bi}^{IVN}}\right)+\left(T^{R^-}_{\tilde{A}_{bi}^{IVN}}-I^{R^-}_{\tilde{A}_{bi}^{IVN}}-F^{R^-}_{\tilde{A}_{bi}^{IVN}}\right)+\\ \left(T^{L^+}_{\tilde{A}_{bi}^{IVN}}-I^{L^+}_{\tilde{A}_{bi}^{IVN}}-F^{L^+}_{\tilde{A}_{bi}^{IVN}}\right)+\left(T^{R^+}_{\tilde{A}_{bi}^{IVN}}-I^{R^+}_{\tilde{A}_{bi}^{IVN}}-F^{R^+}_{\tilde{A}_{bi}^{IVN}}\right)\end{array}\right)\Big/8\right\}$$ then

the score function $s\left(\tilde{A}_{bi}^{IVN}\right)$ and accuracy function $a\left(\tilde{A}_{bi}^{IVN}\right)$ of a bipolar neutrosophic number are defined as follows:

$$s\left(\tilde{A}_{bi}^{IVN}\right)=\frac{1}{32}\left(a_1+b_1+c_1+d_1\right)\times\left\{\begin{array}{r}8+\left(T^{L^-}_{\tilde{A}_{bi}^{IVN}}-I^{L^-}_{\tilde{A}_{bi}^{IVN}}-F^{L^-}_{\tilde{A}_{bi}^{IVN}}\right)+\\ +\left(T^{R^-}_{\tilde{A}_{bi}^{IVN}}-I^{R^-}_{\tilde{A}_{bi}^{IVN}}-F^{R^-}_{\tilde{A}_{bi}^{IVN}}\right)+\\ +\left(T^{L^+}_{\tilde{A}_{bi}^{IVN}}-I^{L^+}_{\tilde{A}_{bi}^{IVN}}-F^{L^+}_{\tilde{A}_{bi}^{IVN}}\right)+\\ +\left(T^{R^+}_{\tilde{A}_{bi}^{IVN}}-I^{R^+}_{\tilde{A}_{bi}^{IVN}}-F^{R^+}_{\tilde{A}_{bi}^{IVN}}\right)\end{array}\right\}$$

$$a\left(\tilde{A}_{bi}^{IVN}\right)=\frac{1}{32}\left(a_1+b_1+c_1+d_1\right)\times\left\{\begin{array}{r}4+\left(T^{L^-}_{\tilde{A}_{bi}^{IVN}}-I^{L^-}_{\tilde{A}_{bi}^{IVN}}+F^{L^-}_{\tilde{A}_{bi}^{IVN}}\right)+\\ +\left(T^{R^-}_{\tilde{A}_{bi}^{IVN}}-I^{R^-}_{\tilde{A}_{bi}^{IVN}}+F^{R^-}_{\tilde{A}_{bi}^{IVN}}\right)+\\ +\left(T^{L^+}_{\tilde{A}_{bi}^{IVN}}-I^{L^+}_{\tilde{A}_{bi}^{IVN}}+F^{L^+}_{\tilde{A}_{bi}^{IVN}}\right)+\\ +\left(T^{R^+}_{\tilde{A}_{bi}^{IVN}}-I^{R^+}_{\tilde{A}_{bi}^{IVN}}+F^{R^+}_{\tilde{A}_{bi}^{IVN}}\right)\end{array}\right\}$$

Definition 2.7. Let

$$\tilde{A}_{bi}^{IVN}=\left\{(a_1,b_1,c_1,d_1),\left\langle\begin{array}{l}\left[T^{L^+}_{\tilde{A}_{bi}^{IVN}},T^{R^+}_{\tilde{A}_{bi}^{IVN}}\right],\left[I^{L^+}_{\tilde{A}_{bi}^{IVN}},I^{R^+}_{\tilde{A}_{bi}^{IVN}}\right],\left[F^{L^+}_{\tilde{A}_{bi}^{IVN}},F^{R^+}_{\tilde{A}_{bi}^{IVN}}\right],\\ \left[T^{L^-}_{\tilde{A}_{bi}^{IVN}},T^{R^-}_{\tilde{A}_{bi}^{IVN}}\right],\left[I^{L^-}_{\tilde{A}_{bi}^{IVN}},I^{R^-}_{\tilde{A}_{bi}^{IVN}}\right],\left[F^{L^-}_{\tilde{A}_{bi}^{IVN}},F^{R^-}_{\tilde{A}_{bi}^{IVN}}\right]\end{array}\right\rangle\right\}$$ and

$$\tilde{B}_{bi}^{IVN}=\left\{(a_2,b_2,c_2,d_2);\left\langle\begin{array}{l}\left[T^{L^+}_{\tilde{A}_{bi}^{IVN}},T^{R^+}_{\tilde{A}_{bi}^{IVN}}\right],\left[I^{L^+}_{\tilde{A}_{bi}^{IVN}},I^{R^+}_{\tilde{A}_{bi}^{IVN}}\right],\left[F^{L^+}_{\tilde{A}_{bi}^{IVN}},F^{R^+}_{\tilde{A}_{bi}^{IVN}}\right],\\ \left[T^{L^-}_{\tilde{A}_{bi}^{IVN}},T^{R^-}_{\tilde{A}_{bi}^{IVN}}\right],\left[I^{L^-}_{\tilde{A}_{bi}^{IVN}},I^{R^-}_{\tilde{A}_{bi}^{IVN}}\right],\left[F^{L^-}_{\tilde{A}_{bi}^{IVN}},F^{R^-}_{\tilde{A}_{bi}^{IVN}}\right]\end{array}\right\rangle\right\}$$ are

two IVBTrNNs on the set of real numbers, then if

(i) $s\left(\tilde{A}_{bi}^{IVN}\right)<s\left(\tilde{B}_{bi}^{IVN}\right)\Rightarrow\tilde{A}_{bi}^{IVN}<\tilde{B}_{bi}^{IVN}$

(ii) $s\left(\tilde{A}_{bi}^{IVN}\right)>s\left(\tilde{B}_{bi}^{IVN}\right)\Rightarrow\tilde{A}_{bi}^{IVN}>\tilde{B}_{bi}^{IVN}$

(iii) $s\left(\tilde{A}_{bi}^{IVN}\right)=s\left(\tilde{B}_{bi}^{IVN}\right)$ then if

(a) $a\left(\tilde{A}_{bi}^{IVN}\right)<a\left(\tilde{B}_{bi}^{IVN}\right)\Rightarrow\tilde{A}_{bi}^{IVN}<\tilde{B}_{bi}^{IVN}$,

(b) $a\left(\tilde{A}_{bi}^{IVN}\right)>a\left(\tilde{B}_{bi}^{IVN}\right)\Rightarrow\tilde{A}_{bi}^{IVN}>\tilde{B}_{bi}^{IVN}$

(c) $a\left(\tilde{A}_{bi}^{IVN}\right)=a\left(\tilde{B}_{bi}^{IVN}\right)\Rightarrow\tilde{A}_{bi}^{IVN}=\tilde{B}_{bi}^{IVN}$

Example 3.1. Let $X=\{x_1,x_2,x_3\}$. The two IVBTrNN in *X* are

$$\tilde{A}_{bi}^{IVN}=\left\langle\begin{array}{l}(0.25,0.50,0.75,0.90);\\ [0.4,0.6],[0.4,0.7],[0.2,0.5],\\ [-0.2,-0.1],[-0.6,-0.1],[-0.3,-0.2]\end{array}\right\rangle$$

$$\tilde{B}_{bi}^{IVN} = \left\langle \begin{array}{l} (0.15, 0.35, 0.65, 0.85); \\ [0.1, 0.2], [0.3, 0.8], [0.2, 0.4], \\ [-0.5, -0.2], [-0.9, -0.3], [-0.6, -0.1] \end{array} \right\rangle$$

then $s\left(\tilde{A}_{bi}^{IVN}\right) = 0.5925$, $s\left(\tilde{B}_{bi}^{IVN}\right) = 0.4875$, $a\left(\tilde{A}_{bi}^{IVN}\right) = 0.3525$, $a\left(\tilde{B}_{bi}^{IVN}\right) = 0.225$. Here $s\left(\tilde{A}_{bi}^{IVN}\right) > s\left(\tilde{B}_{bi}^{IVN}\right)$ implies that $\tilde{A}_{bi}^{IVN} > \tilde{B}_{bi}^{IVN}$.

4. Mathematical Formulation

The mathematical formulation of fully neutrosophic interval valued transportation problem [FNIVTP], where transported units, cost, demands and supplies are in the form of IVBTrNN. The mathematical distribution system of FNIVTP defined as follows:

$$\text{(FNIVTP)} \ \text{Min} \ \tilde{Z}_{bi}^{IVN} = \sum_{i=0}^{m} \sum_{j=0}^{n} \left(\tilde{x}_{bi}^{IVN}\right)_{ij} \left(\tilde{c}_{bi}^{IVN}\right)_{ij}$$

Subject to

$$\sum_{j=0}^{n} \left(\tilde{x}_{bi}^{IVN}\right)_{ij} \approx \left(\tilde{a}_{bi}^{IVN}\right)_{i}, \ i = 1, 2, 3, \dots, m (\text{shipping sources}),$$

$$\sum_{i=0}^{m} \left(\tilde{x}_{bi}^{IVN}\right)_{ij} \approx \left(\tilde{b}_{bi}^{IVN}\right)_{j}, \ j = 1, 2, 3, \dots, n (\text{destination}),$$

$$\left(\tilde{x}_{bi}^{IVN}\right)_{ij} \geq \tilde{0}, \ \forall \ i = 1, 2, 3, \dots, m, \ j = 1, 2, 3, \dots, n.$$

where $\left(\tilde{x}_{bi}^{IVN}\right)_{12}, \dots\dots\left(\tilde{x}_{bi}^{IVN}\right)_{53}$, are according to

$$\left(\tilde{x}_{bi}^{IVN}\right)_{ij} = \left\langle \begin{array}{l} (a_{\tilde{x}_{bi}^{IVN}}, b_{\tilde{x}_{bi}^{IVN}}, c_{\tilde{x}_{bi}^{IVN}}, d_{\tilde{x}_{bi}^{IVN}}), \\ \left[T^{L^+}_{\tilde{x}_{bi}^{IVN}}, T^{R^+}_{\tilde{x}_{bi}^{IVN}}\right], \left[I^{L^+}_{\tilde{x}_{bi}^{IVN}}, I^{R^+}_{\tilde{x}_{bi}^{IVN}}\right], \left[F^{L^+}_{\tilde{x}_{bi}^{IVN}}, F^{R^+}_{\tilde{x}_{bi}^{IVN}}\right], \\ \left[T^{L^-}_{\tilde{x}_{bi}^{IVN}}, T^{R^-}_{\tilde{x}_{bi}^{IVN}}\right], \left[I^{L^-}_{\tilde{x}_{bi}^{IVN}}, I^{R^-}_{\tilde{x}_{bi}^{IVN}}\right], \left[F^{L^-}_{\tilde{x}_{bi}^{IVN}}, F^{R^-}_{\tilde{x}_{bi}^{IVN}}\right] \end{array} \right\rangle; \quad i = 1, 2, 3, 4, 5; \ j = 1, 2, 3.$$

$\left(\tilde{y}_{bi}^{IVN}\right)_{1}, \dots\dots\left(\tilde{y}_{bi}^{IVN}\right)_{4}$ are according to

$$\left(\tilde{y}_{bi}^{IVN}\right)_{k} = \left\langle \begin{array}{l} (a_{\tilde{y}_{bi}^{IVN}}, b_{\tilde{y}_{bi}^{IVN}}, c_{\tilde{y}_{bi}^{IVN}}, d_{\tilde{y}_{bi}^{IVN}}), \\ \left[T^{L^+}_{\tilde{y}_{bi}^{IVN}}, T^{R^+}_{\tilde{y}_{bi}^{IVN}}\right], \left[I^{L^+}_{\tilde{y}_{bi}^{IVN}}, I^{R^+}_{\tilde{y}_{bi}^{IVN}}\right], \left[F^{L^+}_{\tilde{y}_{bi}^{IVN}}, F^{R^+}_{\tilde{y}_{bi}^{IVN}}\right], \\ \left[T^{L^-}_{\tilde{y}_{bi}^{IVN}}, T^{R^-}_{\tilde{y}_{bi}^{IVN}}\right], \left[I^{L^-}_{\tilde{y}_{bi}^{IVN}}, I^{R^-}_{\tilde{y}_{bi}^{IVN}}\right], \left[F^{L^-}_{\tilde{y}_{bi}^{IVN}}, F^{R^-}_{\tilde{y}_{bi}^{IVN}}\right] \end{array} \right\rangle; \quad k = 1, 2, 3, 4.$$

and $\left(\tilde{x}_{bi}^{IVN}\right)_{ij} \geq 0; i = 1, 2, 3, 4, 5; \ j = 1, 2, 3.$

where

$\tilde{x}_{bi}^{IVN} =$ the number of transported neutrosophic units shipped (in thousands) from plant i to distribution center j, for each $i=1,2,3,4,5$ and $j=1,2,3$.

$\tilde{c}_{bi}^{IVN} =$ the shipping neutrosophic cost data of one unit transported from i^{th} source to j^{th} destination.

$\tilde{a}_{bi}^{IVN} =$ available neutrosophic supply quantity from i^{th} plant

$\tilde{b}_{bi}^{IVN} =$ required neutrosophic demand quantity from j^{th} distribution centre. Also

$$\left(\tilde{c}_{bi}^{IVN}\right)_{ij} = \left\langle \begin{array}{l} (a_{\tilde{c}_{bi}^{IVN}}, b_{\tilde{c}_{bi}^{IVN}}, c_{\tilde{c}_{bi}^{IVN}}, d_{\tilde{c}_{bi}^{IVN}}), \\ \left[T^{L^+}_{\tilde{c}_{bi}^{IVN}}, T^{R^+}_{\tilde{c}_{bi}^{IVN}}\right], \left[I^{L^+}_{\tilde{c}_{bi}^{IVN}}, I^{R^+}_{\tilde{c}_{bi}^{IVN}}\right], \left[F^{L^+}_{\tilde{c}_{bi}^{IVN}}, F^{R^+}_{\tilde{c}_{bi}^{IVN}}\right], \\ \left[T^{L^-}_{\tilde{c}_{bi}^{IVN}}, T^{R^-}_{\tilde{c}_{bi}^{IVN}}\right], \left[I^{L^-}_{\tilde{c}_{bi}^{IVN}}, I^{R^-}_{\tilde{c}_{bi}^{IVN}}\right], \left[F^{L^-}_{\tilde{c}_{bi}^{IVN}}, F^{R^-}_{\tilde{c}_{bi}^{IVN}}\right] \end{array} \right\rangle$$

$$\left(\tilde{x}_{bi}^{IVN}\right)_{ij} = \left\langle \begin{array}{l} (a_{\tilde{x}_{bi}^{IVN}}, b_{\tilde{x}_{bi}^{IVN}}, c_{\tilde{x}_{bi}^{IVN}}, d_{\tilde{x}_{bi}^{IVN}}) \\ \left[T^{L^+}_{\tilde{x}_{bi}^{IVN}}, T^{R^+}_{\tilde{x}_{bi}^{IVN}}\right], \left[I^{L^+}_{\tilde{x}_{bi}^{IVN}}, I^{R^+}_{\tilde{x}_{bi}^{IVN}}\right], \left[F^{L^+}_{\tilde{x}_{bi}^{IVN}}, F^{R^+}_{\tilde{x}_{bi}^{IVN}}\right], \\ \left[T^{L^-}_{\tilde{x}_{bi}^{IVN}}, T^{R^-}_{\tilde{x}_{bi}^{IVN}}\right], \left[I^{L^-}_{\tilde{x}_{bi}^{IVN}}, I^{R^-}_{\tilde{x}_{bi}^{IVN}}\right], \left[F^{L^-}_{\tilde{x}_{bi}^{IVN}}, F^{R^-}_{\tilde{x}_{bi}^{IVN}}\right] \end{array} \right\rangle$$

$$\left(\tilde{a}_{bi}^{IVN}\right)_{ij} = \left\langle \begin{array}{l} (a_{\tilde{a}_{bi}^{IVN}}, b_{\tilde{a}_{bi}^{IVN}}, c_{\tilde{a}_{bi}^{IVN}}, d_{\tilde{a}_{bi}^{IVN}}), \\ \left[T^{L^+}_{\tilde{a}_{bi}^{IVN}}, T^{R^+}_{\tilde{a}_{bi}^{IVN}}\right], \left[I^{L^+}_{\tilde{a}_{bi}^{IVN}}, I^{R^+}_{\tilde{a}_{bi}^{IVN}}\right], \left[F^{L^+}_{\tilde{a}_{bi}^{IVN}}, F^{R^+}_{\tilde{a}_{bi}^{IVN}}\right], \\ \left[T^{L^-}_{\tilde{a}_{bi}^{IVN}}, T^{R^-}_{\tilde{a}_{bi}^{IVN}}\right], \left[I^{L^-}_{\tilde{a}_{bi}^{IVN}}, I^{R^-}_{\tilde{a}_{bi}^{IVN}}\right], \left[F^{L^-}_{\tilde{a}_{bi}^{IVN}}, F^{R^-}_{\tilde{a}_{bi}^{IVN}}\right] \end{array} \right\rangle,$$

$$\left(\tilde{b}_{bi}^{IVN}\right)_{ij} = \left\langle \begin{array}{l} (a_{\tilde{b}_{bi}^{IVN}}, b_{\tilde{b}_{bi}^{IVN}}, c_{\tilde{b}_{bi}^{IVN}}, d_{\tilde{b}_{bi}^{IVN}}), \\ \left[T^{L^+}_{\tilde{b}_{bi}^{IVN}}, T^{R^+}_{\tilde{b}_{bi}^{IVN}}\right], \left[I^{L^+}_{\tilde{b}_{bi}^{IVN}}, I^{R^+}_{\tilde{b}_{bi}^{IVN}}\right], \left[F^{L^+}_{\tilde{b}_{bi}^{IVN}}, F^{R^+}_{\tilde{b}_{bi}^{IVN}}\right], \\ \left[T^{L^-}_{\tilde{b}_{bi}^{IVN}}, T^{R^-}_{\tilde{b}_{bi}^{IVN}}\right], \left[I^{L^-}_{\tilde{b}_{bi}^{IVN}}, I^{R^-}_{\tilde{b}_{bi}^{IVN}}\right], \left[F^{L^-}_{\tilde{b}_{bi}^{IVN}}, F^{R^-}_{\tilde{b}_{bi}^{IVN}}\right] \end{array} \right\rangle$$

The above (FNIVTP) may be written as:

$$\text{Min} \ \tilde{Z}_{bi}^{IVN} = \sum_{i=0}^{m} \sum_{j=0}^{n} \left\{ \begin{array}{l} \left\langle \begin{array}{l} (a_{\tilde{c}_{bi}^{IVN}}, b_{\tilde{c}_{bi}^{IVN}}, c_{\tilde{c}_{bi}^{IVN}}, d_{\tilde{c}_{bi}^{IVN}}) \\ \left[T^{L^+}_{\tilde{c}_{bi}^{IVN}}, T^{R^+}_{\tilde{c}_{bi}^{IVN}}\right], \left[I^{L^+}_{\tilde{c}_{bi}^{IVN}}, I^{R^+}_{\tilde{c}_{bi}^{IVN}}\right], \left[F^{L^+}_{\tilde{c}_{bi}^{IVN}}, F^{R^+}_{\tilde{c}_{bi}^{IVN}}\right], \\ \left[T^{L^-}_{\tilde{c}_{bi}^{IVN}}, T^{R^-}_{\tilde{c}_{bi}^{IVN}}\right], \left[I^{L^-}_{\tilde{c}_{bi}^{IVN}}, I^{R^-}_{\tilde{c}_{bi}^{IVN}}\right], \left[F^{L^-}_{\tilde{c}_{bi}^{IVN}}, F^{R^-}_{\tilde{c}_{bi}^{IVN}}\right] \end{array} \right\rangle \cdot \\ \left\langle \begin{array}{l} (a_{\tilde{x}_{bi}^{IVN}}, b_{\tilde{x}_{bi}^{IVN}}, c_{\tilde{x}_{bi}^{IVN}}, d_{\tilde{x}_{bi}^{IVN}}) \\ \left[T^{L^+}_{\tilde{x}_{bi}^{IVN}}, T^{R^+}_{\tilde{x}_{bi}^{IVN}}\right], \left[I^{L^+}_{\tilde{x}_{bi}^{IVN}}, I^{R^+}_{\tilde{x}_{bi}^{IVN}}\right], \left[F^{L^+}_{\tilde{x}_{bi}^{IVN}}, F^{R^+}_{\tilde{x}_{bi}^{IVN}}\right], \\ \left[T^{L^-}_{\tilde{x}_{bi}^{IVN}}, T^{R^-}_{\tilde{x}_{bi}^{IVN}}\right], \left[I^{L^-}_{\tilde{x}_{bi}^{IVN}}, I^{R^-}_{\tilde{x}_{bi}^{IVN}}\right], \left[F^{L^-}_{\tilde{x}_{bi}^{IVN}}, F^{R^-}_{\tilde{x}_{bi}^{IVN}}\right] \end{array} \right\rangle \end{array} \right\}$$

Subject to

$$\sum_{j=0}^{n} \left\langle \begin{array}{l} (a_{\tilde{x}_{bi}^{IVN}}, b_{\tilde{x}_{bi}^{IVN}}, c_{\tilde{x}_{bi}^{IVN}}, d_{\tilde{x}_{bi}^{IVN}}) \\ \left[T^{L^+}_{\tilde{x}_{bi}^{IVN}}, T^{R^+}_{\tilde{x}_{bi}^{IVN}}\right], \left[I^{L^+}_{\tilde{x}_{bi}^{IVN}}, I^{R^+}_{\tilde{x}_{bi}^{IVN}}\right], \left[F^{L^+}_{\tilde{x}_{bi}^{IVN}}, F^{R^+}_{\tilde{x}_{bi}^{IVN}}\right], \\ \left[T^{L^-}_{\tilde{x}_{bi}^{IVN}}, T^{R^-}_{\tilde{x}_{bi}^{IVN}}\right], \left[I^{L^-}_{\tilde{x}_{bi}^{IVN}}, I^{R^-}_{\tilde{x}_{bi}^{IVN}}\right], \left[F^{L^-}_{\tilde{x}_{bi}^{IVN}}, F^{R^-}_{\tilde{x}_{bi}^{IVN}}\right] \end{array} \right\rangle \approx \left\langle \begin{array}{l} (a_{\tilde{a}_{bi}^{IVN}}, b_{\tilde{a}_{bi}^{IVN}}, c_{\tilde{a}_{bi}^{IVN}}, d_{\tilde{a}_{bi}^{IVN}}), \\ \left[T^{L^+}_{\tilde{a}_{bi}^{IVN}}, T^{R^+}_{\tilde{a}_{bi}^{IVN}}\right], \left[I^{L^+}_{\tilde{a}_{bi}^{IVN}}, I^{R^+}_{\tilde{a}_{bi}^{IVN}}\right], \left[F^{L^+}_{\tilde{a}_{bi}^{IVN}}, F^{R^+}_{\tilde{a}_{bi}^{IVN}}\right], \\ \left[T^{L^-}_{\tilde{a}_{bi}^{IVN}}, T^{R^-}_{\tilde{a}_{bi}^{IVN}}\right], \left[I^{L^-}_{\tilde{a}_{bi}^{IVN}}, I^{R^-}_{\tilde{a}_{bi}^{IVN}}\right], \left[F^{L^-}_{\tilde{a}_{bi}^{IVN}}, F^{R^-}_{\tilde{a}_{bi}^{IVN}}\right] \end{array} \right\rangle,$$

$$i = 1, 2, 3, \dots, m (\text{shipping sources}),$$

$$\sum_{i=0}^{m} \left\langle \begin{array}{l} (a_{\tilde{x}_{bi}^{IVN}}, b_{\tilde{x}_{bi}^{IVN}}, c_{\tilde{x}_{bi}^{IVN}}, d_{\tilde{x}_{bi}^{IVN}}) \\ \left[T^{L^+}_{\tilde{x}_{bi}^{IVN}}, T^{R^+}_{\tilde{x}_{bi}^{IVN}}\right], \left[I^{L^+}_{\tilde{x}_{bi}^{IVN}}, I^{R^+}_{\tilde{x}_{bi}^{IVN}}\right], \left[F^{L^+}_{\tilde{x}_{bi}^{IVN}}, F^{R^+}_{\tilde{x}_{bi}^{IVN}}\right], \\ \left[T^{L^-}_{\tilde{x}_{bi}^{IVN}}, T^{R^-}_{\tilde{x}_{bi}^{IVN}}\right], \left[I^{L^-}_{\tilde{x}_{bi}^{IVN}}, I^{R^-}_{\tilde{x}_{bi}^{IVN}}\right], \left[F^{L^-}_{\tilde{x}_{bi}^{IVN}}, F^{R^-}_{\tilde{x}_{bi}^{IVN}}\right] \end{array} \right\rangle \approx \left\langle \begin{array}{l} (a_{\tilde{b}_{bi}^{IVN}}, b_{\tilde{b}_{bi}^{IVN}}, c_{\tilde{b}_{bi}^{IVN}}, d_{\tilde{b}_{bi}^{IVN}}), \\ \left[T^{L^+}_{\tilde{b}_{bi}^{IVN}}, T^{R^+}_{\tilde{b}_{bi}^{IVN}}\right], \left[I^{L^+}_{\tilde{b}_{bi}^{IVN}}, I^{R^+}_{\tilde{b}_{bi}^{IVN}}\right], \left[F^{L^+}_{\tilde{b}_{bi}^{IVN}}, F^{R^+}_{\tilde{b}_{bi}^{IVN}}\right], \\ \left[T^{L^-}_{\tilde{b}_{bi}^{IVN}}, T^{R^-}_{\tilde{b}_{bi}^{IVN}}\right], \left[I^{L^-}_{\tilde{b}_{bi}^{IVN}}, I^{R^-}_{\tilde{b}_{bi}^{IVN}}\right], \left[F^{L^-}_{\tilde{b}_{bi}^{IVN}}, F^{R^-}_{\tilde{b}_{bi}^{IVN}}\right] \end{array} \right\rangle,$$

$$j = 1, 2, 3, \dots, n (\text{destination}),$$

and

$$\left\langle \begin{array}{l} (a_{\tilde{x}_{bi}^{IVN}}, b_{\tilde{x}_{bi}^{IVN}}, c_{\tilde{x}_{bi}^{IVN}}, d_{\tilde{x}_{bi}^{IVN}}) \\ \left[T^{L^+}_{\tilde{x}_{bi}^{IVN}}, T^{R^+}_{\tilde{x}_{bi}^{IVN}}\right], \left[I^{L^+}_{\tilde{x}_{bi}^{IVN}}, I^{R^+}_{\tilde{x}_{bi}^{IVN}}\right], \left[F^{L^+}_{\tilde{x}_{bi}^{IVN}}, F^{R^+}_{\tilde{x}_{bi}^{IVN}}\right], \\ \left[T^{L^-}_{\tilde{x}_{bi}^{IVN}}, T^{R^-}_{\tilde{x}_{bi}^{IVN}}\right], \left[I^{L^-}_{\tilde{x}_{bi}^{IVN}}, I^{R^-}_{\tilde{x}_{bi}^{IVN}}\right], \left[F^{L^-}_{\tilde{x}_{bi}^{IVN}}, F^{R^-}_{\tilde{x}_{bi}^{IVN}}\right] \end{array} \right\rangle \geq \tilde{0},$$

$$\forall \ i = 1, 2, 3, \dots, m, \ j = 1, 2, 3, \dots, n.$$

4.1. Solution Procedure:

The step wise solution procedure of an unbalanced FNIVTP can be summarized as follows:

Step 1: Formulate the FNIVTP form given uncertain data of company setup in new places.

Step 2: Convert the FNIVTP into crisp TP by using score function

Step 3: Solve the crisp TP by using Microsoft excel

Step 4: Find the corresponding solution of FNIVTP

Step 5: Compare the crisp solution and neutrosophic solution

4.2. Steps for Balancing of FNIVTP

For solution of FNIVTP, first we convert all cost, demand and supply, which are in the form IVBTrNNs into crisp values by using score function. If $\sum_{i=0}^{m}(\tilde{a}_{bi}^{IVN})_i < \sum_{j=0}^{n}(\tilde{b}_{bi}^{IVN})_j$ or $\sum_{i=0}^{m}(\tilde{a}_{bi}^{IVN})_i > \sum_{j=0}^{n}(\tilde{b}_{bi}^{IVN})_j$ for all *i, j*, then for balance, make sure as $\sum_{i=0}^{m}(\tilde{a}_{bi}^{IVN})_i = \sum_{j=0}^{n}(\tilde{b}_{bi}^{IVN})_j$, for all *i, j* by adding a row or column with zero IVBTrNNs cost entries in cost matrix. The proposed approach is applied in the following example where the author considered an examples on FNTP i.e. NTP type -3.

5. Numerical Problem

A top rated IT company have a setup in a city S with annual neutrosophic capacities approximately $\left\langle \begin{matrix}(1.7,2.8,3.9,4.8);\\ [0.3,0.6],[0.5,0.7],[0.4,0.7],\\ [-0.5,-0.2],[-0.7,-0.2],[-0.5,-0.1]\end{matrix}\right\rangle$ units. The company shipped these products to the distribution centres located at M, N and O with annual neutrosophic demand are as follows:

Estimated Demands		
Distribution	M	$\left\langle \begin{matrix}(1.5,2.5,4.0,4.5);\\ [0.5,0.6],[0.3,0.7],[0.2,0.8],\\ [-0.4,-0.1],[-0.6,-0.3],[-0.3,-0.1]\end{matrix}\right\rangle$
	N	$\left\langle \begin{matrix}(0.5,1.5,3.0,4.5);\\ [0.2,0.4],[0.5,0.7],[0.4,0.7],\\ [-0.5,-0.2],[-0.7,-0.2],[-0.5,-0.1]\end{matrix}\right\rangle$
	O	$\left\langle \begin{matrix}(1,2,3,4);\\ [0.3,0.5],[0.6,0.7],[0.5,0.8],\\ [-0.5,-0.2],[-0.6,-0.3],[-0.5,-0.3]\end{matrix}\right\rangle$

Due to the more demands of product at different part of country, the company plans to increase the capacities by setting up new branches at cities A, B, C, D. The estimated fixed neutrosophic price and the annual neutrosophic capacities in four cities as follows:

	Estimated fixed value in IVBTrNN	Estimated Expense in IVBTrNN
A	$\left\langle \begin{matrix}(10.5,12.7,19.1,28.8);\\ [0.5,0.8],[0.3,0.6],[0.5,0.7],\\ [-0.4,-0.2],[-0.4,-0.2],[-0.5,-0.2]\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.7,0.9,1.2,1.4);\\ [0.5,0.7],[0.4,0.8],[0.3,0.7],\\ [-0.6,-0.2],[-0.5,-0.4],[-0.5,-0.3]\end{matrix}\right\rangle$
B	$\left\langle \begin{matrix}(20.5,27.5,34.5,39.5);\\ [0.4,0.7],[0.4,0.6],[0.4,0.7],\\ [-0.4,-0.3],[-0.4,-0.1],[-0.7,-0.4]\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(0.75,1.5,2.5,4);\\ [0.4,0.6],[0.4,0.7],[0.6,0.8],\\ [-0.5,-0.1],[-0.6,-0.3],[-0.5,-0.4]\end{matrix}\right\rangle$
C	$\left\langle \begin{matrix}(21.5,33.5,43.5,52.5);\\ [0.4,0.8],[0.2,0.6],[0.5,0.8],\\ [-0.4,-0.3],[-0.6,-0.2],[-0.4,-0.3]\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(1,2.5,4,5);\\ [0.5,0.7],[0.4,0.7],[0.5,0.6],\\ [-0.4,-0.1],[-0.6,-0.3],[-0.4,-0.1]\end{matrix}\right\rangle$
D	$\left\langle \begin{matrix}(35.5,45.5,55.5,65.5);\\ [0.6,0.7],[0.4,0.8],[0.4,0.6],\\ [-0.7,-0.31],[-0.5,-0.3],[-0.7,-0.4]\end{matrix}\right\rangle$	$\left\langle \begin{matrix}(1.8,3.8,4.9,5.8);\\ [0.6,0.7],[0.3,0.7],[0.5,0.6],\\ [-0.4,-0.1],[-0.6,-0.3],[-0.2,-0.1]\end{matrix}\right\rangle$

If company plan to setup the branch at city A, then $\left(\tilde{y}_{bi}^{IVN}\right)_1 = \left\langle \begin{matrix}(1,1,1,1);\\ [1,0],[1,0],[1,0],\\ [0,-1],[,-1],[0,1]\end{matrix}\right\rangle$ and the total cost shipped from city A to the three cities i.e. at M, N and O must be less than or equal to $\left\langle \begin{matrix}(0.7,0.9,1.2,1.4);\\ [0.5,0.7],[0.4,0.8],[0.3,0.7],\\ [-0.6,-0.2],[-0.5,-0.4],[-0.5,-0.3]\end{matrix}\right\rangle$ units, otherwise it will be $\left(\tilde{y}_{bi}^{IVN}\right)_1 = \left\langle \begin{matrix}(0,0,0,0);\\ [1,0],[1,0],[1,0],\\ [0,-1],[,-1],[0,1]\end{matrix}\right\rangle$. In the similar fashion $\left(\tilde{y}_{bi}^{IVN}\right)_2, \left(\tilde{y}_{bi}^{IVN}\right)_3$ and $\left(\tilde{y}_{bi}^{IVN}\right)_4$ are equal to $\left\langle \begin{matrix}(1,1,1,1);\\ [1,0],[1,0],[1,0],\\ [0,-1],[,-1],[0,1]\end{matrix}\right\rangle$, if the company plans to setup the branch at city B, city C or city D respectively, otherwise $\left(\tilde{y}_{bi}^{IVN}\right)_2, \left(\tilde{y}_{bi}^{IVN}\right)_3$ and $\left(\tilde{y}_{bi}^{IVN}\right)_4$ are equal to $\left\langle \begin{matrix}(0,0,0,0);\\ [1,0],[1,0],[1,0],\\ [0,-1],[,-1],[0,1]\end{matrix}\right\rangle$. The annual fixed price of operating the new branch of company is written as:

$$\left\langle \begin{matrix}(10.5,12.7,19.1,28.8);\\ [0.5,0.8],[0.3,0.6],[0.5,0.7],\\ [-0.4,-0.2],[-0.4,-0.2],[-0.5,-0.2]\end{matrix}\right\rangle.\left(\tilde{y}_{bi}^{IVN}\right)_1 +$$
$$+\left\langle \begin{matrix}(20.7,27.5,34.5,39.5);\\ [0.4,0.7],[0.4,0.6],[0.4,0.7],\\ [-0.4,-0.3],[-0.4,-0.1],[-0.7,-0.4]\end{matrix}\right\rangle.\left(\tilde{y}_{bi}^{IVN}\right)_2 +$$
$$+\left\langle \begin{matrix}(21.5,33.5,43.5,52.5);\\ [0.4,0.8],[0.2,0.6],[0.5,0.8],\\ [-0.4,-0.3],[-0.6,-0.2],[-0.4,-0.3]\end{matrix}\right\rangle.\left(\tilde{y}_{bi}^{IVN}\right)_3 +$$
$$+\left\langle \begin{matrix}(35.5,45.5,55.5,65.5);\\ [0.6,0.7],[0.4,0.8],[0.4,0.6],\\ [-0.7,-0.31],[-0.5,-0.3],[-0.7,-0.4]\end{matrix}\right\rangle.\left(\tilde{y}_{bi}^{IVN}\right)_4$$

The above problem formulated as

$$\text{Min } \tilde{Z}_{bi}^{IVN}=\left(\tilde{c}_{bi}^{IVN}\right)_{11}\left(\tilde{x}_{bi}^{IVN}\right)_{11}+\left(\tilde{c}_{bi}^{IVN}\right)_{12}\left(\tilde{x}_{bi}^{IVN}\right)_{12}+\left(\tilde{c}_{bi}^{IVN}\right)_{13}\left(\tilde{x}_{bi}^{IVN}\right)_{13}$$
$$+\left(\tilde{c}_{bi}^{IVN}\right)_{21}\left(\tilde{x}_{bi}^{IVN}\right)_{21}+\left(\tilde{c}_{bi}^{IVN}\right)_{22}\left(\tilde{x}_{bi}^{IVN}\right)_{22}+\left(\tilde{c}_{bi}^{IVN}\right)_{23}\left(\tilde{x}_{bi}^{IVN}\right)_{23}+$$
$$+\left(\tilde{c}_{bi}^{IVN}\right)_{31}\left(\tilde{x}_{bi}^{IVN}\right)_{31}+\left(\tilde{c}_{bi}^{IVN}\right)_{32}\left(\tilde{x}_{bi}^{IVN}\right)_{32}+\left(\tilde{c}_{bi}^{IVN}\right)_{33}\left(\tilde{x}_{bi}^{IVN}\right)_{33}+$$
$$+\left(\tilde{c}_{bi}^{IVN}\right)_{41}\left(\tilde{x}_{bi}^{IVN}\right)_{41}+\left(\tilde{c}_{bi}^{IVN}\right)_{42}\left(\tilde{x}_{bi}^{IVN}\right)_{42}+\left(\tilde{c}_{bi}^{IVN}\right)_{43}\left(\tilde{x}_{bi}^{IVN}\right)_{43}+$$
$$+\left(\tilde{c}_{bi}^{IVN}\right)_{51}\left(\tilde{x}_{bi}^{IVN}\right)_{51}+\left(\tilde{c}_{bi}^{IVN}\right)_{52}\left(\tilde{x}_{bi}^{IVN}\right)_{52}+\left(\tilde{c}_{bi}^{IVN}\right)_{53}\left(\tilde{x}_{bi}^{IVN}\right)_{53}+$$
$$+\left(\tilde{C}_{bi}^{IVN}\right)_{1}\left(\tilde{y}_{bi}^{IVN}\right)_{1}+\left(\tilde{C}_{bi}^{IVN}\right)_{2}\left(\tilde{y}_{bi}^{IVN}\right)_{2}+\left(\tilde{C}_{bi}^{IVN}\right)_{3}\left(\tilde{y}_{bi}^{IVN}\right)_{3}+\left(\tilde{C}_{bi}^{IVN}\right)_{4}\left(\tilde{y}_{bi}^{IVN}\right)_{4}$$

Subject to

$$\left(\tilde{x}_{bi}^{IVN}\right)_{11}+\left(\tilde{x}_{bi}^{IVN}\right)_{12}+\left(\tilde{x}_{bi}^{IVN}\right)_{13} \le \left\langle \begin{array}{l}(0.7,0.9,1.2,1.4);\\ [0.5,0.7],[0.4,0.8],[0.3,0.7],\\ [-0.6,-0.2],[-0.5,-0.4],[-0.5,-0.3]\end{array}\right\rangle \left(\tilde{y}_{bi}^{IVN}\right)_{1}$$

$$\left(\tilde{x}_{bi}^{IVN}\right)_{21}+\left(\tilde{x}_{bi}^{IVN}\right)_{22}+\left(\tilde{x}_{bi}^{IVN}\right)_{23} \le \left\langle \begin{array}{l}(0.75,1.5,2.5,4);\\ [0.4,0.6],[0.4,0.7],[0.6,0.8],\\ [-0.5,-0.1],[-0.6,-0.3],[-0.5,-0.4]\end{array}\right\rangle \left(\tilde{y}_{bi}^{IVN}\right)_{2}$$

$$\left(\tilde{x}_{bi}^{IVN}\right)_{41}+\left(\tilde{x}_{bi}^{IVN}\right)_{42}+\left(\tilde{x}_{bi}^{IVN}\right)_{43} \le \left\langle \begin{array}{l}(1.8,3.8,4.9,5.8);\\ [0.6,0.7],[0.3,0.7],[0.5,0.6],\\ [-0.4,-0.1],[-0.6,-0.3],[-0.2,-0.1]\end{array}\right\rangle \left(\tilde{y}_{bi}^{IVN}\right)_{4}$$

while for city S it is written as

$$\left(\tilde{x}_{bi}^{IVN}\right)_{51}+\left(\tilde{x}_{bi}^{IVN}\right)_{52}+\left(\tilde{x}_{bi}^{IVN}\right)_{53} \le \left\langle \begin{array}{l}(1.7,2.8,3.9,4.8);\\ [0.3,0.6],[0.5,0.7],[0.4,0.7],\\ [-0.5,-0.2],[-0.7,-0.2],[-0.5,-0.1]\end{array}\right\rangle$$

In the same manner the capacity (demand) at M, N and O city respectively is as follows:

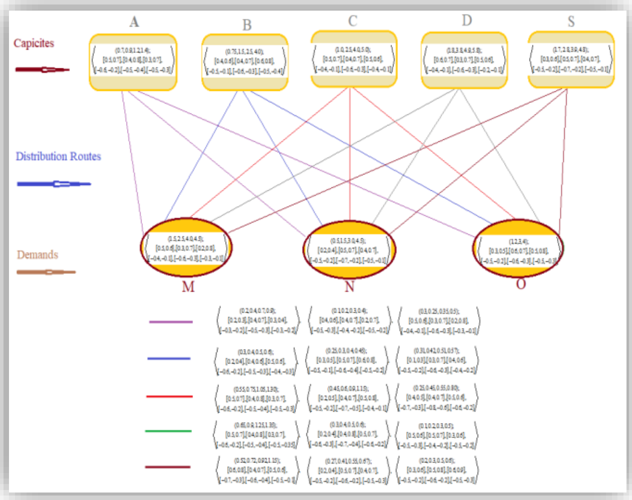
$$\left(\tilde{x}_{bi}^{IVN}\right)_{11}+\left(\tilde{x}_{bi}^{IVN}\right)_{21}+\left(\tilde{x}_{bi}^{IVN}\right)_{31}+\left(\tilde{x}_{bi}^{IVN}\right)_{41}+\left(\tilde{x}_{bi}^{IVN}\right)_{51} = \left\langle \begin{array}{c}(1.5,2.5,4,4.5);\\ [0.5,0.6],[0.3,0.7],[0.2,0.8],\\ [-0.4,-0.1],[-0.6,-0.3],[-0.5,-0.3]\end{array}\right\rangle$$

$$\left(\tilde{x}_{bi}^{IVN}\right)_{12}+\left(\tilde{x}_{bi}^{IVN}\right)_{22}+\left(\tilde{x}_{bi}^{IVN}\right)_{32}+\left(\tilde{x}_{bi}^{IVN}\right)_{42}+\left(\tilde{x}_{bi}^{IVN}\right)_{52} = \left\langle \begin{array}{c}(0.5,1.5,3,4.5);\\ [0.2,0.4],[0.5,0.7],[0.4,0.7],\\ [-0.5,-0.2],[-0.7,-0.2],[-0.5,-0.1]\end{array}\right\rangle$$

$$\left(\tilde{x}_{bi}^{IVN}\right)_{13}+\left(\tilde{x}_{bi}^{IVN}\right)_{23}+\left(\tilde{x}_{bi}^{IVN}\right)_{33}+\left(\tilde{x}_{bi}^{IVN}\right)_{43}+\left(\tilde{x}_{bi}^{IVN}\right)_{53} = \left\langle \begin{array}{c}(1,2,3,4);\\ [0.3,0.5],[0.6,0.7],[0.5,0.8],\\ [-0.5,-0.2],[-0.6,-0.3],[-0.5,-0.3]\end{array}\right\rangle$$

and $\left(\tilde{x}_{bi}^{IVN}\right)_{ij} \ge 0; i=1,2,3,4,5; j=1,2,3.$

Distribution of NTP type-3 shown in figure 3 as follows:



6. IVBTrNN cost, demand and supply with Score Function

Using the score function, one convert each IVBTrNN cost, demand and supply into crisp numbers as follows in table 2:

Table 2

	a	b	c	d	$\frac{a+b+c+d}{4}$	$T^{L^+}_{\tilde{x}^{IVN}_{bi}}$	$T^{R^+}_{\tilde{x}^{IVN}_{bi}}$	$I^{L^+}_{\tilde{x}^{IVN}_{bi}}$	$I^{R^+}_{\tilde{x}^{IVN}_{bi}}$	$F^{L^+}_{\tilde{x}^{IVN}_{bi}}$	$F^{R^+}_{\tilde{x}^{IVN}_{bi}}$	$T^{L^-}_{\tilde{x}^{IVN}_{bi}}$	$T^{R^-}_{\tilde{x}^{IVN}_{bi}}$	$I^{L^-}_{\tilde{x}^{IVN}_{bi}}$	$I^{R^-}_{\tilde{x}^{IVN}_{bi}}$	$F^{L^-}_{\tilde{x}^{IVN}_{bi}}$	$F^{R^-}_{\tilde{x}^{IVN}_{bi}}$	Score function
$\left(\tilde{c}^{IVN}_{bi}\right)_{11}$	0.2	0.4	0.7	0.9	0.55	0.2	0.3	0.4	0.7	0.3	0.4	-0.3	-0.2	-0.5	-0.3	-0.3	-0.2	0.515625
$\left(\tilde{c}^{IVN}_{bi}\right)_{12}$	0.1	0.2	0.3	0.4	0.25	0.4	0.6	0.4	0.7	0.2	0.7	-0.5	-0.3	-0.4	-0.2	-0.5	-0.2	0.234375
$\left(\tilde{c}^{IVN}_{bi}\right)_{13}$	0.3	0.25	0.35	0.5	0.35	0.5	0.6	0.3	0.7	0.2	0.8	-0.4	-0.1	-0.6	-0.3	-0.3	-0.1	0.345625
$\left(\tilde{c}^{IVN}_{bi}\right)_{21}$	0.3	0.4	0.5	0.6	0.45	0.2	0.4	0.4	0.6	0.5	0.6	-0.6	-0.2	-0.5	-0.3	-0.4	-0.3	0.405
$\left(\tilde{c}^{IVN}_{bi}\right)_{22}$	0.25	0.3	0.4	0.45	0.35	0.3	0.5	0.5	0.7	0.6	0.8	-0.5	-0.1	-0.6	-0.4	-0.5	-0.2	0.319375
$\left(\tilde{c}^{IVN}_{bi}\right)_{23}$	0.31	0.42	0.51	0.57	0.4525	0.1	0.3	0.3	0.7	0.4	0.6	-0.5	-0.2	-0.6	-0.3	-0.4	-0.2	0.40725
$\left(\tilde{c}^{IVN}_{bi}\right)_{31}$	0.55	0.75	1.05	1.3	0.9125	0.5	0.7	0.4	0.8	0.3	0.7	-0.6	-0.2	-0.5	-0.4	-0.5	-0.3	0.901094
$\left(\tilde{c}^{IVN}_{bi}\right)_{32}$	0.45	0.6	0.9	1.15	0.775	0.2	0.5	0.4	0.7	0.5	0.8	-0.5	-0.2	-0.7	-0.5	-0.4	-0.1	0.707188
$\left(\tilde{c}^{IVN}_{bi}\right)_{33}$	0.25	0.45	0.55	0.8	0.5125	0.4	0.5	0.4	0.7	0.5	0.6	-0.7	-0.3	-0.8	-0.6	-0.6	-0.2	0.506094
$\left(\tilde{c}^{IVN}_{bi}\right)_{41}$	0.65	0.9	1.15	1.35	1.0125	0.5	0.7	0.4	0.8	0.3	0.7	-0.6	-0.2	-0.5	-0.4	-0.5	-0.35	1.006172
$\left(\tilde{c}^{IVN}_{bi}\right)_{42}$	0.3	0.4	0.5	0.6	0.45	0.2	0.4	0.4	0.8	0.5	0.7	-0.6	-0.3	-0.7	-0.4	-0.6	-0.2	0.405
$\left(\tilde{c}^{IVN}_{bi}\right)_{43}$	0.1	0.2	0.3	0.5	0.275	0.5	0.6	0.5	0.7	0.3	0.6	-0.5	-0.3	-0.4	-0.2	-0.5	-0.2	0.257813
$\left(\tilde{c}^{IVN}_{bi}\right)_{51}$	0.52	0.72	0.92	1.15	0.8275	0.6	0.8	0.4	0.7	0.5	0.6	-0.7	-0.3	-0.6	-0.4	-0.5	-0.1	0.806813
$\left(\tilde{c}^{IVN}_{bi}\right)_{52}$	0.27	0.41	0.55	0.67	0.475	0.2	0.4	0.5	0.7	0.4	0.7	-0.5	-0.2	-0.7	-0.2	-0.5	-0.1	0.421563
$\left(\tilde{c}^{IVN}_{bi}\right)_{53}$	0.2	0.3	0.5	0.6	0.4	0.3	0.6	0.5	0.8	0.6	0.9	-0.5	-0.2	-0.6	-0.2	-0.5	-0.3	0.35
$(\tilde{b}^{IVN}_{bi})_1$	1.5	2.5	4	4.5	3.125	0.5	0.6	0.3	0.7	0.2	0.8	-0.4	-0.1	-0.6	-0.3	-0.3	-0.1	3.085938
$(\tilde{b}^{IVN}_{bi})_2$	0.5	1.5	3	4.5	2.375	0.2	0.4	0.5	0.7	0.4	0.7	-0.5	-0.2	-0.7	-0.2	-0.5	-0.1	2.107813
$(\tilde{b}^{IVN}_{bi})_3$	1	2	3	4	2.5	0.3	0.5	0.6	0.7	0.5	0.8	-0.5	-0.2	-0.6	-0.3	-0.5	-0.3	2.25
$(\tilde{a}^{IVN}_{bi})_1$	0.7	0.9	1.2	1.4	1.05	0.5	0.7	0.4	0.8	0.3	0.7	-0.6	-0.2	-0.5	-0.4	-0.5	-0.3	1.036875
$(\tilde{a}^{IVN}_{bi})_2$	0.75	1.5	2.5	4	2.1875	0.4	0.6	0.4	0.7	0.6	0.8	-0.5	-0.1	-0.6	-0.3	-0.5	-0.4	2.105469
$(\tilde{a}^{IVN}_{bi})_3$	1	2.5	4	5	3.125	0.5	0.7	0.4	0.7	0.5	0.6	-0.4	-0.1	-0.6	-0.3	-0.4	-0.1	3.085938
$(\tilde{a}^{IVN}_{bi})_4$	1.8	3.8	4.9	5.8	4.075	0.6	0.7	0.3	0.7	0.5	0.6	-0.4	-0.1	-0.6	-0.3	-0.2	-0.1	4.024063
$\tilde{s}^{IVN}_{bi}$	1.7	2.8	3.9	4.8	3.3	0.3	0.6	0.5	0.7	0.4	0.7	-0.5	-0.2	-0.7	-0.2	-0.5	-0.1	3.0525

On the basis of score function the CTP is as follows:

$$\begin{aligned}\text{Min } \tilde{Z}_{bi}^{IVN} &= 0.515625\left(\tilde{x}_{bi}^{IVN}\right)_{11} + 0.234375\left(\tilde{x}_{bi}^{IVN}\right)_{12} + \\ &+0.345625\left(\tilde{x}_{bi}^{IVN}\right)_{13} + 0.405\left(\tilde{x}_{bi}^{IVN}\right)_{21} + 0.319375\left(\tilde{x}_{bi}^{IVN}\right)_{22} + \\ &+0.40725\left(\tilde{x}_{bi}^{IVN}\right)_{23} + 0.901094\left(\tilde{x}_{bi}^{IVN}\right)_{31} + 0.707188\left(\tilde{x}_{bi}^{IVN}\right)_{32} + \\ &+0.506094\left(\tilde{x}_{bi}^{IVN}\right)_{33} + 1.006172\left(\tilde{x}_{bi}^{IVN}\right)_{41} + 0.405\left(\tilde{x}_{bi}^{IVN}\right)_{42} + \\ &+0.257813\left(\tilde{x}_{bi}^{IVN}\right)_{43} + 0.806813\left(\tilde{x}_{bi}^{IVN}\right)_{51} + 0.421563\left(\tilde{x}_{bi}^{IVN}\right)_{52} + \\ &+0.35\left(\tilde{x}_{bi}^{IVN}\right)_{53} + 17.55281\left(\tilde{y}_{bi}^{IVN}\right)_{1} + 30.11875\left(\tilde{y}_{bi}^{IVN}\right)_{2} + \\ &+37.27813\left(\tilde{y}_{bi}^{IVN}\right)_{3} + 50.43688\left(\tilde{y}_{bi}^{IVN}\right)_{3}\end{aligned}$$

subject to

$$\left(\tilde{x}_{bi}^{IVN}\right)_{11} + \left(\tilde{x}_{bi}^{IVN}\right)_{12} + \left(\tilde{x}_{bi}^{IVN}\right)_{13} + \left(\tilde{y}_{bi}^{IVN}\right)_{1} \le 1.036875$$
$$\left(\tilde{x}_{bi}^{IVN}\right)_{21} + \left(\tilde{x}_{bi}^{IVN}\right)_{22} + \left(\tilde{x}_{bi}^{IVN}\right)_{23} + \left(\tilde{y}_{bi}^{IVN}\right)_{2} = 2.105469$$
$$\left(\tilde{x}_{bi}^{IVN}\right)_{31} + \left(\tilde{x}_{bi}^{IVN}\right)_{32} + \left(\tilde{x}_{bi}^{IVN}\right)_{33} + \left(\tilde{y}_{bi}^{IVN}\right)_{3} \le 3.085938$$
$$\left(\tilde{x}_{bi}^{IVN}\right)_{41} + \left(\tilde{x}_{bi}^{IVN}\right)_{42} + \left(\tilde{x}_{bi}^{IVN}\right)_{43} + \left(\tilde{y}_{bi}^{IVN}\right)_{4} \le 4.024063$$
$$\left(\tilde{x}_{bi}^{IVN}\right)_{51} + \left(\tilde{x}_{bi}^{IVN}\right)_{52} + \left(\tilde{x}_{bi}^{IVN}\right)_{53} \le 3.0525$$
$$\left(\tilde{x}_{bi}^{IVN}\right)_{11} + \left(\tilde{x}_{bi}^{IVN}\right)_{21} + \left(\tilde{x}_{bi}^{IVN}\right)_{31} + \left(\tilde{x}_{bi}^{IVN}\right)_{41} + \left(\tilde{x}_{bi}^{IVN}\right)_{51} = 3.085938$$
$$\left(\tilde{x}_{bi}^{IVN}\right)_{12} + \left(\tilde{x}_{bi}^{IVN}\right)_{22} + \left(\tilde{x}_{bi}^{IVN}\right)_{32} + \left(\tilde{x}_{bi}^{IVN}\right)_{42} + \left(\tilde{x}_{bi}^{IVN}\right)_{52} = 2.107813$$
$$\left(\tilde{x}_{bi}^{IVN}\right)_{13} + \left(\tilde{x}_{bi}^{IVN}\right)_{23} + \left(\tilde{x}_{bi}^{IVN}\right)_{33} + \left(\tilde{x}_{bi}^{IVN}\right)_{43} + \left(\tilde{x}_{bi}^{IVN}\right)_{53} = 2.25$$

With help of excel solver, the solution of CTP is $\left(\tilde{x}_{bi}^{IVN}\right)_{11} = 0.980469$, $\left(\tilde{x}_{bi}^{IVN}\right)_{12} = 0.056406$, $\left(\tilde{x}_{bi}^{IVN}\right)_{21} = 2.105469$, $\left(\tilde{x}_{bi}^{IVN}\right)_{42} = 1.774063$, $\left(\tilde{x}_{bi}^{IVN}\right)_{43} = 2.25$, $\left(\tilde{x}_{bi}^{IVN}\right)_{52} = 0.277344$ and hence

$$\begin{aligned}\text{Min } \tilde{Z}_{bi}^{IVN} &= 0.515625 \times 0.980469 + \\ &+0.234375 \times 0.056406 + 0.405 \times 2.105469 + \\ &+0.405 \times 1.774063 + 0.257813 \times 2.25 + \\ &+0.421563 \times 0.277344 \\ &= 2.786982163\end{aligned}$$

$$\left(\tilde{x}_{bi}^{IVN}\right)_{11} = \left\langle \begin{matrix}(-3, 0, 2.5, 4.25); \\ [0.2, 0.4], [0.45, 0.7], [0.7, 0.5], \\ [-0.4, -0.1], [-0.65, -0.25], [-0.4, -0.1]\end{matrix} \right\rangle,$$

$$\left(\tilde{x}_{bi}^{IVN}\right)_{12} = \left\langle \begin{matrix}(-3.55, -1.6, 1.2, 4.4); \\ [0.2, 0.4], [0.425, 0.75], [0.5, 0.7], \\ [-0.4, -0.1], [-0.55, -0.35], [-0.5, -0.3]\end{matrix} \right\rangle,$$

$$\left(\tilde{x}_{bi}^{IVN}\right)_{21} = \left\langle \begin{matrix}(0.75, 1.5, 2.5, 4); \\ [0.4, 0.6], [0.4, 0.7], [0.6, 0.8], \\ [-0.5, -0.1], [-0.6, -0.3], [-0.5, -0.4]\end{matrix} \right\rangle,$$

$$\left(\tilde{x}_{bi}^{IVN}\right)_{42} = \left\langle \begin{matrix}(-2.2, 0.8, 2.9, 4.8); \\ [0.3, 0.5], [0.45, 0.7], [0.5, 0.8], \\ [-0.4, -0.1], [-0.6, -0.3], [-0.5, -0.3]\end{matrix} \right\rangle,$$

$$\left(\tilde{x}_{bi}^{IVN}\right)_{43} = \left\langle \begin{matrix}(1, 2, 3, 4); \\ [0.3, 0.5], [0.6, 0.7], [0.5, 0.8], \\ [-0.5, -0.2], [-0.6, -0.3], [-0.5, -0.3]\end{matrix} \right\rangle,$$

$$\left(\tilde{x}_{bi}^{IVN}\right)_{52} = \left\langle \begin{matrix}(-15.3, -5.2, 6.4, 16.85); \\ [0.3, 0.6], [0.4875, 0.7025], [0.5, 0.7], \\ [-0.4, -0.1], [-0.5644, -0.2458], [-0.5, -0.1]\end{matrix} \right\rangle.$$

The company should be enhance their product by setup new plants at city D from where $\left\langle \begin{matrix}(-2.2, 0.8, 2.9, 4.8); \\ [0.3, 0.5], [0.45, 0.7], [0.5, 0.8], \\ [-0.4, -0.1], [-0.6, -0.3], [-0.5, -0.3]\end{matrix} \right\rangle$ units shipped to city N and $\left\langle \begin{matrix}(1, 2, 3, 4); \\ [0.3, 0.5], [0.6, 0.7], [0.5, 0.8], \\ [-0.5, -0.2], [-0.6, -0.3], [-0.5, -0.3]\end{matrix} \right\rangle$ units shipped to city O.

This solution in tabular form shown in the following table 3,

Table 3

	a	b	c	d	$T^{L^+}_{\tilde{x}_{bi}^{IVN}}$	$T^{R^+}_{\tilde{x}_{bi}^{IVN}}$	$I^{L^+}_{\tilde{x}_{bi}^{IVN}}$	$I^{R^+}_{\tilde{x}_{bi}^{IVN}}$	$F^{L^+}_{\tilde{x}_{bi}^{IVN}}$	$F^{R^+}_{\tilde{x}_{bi}^{IVN}}$	$T^{L^-}_{\tilde{x}_{bi}^{IVN}}$	$T^{R^-}_{\tilde{x}_{bi}^{IVN}}$	$I^{L^-}_{\tilde{x}_{bi}^{IVN}}$	$I^{R^-}_{\tilde{x}_{bi}^{IVN}}$	$F^{L^-}_{\tilde{x}_{bi}^{IVN}}$	$F^{R^-}_{\tilde{x}_{bi}^{IVN}}$	**Score function**
$\left(\tilde{c}_{bi}^{IVN}\right)_{11} \cdot \left(\tilde{x}_{bi}^{IVN}\right)_{11}$	-0.6	0	1.75	3.825	0.04	0.12	0.67	0.91	0.79	0.7	-0.82	-0.32	-0.325	-0.075	-0.12	-0.02	1.061852
+																	
$\left(\tilde{c}_{bi}^{IVN}\right)_{12} \cdot \left(\tilde{x}_{bi}^{IVN}\right)_{12}$	-0.355	-0.32	0.36	1.76	0.08	0.24	0.655	0.925	0.6	0.91	-1.1	-0.43	-0.22	-0.07	-0.25	-0.06	0.334608
+																	
$\left(\tilde{c}_{bi}^{IVN}\right)_{21} \cdot \left(\tilde{x}_{bi}^{IVN}\right)_{21}$	0.225	0.6	1.25	2.4	0.08	0.24	0.64	0.88	0.8	0.92	-1.4	-0.32	-0.3	-0.09	-0.2	-0.12	1.089383
+																	
$\left(\tilde{c}_{bi}^{IVN}\right)_{42} \cdot \left(\tilde{x}_{bi}^{IVN}\right)_{42}$	-0.66	0.32	1.45	2.88	0.06	0.2	0.67	0.94	0.75	0.94	-1.24	-0.43	-0.42	-0.12	-0.3	-0.06	0.948872
+																	
$\left(\tilde{c}_{bi}^{IVN}\right)_{43} \cdot \left(\tilde{x}_{bi}^{IVN}\right)_{43}$	0.1	0.4	0.9	2	0.15	0.3	0.8	0.91	0.65	0.92	-1.25	-0.56	-0.24	-0.06	-0.25	-0.06	0.785188
+																	
$\left(\tilde{c}_{bi}^{IVN}\right)_{52} \cdot \left(\tilde{x}_{bi}^{IVN}\right)_{52}$	-4.131	-2.132	3.52	11.2895	0.06	0.24	0.74375	0.91075	0.7	0.91	-1.1	-0.32	-0.33864	-0.04916	-0.25	-0.03	1.974589
Z_{NTP}	-5.421	-1.132	9.23	24.1545	0.15	0.3	0.696458	0.912625	0.6	0.7	-1.25	-0.56	-0.30727	-0.07736	-0.12	-0.02	6.251362

By using the algebraic operations on neutrosophic numbers, the corresponding neutrosophic solution of FNTP as follows:

7. **Conclusion**

The above research article accord a new concept to design a fully neutrosophic mathematical model of distribution system that determines the economical and best site selection that provide minimum transportation cost for shipping the products to the issuing nodes in unsettled domain of supply chain. We use here IVBTrNN in place of crisp numbers that can handle the uncertain information more flexibly in the optimization. In example a neutrosophic value is $\left\langle \begin{matrix} (1,2,3,4); \\ [0.3,0.5],[0.6,0.7],[0.5,0.8], \\ [-0.5,-0.2],[-0.6,-0.3],[-0.5,-0.3] \end{matrix} \right\rangle$, which is equivalent to 2.5 as an average, but due to variance of different conditions in all poles like $[0.3,0.5],[0.6,0.7],[0.5,0.8]$, and in $[-0.5,-0.2],[-0.6,-0.3],[-0.5,-0.3]$ the value is finally equivalent to 2.25. During the covid-19 pandemic, to maintain physical distance among, human, used & unused equipments and researchers may be considered as IVBTrNN in place of crisp numbers that are much effective to address the uncertainty & hesitation in real world situations.

Conflict of Interest

No potential conflict of interest was reported by the author(s).

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