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M$\mathcal{B}$J-FILTERS ON LATTICE IMPLICATION ALGEBRAS

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Abstract. We explored the some equivalent conditions and properties of M$\mathcal{B}$J-filters of Lattice implication algebras. As well we define M$\mathcal{B}$J-lattice filters of Lattice implication algebras. Finally, it was established that M$\mathcal{B}$J-Lattice filter is an M$\mathcal{B}$J-filter, but not the other way around.

Key words: M$\mathcal{B}$J-filter, M$\mathcal{B}$J-lattice filter, Lattice implication algebra(LIA)

1.Introduction

Xu [1] proposed a logical algebra—lattice implication algebra in 1993. The lattice is defined in lattice implication algebra to explain uncertainties and particularly beyond comparison, and supposed to describe how humans reasoning. In lattice implication algebra, Xu and Qin [2] established the notion of filters and examined its properties. Xu and Qin define the fuzzy filter in lattice implication algebra and discuss some of its characteristics in [3]. Neutrosophic set was defined by Smarandache in 1998 ([4]). Neutrosophic sets are a new tool for finding uncertainty that avoids many of the problems that have plagued traditional theoretical approaches. Neutosophic set theory is being studied for a variety of applications, including information theory, probability theory, control theory, decision making, measurement theory, and so on. Since then, numerous researchers have researched in this field, and a large body of literature on the theory of neutrosophic sets has been published. The notion of M$\mathcal{B}$J-neutrosophic sets is introduced as another generalization of neutrosophic set.

In this paper, We explored the some equivalent conditions and properties of M$\mathcal{B}$J-filters of Lattice implication algebras. As well we define M$\mathcal{B}$J-lattice filters of Lattice implication algebras. Finally, it was established that M$\mathcal{B}$J-Lattice filter is an M$\mathcal{B}$J-filter, but not the other way around

2. Preliminaries

2.1 *Definition[8]:*
Let X be a non-empty set. A MBJ-neutrosophic set of the form $A = \{\langle \varsigma; M_A(\varsigma), \widetilde{B_A}(\varsigma), J_A(\varsigma)/\varsigma \in X\}\rangle\}$ where M_A and J_A are fuzzy sets in X, which are called a truth membership function and a false membership function, respectively and $\widetilde{B_A}$ is an IVF set in X which is called an indeterminate interval valued membership function. For the sake of simplicity, we shall use the symbol$A = (M_A, \widetilde{B_A}, J_A)$ for the M$\mathcal{B}$J-Neutrosophic set$A = \{\langle \varsigma; M_A(\varsigma), \widetilde{B_A}(\varsigma), J_A(\varsigma)/\varsigma \in X\}\rangle\}$.
In an M$\mathcal{B}$J-Neutrosophic set$A = (M_A, \widetilde{B_A}, J_A)$ in X we take $\widetilde{B_A}: X \longrightarrow [I], \varsigma \longrightarrow [B_A^-(\varsigma), B_A^+(\varsigma)]$ with $B_A^-(\varsigma) = B_A^+(\varsigma)$then $A = (M_A, \widetilde{B_A}, J_A)$ is aneutrosophic set in X.

2.2*Definition[1]*:
A bounded lattice (L, ∨, ∧, 0, 1) satisfy the following axioms:
(C1) $\varsigma \to (\varrho \to \varphi) = \varrho \to (\varsigma \to \varphi)$
(C2) $\varsigma \to \varsigma = 1$,
(C3) $\varsigma \to \varrho = \varrho' \to \varsigma'$,
(C4) $\varsigma \to \varrho = \varrho \to \varsigma = 1 \Rightarrow \varsigma = \varrho$,
(C5) $(\varsigma \to \varrho) \to \varrho = (\varrho \to \varsigma) \to \varsigma$,
(D1) $(\varsigma \vee \varrho) \to \varphi = (\varsigma \to \varphi) \wedge (\varrho \to \varphi)$,
(D2) $(\varsigma \wedge \varrho) \to \varphi = (\varsigma \to \varphi) \vee (\varrho \to \varphi)$, for all $\varsigma, \varrho, \varphi \in$ L.
If (L, ∨, ∧, 0, 1) satisfies the conditions (C1), (C2), (C3), (C4), and (C5), is called quasi-LIA.
We can define a partial ordering ≤ on a LIA L by $\varsigma \le \varrho$ if and only if $\varsigma \to \varrho = 1$.
In a LIA L, the following hold: for all $\varsigma, \varrho, \varphi \in$ L,
(1) $0 \to \varsigma = 1$, $1 \to \varsigma = \varsigma$, and $\varsigma \to 1 = 1$,
(2) $\varsigma \le \varrho \Longrightarrow \varphi \to \varsigma \le \varphi \to \varrho$ and $\varsigma \to \varphi \ge \varrho \to \varphi$,
(3) $(\varsigma \to \varrho) \to ((\varrho \to \varphi) \to (\varsigma \to \varphi)) = 1$,
(4) $\varsigma \to ((\varsigma \to \varrho) \to \varrho) = 1$,
(5) $((\varsigma \to \varrho) \to \varrho) \to \varrho = \varsigma \to \varrho$.

2.3*Definition[7]*:
Let χbe the universe. A neutrosophic set $\mathbb{A}$ in χ is characterized by a truth membership function T_A, indeterminate membership function I_A and falsity membership function F_A where T_A, I_A and F_A are real standard elements of [0,1]. It can be written as
A= $\{\langle \varsigma, (T_A(\varsigma), I_A(\varsigma), F_A(\varsigma)/\varsigma \in E, T_A, I_A, F_A \in]0^-, 1^+[\rangle\}$. There is no restriction on the sum of $T_A(\varsigma), I_A(\varsigma)$ *and* $F_A(\varsigma)$ and so$0^- \le T_A(\varsigma) + I_A(\varsigma) + F_A(\varsigma) \le 3^+$.

3. M$\mathcal{B}$J-filters

3.1 *Definition*:
Let χbe the universe. A M $\mathcal{B}$J-neutrosophic set
$\mathbb{A} = \{\langle \varsigma; M_A(\varsigma), \widetilde{\mathcal{B}_A}(\varsigma), J_A(\varsigma)/\varsigma \in X\}\rangle$ of a LIA L is called a M $\mathcal{B}$J-filter of L if it satisfies

(i) $M_A(1) \ge M_A(\varsigma)$, $\widetilde{\mathcal{B}_A}(1) \ge* \widetilde{\mathcal{B}_A}(\varsigma)$, $J_A(1) \le J_A(\varsigma)$
(ii) $M_A(\varrho) \ge \min\{M_A(\varsigma), M_A(\varsigma \longrightarrow \varrho)\}$
(iii) $\widetilde{\mathcal{B}_A}(\varrho) \ge \text{rmin}\{\widetilde{\mathcal{B}_A}(\varsigma), \widetilde{\mathcal{B}_A}(\varsigma \longrightarrow \varrho)\}$
(iv) $J_A(\varrho) \le \max\{J_A(\varsigma), J_A(\varsigma \longrightarrow \varrho)\}$

3.2 *Example*:
Let L= {0, a, $\mathscr{b}$, c, 1}. Define partial order on L as 0<a<$\mathscr{b}$<c<1 and define $\varsigma \wedge \varrho = \min\{\varsigma, \varrho\}$ and $\varsigma \vee \varrho = \max\{\varsigma, \varrho\}$ for all $\varsigma, \varrho \in L$ and " ' "and "⟶" as follows

ς	ς'
0	1
a	c
b	b
c	a
1	0

$\longrightarrow$	0	a	b	c	1
0	1	1	1	1	1
a	c	1	1	1	1
b	b	c	1	1	1
c	a	b	c	1	1
1	0	a	b	c	1

Then (L,∨,∧,′ , ⟶) is LIA.
Define M$\mathcal{B}$J-neutrosophic set A in L by

	0	a	b	c	1
M_A	0.1	0.2	0.3	0.4	0.5
$\widetilde{\mathcal{B}_A}$	[0.2,0.3]	[0.4,0.5]	[0.3,0.4]	[0.2,0.5]	[0.2,0.4]
J_A	0.5	0.4	0.3	0.2	0.1

Then A is a M$\mathcal{B}$J-filter on L.

3.3 *Theorem*:
Let L be a LIA and $A = \{\langle\varsigma; M_A(\varsigma), \widetilde{\mathcal{B}_A}(\varsigma), J_A(\varsigma)/\varsigma \in X\}\rangle$ is a M$\mathcal{B}$J-filter of L, then for all$\varsigma, \varrho \in L$, $\beta \geq \varsigma$ implies $M_A(\varrho) \geq M_A(\varsigma), \widetilde{\mathcal{B}_A}(\varrho) \geq \widetilde{\mathcal{B}_A}(\varsigma), J_A(\varrho) \leq J_A(\varsigma)$.
Proof: Consider $M_A(\varrho) \geq \min\{M_A(\varsigma), M_A(\varsigma \longrightarrow \varrho)\} \geq \min\{M_A(\varsigma), M_A(1)\} = M_A(\varsigma)$
Consider $\widetilde{\mathcal{B}_A}(\varrho) \geq \text{rmin}\{\widetilde{\mathcal{B}_A}(\varsigma), \widetilde{\mathcal{B}_A}(\varsigma \longrightarrow \varrho)\} \geq r\min\{\widetilde{\mathcal{B}_A}(\varsigma), \widetilde{\mathcal{B}_A}(1)\} = \widetilde{\mathcal{B}_A}(\varsigma)$
Consider $J_A(\varrho) \leq \max\{J_A(\varsigma), J_A(\varsigma \longrightarrow \varrho)\} \leq \max\{J_A(\varsigma), J_A(1)\} = J_A(\varsigma)$

3.4 *Theorem*:
Let L be LIA and$A = \{\langle\varsigma; M_A(\varsigma), \widetilde{\mathcal{B}_A}(\varsigma), J_A(\varsigma)/\varsigma \in X\}\rangle$ is a M$\mathcal{B}$J-filter of L iff it satisfies
(i) $M_A(1) \geq M_A(\varsigma)$, $\widetilde{\mathcal{B}_A}(1) \geq \widetilde{\mathcal{B}_A}(\varsigma), J_A(1) \leq J_A(\varsigma)$
(ii) $M_A(\varsigma \longrightarrow \varphi) \geq \min\{M_A(\varsigma \longrightarrow \varrho), M_A(\varrho \longrightarrow \varphi)\}$
(iii) $\widetilde{\mathcal{B}_A}(\varsigma \longrightarrow \varphi) \geq r\min\{\widetilde{\mathcal{B}_A}(\varsigma \longrightarrow \varrho), \widetilde{\mathcal{B}_A}(\varrho \longrightarrow \varphi)\}$

(iv) $J_A(\varsigma \longrightarrow \varphi) \leq \max\{J_A(\varsigma \longrightarrow \varrho), J_A(\varrho \longrightarrow \varphi)\}$

Proof: Assume that A is a M$\mathcal{B}$J-filter of L.

Then (i) is trivial.

To prove $M_A(\varsigma \longrightarrow \varphi) \geq \min\{M_A(\varsigma \longrightarrow \varrho), M_A(\varrho \longrightarrow \varphi)\}$

$M_A(\varsigma \longrightarrow \varphi) \geq \min\{M_A(\varrho \longrightarrow \varphi), M_A((\varrho \longrightarrow \varphi) \longrightarrow (\varsigma \longrightarrow \varphi))\}$…….(1)

Consider$(\varsigma \longrightarrow \varrho) \longrightarrow ((\varrho \longrightarrow \varphi) \longrightarrow (\varrho \longrightarrow \varphi))$

$= (\varsigma \longrightarrow \varrho) \longrightarrow ((\varphi \longrightarrow \varrho) \longrightarrow (\varsigma \longrightarrow \varrho))$

$=(\varphi \longrightarrow \varrho) \longrightarrow ((\varsigma \longrightarrow \varrho) \longrightarrow (\varsigma \longrightarrow \varrho))$

$=(\varphi \longrightarrow \varrho) \longrightarrow 1$

$=1$

Then$(\varsigma \longrightarrow \varrho) \leq ((\varrho \longrightarrow \varphi) \longrightarrow (\varsigma \longrightarrow \varphi))$

$\Longrightarrow M_A(\varsigma \longrightarrow \varrho) \leq M_A((\varrho \longrightarrow \varphi) \longrightarrow (\varsigma \longrightarrow \varphi))$……..(2)

From (1) and (2) $M_A(\varsigma \longrightarrow \varphi) \geq \min\{M_A(\varsigma \longrightarrow \varrho), M_A(\varrho \longrightarrow \varphi)\}$

To prove $\widetilde{\mathcal{B}_A}(\varsigma \longrightarrow \varphi) \geq r\min\{\widetilde{\mathcal{B}_A}(\varsigma \longrightarrow \varrho), \widetilde{\mathcal{B}_A}(\varrho \longrightarrow \varphi)\}$

$\widetilde{\mathcal{B}_A}(\varsigma \longrightarrow \varphi) \geq r\min\{\widetilde{\mathcal{B}_A}(\varrho \longrightarrow \varphi), \widetilde{\mathcal{B}_A}((\varrho \longrightarrow \varphi) \longrightarrow (\varsigma \longrightarrow \varphi))\}$……..(3)

Since$(\varsigma \longrightarrow \varrho) \leq ((\varrho \longrightarrow \varphi) \longrightarrow (\varsigma \longrightarrow \varphi))$

$\Longrightarrow \widetilde{\mathcal{B}_A}(\varsigma \longrightarrow \varrho) \leq \widetilde{\mathcal{B}_A}((\varrho \longrightarrow \varphi) \longrightarrow (\varsigma \longrightarrow \varphi))$

Therefore $\widetilde{\mathcal{B}_A}(\varsigma \longrightarrow \varphi) \geq$ rmin$\{\widetilde{\mathcal{B}_A}(\varsigma \longrightarrow \varrho), \widetilde{\mathcal{B}_A}(\varrho \longrightarrow \varphi)\}$

To prove $J_A(\varsigma \longrightarrow \varphi) \leq \max\{J_A(\varsigma \longrightarrow \varrho), J_A(\varrho \longrightarrow \varphi)\}$

$J_A(\varsigma \longrightarrow \varphi) \leq \max\{J_A(\varrho \longrightarrow \varphi), J_A((\varrho \longrightarrow \varphi) \longrightarrow (\varsigma \longrightarrow \varphi))\}$

Since $(\varsigma \longrightarrow \varrho) \leq ((\varrho \longrightarrow \varphi) \longrightarrow (\varrho \longrightarrow \varphi))$

$\Longrightarrow J_A(\varsigma \longrightarrow \varrho) \geq J_A((\varrho \longrightarrow \varphi) \longrightarrow (\varsigma \longrightarrow \varphi))$

Therefore $J_A(\varsigma \longrightarrow \varphi) \leq \max\{J_A(\varsigma \longrightarrow \varrho), J_A(\varrho \longrightarrow \varphi)\}$.

3.5 *Theorem*:

Let A be a Neutrosophic set and L is a LIA.

Then $A = \{\langle \varsigma; M_A(\varsigma), \widetilde{\mathcal{B}_A}(\varsigma), J_A(\varsigma)/\varsigma \in X\}\rangle$ is a M$\mathcal{B}$J-filter of L iff it satisfies

(i) $M_A(1) \geq M_A(\mathfrak{p})$, $\widetilde{\mathcal{B}_A}(1) \geq \widetilde{\mathcal{B}_A}(\mathfrak{p}), J_A(1) \leq J_A(\mathfrak{p})$

(ii) $M_A(\varphi) \geq \min\{M_A(\varsigma), M_A(\varrho), M_A(\varsigma \longrightarrow (\varrho \longrightarrow \varphi)\}$

(iii) $\widetilde{\mathcal{B}_A}(\varphi) \geq$ rmin$\{\widetilde{\mathcal{B}_A}(\varsigma), \widetilde{\mathcal{B}_A}(\varrho), \widetilde{\mathcal{B}_A}(\varsigma \longrightarrow (\varrho \longrightarrow \varphi)\}$

(iv) $J_A(\varphi) \leq \max\{J_A(\varsigma), J_A(\varrho), J_A(\varsigma \longrightarrow (\varrho \longrightarrow \varphi)\}$

Proof: Suppose A is a M$\mathcal{B}$J-filter of L.

Then (i) is trivial for any $\varsigma, \varrho, \varphi \in L$.

Since $M_A(\varphi) \geq \min\{M_A(\varsigma), M_A(\varsigma \longrightarrow \varphi)\}$

And $M_A(\varsigma \longrightarrow \varphi) \geq \min\{M_A(\varrho), M_A(\varrho \longrightarrow (\varsigma \longrightarrow \varphi)\}$

Hence$M_A(\varphi) \geq \min\{M_A(\varsigma), M_A(\varrho), M_A(\varsigma \longrightarrow (\varrho \longrightarrow \varphi)\}$

Since$\widetilde{\mathcal{B}_A}(\varphi) \geq r\min\{\widetilde{\mathcal{B}_A}(\varsigma), \widetilde{\mathcal{B}_A}(\varsigma \longrightarrow \varphi)\}$

And $\widetilde{\mathcal{B}_A}(\varsigma \longrightarrow \varphi) \geq r\min\{\widetilde{\mathcal{B}_A}(\varrho), \widetilde{\mathcal{B}_A}(\varrho \longrightarrow (\varsigma \longrightarrow \varphi)\}$

Hence $\widetilde{\mathcal{B}_A}(\varphi) \geq$ rmin$\{\widetilde{\mathcal{B}_A}(\varsigma), \widetilde{\mathcal{B}_A}(\varrho), \widetilde{\mathcal{B}_A}(\varsigma \longrightarrow (\varrho \longrightarrow \varphi)\}$

Since $J_A(\varphi) \geq max\{J_A(\varsigma), J_A(\varsigma \longrightarrow \varphi)\}$

And $J_A(\varsigma \longrightarrow \varphi) \geq \max\{J_A(\varrho), J_A(\varrho \longrightarrow (\varsigma \longrightarrow \varphi)\}$

Hence $J_A(\varphi) \geq max\{J_A(\varsigma), J_A(\varrho), J_A(\varsigma \longrightarrow (\varrho \longrightarrow \varphi)\}$.

Conversely

Since $M_A(\varphi) \geq \min\{M_A(\varsigma), M_A(\varrho), M_A(\varsigma \longrightarrow (\varrho \longrightarrow \varphi)\}$

If we take $\varsigma = \varrho$ then$M_A(\varphi) \geq \min\{M_A(\varsigma), M_A(\varsigma), M_A(\varsigma \longrightarrow (\varsigma \longrightarrow \varphi)\}$

$= \min\{M_A(\varsigma), M_A(\varsigma \longrightarrow (\varsigma \longrightarrow \varphi)\}$

Since $((\varsigma \longrightarrow \varphi) \longrightarrow (\varsigma \longrightarrow (\varsigma \longrightarrow \varphi)) = I$ then $((\varsigma \longrightarrow \varphi) \leq (\varsigma \longrightarrow (\varsigma \longrightarrow \varphi)$

Implies $M_A(\varsigma \longrightarrow \varphi) \leq M_A(\varsigma \longrightarrow (\varsigma \longrightarrow \varphi)$

Therefore $M_A(\varphi) \geq \min\{M_A(\varsigma), M_A(\varsigma \longrightarrow \varphi)\}$
$\widetilde{\mathcal{B}_A}(\varphi) \geq \min\{\widetilde{\mathcal{B}_A}(\varsigma), \widetilde{\mathcal{B}_A}(\varsigma \longrightarrow \varphi)\}$
$J_A(\varphi) \geq max\{J_A(\varsigma), J_A(\varsigma \longrightarrow \varphi)\}$.
Then A is a M$\mathcal{B}$J-filter of L.

3.6 *Theorem*:
Let A be a neutrosophic set of L. A is a M$\mathcal{B}$J-filter of L if and only if
(i) $M_A(1) \geq M_A(\varsigma)$, $\widetilde{\mathcal{B}_A}(1) \geq \widetilde{\mathcal{B}_A}(\varsigma), J_A(1) \leq J_A(\varsigma)$
(ii) $M_A(\varsigma \longrightarrow \varphi) \geq \min\{M_A((\varphi \longrightarrow \varrho) \longrightarrow \varsigma), M_A(\varrho)\}$
(iii) $\widetilde{\mathcal{B}_A}(\varsigma \longrightarrow \varphi) \geq \min\{\widetilde{\mathcal{B}_A}((\varphi \longrightarrow \varrho) \longrightarrow \varsigma), \widetilde{\mathcal{B}_A}(\varrho)\}$
(iv) $J_A(\varsigma \longrightarrow \varphi) \leq \max\{J_A((\varphi \longrightarrow \varrho) \longrightarrow \varsigma), J_A(\varrho)\}$

for any $\mathfrak{p}, \mathfrak{q}, \varphi \in L$.
Proof: Assume that A is a M$\mathcal{B}$J-filter of L then for any $\varsigma, \varrho, \varphi \in L$
$M_A(\varphi \longrightarrow \varsigma) \geq \min\{M_A(\varrho \longrightarrow (\varphi \longrightarrow \varsigma)), M_A(\varrho)\}$
$\widetilde{\mathcal{B}_A}(\varphi \longrightarrow \varsigma) \geq \mathrm{r}\min\{\widetilde{\mathcal{B}_A}(\varrho \longrightarrow (\varphi \longrightarrow \varsigma)), \widetilde{\mathcal{B}_A}(\varrho)\}$
$J_A(\varphi \longrightarrow \varsigma) \leq \max\{J_A(\varrho \longrightarrow (\varphi \longrightarrow \varsigma)), J_A(\varrho)\}$
Notice that for any $\varsigma, \varrho, \varphi \in L$
$((\varphi \longrightarrow \varrho) \longrightarrow \varsigma) \longrightarrow ((\varphi \longrightarrow \varrho) \longrightarrow (\varphi \longrightarrow \varsigma))$
$=((\varphi \longrightarrow \varrho) \longrightarrow \varsigma) \longrightarrow (\varphi \longrightarrow ((\varphi \longrightarrow \varrho) \longrightarrow \varsigma))$
$= \varphi \longrightarrow (((\varphi \longrightarrow \varrho) \longrightarrow \varsigma) \longrightarrow ((\varphi \longrightarrow \varrho) \longrightarrow \varsigma)) = I$
Implies$((\varphi \longrightarrow \varrho) \longrightarrow \varsigma) \leq ((\varphi \longrightarrow \varrho) \longrightarrow (\varphi \longrightarrow \varsigma))$
Implies $M_A((\varphi \longrightarrow \varrho) \longrightarrow \varsigma) \leq M_A((\varphi \longrightarrow \varrho) \longrightarrow (\varphi \longrightarrow \varsigma))$
$\widetilde{\mathcal{B}_A}((\varphi \longrightarrow \varrho) \longrightarrow \varsigma) \leq \widetilde{\mathcal{B}_A}((\varphi \longrightarrow \varrho) \longrightarrow (\varphi \longrightarrow \varsigma))$
$J_A((\varphi \longrightarrow \varrho) \longrightarrow \varsigma) \geq J_A((\varphi \longrightarrow \varrho) \longrightarrow (\varphi \longrightarrow \varsigma))$
We know that $((\varphi \longrightarrow (\varrho \longrightarrow \varsigma)) \leq ((\varphi \longrightarrow \varrho) \longrightarrow (\varphi \longrightarrow \varsigma))$
Implies $T_A((\varphi \longrightarrow (\varrho \longrightarrow \varsigma)) \leq T_A((\varphi \longrightarrow \varrho) \longrightarrow (\varphi \longrightarrow \varsigma))$
$\widetilde{\mathcal{B}_A}((\varphi \longrightarrow (\varrho \longrightarrow \varsigma)) \leq \widetilde{\mathcal{B}_A}((\varphi \longrightarrow \varrho) \longrightarrow (\varphi \longrightarrow \varsigma))$
$F_A((\varphi \longrightarrow (\varrho \longrightarrow \varsigma)) \geq F_A((\varphi \longrightarrow \varrho) \longrightarrow (\varphi \longrightarrow \varsigma))$
Hence $M_A((\mathrm{r}\varphi(\varrho \longrightarrow \varsigma) \geq M_A((\varphi \longrightarrow \varrho) \longrightarrow \varsigma)$
$\widetilde{\mathcal{B}_A}((\varphi \longrightarrow (\varrho \longrightarrow \varsigma)) \geq \widetilde{\mathcal{B}_A}((\varphi \longrightarrow \varrho) \longrightarrow \varsigma))$
$J_A((\varphi \longrightarrow (\varrho \longrightarrow \varsigma)) \leq J_A((\varphi \longrightarrow \varrho) \longrightarrow \varsigma))$
Therefore $M_A(\varsigma \longrightarrow \varphi) \geq \min\{M_A((\varphi \longrightarrow \varrho) \longrightarrow \varsigma), M_A(\varrho)\}$
$\widetilde{\mathcal{B}_A}(\varsigma \longrightarrow \varphi) \geq \min\{\widetilde{\mathcal{B}_A}((\varphi \longrightarrow \varrho) \longrightarrow \varsigma), \widetilde{\mathcal{B}_A}(\varrho)\}$
$J_A(\varsigma \longrightarrow \varphi) \leq \max\{J_A((\varphi \longrightarrow \varrho) \longrightarrow \varsigma), J_A(\varrho)\}$
Conversely put z = I in (ii), (iii) and (iv) and from (i)
(i) Then $M_A(1) \geq M_A(\varsigma)$, $I_A(1) \geq I_A(\varsigma), J_A(1) \leq J_A(\varsigma)$
(ii) $M_A(\varrho) \geq \min\{M_A(\varsigma), M_A(\varsigma \longrightarrow \varrho)\}$
(iii) $\widetilde{\mathcal{B}_A}(\varrho) \geq r\min\{\widetilde{\mathcal{B}_A}(\varsigma), \widetilde{\mathcal{B}_A}(\varsigma \longrightarrow \varrho)\}$
(iv) $J_A(\varrho) \leq \max\{J_A(\varsigma), J_A(\varsigma \longrightarrow \varrho)\}$

3.7 *Theorem*:
Let N be a M$\mathcal{B}$J-filter of L if $\varsigma \leq \varrho \longrightarrow \varphi$ for any $\varsigma, \varrho, \varphi \in L$ then
(i) $M_A(\varphi) \geq \min\{M_A(\varsigma), M_A(\varrho)\}$
(ii) $\widetilde{\mathcal{B}_A}(\varphi) \geq \min\{\widetilde{\mathcal{B}_A}(\varsigma), \widetilde{\mathcal{B}_A}(\varrho)\}$
(iii) $J_A(\varphi) \leq \max\{J_A(\varsigma), J_A(\varrho)\}$.

Proof: Let N be a M$\mathcal{B}$J-filter of L.
$M_A(\varphi) \geq \min\{M_A(\varrho), M_A(\varrho \longrightarrow \varphi)\}$

$\widetilde{\mathcal{B}_A}(\varphi) \geq \min\{\widetilde{\mathcal{B}_A}(\varrho), \widetilde{\mathcal{B}_A}(\varrho \longrightarrow \varphi)\}$
$J_A(\varphi) \leq \max\{J_A(\varrho), J_A(\varrho \longrightarrow \varphi)\}$
If $\varsigma \leq \varrho \longrightarrow \varphi$ then $M_A(\varsigma) \leq M_A(\varrho \longrightarrow \varphi)$
$\widetilde{\mathcal{B}_A}(\varsigma) \leq \widetilde{\mathcal{B}_A}(\varrho \longrightarrow \varphi)$
$J_A(\varsigma) \geq J_A(\varrho \longrightarrow \varphi)$
Hence $M_A(\varphi) \geq \min\{M_A(\varsigma), M_A(\varrho)\}$
$\widetilde{\mathcal{B}_A}(\varphi) \geq \min\{\widetilde{\mathcal{B}_A}(\varsigma), \widetilde{\mathcal{B}_A}(\varrho)\}$
$J_A(\varphi) \leq \max\{J_A(\varsigma), J_A(\varrho)\}$.

3.8 *Corollary*:
Let N be a M$\mathcal{B}$J-filter of L. If $(\varsigma \otimes \varrho) \longrightarrow \varphi$= I for any $\varsigma, \varrho, \varphi \in L$ then
(i) $M_A(\varphi) \geq \min\{M_A(\varsigma), M_A(\varrho)\}$
(ii) $\widetilde{\mathcal{B}_A}(\varphi) \geq \min\{\widetilde{\mathcal{B}_A}(\varsigma), \widetilde{\mathcal{B}_A}(\varrho)\}$
(iii) $J_A(\varphi) \leq \max\{J_A(\varsigma), J_A(\varrho)\}$.

3.9 *Theorem*:
Let A be a neutrosophic set of L. A is a M$\mathcal{B}$J-filte of L if and only if it satisfies the following conditions for any $\varsigma, \varrho \in L$
(i) If $\varsigma \leq \varrho \Longrightarrow M_A(\varsigma) \leq M_A(\varrho)$
(ii) $M_A(\varsigma \otimes \varrho) \geq \min\{M_A(\varsigma), M_A(\varrho)\}$
(iii) $\widetilde{\mathcal{B}_A}(\varsigma \otimes \varrho) \geq \min\{\widetilde{\mathcal{B}_A}(\varsigma), \widetilde{\mathcal{B}_A}(\varrho)\}$
(iv) $J_A(\varsigma \otimes \varrho) \leq \max\{J_A(\varsigma), J_A(\varrho)\}$
Proof: Suppose N is a M$\mathcal{B}$J-filter of L.
Then (i) hold clearly for any $\varsigma, \varrho \in L$
$M_A(\varsigma \otimes \varrho) \geq \min\{M_A(\varsigma), M_A(\varsigma \longrightarrow (\varsigma \otimes \varrho))\}$
$\widetilde{\mathcal{B}_A}(\varsigma \otimes \varrho) \geq \min\{\widetilde{\mathcal{B}_A}(\varsigma), \widetilde{\mathcal{B}_A}(\varsigma \longrightarrow (\varsigma \otimes \varrho))\}$
$J_A(\varsigma \otimes \varrho) \leq \max\{J_A(\varsigma), J_A(\varsigma \longrightarrow (\varsigma \otimes \varrho))\}$
ꝗ $\longrightarrow(\varsigma \longrightarrow (\varsigma \otimes \varrho)) = \varrho \longrightarrow (\varsigma' \vee \varrho) = (\varrho \longrightarrow \varsigma_1) \vee (\varrho \longrightarrow \varrho) = (\varrho \longrightarrow \varsigma)) \vee I$=I
$(\varsigma \longrightarrow (\varsigma \otimes \varrho))) \geq \varrho$
Implies $M_A\ (\varsigma \longrightarrow (\varsigma \otimes \varrho)) \geq M_A(\varrho)$
$\widetilde{\mathcal{B}_A}\ (\varsigma \longrightarrow (\varsigma \otimes \varrho)) \geq \widetilde{\mathcal{B}_A}(\varrho)$
$J_A\ (\varsigma \longrightarrow (\varsigma \otimes \varrho)) \leq J_A(\varrho)$
Hence $M_A(\varsigma \otimes \varrho) \geq \min\{M_A(\varsigma), M_A(\varrho)\}$
$\widetilde{\mathcal{B}_A}(\varsigma \otimes \varrho) \geq \min\{\widetilde{\mathcal{B}_A}(\varsigma), \widetilde{\mathcal{B}_A}(\varrho)\}$
$J_A(\varsigma \otimes \varrho) \leq \max\{J_A(\varsigma), J_A(\varrho)\}$
Conversely if A is satisfied 1, 2, 3 and 4 then $M_A(\varsigma \otimes \varrho) \geq \min\{M_A(\varsigma), M_A(\varrho)\}$
$\widetilde{\mathcal{B}_A}(\varsigma \otimes \varrho) \geq \min\{\widetilde{\mathcal{B}_A}(\varsigma), \widetilde{\mathcal{B}_A}(\varrho)\}$
$J_A(\varsigma \otimes \varrho) \leq \max\{J_A(\varsigma), J_A(\varrho)\}$
Implies min $\{M_A(\varsigma), T_A(\varsigma \longrightarrow \varrho)\} \leq M_A(\varsigma \otimes (\varsigma \longrightarrow \varrho))$
$(\varsigma \otimes (\varsigma \longrightarrow \varrho)) \longrightarrow \varrho = (\varsigma \wedge \varrho) \longrightarrow \varrho = (\varsigma \longrightarrow \varrho) \vee (\varrho \longrightarrow \varrho)$=I
$\varrho \geq (\varsigma \otimes (\varsigma \longrightarrow \varrho))$ then $M_A(\varrho) \geq M_A\ (\varsigma \otimes (\varsigma \longrightarrow \varrho))$
It follows that min $\{M_A(\varsigma), T_A(\varsigma \longrightarrow \varrho)\} \leq M_A(\varsigma \otimes (\varsigma \longrightarrow \varrho)) \leq M_A(\varrho)$
i.e $M_A(\varrho) \geq \min\{M_A(\varsigma), M_A(\varsigma \longrightarrow \varrho)\}$
Hence A is a M$\mathcal{B}$J-filter of A

4. M $\mathcal{B}$ J-lattice filters

4.1 *Definition*:
A M $\mathcal{B}$J-Neutrosophic set A== $\{\langle \varsigma; M_A(\varsigma), \widetilde{\mathcal{B}_A}(\varsigma), J_A(\varsigma) / \varsigma \in X\}\rangle$ is on L is called a M $\mathcal{B}$ J-lattice filter if it satisfies for all $\varsigma, \varrho \in L$,

$M_A(\varsigma \wedge \varrho) \geq \min\{ M_A(\varsigma),\ M_A(\varrho)\}$
$\widetilde{\mathcal{B}_A}(\varsigma \wedge \varrho) \geq * \, rmin\{\widetilde{\mathcal{B}_A}(\varsigma), \widetilde{\mathcal{B}}(\varrho)\}$
$J_A(\varsigma \wedge \varrho) \leq \max\{J_A(\varsigma), J_A(\varrho)\}$

4.2 *Example*:
Let L= {0, a, b, c, 1}. Define partial order on L as 0<a<b<c<1 and define $\varsigma \wedge \varrho = \min\{\varsigma, \varrho\}$ and $\varsigma \vee \varrho = \max\{\varsigma, \varrho\}$ for all $\varsigma, \varrho \in L$ and "ʼ "and "⟶" as follows

ς	ς'
0	1
a	c
b	b
c	a
1	0

⟶	0	a	b	c	1
0	1	1	1	1	1
a	c	1	1	1	1
b	b	c	1	1	1
c	a	b	c	1	1
1	0	a	b	c	1

Then (L,∨,∧,ʹ , ⟶) is LIA.
Define M $\mathcal{B}$ J-neutrosophic set A in L by

	0	a	b	c	1
M_A	0.1	0.2	0.3	0.4	0.5
$\widetilde{\mathcal{B}_A}$	[0.2,0.3]	[0.4,0.5]	[0.3,0.4]	[0.2,0.5]	[0.2,0.4]
J_A	0.5	0.4	0.3	0.2	0.1

Then A is a M $\mathcal{B}$ J lattice filter on L.

4.3*Theorem*:
A M$\mathcal{B}$J-Neutrosophic set A={⟨ς; $M_A(\varsigma), \widetilde{\mathcal{B}_A}(\varsigma), J_A(\varsigma)/\varsigma \in X\}$⟩ is on L is a M$\mathcal{B}$J-filte then A is a M $\mathcal{B}$ J-lattice filter on L.
Proof: Consider $M_A(\varsigma \wedge \varrho) \geq \min\{ M_A(\varsigma),\ M_A((\varsigma \wedge \varrho) \longrightarrow \varrho)\}$
$= \min\{ M_A(\varsigma),\ M_A((\varsigma \longrightarrow \varrho) \vee (\varrho \longrightarrow \varrho)\}$
$= \min\{ M_A(\varsigma),\ M_A((\varsigma \longrightarrow \varrho) \vee 1\}$
$= \min\{ M_A(\varsigma),\ M_A(1)\}$
$\geq \min\{ M_A(\varsigma),\ M_A(\varrho)\}$

$\widetilde{\mathcal{B}_A}(\varsigma \wedge \varrho) \geq * \, r \min\{ \widetilde{\mathcal{B}_A}(\varsigma), \widetilde{\mathcal{B}_A}((\varsigma \wedge \varrho) \longrightarrow \varrho)\}$
$= r \min\{ \widetilde{\mathcal{B}_A}(\varsigma), \widetilde{\mathcal{B}_A}((\varsigma \longrightarrow \varrho) \vee (\varrho \longrightarrow \varrho)\}$
$= \mathrm{rmin}\{ \widetilde{\mathcal{B}_A}(\varsigma), \widetilde{\mathcal{B}_A}((\varsigma \longrightarrow \varrho) \vee 1\}$
$= r\min\{\widetilde{\mathcal{B}_A}(\varsigma), \widetilde{\mathcal{B}_A}(1)\}$
$\geq r\min\{\widetilde{\mathcal{B}_A}(\varsigma), \widetilde{\mathcal{B}_A}(\varrho)\}$

$J_A(\varsigma \wedge \varrho) \leq \max\{ J_A(\varsigma),\ J_A((\varsigma \wedge \varrho) \longrightarrow \varrho)\}$
$= \max\{ J_A(\varsigma),\ J_A((\varsigma \longrightarrow \varrho) \vee (\varrho \longrightarrow \varrho)\}$
$= \max\{ J_A(\varsigma),\ J_A((\varsigma \longrightarrow \varrho) \vee 1\}$
ax$\{ J_A(\varsigma),\ J_A(1)\}$
$\leq \max\{ J_A(\varsigma),\ J_A(\varrho)\}$
Then A is a M $\mathcal{B}$J-lattice filter on L.

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