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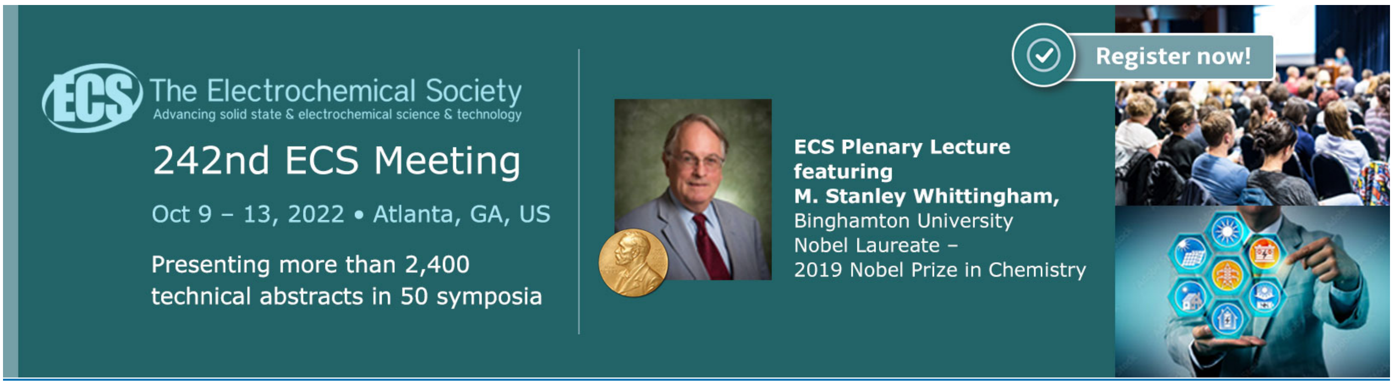

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ON FNS - COMPACTNESS IN FUZZY NEUTROSOPHIC SUPRA TOPOLOGICAL SPACES

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Abstract: In this paper, we introducing fuzzy neutrosophic supra first countable (FNS - FCS), fuzzy neutrosophic supra second countable (FNS- SCS) and fuzzy neutrosophic supra compactness (FNS- compactness), fuzzy neutrosophic supra Q^{μ} - compactness (FNS - Q^{μ} - compactness) in fuzzy neutrosophic supra topological space (FNSTS). we derive the union of two FNS – compact spaces is also FNS – compact space and similarly the union of two FNS -Q^{μ} - compact spaces is also FNS - Q^{μ} - compact space. Also we define some theorems using finite intersection property and productive property. Finally we observe that our notions preserve under one – one, onto and continuous mapping.

1. Introduction: In 1965, Zadeh [9] introduced the notion of fuzzy sets. The study of fuzzy topological spaces was first initiated by chang [4] in 1968. K. Atanassov [8] introduced the notion of intuitionistic fuzzy sets (Ifs) in 1986. Later Coker [5,6] introduced intuitionistic fuzzy topological space. In 1983[2], Mashhour et al. introduced the concepts of supra topological spaces, supra open sets& supra closed sets. Later on 1987, ME Abd El-Monsef et al [10] introduced the concept of fuzzy supra topological as a natural generalization of the notion of supra topological spaces.

In 2002, Florentin Smarandache [15] introduced the extend the IFsets into Nsets. Nset is classified into 3 independent functions. In 2012, Salama, Alblowi [1,13,14] introduced the NTY. NTSs are very natural generalizations of FTSs. In 2014, Salama, Smarandache and valeri [14] introduced the concept of neutrosophic closed sets and continuous functions. Dogen Coker [3,7]& Bayhan introduced Fuzzy-compactness in IFTSs. In 2018,2019,[11,12] Md.Aman Mahbub introduced some properties of compactness in IFTSs and On Q-Compactness in IFTSs.

In this paper two definitions of FNS - compactness in ‘fuzzy ‘neutrosophic supra ‘topological ‘space and some of their features are defined.

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2. Preliminaries:

2.1. Definition:[15]Let X be a nonempty set and P is a fuzzy neutrosophic set (FNS) is an object having the form $P=\{\langle l, T_P(l), I_P(l), F_P(l)\rangle : l \in X\}$,where the functions

$T_P : X \to]^-0,1^+[$, $I_P : X \to]^-0,1^+[$, $F_P : X \to]^-0,1^+[$

denote the degree of membership function $(T_P(l))$, the degree of indeterminacy function ($I_P(l)$), and the degree of non membership ($F_P(l)$) respectively, each element $l \in X$ to the set P and $^+0 \le T_P(l) \le I_P(l) \le F_P(l) \le 1^+$, for each $l \in X$.

A FNS $P=\{\langle l, T_P(l), I_P(l), F_P(l)\rangle : l \in X\}$ can be identified to an ordered triple $\langle l, T_P, I_P, F_P\rangle$ on $]^-0,1^+[$ on X.

2.2. Definition: [1,15] Let X be a nonempty set and the 'FNSs P & R be in the form $P=\{\langle l, T_P(l), I_P(l), F_P(l)\rangle : l \in X\}$ & $R=\{\langle l, T_R(l), I_R(l), F_R(l)\rangle : l \in X\}$ in X

1. $CO(P)=\{\langle 1-T_P(l), 1-I_P(l), 1-F_P(l)\rangle : l \in X\}$

2. $P \subseteq R \Leftrightarrow T_P(l) \le T_R(l), I_P(l) \le I_R(l), F_P(l) \ge F_R(l)$, for all $l \in X$,

3. $P-R=\{\langle l, T_P(l) \wedge F_R(l), I_P(l) \wedge (1-I_R(l)), F_P(l) \vee T_R(l)\rangle : l \in X\}$

4. $[]P=\{\langle l, T_P(l), I_P(l), 1-T_P(l)\rangle : l \in X\}$

5. $\langle\ \rangle P=\{\langle l, 1-F_P(l), I_P(l), F_P(l)\rangle : l \in X\}$

6. $0_{FN}=\langle l,0,0,1\rangle$ and $1_{FN}=\langle l,1,1,0\rangle$

2.3. Definition: [15] Let $\{P_j : j \in K\}$ be an arbitrary family of FNSs in X, where $P_j=\{\langle l, T_{P_j}(l), I_{P_j}(l), F_{P_j}(l)\rangle : l \in X\}$ then

1. $\cap P_j=\{\langle l, \wedge_{j\in K} T_{P_j}(l), \wedge_{j\in K} I_{P_j}(l), \vee_{j\in K} F_{P_j}(l)\rangle : l \in X\}$

2. $\cup P_j=\{\langle l, \vee_{j\in K} T_{P_j}(l), \vee_{j\in K} I_{P_j}(l), \wedge_{j\in K} F_{P_j}(l)\rangle : l \in X\}$

2.4. Definition: [13,17] A fuzzy neutrosophic supra topology (FNST), Let X be a nonempty set & τ^μ be a family of FNS subsets in X, satisfying the following axioms.

$0_{FN}, 1_{FN} \in \tau^{\mu}$

$\cup M_i \in \tau^{\mu}, \forall \{M_i : i \in K\} \subseteq \tau^{\mu}$

In the pair, (X, τ^{μ}) is said to be a Fuzzy neutrosophic supra topological space (FNSTS) and any FNSS in τ^{μ} is known as Fuzzy neutrosophic supra open set (FNSOS) in X. The element of τ^{μ} are called FNSOSs. The complement of FNSOS in the FNSTS (X, τ^{μ}) is called fuzzy neutrosophic supra closed set (FNSCS).

2.5. Definition: [3] Let M and N are IFSs on X and Y. Then the product of IFSs M and N denoted by $M \times N$ is defined by $M \times N = \{(r,t), T_M \times T_N, F_M \times F_N\}$ where $(T_M \times T_N)(r,t) = \min(T_M(r), T_N(t))$ and $(F_M \times F_N)(r,t) = \max(F_M(r), F_N(t))$ for all $(r,t) \in X \times Y$. Obviously $0 \le (T_M \times T_N) + (F_M \times F_N) \le 1$. This definition can be extended to an arbitrary family of IFSs.

2.6 .Definition: [3] Let $(X_i, Y_i), i = 1,2$ be two IFTSs. The product topology $\tau_1 \times \tau_2$ on $X_1 \times X_2$ is the IFT generated by $\{p_i^{-1}(V_i) : V_i \in \tau_i, i = 1,2\}$ where $p_i : X_1 \times X_2 \to X_i, i = 1,2$ are the projection maps and IFTS $\{X_1 \times X_2, \tau_1 \times \tau_2\}$ is called the product IFTS of $(X_i, Y_i), i = 1,2$. In this case $\delta = \{p_i^{-1}(V_i) : V_i \in \tau_i, i \in K\}$ is a sub base and $B = \{V_1 \times V_2 : V_i \in \tau_i, i = 1,2\}$ is base for $\tau_1 \times \tau_2$ in $X_1 \times X_2$

2.7. Definition: [14] (i) If $M = \langle T_M, I_M, F_M \rangle$ is a NS on Y, then the preimage of M under *h*, denoted by $h^{-1}(M)$, is a NS in X defined by $h^{-1}(M) = \langle h^{-1}(T_M), h^{-1}(I_M).h^{-1}(F_M) \rangle$.
(ii) If $N = \langle T_N, I_N, F_N \rangle$ is a NS in X, then the image of N under *h*, denoted by $h(N)$, is a NS in Y defined by $h(N) = \langle h(T_N), h(I_N).h(F_N) \rangle$.

2.8 .Definition: [14] Let $(X, \tau^{\mu}), (Y, \sigma^{\mu})$ be FNTSs. A function $h : X \to Y$ is called neutrosophic Continuous, if $h^{-1}(M) \in \tau^{\mu}$ for all $M \in \sigma^{\mu}$ and *h* is called Neutrosophic Open, if $h(N) \in \sigma^{\mu}$ for all $N \in \tau^{\mu}$.

2.9. Definition: [16] A collection ϑ of NS on a set X is called basis (or base) for a NTS on X, if
(i) For each $p_N^x \in X$, there exists $E \in \vartheta$ such that $p_N^x \in E$,
(ii) If $p_N^x \in E_1 \cap E_2$, where $E_1, E_2 \in \vartheta$ then $\exists E_3 \in \vartheta$ such that $p_N^x \in E_3 \subseteq E_1 \cap E_2$.

2.10. Definition: [11,12](i) Let (X, τ^{μ}) be a NTS and E be a NS on X. If a family $\{\langle l, T_{E_i}, I_{E_i}, F_{E_i} \rangle : l \in K\}$ of NOSs on X satisfies the condition, $\cup \{\langle l, T_{E_i}, I_{E_i}, F_{E_i} \rangle : l \in K\} = 1_N$, then it is said to be a neutrosophic open cover of X. A finite sub family of neutrosophic open cover

$\{\langle l, T_{E_i}, I_{E_i}, F_{E_i}\rangle : l \in K\}$ of X, which is also NOC of X is called a finite sub cover of $\{\langle l, T_{E_i}, I_{E_i}, F_{E_i}\rangle : l \in K\}$.

3. FNS – First and Second countable space in FNSTS

3.1. Definition: A FNSTS (X, τ^{μ}) is called fuzzy neutrosophic supra first countable space (FNS – FCS) if for every FNP P_{FN} there exists a countable local base.

3.2. Example: Let $X = \{l, m, n\}$ and $\tau^{\mu} = \{0_{FN}, 1_{FN}, M_1, M_2, M_3\}$, where

$$M_1 = \left\{\left\langle \frac{l}{0.25}, \frac{l}{0.45}, \frac{l}{0.75}\right\rangle \left\langle \frac{m}{0.28}, \frac{m}{0.45}, \frac{m}{0.72}\right\rangle \left\langle \frac{n}{0.35}, \frac{n}{0.55}, \frac{n}{0.65}\right\rangle\right\},$$

$$M_2 = \left\{\left\langle \frac{l}{0.45}, \frac{l}{0.45}, \frac{l}{0.55}\right\rangle \left\langle \frac{m}{0.48}, \frac{m}{0.45}, \frac{m}{0.52}\right\rangle \left\langle \frac{n}{0.55}, \frac{n}{0.55}, \frac{n}{0.45}\right\rangle\right\}$$ and

$$M_3 = \left\{\left\langle \frac{l}{0.55}, \frac{l}{0.45}, \frac{l}{0.45}\right\rangle \left\langle \frac{m}{0.65}, \frac{m}{0.45}, \frac{m}{0.35}\right\rangle \left\langle \frac{n}{0.75}, \frac{n}{0.55}, \frac{n}{0.25}\right\rangle\right\}.$$ Then (X, τ^{μ}) is a FNSTS.

Let $P_{FN} = \left\{\left\langle \frac{l}{0.35}, \frac{l}{0.45}, \frac{l}{0.65}\right\rangle \left\langle \frac{m}{0.38}, \frac{m}{0.45}, \frac{m}{0.62}\right\rangle \left\langle \frac{n}{0.45}, \frac{n}{0.55}, \frac{n}{0.55}\right\rangle\right\}$ be a FNP.

H $(P_{FN}) = \{V_1, V_2\}$, where $V_1 = \left\{\left\langle \frac{l}{0.45}, \frac{l}{0.45}, \frac{l}{0.55}\right\rangle \left\langle \frac{m}{0.48}, \frac{m}{0.45}, \frac{m}{0.52}\right\rangle \left\langle \frac{n}{0.55}, \frac{n}{0.55}, \frac{n}{0.45}\right\rangle\right\}$,

$$V_2 = \left\{\left\langle \frac{l}{0.55}, \frac{l}{0.45}, \frac{l}{0.45}\right\rangle \left\langle \frac{m}{0.65}, \frac{m}{0.45}, \frac{m}{0.35}\right\rangle \left\langle \frac{n}{0.75}, \frac{n}{0.55}, \frac{n}{0.25}\right\rangle\right\}.$$

Write $\mathrm{N}^* = \left\{\left\langle \frac{l}{0.85}, \frac{l}{0.55}, \frac{l}{0.25}\right\rangle \left\langle \frac{m}{0.88}, \frac{m}{0.55}, \frac{m}{0.22}\right\rangle \left\langle \frac{n}{0.95}, \frac{n}{0.55}, \frac{n}{0.15}\right\rangle\right\}$ is a FNSNHD of FNP P_{FN}.

Then $V_1, V_2 \subseteq \mathrm{N}^*$. Therefore H (P_{FN}) is local base of FNP P_{FN}. Hence (X, τ^{μ}) is FNS – FCS.

3.3. Theorem: Let (X, τ^{μ}) & (Y, δ^{μ}) are any FNSTSs and $h : X \to Y$ is onto Open and FNS – continuous mapping. If (X, τ^{μ}) is FNS – FCS then (Y, δ^{μ}) is also FNS – FCS.

Proof: Suppose ${P_{FN}}^{y}$ is a FNP on Y. Since $h : (X, \tau^{\mu}) \to (Y, \delta^{\mu})$ is onto then ∃ FNP ${P_{FN}}^{y} \in Y \ni h({P_{FN}}^{x}) = {P_{FN}}^{y}$. Since (X, τ^{μ}) is FNS – FCS then ∃ a countable local base. Say H of ${P_{FN}}^{x}$. We have to prove that *h* (H) has a countable local base in Y at ${P_{FN}}^{y}$. Since H is countable, so *h* (H) is countable. Let N be a FNSNHD of ${P_{FN}}^{y}$.

Since *h* is FNS – continuous then h^{-1} (N) $\in \tau^{\mu}$ such that ${P_{FN}}^{x} \in h^{-1}$ (N) & ∃ $\mathrm{H}^* \in$ H such that ${P_{FN}}^{x} \in \mathrm{H}^* \subseteq$

h^{-1} (N). Since h is open, $h({P_{FN}}^{x}) \in h$ (H) $\subseteq$ N. So ${P_{FN}}^{y} \in h(\mathrm{H}^{*}) \subseteq$ G. Hence h (H) is countable local base for Y. Therefore (Y, δ^{μ}) is also FNS – FCS.

3.4. Definition: A FNSTS (X, τ^{μ}) is called fuzzy neutrosophic supra second countable space (FNS – SCS) if it has a countable base.

3.5. Example: Let $X = \{p, q, r\}$ and $\tau^{\mu} = \{0_{FN}, K, L, M, 1_{FN}\}$, where

$$K = \left\{\left\langle \frac{l}{0.24}, \frac{l}{0.45}, \frac{l}{0.76} \right\rangle \left\langle \frac{m}{0.34}, \frac{m}{0.45}, \frac{m}{0.66} \right\rangle \left\langle \frac{n}{0.44}, \frac{n}{0.55}, \frac{n}{0.56} \right\rangle\right\},$$

$$L = \left\{\left\langle \frac{l}{0.46}, \frac{l}{0.45}, \frac{l}{0.54} \right\rangle \left\langle \frac{m}{0.48}, \frac{m}{0.45}, \frac{m}{0.52} \right\rangle \left\langle \frac{n}{0.54}, \frac{n}{0.55}, \frac{n}{0.46} \right\rangle\right\} \text{ and}$$

$$M_3 = \left\{\left\langle \frac{l}{0.58}, \frac{l}{0.45}, \frac{l}{0.42} \right\rangle \left\langle \frac{m}{0.64}, \frac{m}{0.45}, \frac{m}{0.36} \right\rangle \left\langle \frac{n}{0.74}, \frac{n}{0.55}, \frac{n}{0.26} \right\rangle\right\}.$$ Then (X, τ^{μ}) is a FNSTS.

Let $$P_{FN} = \left\{\left\langle \frac{l}{0.34}, \frac{l}{0.45}, \frac{l}{0.66} \right\rangle \left\langle \frac{m}{0.38}, \frac{m}{0.45}, \frac{m}{0.62} \right\rangle \left\langle \frac{n}{0.44}, \frac{n}{0.55}, \frac{n}{0.56} \right\rangle\right\}$$ be a FNP.

Here $P_{FN} \in L \subseteq \mathrm{N}^{*}$ and $P_{FN} \in M \subseteq \mathrm{N}^{*}$, where N^{*} = {N / N $\supseteq$ L & M}. Then H = {L & M}is countable base for τ^{μ}. Hence (X, τ^{μ}) is FNS – SCS.

3.6. Theorem: Let (X, τ^{μ}) & (Y, δ^{μ}) are any FNSTSs and $h : X \to Y$ is onto Open and FNS – continuous mapping. If (X, τ^{μ}) is FNS – SCS then (Y, δ^{μ}) is also FNS – SCS.

Proof: Suppose (X, τ^{μ}) is a FNS – SCS. Then H is countable base for τ^{μ}, h (H) is countable collection in Y. Now show that h (H) is countable base for τ^{μ}. Let V be a FNSNHD of FNP ${P_{FN}}^{y} \in Y$. So ${P_{FN}}^{y} \in V$ and since h is onto $\exists {P_{FN}}^{x} \in X \ni h\left({P_{FN}}^{x}\right) = {P_{FN}}^{y} \in V$. So ${P_{FN}}^{x} \in h^{-1}(V)$ is FNSNHD of (X, τ^{μ}) and it is FNS – SCS then $\exists$ $\mathrm{H}^{*} \in$ H & H $\subseteq \tau^{\mu}$ such that ${P_{FN}}^{x} \in$ H$\subseteq h^{-1}$ (V). So $h\left({P_{FN}}^{x}\right) \in h\,(\mathrm{H}^{*}) \subseteq$ V. So ${P_{FN}}^{y} \in h(\mathrm{H}) \subseteq$ V. Hence h (H) is countable base for τ^{μ}. Therefore (Y, δ^{μ}) is also FNS – SCS.

4. FNS - Compactness in FNSTS

In this section we define two definitions of fuzzy neutrosophic supra compactness (FNS – Compact) in fuzzy neutrosophic supra topological space (FNSTS) and established several 'properties of such notions'.

4.1. Definition: Let (X, τ^{μ}) be a FNSTS. A family $\{G_k : k \in K\}$ of 'fuzzy neutrosophic supra open sets in X satisfies the condition, $\bigcup \{G_k : k \in K\} = 1_{FN}$. Then it is called fuzzy neutrosophic supra open cover (FNS – O – C) of X.

4.2. Example: Let $X=\{l,m\}$ and $\tau^{\mu}=\{0_{FN},1_{FN},\gamma_1,\gamma_2,\gamma_3,\gamma_4\}$, where

$$\gamma_1=\left\{\left\langle\frac{l}{1},\frac{l}{1},\frac{l}{0.15}\right\rangle\left\langle\frac{m}{1},\frac{m}{1},\frac{m}{0.10}\right\rangle\right\},\ \gamma_2=\left\{\left\langle\frac{l}{0.66},\frac{l}{0.45},\frac{l}{0.34}\right\rangle\left\langle\frac{m}{0.74},\frac{m}{0.45},\frac{m}{0.26}\right\rangle\right\},$$

$$\gamma_3=\left\{\left\langle\frac{l}{0.77},\frac{l}{0.54},\frac{l}{0.23}\right\rangle\left\langle\frac{m}{0.82},\frac{m}{0.54},\frac{m}{0.25}\right\rangle\right\}\text{ and }\gamma_4=\left\{\left\langle\frac{l}{0.85},\frac{l}{0.55},\frac{l}{0}\right\rangle\left\langle\frac{m}{0.95},\frac{m}{0.55},\frac{m}{0}\right\rangle\right\}.$$

Then (X,τ^{μ}) is a FNSTS and $\bigcup\{\gamma_k : k=1,2,3,4\}=1_{FN}$. Hence $\{\gamma_1,\gamma_2,\gamma_3,\gamma_4\}$ is FNS – open cover in X.

4.3. Definition: Let (X,τ^{μ}) be a FNSTS. A finite sub family of fuzzy neutrosophic supra open cover $\{G_k : k\in K\}$ on X, which is also a fuzzy neutrosophic supra open cover of X, is called fuzzy neutrosophic supra finite sub cover (FNS – F – SC) of X.

4.4. Definition: A FNSTS (X,τ^{μ}) is called FNS – Compact if each FNSO – cover of X has a FNS finite sub cover.

4.5. Example: Let $X=\{l,m\}$ and $\tau^{\mu}=\{0_{FN},1_{FN},\gamma_1,\gamma_2,\gamma_3,\gamma_4\}$, where

$$\gamma_1=\left\{\left\langle\frac{l}{1},\frac{l}{1},\frac{l}{0.15}\right\rangle\left\langle\frac{m}{1},\frac{m}{1},\frac{m}{0.10}\right\rangle\right\},\ \gamma_2=\left\{\left\langle\frac{l}{0.66},\frac{l}{0.45},\frac{l}{0.34}\right\rangle\left\langle\frac{m}{0.74},\frac{m}{0.45},\frac{m}{0.26}\right\rangle\right\},$$

$$\gamma_3=\left\{\left\langle\frac{l}{0.77},\frac{l}{0.54},\frac{l}{0.23}\right\rangle\left\langle\frac{m}{0.82},\frac{m}{0.54},\frac{m}{0.25}\right\rangle\right\}\text{ and }\gamma_4=\left\{\left\langle\frac{l}{0.85},\frac{l}{0.55},\frac{l}{0}\right\rangle\left\langle\frac{m}{0.95},\frac{m}{0.55},\frac{m}{0}\right\rangle\right\}.$$

Then (X,τ^{μ}) is a FNSTS and $\bigcup\{\gamma_k : k=1,2,3,4\}=1_{FN}$. Hence $\{\gamma_1,\gamma_2,\gamma_3,\gamma_4\}$ is FNS – open cover in X. And also $\{\gamma_1,\gamma_2,\gamma_4\}$ is FNS – F – SC of X. Therefore (X,τ^{μ}) is a FNS – compact.

4.6. Remark: Every fuzzy neutrosophic supra topological space need not be a FNS – compact following the example 4.7.

4.7. Example: Let $X=\{l,m\}$ and $\tau^{\mu}=\{0_{FN},1_{FN},\gamma_1,\gamma_2,\gamma_3\}$, where

$$\gamma_1=\left\{\left\langle\frac{l}{0.45},\frac{l}{0.25},\frac{l}{0.65}\right\rangle\left\langle\frac{m}{0.35},\frac{m}{0.55},\frac{m}{0.75}\right\rangle\right\},\ \gamma_2=\left\{\left\langle\frac{l}{0.65},\frac{l}{0.35},\frac{l}{0.45}\right\rangle\left\langle\frac{m}{0.75},\frac{m}{0.65},\frac{m}{0.55}\right\rangle\right\}\text{ and}$$

$$\gamma_3=\left\{\left\langle\frac{l}{0.55},\frac{l}{0.25},\frac{l}{0.45}\right\rangle\left\langle\frac{m}{0.45},\frac{m}{0.55},\frac{m}{0.65}\right\rangle\right\}.$$ Then (X,τ^{μ}) is a FNSTS but $\bigcup\{\gamma_k : k=1,2,3\}=\gamma_2\neq 1_{FN}$. Hence (X,τ^{μ}) is not a FNS – compact of X.

4.8. Theorem: A FNSTS (X,τ^{μ}) is FNS – compact. Let $\eta^{\mu}\subseteq\tau^{\mu}$, then (X,η^{μ}) is also FNS – compact.
Proof: Let $\{G_k : k\in K\}$ be FNS – O – C of X. Since $\eta^{\mu}\subseteq\tau^{\mu}$, $\{G_k : k\in K\}$ be FNS – O – C of X.

Since X is FNS – compact, $\exists$ FNS – F – SC $\{G_k : k \in K\}$ of X.
This turn to show that (X, η^{μ}) is also FNS – compact.

4.9. Theorem: Let (X, τ^{μ}) be a FNSTS and M, N are FNS – compact subsets of X. Then $M \cup N$ is also FNS – compact.
Proof: Let (X, τ^{μ}) be a FNSTS and M, N are FNS – compact subsets of X. To show that $M \cup N$ is also FNS – compact. Let $\{G_k : k \in K\}$ be a FNS – O – C of both M, N and $M \cup N$. Since M, N are FNS – compact subsets of X. Then M and N have FNS – F – SC. Let $\{G_k : k = 1 \text{ to } K\}$ be FNS – F – SC of M and $\{G_l : l = 1 \text{ to } L\}$ be FNS – F – SC of N. Then $\bigcup\{G_k : k = 1 \text{ to } K\} = 1_{FN}, \bigcup\{G_l : l = 1 \text{ to } L\} = 1_{FN}$.
$\bigcup\{G_k : k = 1 \text{ to } K + L\} = 1_{FN}$. Hence this FNS – O – C $\{G_k : k \in K\}$ contains FNS – F – SC of $M \cup N$. Therefore $M \cup N$ is FNS – compact.

4.10. Theorem: Let (X, τ^{μ}), (Y, δ^{μ}) be FNSTSs and let $h : X \to Y$ be a FNS – continuous surjection. If (X, τ^{μ}) is FNS – compact, then (Y, δ^{μ}) is also FNS – compact.
Proof: Given that h is FNS – continuous and onto and (X, τ^{μ}) is FNS – compact.
Let us consider $\gamma_k = \{G_k : k \in K\}$ be a FNS – O – C for Y, then $\bigcup \gamma_k = 1_{FN}$.
Since *h* is FNS – continuous, $h^{-1}(\bigcup \gamma_k) = h^{-1}(1_{FN}) \Rightarrow \bigcup h^{-1}(\gamma_k) = 1_{FN}$.
Since γ_k is FNSO in Y, for every $k \in K$ as the map *h* is FNS – continuous.
Thus the family $\{h^{-1}(\gamma_k) : k \in K\}$ is a FNS – O – C for X and since X is FNS – compact.
Then $\exists$ a FNS – F – SC $\{h^{-1}(\gamma_j) : j = 1 \text{ to } n\} \ni \bigcup\{h^{-1}(\gamma_j) : j = 1 \text{ to } n\} = 1_{FN}$.
Now, $h\left(\bigcup_{j=1}^{n} h^{-1}(\gamma_k)\right) = h(1_{FN}) \Rightarrow \bigcup_{j=1}^{n} h\left(h^{-1}(\gamma_k)\right) = h(1_{FN}) \Rightarrow \bigcup_{j=1}^{n}(\gamma_k) = 1_{FN}$, (as the map *h* is surjective). Therefore Y is FNS – compact space.

4.11. Theorem: Let (X, τ^{μ}), (Y, δ^{μ}) be FNSTSs and let $h : X \to Y$ be a FNS – open function and (Y, δ^{μ}) be a FNS – compact. Then (X, τ^{μ}) is FNS – compact.
Proof: Let $h : X \to Y$ be a FNS – open function and (Y, δ^{μ}) be a FNS – compact.
Let $\gamma_k = \{G_k : k \in K\}$ be a FNS – O – C for X. Since *h* is FNS – open function, $h(\gamma_k)$ is a FNS – O – C of Y. Since Y is FNS – compact, $h(\gamma_k)$ contains a FNS – F – SC, $\{h(\gamma_{k1}, \gamma_{k2}, \gamma_{k3}, ...\gamma_{kn})\}$. Then $\{\gamma_{k1}, \gamma_{k2}, \gamma_{k3}, ...\gamma_{kn}\}$ is a FNS – F – SC for X. Thus X is FNS – compact.

4.12. Theorem: The image of FNS – compact space under a FNS – continuous map is FNS – compact space.
Proof: Let $h : X \to Y$ be a FNS – continuous map from a FNS – compact space (X, τ^{μ}) onto a FNSTS (Y, δ^{μ}). Let $\gamma_k = \{G_k : k \in K\}$ be a FNS – O – C for (Y, δ^{μ}). Since *h* is FNS – continuous, $\{h^{-1}(\gamma_k) : k \in K\}$ is a FNS – O – C of (X, τ^{μ}). We know that (X, τ^{μ}) is FNS – compact, the FNS – O –

C $\{h^{-1}(\gamma_k) : k \in K\}$ of (X, τ^{μ}) has a FNS – F – SC $\{h^{-1}(\gamma_k) : k = 1,2,....n\}$. Therefore $P = \bigcup_{k\in K}^{n} h^{-1}(\gamma_k)$. Then $h(p) = \bigcup_{k\in K}^{n} (\gamma_k)$, that is $Q = \bigcup_{k\in K}^{n} (\gamma_k)$. Thus $\{\gamma_{k1}, \gamma_{k2}, \gamma_{k3}, ...\gamma_{kn}\}$ is a FNS – F – SC of $\{(\gamma_k) : k \in K\}$ for (Y, δ^{μ}). Hence (Y, δ^{μ}) is a FNS – compact.

4.13. Definition: (*a*) Let (X, τ^{μ}) be a FNSTSs and *H* is FNSS on X. If a family $\{G_k : k \in K\}$ of FNSOSs on X satisfies the condition, $H \subseteq \bigcup \{G_k : k \in K\}$, then it is said to be a FNS – O – C of *H.* A finite subfamily of the FNS – O – C $\{G_k : k \in K\}$ of *H*, which is also a FNS – O – C of *H*, is said to be a FNS – F – SC of $\{G_k : k \in K\}$.
(*b*) A FNS *H* in FNSTS (X, τ^{μ}) is said to be FNS – compact iff every FNS – O – C of *H* has a FNS – F – SC.

4.14. Theorem: Let (X, τ^{μ}), (Y, δ^{μ}) be FNSTSs and let $h : X \to Y$ be a FNS – continuous function. If *H* is FNS – compact in (X, τ^{μ}) and then *h (H)* is also FNS – compact in (Y, δ^{μ}).
Proof: Let $Q = \{\gamma_k : k \in K\}$, where $\{\gamma_k = \langle \gamma_{k1}, \gamma_{k2}, \gamma_{k3} \rangle : k \in K\}$ be a FNS – O – C of *h (H).*
Then, by the definition of FNS – continuity $P = \{h^{-1}(\gamma_k) : k \in K\}$ is a FNS – O – C of *H*. Since *H* is FNS – compact, $\exists$ a FNS – F – SC of P, i.e., there exists $\{\gamma_{k1}, \gamma_{k2}, \gamma_{k3}, ...\gamma_{kn}\}$ such that $H \subseteq \bigcup_{k=1}^{n} h^{-1}(\gamma_k)$. Hence *h (H)* $\subseteq h(\bigcup_{k=1}^{n} h^{-1}(\gamma_k)) \subseteq \bigcup_{k=1}^{n} h(h^{-1}(\gamma_k)) \subseteq \bigcup_{k=1}^{n} (\gamma_k)$. Therefore, *h (H)* is also FNS – compact.

4.15. Definition: Let (X, τ^{μ}) be a FNSTS then the family $\{G_k : k \in K\}$ of fuzzy neutrosophic supra closed sets in X satisfies finite intersection property (FIP) if every finite sub family $\{G_k : k = 1,2,..., m\}$ of the family satisfies the condition, $\bigcap \{G_k : k \in K\} \neq 0_{FN}$.

4.16. Theorem: A FNSTS (X, τ^{μ}) is a FNS – compact iff the collection of FNSCSs in X having the FIP has a non empty intersection.
Proof: Suppose that X is fuzzy neutrosophic supra compact.Let $\{\gamma_k : k \in K\}$ be a family of FNSCSs in X. Also assume $\{\gamma_k : k \in K\}$ has finite intersection property. Now we have to show that $\bigcap \{\gamma_k : k \in K\} \neq 0_{FN}$. On the contrary way suppose that $\bigcap \{\gamma_k : k \in K\} = 0_{FN} \Rightarrow \overline{\bigcap_{k\in K} \gamma_k} = \overline{0_{FN}}$ $\Rightarrow \bigcup_{k\in K} \overline{\gamma_k} = 1_{FN}$. Clearly $\bigcup_{k\in K} \overline{\gamma_k} = \bigcup \{\gamma_k : k \in K\} = 1_{FN}$. For every $k \in K$, γ_k is FNSCS of X. Therefore $\overline{\gamma_k}$ is FNSOS of X. Thus $\{\overline{\gamma_k} : k \in K\}$ is FNS – O – C of X. Since X is FNS – compact, therefore this FNS – O – C has FNS – F – SC, say, $\bigcup \{\overline{\gamma_k} : k = 1,2,...., m\} = 1_{FN}$.
Then $\bigcap \{\gamma_k : k = 1,2,...., m\} = 0_{FN}$. Thus the above considered family does not satisfy FIP, which is a contradiction. Therefore, $\bigcap \{\gamma_k : k \in K\} \neq 0_{FN}$. Conversely, assume that the family of FNSCS of X having FIP has nonempty intersection. To show that X is fuzzy neutrosophic supra compact.
Let $\{G_k : k \in K\}$ be a FNS – O – C of X. Suppose that this FNS – O – C has no FNS – F – Sc. That is for

every finite sub collection of this cover, say $\bigcup\{G_k : k \in K\} \neq 1_{FN} \Rightarrow \bigcap\{\overline{G_k} : k \in K\} \neq 0_{FN}$. As each G_k is FNSOS of X and each $\overline{G_k}$ is FNSCS of X. Thus, $\{\overline{G_k} : k \in K\}$ is a family of FNSCS of X having FIP. So by the hypothesis it has nonempty intersection, that is $\bigcap\{\overline{G_k} : k \in K\} \neq 0_{FN} \Rightarrow \bigcup\{G_k : k \in K\} \neq 1_{FN}$. This shows that $\{G_k : k \in K\}$ is not FNS – O – C for X. which is a contradiction. Therefore the given family should have a FNS – F – SC. This shows that X is fuzzy neutrosophic supra compact.

4.17. Theorem: Show that the following statements are equivalent
(I) X is FNS – compact
(II) For every $\{F_i\}$, where $F_i = \langle T_{F_i}, I_{F_i}, F_{F_i} \rangle$ of FNSC subsets of X, with $\bigcap F_i = 0_{FN}$ implies $\{F_i\}$ contains FNS – finite subclass $\{F_{i1}, F_{i2}, ..., F_{in}\}$ with $F_{i1} \cap F_{i2} \cap ...... \cap F_{in} = 0_{FN}$.
Proof: $(I) \Rightarrow (II)$. Suppose $\bigcap F_i = 0_{FN}$. Then by De Morgan's Law, $(\bigcap F_i)^C = (0_{FN})^C \Rightarrow \bigcup F_i^C = 1_{FN}$. So $\{F_i^C\}$ is a FNS – O – cover of X. Since X is FNS – compact then $\exists F_{i1}^C, F_{i2}^C, ..., F_{in}^C \in \{F_i^C\}$ such that $F_{i1}^C \cup F_{i2}^C \cup ...... \cup F_{in}^C = 1_{FN}$. Then
$0_{FN} = (1_{FN})^C = (F_{i1}^C \cup F_{i2}^C \cup ..... \cup F_{in}^C)^C = (F_{i1}^C)^C \cap (F_{i2}^C)^C \cap ..... \cap (F_{in}^C)^C$ by De Morgan's Law $F_{i1} \cap F_{i2} \cap ...... \cap F_{in}$. Therefore $F_{i1} \cap F_{i2} \cap ...... \cap F_{in} = 0_{FN}$. So we have to prove that $(I) \Rightarrow (II)$.
$(II) \Rightarrow (I)$. Let $\{G_i\}$ be a FNS – O – C of X, where $G_i = \langle T_{G_i}, I_{G_i}, F_{G_i} \rangle$. i.e. $\bigcup G_i = 1_{FN}$.
By De Morgan's Law, $0_{FN} = (1_{FN})^C = (\bigcup G_i)^C = \bigcap G_i^C$.
Since, each G_i is FNSO, so $\{G_i^C\}$ is a class of FNSCSs & by (II). $\exists G_{i1}^C, G_{i2}^C, ..., G_{in}^C \in \{G_i^C\}$ Such that $G_{i1}^C \cap G_{i2}^C \cap ...... \cap G_{in}^C = 0_{FN}$. So by De Morgan's Law,
$1_{FN} = (0_{FN})^C = (G_{i1}^C \cap G_{i2}^C \cap ..... \cap G_{in}^C)^C = G_{i1} \cup G_{i2} \cup ...... \cup G_{in}$. Therefore X is FNS – compact. So we have to prove that $(II) \Rightarrow (I)$.

4.18. Theorem: Let FNSTSs (X_1, τ_1^μ) and (X_2, τ_2^μ) be a FNS – compact. Then the product FNST $\tau_1^\mu \times \tau_2^\mu$ on $X_1 \times X_2$ is FNS – compact.
Proof: Consider (X_1, τ_1^μ) and (X_2, τ_2^μ) be a FNS – compact. Let $M_i = (T_{M_i}, I_{M_i}, F_{M_i}) \in \tau_1^\mu$ with $\bigcup M_i = 1_{FN}$ and $N_i = (T_{N_i}, I_{N_i}, F_{N_i}) \in \tau_2^\mu$ with $\bigcup N_i = 1_{FN}$.
Now $M_i \times N_i = (T_{M_i}, I_{M_i}, F_{M_i}) \times (T_{N_i}, I_{N_i}, F_{N_i}) = (T_{M_i} \times T_{N_i}, I_{M_i} \times I_{N_i}, F_{M_i} \times F_{N_i})$, where
$(T_{M_i} \times T_{N_i})(r,t) = \min(T_{M_i}(r), T_{N_i}(t)), (I_{M_i} \times I_{N_i})(r,t) = \min(I_{M_i}(r), I_{N_i}(t)), (F_{M_i} \times F_{N_i})(r,t) = \max(F_{M_i}(r), F_{N_i}(t))$
where $r \in X_1, t \in X_2$. So, $M_i \times N_i = 1_{FN}$. But by definition of product topology, $M_i \times N_i \in \tau_1^\mu \times \tau_2^\mu$. i.e $\{M_i \times N_i\}$ is family of FNSOSs in $X_1 \times X_2$. Choose $\bigcup (M_i \times N_i) = 1_{FN}$. Since (X_1, τ_1^μ) is FNS – compact then $\{M_i\}$ has FNS – finite subclass $\{M_{ij}\}$ such that $\bigcup M_{ij} = 1_{FN}$ and also (X_2, τ_2^μ) is FNS –

compact then $\{N_i\}$ has FNS – finite subclass $\{N_{ij}\}$ such that $\bigcup N_{ij} = 1_{FN}$, where $j = 1,2,...,n$.
Hence $\bigcup M_{ij} \times \bigcup N_{ij} = 1_{FN}$. Therefore the product FNST $\left(X_1 \times X_2, \tau_1^{\mu} \times \tau_2^{\mu}\right)$ is also FNS – compact.

5. FNS – Q^{μ} – compactness in FNSTS

In this section two definitions of FNS – Q^{μ} – compactness in fuzzy neutrosophic supra topological space and established several properties of such notions are established.

5.1. Definition: Let $\left(X, \tau^{\mu}\right)$ be a FNSTS and E be a FNSS in X. A family M $= \{P_i : i \in I\}$ of FNS sets in X, where $P_i = \left\langle T_{P_i}, I_{P_i}, F_{P_i} \right\rangle$. Then M is said to be FNS – Q^{μ} – cover of E, if $E \subseteq \bigcup P_i$,
$T_E(l) + T_{P_i}(l) \geq 1_{FN}$, for each T_{P_i} and some $l \in X$. If each P_i is FNSO then M is said to be FNS – Q^{μ} – Open cover of E.
A sub family of FNS – Q^{μ} – cover of FNSS E in X which is also FNS – Q^{μ} – cover of E is said to be FNS – Q^{μ} – sub cover of E.

5.2. Definition: A FNSS E in X is called a FNS – Q^{μ} – compact if every FNS – Q^{μ} – open cover of E has a FNS – Q^{μ} – finite sub cover. i.e. $\exists P_{i1}, P_{i2}, ..., P_{in} \in$ M such that $E \subseteq \bigcup P_{ij}$, $T_E(l) + T_{P_{ij}}(l) \geq 1_{FN}$, for each $T_{P_{ij}}$ and some $l \in X$, j = 1,2,…..,m.

5.3. Example: Let $X = (r, s, t)$ & $\tau^{\mu} = \{0_{FN}, 1_{FN}, P_1, P_2\}$, where

$$P_1 = \left\{\left\langle \frac{r}{0.55,0.55,0.45} \right\rangle \left\langle \frac{s}{0.65,0.55,0.35} \right\rangle \left\langle \frac{t}{0.75,0.55,0.25} \right\rangle\right\},$$

$$P_2 = \left\{\left\langle \frac{r}{0.58,0.55,0.42} \right\rangle \left\langle \frac{s}{0.68,0.55,0.32} \right\rangle \left\langle \frac{t}{0.78,0.55,0.22} \right\rangle\right\}, \text{ Then } \left(X, \tau^{\mu}\right) \text{ is FNSTS.}$$

Write $E = \left\{\left\langle \frac{r}{0.48,0.55,0.52} \right\rangle \left\langle \frac{s}{0.52,0.55,0.48} \right\rangle \left\langle \frac{t}{0.63,0.55,0.37} \right\rangle\right\}$ be a FNSS in X.
Here $E(r) \subseteq \bigcup P_i(r)$, $T_E(r) + T_{P_i}(r) \geq 1$, $E(s) \subseteq \bigcup P_i(s)$, $T_E(s) + T_{P_i}(s) \geq 1$ and $E(t) \subseteq \bigcup P_i(t)$, $T_E(t) + T_{P_i}(t) \geq 1_{FN}$. Therefore $\{P_1, P_2\}$ is a FNS – Q^{μ} – open cover of E and P_1 or P_2 is a FNS – Q^{μ} – finite sub cover of E.

5.4. Theorem: Let $\left(X, \tau^{\mu}\right)$ be a FNSTS and P, R are FNS – Q^{μ} – compact subsets of $\left(X, \tau^{\mu}\right)$. Then $P \cup R$ is FNS – Q^{μ} – compact in $\left(X, \tau^{\mu}\right)$.
Proof: Let $\left(X, \tau^{\mu}\right)$ be a FNSTS and P, R are FNS – Q^{μ} – compact subsets of $\left(X, \tau^{\mu}\right)$. To show that

$P \cup R$ is FNS – Q^{μ} – compact in (X, τ^{μ}). Let M $= \{E_i : i \in I\}$ be a FNS – Q^{μ} – open cover of P and N $= \{F_i : i \in I\}$ be a FNS – Q^{μ} – open cover of R in (X, τ^{μ}). Now $P \subseteq \bigcup E_i$ and $R \subseteq \bigcup F_i$.
$\Rightarrow P \cup R \subseteq \bigcup E_{ij} \cup \bigcup F_{ij} \Rightarrow P \cup R \subseteq \bigcup (E_{ij} \cup F_{ij})$, j = 1,2,…,m. By definition of FNS – Q^{μ} – compactness, $T_P(r) + T_{E_i}(r) \geq 1_{FN}$ and $T_R(r) + T_{F_i}(r) \geq 1_{FN}$, for some $r \in X$.
$\Rightarrow T_{P \cup R}(r) + T_{E_i \cup F_i}(r) \geq 1_{FN}$. i.e. M $\cup$ N $= \{E_i \cup F_i\}$ is FNS – Q^{μ} – cover of $P \cup R$. Since P is FNS – Q^{μ} – compact in (X, τ^{μ}). Then P has FNS – Q^{μ} – finite sub cover, $\exists P_{i1}, P_{i2}, \ldots\ldots, P_{in} \in E_i$ such that $P \subseteq \bigcup E_{ij}$, $T_P(r) + T_{E_{ij}}(r) \geq 1_{FN}$ for some $r \in X, j = 1,2,\ldots, m$ and also R is FNS – Q^{μ} – compact in (X, τ^{μ}). Then R has FNS – Q^{μ} – finite sub cover, $\exists R_{i1}, R_{i2}, \ldots\ldots, R_{in} \in F_i$ such that $R \subseteq \bigcup F_{ij}$, $T_R(r) + T_{F_{ij}}(r) \geq 1_{FN}$ for some $r \in X, j = 1,2,\ldots, m$. Now $P \subseteq \bigcup E_{ij}$ and $R \subseteq \bigcup F_{ij}$,
$\Rightarrow P \cup R \subseteq \bigcup (E_{ij} \cup F_{ij})$, Also $T_P(r) + T_{E_{ij}}(r) \geq 1_{FN}$ and $T_R(r) + T_{F_{ij}}(r) \geq 1_{FN}$, for some $r \in X$.
$\Rightarrow T_{P \cup R}(r) + T_{E_{ij} \cup F_{ij}}(r) \geq 1_{FN}$. Hence $\{E_{ij} \cup F_{ij}\}$ is a FNS – Q^{μ} – finite sub cover of $P \cup R$. Therefore $P \cup R$ is FNS – Q^{μ} – compact in (X, τ^{μ}).

5.5. Theorem: Let (X, τ^{μ}) & (Y, δ^{μ}) be FNSTSs and $h : X \rightarrow Y$ is bijective, FNSO and FNS – continuous. If FNSS E is FNS – Q^{μ} – compact in (X, τ^{μ}) and then $h(E)$ is FNS – Q^{μ} – compact in (Y, δ^{μ}).

Proof: Let N $= \{P_i \in \delta^{\mu}\}$ be a FNS – Q^{μ} – open cover of $h(E)$ with $h(E) \subseteq \bigcup P_i$ and $T_{h(E)}(r) + T_{P_{ij}}(r) \geq 1_{FN}$, for some $r \in X$. Since $P_i \in \delta^{\mu}$ then $h^{-1}(P_i) \in \tau^{\mu}$ and $h(E) \subseteq \bigcup P_i$
$\Rightarrow E \subseteq h^{-1}(\bigcup P_i)$, for some $r \in X \Rightarrow T_E(r) + T_{h^{-1}(P_{ij})}(r) \geq 1_{FN}$. Since E is FNS – Q^{μ} – compact then a family H $= \{h^{-1}(P_i) : i \in I\}$ is FNS – Q^{μ} – open cover of E. Further since E is FNS – Q^{μ} – compact in (X, τ^{μ}) then $\exists h^{-1}(P_{i1}), h^{-1}(P_{i2}), \ldots\ldots, h^{-1}(P_{in}) \in \tau^{\mu}$ such that $E \subseteq \bigcup h^{-1}(P_{ij})$ and $T_E(r) + T_{h^{-1}(P_{ijj})}(r) \geq 1_{FN}$, for some $r \in X$, j = 1,2,…..,m.
$\Rightarrow h(T_E(r)) + h(T_{h^{-1}(P_{ij})})(r) \geq h(1_{FN}) \Rightarrow T_{h(E)}(r) + T_{P_{ij}}(r) \geq 1_{FN}$, as h is FNS – continuous. But $E \subseteq \bigcup h^{-1}(P_{ij}) \Rightarrow h(E) \subseteq h(\bigcup h^{-1}(P_{ij})) \Rightarrow h(E) \subseteq \bigcup P_{ij}$. Hence $P_{ij} \in \delta^{\mu}$ such that $h(E) \subseteq \bigcup P_{ij}$ & $T_{h(E)}(r) + T_{P_{ij}}(r) \geq 1_{FN}$. Therefore $h(E)$ is FNS – Q^{μ} – compact in (Y, δ^{μ}).

5.6. Theorem: Let (X, τ^{μ}) be a FNSTS and E be a FNSS on X. if a family $\{F_i : i \in I\}$ of FNSC sub sets of X with $\bigcap F_i = 0_{FN}$ implies $\{F_i\}$ contains finite subclass $\{F_{i1}, F_{i2}, \ldots.., F_{in}\}$ with

$F_{i1} \cap F_{i2} \cap .... \cap F_{in} = 0_{FN}$. Then E is FNS – Q^{μ} – compact in $\left(X, \tau^{\mu}\right)$.

Proof: Given that $\bigcap F_i = 0_{FN}$. Then by De Morgan's Law, $(\bigcap F_i)^C = (0_{FN})^C$.

$\Rightarrow \bigcup F_i^C = 1_{FN}$. Let M $= \{H_i : i \in I\}$ be a FNS – Q^{μ} – open cover of E in $\left(X, \tau^{\mu}\right)$.

So $E \subseteq \bigcup H_i, T_E(r) + T_{H_i}(r) \geq 1_{FN}$ for some $r \in X$. Since each H_i is FNSO then $\{H_i^C\}$ is a class of FNSCSs and by given condition, $\exists H_{i1}^C, H_{i2}^C, ..., H_{in}^C \in \{H_i^C\}$ such that $H_{i1}^C \cap H_{i2}^C \cap ...... \cap H_{in}^C = 0_{FN}$.

So by De Morgan's Law, $1_{FN} = (1_{FN}^C) = \left(H_{i1}^C \cap H_{i2}^C \cap ...... \cap H_{in}^C\right)^C = H_{i1} \cup H_{i2} \cup .... \cup H_{in}$.

Hence $E \subseteq \bigcup H_{ij}, T_E(r) + T_{H_{ij}}(r) \geq 1_{FN}, j = 1,2,....,n$, for some $r \in X$. Therefore E is FNS – Q^{μ} – compact in $\left(X, \tau^{\mu}\right)$.

5.7. Theorem: Let $\left(X, \tau^{\mu}\right)$ be a FNSTS and M, N are FNS – Q^{μ} – compact on $\left(X, \tau^{\mu}\right)$. Then $(M \times N)$ is FNS – Q^{μ} – compact in $\left(X \times X, \tau^{\mu} \times \tau^{\mu}\right)$.

Proof: Let K $= \{H_i : i \in I\}$, where $H_i \in \tau^{\mu} \times \tau^{\mu}$ be a FNS – Q^{μ} – cover of $M \times N$ in $\left(X \times X, \tau^{\mu} \times \tau^{\mu}\right)$. Then $M \times N \subseteq \bigcup H_i$ and $T_{M \times N}(r,t) + T_{H_i}(r,t) \geq 1_{FN}$, for some $(r,t) \in X \times X$. Now write $H_i = P_i \times R_i$, where $P_i \times R_i \in \tau^{\mu}$. Thus $M \times N \subseteq \bigcup H_i$
$\Rightarrow M \times N \subseteq \bigcup(P_i \times R_i) \Rightarrow M \subseteq \bigcup P_i, N \subseteq \bigcup R_i$. Also $T_{M \times N}(r,t) + T_{P_i \times R_i}(r,t) \geq 1_{FN}$, for some $(r,t) \in X \times X$. Hence it is clear that $T_M(r) + T_{P_i}(r) \geq 1_{FN}$, for some $r \in X$ and $T_N(t) + T_{R_i}(t) \geq 1_{FN}$, for some $t \in X$. Therefore $\{P_i : i \in I\}$ and $\{R_i : i \in I\}$ are FNS – Q^{μ} – open cover of M and N. Since M & N are compacts then $\{P_i : i \in I\}$ and $\{R_i : i \in I\}$ have FNS – Q^{μ} – finite sub covers of M and N. Say, $\{P_{ij} : j \in I_n\}$ and $\{R_{ij} : j \in I_n\}$ such that $M \subseteq \bigcup P_{ij}, T_M(r) + T_{P_{ij}}(r) \geq 1_{FN}$, for some $r \in X$ and $N \subseteq \bigcup R_{ij}, T_N(t) + T_{R_{ij}}(t) \geq 1_{FN}$, for some $t \in X$. Thus $M \times N \subseteq \bigcup\left(P_{ij} \times R_{ij}\right)$. $\Rightarrow M \times N \subseteq \bigcup H_{ij}$ and $T_{M \times N}(r,t) + T_{H_i j}(r,t) \geq 1_{FN}$, for some $(r,t) \in X \times X$. Therefore $(M \times N)$ is also FNS – Q^{μ} – compact on $\left(X \times X, \tau^{\mu} \times \tau^{\mu}\right)$.

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