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Research Article

On a Matrix over NC and Multiset NC Semigroups

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In this paper, we define a matrix over neutrosophic components (NCs), which was built using the four different intervals $(0, 1)$, $[0, 1)$, $t(0, 1]$, and $[0, 1]$. This definition was made clear by introducing some examples. Then, the study of the algebraic structure of matrices over NC under addition modulo 1, the usual product, and product by using addition modulo 1 was introduced, from which it was found that the matrix over NC built using interval $[0, 1)$ happens to be an abelian group under addition modulo 1. Furthermore, it is proved that the matrix over NC defined on the interval $[0, 1)$ is not a regular semiring. Also, we define a matrix over multiset NC semigroup using the interval $[0, 1)$. Moreover, we define a matrix over m-multiplicity multiset NC semigroup for finite m. Several interesting properties are discussed for the three structures. It was concluded that the last two structures are semigroups and semirings under addition modulo 1 and usual product, respectively.

1. Introduction

Semigroups play main roles in algebraic structures [1–9]. Neutrosophic sets were introduced by Smarandache [10]. Every element in a neutrosophic set has three associated functions, namely, the membership function, the non-membership function, and the indeterminacy function, all of which are defined on the universe of discourse X. Moreover, these three functions are completely independent. Since then, this concept has become an interesting area of major research both in the area of algebraic structures [11–16] and analysis [17] and in applications ranging from medical diagnosis to sentiment analysis [18, 19]. Also, matrices always play a significant role in technology and science. However, in some situations, the classical matrix theory fails to solve problems involving uncertainties that emerge in an unpredictable world. Some researchers have studied and applied neutrosophic matrix [20–22]. For instance, they introduced some novel operations on neutrosophic matrices, that is, type-1 product operation of the SVN-matrices is a similar Hadamard product of matrices. Therefore, the type-1 product allows making some applications related to Hadamard product under the SVN environment. On the other hand, there is no study for neutrosophic matrix under the usual addition and product operations. Motivated by the previous study of Vasantha et al. [23], where they defined the NC over the intervals $(0, 1)$, $(0, 1]$, $t[0, 1)$, and $[0, 1]$ under the usual addition, addition modulo 1, and classic product, here we venture to study matrix over NC on all the four mentioned intervals results in several interesting algebraic structures. Indeed, it is proven that there is no algebraic structure under addition modulo 1 and the new type of product for matrix over NC over the intervals $(0, 1)$, $t(0, 1]$, and $[0, 1]$. It is shown that the interval $[0, 1)$ provides an interesting algebraic structure. Thus, the aim of the present paper is to define a matrix over neutrosophic components and multiset neutrosophic components. Also, we define some operations between a matrix over neutrosophic components and multiset neutrosophic components to investigate some of their properties. The remaining part of the paper is organized as follows. Section 2 gives a brief summary of neutrosophic components, multiset neutrosophic components, and operations on these sets. In Section 3, we introduce and study a matrix over neutrosophic components. In Section 4, we define and study matrix multiset NC built using the interval $[0, 1)$ semigroups under semigroup under addition modulo 1 and product $\times_{\oplus}$. Section 5 is devoted to studying matrix over m-multiplicity multiset NC on the interval $[0, 1)$ and obtaining several interesting properties. In Section 6, we draw some conclusions.

2. Basic Concepts

This section introduces the fundamental principles needed to make this paper self-contained.

Now, we recall the definition of semigroup in the following form.

Definition 1 (see [6]). A couple $(S, \times)$ is said to be a semigroup if the operation $\times$ is closed and associative.

Definition 2 (see [6]). A couple $(S, \times)$ is said to be a regular semigroup if for each element $a \in S$, there exists $x \in S$ such that $a = a \cdot x \cdot a$.

Definition 3. A semigroup $(S, \times)$ is said to be a torsion-free semigroup if for $a, b \in S, a \neq b, a^n \neq b^n$ for any $1 \leq n < \infty$.

Definition 4 (see [23]). *The neutrosophic component (NC) is a triplet (a, b, c), where a is the truth membership function, b is the indeterminacy membership function, and c is the falsity membership function, and all of them are from the unit interval* $[0, 1]$.

Definition 5 (see [10]). *A neutrosophic multiset is a neutrosophic set where one or more elements are repeated with same neutrosophic components or with different neutrosophic components.*

Now, we define four sets of neutrosophic components as follows.

Definition 6 (see [23]). Let $S_l, l = 1, 2, 3, 4$. Then, we define four sets of NC as follows: $S_1 = \{(x, y, z)\colon x, y, z \in (0, 1)\}, S_2 = \{(x, y, z)\colon x, y, z \in [0, 1)\}, t \quad S_3 n = q\{(x, y, z)\colon x, y, z \in (0, 1]\}$, and $S_4 = \{(x, y, z)\colon x, y, z \in [0, 1]\}$.

Definition 7 (see [23]). Let $(S_2, \times)$ be a multiset NC semigroup under $\times$; then, the elements of the form $(a, b, 0)$, $(0, 0, c)$, and so on which are infinite in number with $a, b, c \in S_2$ contribute to zero divisors. Hence, multisets using these types of elements contribute to zeros of the form $n(0, 0, 0), 1 < n < \infty$. As the zeros are of varying multiplicity, we call these zero divisors as special type of zero divisors.

Remark 1. We denote the set of the multisets of NC using elements of $S_l, l = 1, 2, 3, 4$ by $M(S_l)$.

Definition 8 (see [23]). The collection of all multisets with entries from $S_l, l = 1, 2, 3, 4$, with at most multiplicity m, $2 \leq m < \infty$, is called m-multiset NC and we denote it by $m\text{-}M(S_l)$.

3. Matrix over NC

In this section, we introduce matrices over a set of NC (MNC for short) and support them by presenting concrete examples. Also, we give the definition of usual addition modulo 1 and two types of multiplication of NC to obtain some algebraic structures for MNC. Furthermore, we give some examples and some results on MNC. Let $S_l, l = 1, 2, 3, 4$, be the four sets of NC and $M_{m\times n}(S_l)$ be the set of all $m \times n$ matrices defined on S_l.

Definition 9. A matrix over NC (MNC) of order $m \times n$ is defined as $\mathscr{A} = (A_{ij})$ where $A_{ij} = (a_{ij}, b_{ij}, c_{ij}) \in S_l$ are called truth-membership, indeterminacy-membership, and falsity-membership values of the i^{th} and j^{th} elements in $\mathscr{A}$. Let $\mathscr{M}_{m\times n}$ denote the set of all MNC of order $m \times n$. In particular, $\mathscr{M}_n(S_l)$ denotes the set of all square MNCs of order n.

Definition 10. Let $\mathscr{A} = (A_{ij})$ be an $m \times n$ MNC. If all of its entries are $A_{ij} = (0, 0, 0), \forall i, j$, then $\mathscr{A}$ is called the zero MNC and denoted by $\mathscr{O}$. If $m = n$, $A_{ij} = (0, 0, 0), \forall i \neq j$, and $A_{ij} = (1, 1, 1), \forall i = j$, then $\mathscr{A}$ is called the identity MNC and denoted by $\mathscr{I}$.

Definition 11. Let $\mathscr{A} = (A_{ij})$ and $\mathscr{B} = (B_{ij})$ be two $m \times n$ MNC; then, addition modulo 1 is defined as follows:

$$\mathscr{A} \oplus_1 \mathscr{B} = \left(A_{ij} \oplus_1 B_{ij}\right). \tag{1}$$

It is well known that for two n-square MMs $\mathscr{A} = (A_{ij})$ and $\mathscr{B} = (B_{ij})$, the usual product is defined as $\mathscr{A} \times \mathscr{B} = (\sum_{j=1}^{n} A_{ij} B_{jk})$. Now, we introduce the new type of the product of two n-square MNCs by using addition modulo 1 as in the following definition.

Definition 12. Let $\mathscr{A} = (A_{ij})$ and $\mathscr{B} = (B_{ij})$ be n-square MNCs; then, we define the product of MNC by using addition modulo 1 as follows:

$$\mathscr{A} \times_{\oplus_1} \mathscr{B} = \left(\oplus_{j=1}^{n} A_{ij} B_{jk}\right), \tag{2}$$

where $\oplus_{j=1}^{n}$ represents the summation using addition modulo 1.

Next, we illustrate how addition modulo 1, usual product, and a new type of product are performed on any two MNCs.

Example 1. Let $\mathscr{A}, \mathscr{B}, \mathscr{C}, \mathscr{D} \in \mathscr{M}_2(S_i)$, $l = 1, 2, 3, 4$, where

$$\mathscr{A} = \begin{pmatrix} (0.3, 0.2, 0.4) & (0.6, 0.7, 0.1) \\ (0.6, 0.7, 0.1) & (0.1, 0.3, 0.2) \end{pmatrix},$$
$$\mathscr{B} = \begin{pmatrix} (0.1, 0.2, 0.5) & (0.8, 0.4, 0.5) \\ (0.1, 0.3, 0.6) & (0.7, 0.4, 0.5) \end{pmatrix},$$
$$\mathscr{C} = \begin{pmatrix} (0.7, 0.8, 0.6) & (0.4, 0.3, 0.9) \\ (0.4, 0.3, 0.9) & (0.9, 0.7, 0.8) \end{pmatrix}, \quad (3)$$
$$\mathscr{D} = \begin{pmatrix} (0.9, 0.3, 0.4) & (0.8, 0.9, 0.6) \\ (0.1, 0.2, 0.1) & (0.2, 0.4, 0.5) \end{pmatrix}.$$

Then, we have, for example, that $\mathscr{A} + \mathscr{C} = \begin{pmatrix} (1,1,1) & (1,1,1) \\ (1,1,1) & (1,1,1) \end{pmatrix}$ and $\mathscr{A} \oplus_1 \mathscr{C} = \mathscr{O}$.

Also,

$$\mathscr{A} + \mathscr{B} = \begin{pmatrix} (0.4, 0.4, 0.9) & (1.4, 1.1, 0.6) \\ (0.7, 1, 0.7) & (0.8, 0.7, 0.7) \end{pmatrix},$$
$$\mathscr{A} \oplus_1 \mathscr{B} = \begin{pmatrix} (0.4, 0.4, 0.9) & (0.4, 0.1, 0.6) \\ (0.7, 0, 0.7) & (0.8, 0.7, 0.7) \end{pmatrix}. \quad (4)$$

Since $(0,0,0) \notin S_l, l = 1, 3$, *then it is clear that the two operations* $+$ *and* $\oplus_1$ *are not closed in* $\mathscr{M}_2(S_l), l = 1, 3$. *Furthermore, the two operations* $\times$ *and* $\oplus_{j=1}^n$ *are not closed.*

In case $\mathscr{A}, \mathscr{D} \in \mathscr{M}_2(S_4)$,

$$\mathscr{A} + \mathscr{D} = \begin{pmatrix} (1.2, 0.5, 0.8) & (1.4, 1.6, 0.7) \\ (0.7, 0.9, 0.2) & (0.3, 0.7, 0.7) \end{pmatrix} \notin \mathscr{M}_2(S_4), \quad (5)$$

and we cannot define addition modulo 1 since $(1,1,1) \in S_4$.

Since the operations $\oplus_1$ and $+$ are not closed, then the operations $\times$ and $\oplus_{j=1}^n$ are not closed. The following example explains the new type of the product defined in Definition 12.

Example 2. Let $\mathscr{A}, \mathscr{B} \in \mathscr{M}_2(S_2)$, *where* $\mathscr{A} = \begin{pmatrix} (0.9, 0.3, 0.4) & (0.9, 0.9, 0.9) \\ (0.1, 0.2, 0.1) & (0.2, 0.1, 0.3) \end{pmatrix}$ *and* $\mathscr{B} = \begin{pmatrix} (0.8, 0.8, 0.2) & (0.2, 0.3, 0.1) \\ (0.7, 0.2, 0.1) & (0.2, 0.4, 0.1) \end{pmatrix}$.

Hence, $\mathscr{A} \times \mathscr{B} = \begin{pmatrix} (1.35, 0.42, 0.17) & (0.36, 0.45, 0.13) \\ (0.22, 0.18, 0.05) & (0.06, 0.1, 0.04) \end{pmatrix} \notin \mathscr{M}_2(S_1)$. But the new type of the product by using addition modulo 1 gives the following:

$$\mathscr{A} \times_{\oplus_1} \mathscr{B} = \begin{pmatrix} (0.35, 0.42, 0.17) & (0.36, 0.45, 0.13) \\ (0.22, 0.18, 0.05) & (0.06, 0.1, 0.04) \end{pmatrix}. \quad (6)$$

Remark 2. Since S_1 *and* S_3 *do not include the elements* $(0,0,0)$ *and* $(1,1,1)$, *then* $\mathscr{M}_{n\times n}(S_1)$ *and* $\mathscr{M}_{n\times n}(S_3)$ *do not include the matrices* $\mathscr{O}$ *and* $\mathscr{I}$.

Lemma 1. *The above two examples explain that* $\mathscr{M}_n(S_l), l = 1, 2, 3$, *are not algebraic structures under usual product and under addition modulo 1. Moreover, they are not semigroups under usual product and under addition modulo 1.*

Theorem 1. *Let* $S_2 = \{(x, y, z): x, y, z \in [0, 1)\}$ *be the collection of NC and* $\mathscr{M}_{m\times n}(S_2)$ *be a set of* $m \times n$ *matrices over* S_2; *then,* $(\mathscr{M}_{m\times n}(S_2), \oplus_1)$ *is a commutative group under addition modulo 1.*

Proof. Assume that $\mathscr{A}, \mathscr{B} \in \mathscr{M}(S_2)$; then, $\mathscr{A} \oplus_1 \mathscr{B} \in \mathscr{M}(S_2)$ and associative law hold. $\mathscr{O} \in \mathscr{M}(S_2)$ is a zero identity. Also, for every $\mathscr{A} \in \mathscr{M}(S_2)$, there is a unique $\mathscr{B} \in \mathscr{M}(S_2)$ such that $\mathscr{A} \oplus_1 \mathscr{B} = \mathscr{O}$. Further, we find $\mathscr{A} \oplus_1 \mathscr{B} = \mathscr{B} \oplus_1 \mathscr{A}$. Thus, $(\mathscr{M}(S_2), \oplus_1)$ is a commutative group. □

Theorem 2. $(\mathscr{M}_n(S_2), \times_{\oplus_1})$ *is only a semigroup and not a monoid. It has infinite number of zero divisors.*

Proof. Suppose that $\mathscr{A} = (A_{ij}), \mathscr{B} = (B_{jk}) \in \mathscr{M}_n(S_2)$; then, $\mathscr{A} \times_{\oplus_1} \mathscr{B} = (\oplus_{j=1}^n A_{ij} B_{jk})$, and since $(\mathscr{M}_n(S_2), \oplus_1)$ is a group, $\mathscr{A} \times_{\oplus_1} \mathscr{B} \in \mathscr{M}_n(S_2)$. Hence, $(\mathscr{M}_n(S_2), \times_{\oplus_1})$ is a semigroup under the $\times_{\oplus_1}$ product. It is obvious that $\mathscr{I} \notin \mathscr{M}_n(S_2)$, and we find $(\mathscr{M}(S_2), \times_{\oplus_1})$ is not a monoid. Finally, $\mathscr{O} \in \mathscr{M}_n(S_2)$, so $(\mathscr{M}_n(S_2), \times_{\oplus_1})$ has infinite number of zero divisors, and hence the claim is satisfied. □

Theorem 3. $(\mathscr{M}_n(S_2), \oplus_1, \times_{\oplus_1})$ *is a ring with infinite number of zero divisors and has no multiplicative identity* $\mathscr{I}$.

Proof. Since $(\mathscr{M}_n(S_2), \oplus_1)$ is a group from Theorem 1 and $(\mathscr{M}_n(S_2), \times_{\oplus_1})$ is a semigroup from Theorem 2, the distributive property is acquired from the number theoretic properties of modulo integers, hence the result.

Next result shows that $(S_2, \oplus_1, \times)$ is not regular semiring. □

Lemma 2. $(S_2, \oplus_1, \times)$ *is not regular semiring.*

Proof. To prove this result, we use the method contradiction. Let $(S_2, \oplus_1, \times)$ be regular semiring, and then for all $(a, b, c) \in S_2$, there exists $(e, f, g) \in S_2$ such that $(aea, bfb, cgc) = (a, b, c)$, and we thus observe that $ea = 1$, $fb = 1$ and $gc = 1$. This completes the proof. □

Remark 3. Since $(S_2, \oplus_1, \times)$ is not regular semiring, then $(M(S_2), \oplus_1, \times)$ is not regular.

4. Matrix over Multiset NC Semigroups

In this section, we introduce and study matrices over multiset NC semigroups using S_2. We are referring to the collection of such matrices by $\mathscr{M}_n(M(S_2))$, where $M(S_2)$ is a collection of all multisets built using S_2. Also, we prove that the algebraic structure $\mathscr{M}_n(\mathscr{M}(S_2), \oplus_1, \times_{\oplus_1})$ is a semiring of infinite order.

Now, we illustrate how the addition and the multiplication operations are performed on any two matrices over multiset NC in $M(S_2)$.

Definition 13 (see [23]). Let $A, B \in M(S_2)$, where $A = \{n(a,b,c): a,b,c \in S_2, n \in \mathbb{N}\}$ and $B = \{m(x,y,x): x,y,z \in S_2, m \in \mathbb{N}\}$. Then, we define the product and the addition modulo 1 as follows:

$$A \times B = \{nm(ax, by, cz): a,b,c,x,y,z \in S_2, n,m \in \mathbb{N}\},$$
$$A \oplus_1 B = \{nm(a \oplus_1 x, b \oplus_1 y, c \oplus_1 z): a,b,c,x,y,z \in S_2, n,m \in \mathbb{N}\}. \quad (7)$$

Now, we illustrate how the two operations $\oplus_1$ and $\times_{\oplus_1}$ are performed in $\mathcal{M}_n(M(S_2))$. To define these operations, we give the following example.

Example 3. Let $\mathcal{MA}_1 = \{\mathcal{A}, \mathcal{B}, \mathcal{C}\}$ and $\mathcal{MB}_1 = \{\mathcal{D}, \mathcal{E}\}$ in $\mathcal{M}_n(M(S_2))$. Then, the sum of $\mathcal{MA}_1$ with $\mathcal{MB}_1$ under addition modulo 1 is given by

$$\mathcal{MA}_1 \oplus_1 \mathcal{MB}_1 = \{\mathcal{A} \oplus_1 \mathcal{D}, \mathcal{A} \oplus_1 \mathcal{E}, \mathcal{B} \oplus_1 \mathcal{D}, \mathcal{B} \oplus_1 \mathcal{E}, \mathcal{C} \oplus_1 \mathcal{D}, \mathcal{C} \oplus_1 \mathcal{E}\}, \quad (8)$$

which is in $\mathcal{M}_n(M(S_2))$. Also, the product of $\mathcal{MA}_1$ and $\mathcal{MB}_1$ is given by

$$\mathcal{MA}_1 \times_{\oplus_1} \mathcal{MB}_1 = \left\{\mathcal{A} \times_{\oplus_1} \mathcal{D}, \mathcal{A} \times_{\oplus_1} \mathcal{E}, \mathcal{B} \times_{\oplus_1} \mathcal{D}, \mathcal{B} \times_{\oplus_1} \mathcal{E}, \mathcal{C} \times_{\oplus_1} \mathcal{D}, \mathcal{C} \times_{\oplus_1} \mathcal{E}\right\}, \quad (9)$$

which is in $\mathcal{M}_n(M(S_2))$.

Example 4. Let $\mathcal{A}, \mathcal{B} \in \mathcal{M}_2(M(S_2))$, where

$$\mathcal{A} = \begin{pmatrix} 9(0.3, 0.2, 0.4) & 2(0.6, 0.7, 0.1) \\ 2(0.6, 0.7, 0.1) & (0.1, 0.3, 0.2) \end{pmatrix},$$
$$\mathcal{B} = \begin{pmatrix} 5(0.1, 0.2, 0.5) & 10(0.8, 0.4, 0.5) \\ 5(0.1, 0.2, 0.5) & 10(0.8, 0.4, 0.5) \end{pmatrix}. \quad (10)$$

Then, we define the addition modulo 1 of $\mathcal{A}$ and $\mathcal{B}$ as follows:

$$\mathcal{A} \oplus_1 \mathcal{B} = \begin{pmatrix} 45(0.4, 0.4, 0.9) & 20(0.4, 0.1, 0.6) \\ 10(0.7, 0.9, 0.6) & 10(0.9, 0.7, 0.7) \end{pmatrix}. \quad (11)$$

Also, we define the new type of product $\times_{\oplus_1}$ of $\mathcal{A}$ and $\mathcal{B}$ as follows:

$$\mathcal{A} \times_{\oplus_1} \mathcal{B} = \begin{pmatrix} 450(0.09, 0.18, 0.25) & 1800(0.72, 0.36, 0.25) \\ 50(0.07, 0.2, 0.15) & 200(0.56, 0.4, 0.15) \end{pmatrix}. \quad (12)$$

Theorem 4. *Let $\mathcal{M}_n(M(S_2))$ be the matrix over multiset NC built using S_2; then, $(\mathcal{M}_n(M(S_2)), \oplus_1)$ is a semigroup under addition modulo 1.*

Proof. Since $\mathcal{M}_n(M(S_2))$ is closed under the binary operation addition modulo 1, $(\mathcal{M}_n(M(S_2)), \oplus_1)$ is a semigroup. □

Theorem 5. $(\mathcal{M}_n(M(S_2)), \times_{\oplus_1})$ *is an infinite non-commutative matrix over multiset NC semigroup, which is not a monoid and has special type of zero divisors.*

Proof. Since $\mathcal{M}_n(M(S_2))$is closed under the binary operation product with associative, let $\mathcal{M}_n(M(S_2))$be non-commutative. Thus, $(\mathcal{M}_n(M(S_2)), \times_{\oplus_1})$ is a non-commutative semigroup of infinite order. Furthermore, $\mathcal{I} \notin \mathcal{M}_n(S_2)$, so $(\mathcal{M}_n(M(S_2)), \times_{\oplus_1})$ is not a monoid. From Definition 7 of special zero divisors, we find that $(\mathcal{M}_n(M(S_2)), \times_{\oplus_1})$ has infinite number of special type of zero divisors.

Using Theorems 4 and 5, we have the following. □

Theorem 6. *Let $(\mathcal{M}_n(M(S_2)), \oplus_1, \times_{\oplus_1})$ be an algebraic structure of matrices over multiset NC. Then, it is a non-commutative semiring of infinite order. Moreover, it has infinite numbers of special type of zero divisors.*

Definition 14. Any subset of c containing elements of the form $m_1(a, 0, 0)$, $m_2(0, b, c), a, b, c \in S_2$, which are infinite numbers, contributes to zero divisor, $1 < m_1, m_2 < \infty$. Hence, any matrix over this subset contributes to zero matrix with entries of the form $m(0, 0, 0), 1 < m < \infty$. As the zeros are of varying multiplicity, we call these divisors as special types of zero divisors.

Example 5. Let $\mathcal{A}, \mathcal{B} \in \mathcal{M}_2(M(S_2))$, where

$$\mathcal{A} = \begin{pmatrix} 3(0.6, 0, 0) & 2(0, 0.2, 0) \\ 4(0, 0.2, 0) & 5(0, 0.2, 0.3) \end{pmatrix},$$
$$\mathcal{B} = \begin{pmatrix} 2(0, 0, 0.1) & 2(0, 0, 0.2) \\ 5(0.1, 0, 0) & 2(0.3, 0, 0) \end{pmatrix}. \quad (13)$$

Then,

$$\mathcal{A} \times_{\oplus_1} \mathcal{B} = \begin{pmatrix} 60(0, 0, 0) & 24(0, 0, 0) \\ 200(0, 0, 0) & 80(0, 0, 0) \end{pmatrix} \quad (14)$$

is a special type of zero divisor of $\mathcal{M}_2(M(S_2))$.

Theorem 7. $\mathcal{M}_n(M(S_2))$ *has infinite number of special types of zero divisor and non-trivial idempotents.*

Proof. Since $M(S_2)$ has infinite number of special types of zero divisor and non-trivial idempotents, we get the claim of the theorem. □

5. Matrix over m-Multiplicity NC Semigroups

In this section, we define the matrix over m-multiplicity NC semigroups. Also, we explain that the matrix over m-multiplicity NC under $\oplus_1$ and $\times_{\oplus_1}$ is a semiring of infinite order.

Definition 15. Let m-$M(S_2)$ be the collection of all multisets with entries in S_2 of at most multiplicity m, where $2 \leq m < \infty$. Then, we denote the n-square matrix over m-$M(S_2)$ by $\mathscr{M}_n(m - M(S_2))$.

Before we build a strong structure using the interval $[0,1)$, it is useful to give this example.

Example 6. Let 3-$M(S_2)$ be the collection of all multisets with entries in S_2 where

$$\begin{aligned} \mathscr{A} &= \left\{ \begin{pmatrix} 3(0.5,0.7,0.4) & 3(0.1,0.9,0.7) \\ 2(0.1,0.2,0.3) & (0.8,0.8,0.8) \end{pmatrix}, \begin{pmatrix} 3(0.1,0.2,0.7) & 3(0.1,0.1,0.1) \\ 2(0.1,0.4,0.5) & (0.2,0.3,0.4) \end{pmatrix} \right\}, \\ \mathscr{B} &= \left\{ \begin{pmatrix} (0.1,0.2,0.3) & 2(0.1,0.4,0.3) \\ 3(0.3,0.2,0.1) & (0.2,0.1,0.1) \end{pmatrix}, \begin{pmatrix} (0.1,0.2,0.3) & (0.3,0.2,0.1) \\ 3(0.4,0.5,0.5) & 2(0.3,0.3,0.2) \end{pmatrix} \right\}. \end{aligned} \tag{15}$$

where $\mathscr{A}, \mathscr{B} \in \mathscr{M}_2(3 - M(S_2))$

Now, we arrive at

$$\begin{aligned} \mathscr{A}\oplus_1\mathscr{B} &= \left\{ \begin{pmatrix} 3(0.6,0.9,0.7) & 3(0.2,0.3,0) \\ 3(0.4,0.4,0.4) & (0,0.9,0.9) \end{pmatrix}, \begin{pmatrix} 3(0.6,0.9,0.7) & 3(0.4,0.1,0.8) \\ 3(0.5,0.7,0.8) & 2(0.1,0.1,0) \end{pmatrix}, \right. \\ &\left. \begin{pmatrix} 3(0.2,0.4,0) & 3(0.2,0.5,0.4) \\ 3(0.4,0.6,0.6) & (0.4,0.4,0.5) \end{pmatrix}, \begin{pmatrix} 3(0.2,0.4,0) & 3(0.4,0.3,0.2) \\ 3(0.5,0.9,0) & 2(0.5,0.6,0.6) \end{pmatrix} \right\}, \\ &\text{where } \mathscr{A}\oplus_1\mathscr{B} \in \mathscr{M}_2(3 - M(S_2)), \\ \mathscr{A}\times_{\oplus_1}\mathscr{B} &= \left\{ \begin{pmatrix} 3(0.08,0.32,0.19) & 3(0.07,0.37,0.19) \\ 3(0.25,0.2,0.17) & 3(0.17,0.16,0.17) \end{pmatrix}, \begin{pmatrix} 3(0.09,0.59,0.47) & 3(0.18,0.41,0.18) \\ 3(0.33,0.44,0.49) & 3(0.27,0.28,0.19) \end{pmatrix}, \right. \\ &\left. \begin{pmatrix} 3(0.04,0.06,0.22) & 3(0.03,0.09,0.22) \\ 3(0.07,0.14,0.19) & 3(0.05,0.19,0.19) \end{pmatrix}, \begin{pmatrix} 3(0.05,0.09,0.26) & 3(0.06,0.07,0.09) \\ 3(0.09,0.23,0.35) & 3(0.09,0.17,0.13) \end{pmatrix} \right\}, \\ &\text{where } \mathscr{A}\times_{\oplus_1}\mathscr{B} \in \mathscr{M}_2(3 - M(S_2)). \end{aligned} \tag{16}$$

Theorem 8. *Let $\mathscr{M}_n(m - M(S_2))$ be a set of $n \times n$ matrices over a collection of all m-multisets of S_2, where $2 \leq m < \infty$. Then, the following axioms are satisfied:*

(1) $(\mathscr{M}_n(m - M(S_2)), +)$ is not a semigroup under usual addition.

(2) $(\mathscr{M}_n(m - M(S_2)), \times)$ is not a semigroup under usual addition.

(3) $(\mathscr{M}_n(m - M(S_2)), \oplus_1)$ is a semigroup.

(4) $(\mathscr{M}_n(m - M(S_2)), \times_{\oplus_1})$ is an infinite non-commutative semigroup and is not a monoid.

Proof. For (1) and (2), suppose that $\mathscr{A}, \mathscr{B} \in \mathscr{M}_2(3 - M(S_2))$, where

$$\begin{aligned} \mathscr{A} &= \begin{pmatrix} 3(0.5,0.7,0.4) & 3(0.1,0.9,0.7) \\ 2(0.1,0.2,0.3) & (0.8,0.8,0.8) \end{pmatrix}, \\ \mathscr{B} &= \begin{pmatrix} 3(0.1,0.2,0.7) & 3(0.1,0.1,0.1) \\ 2(0.1,0.4,0.5) & (0.2,0.3,0.4) \end{pmatrix}. \end{aligned} \tag{17}$$

Then, $\mathscr{A} + \mathscr{B}$ and $\mathscr{A} \times \mathscr{B}$ are not in $\mathscr{M}_2(3 - M(S_2))$. Thus, $(\mathscr{M}_2(3 - M(S_2)), +)$ is not a semigroup under the usual addition and the usual product.

For (3) and (4), since $(\mathscr{M}_n(M(S_2)), \oplus_1)$ and $(\mathscr{M}_n(M(S_2)), \times_{\oplus_1})$ are closed under $\oplus_1$ and $\times_{\oplus_1}$, respectively, then $(\mathscr{M}_n(m - M(S_2)), \oplus_1)$ and $(\mathscr{M}_n(m - M(S_2)), \times_{\oplus_1})$, replacing the numbers greater than m by m in the resultant product, are semigroups as claimed. Also, $(\mathscr{M}_n(m - M(S_2)), \times_{\oplus_1})$ is non-commutative under $\times_{\oplus_1}$. Thus, it is a non-commutative semigroup of infinite order. Furthermore, since $\mathscr{I} \notin \mathscr{M}_n(S_2)$, then it is not a monoid. □

Theorem 9. $(\mathscr{M}_n(m - M(S_2)), \oplus_1, \times_{\oplus_1})$ *is a non-commutative semiring of infinite order and has no unit.*

Proof. The claim follows from (3) and (4) of Theorem 8. □

6. Conclusions

In this paper, we study the matrices over NC which are built using the intervals $(0,1)$, $(0,1]$, $t[0,1)$, and $[0,1]$. We define the usual addition, addition modulo 1, multiplication, and a new type of product by using addition modulo 1 on each of these intervals that are different from the studies conducted so far.

The main interesting properties for matrices were developed for the current structures which are also totally different from the usual ones. In addition, the interval [0; 1) gives the algebraic structure of the commutative group and the semigroup under addition modulo 1 and usual multiplication, respectively. This result leads to constructing a structure of matrices over NC which is a non-commutative ring under usual addition modulo 1 and the new type of the product. Moreover, it has an infinite number of zero divisors.

Furthermore, the notion of the matrix over multiset of NC was introduced by using the interval $[0,1)$ under the new type of product and addition modulo 1. Finally, the matrix over m-multiplicity multiset of NC was also presented.

Data Availability

No data were used to support the study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] D. D. Anderson, S. Chun, and J. R. Juett, "Weakly prime ideals in commutative semigroups," *Bulletin Korean Mathematical Society*, vol. 56, no. 4, pp. 829–839, 2019.

[2] J. Araújo, M. Kinyon, and J. Konieczny, "Conjugacy in inverse semigroups," *Journal of Algebra*, vol. 533, pp. 142–173, 2019.

[3] J. East, A. Egri-Nagy, J. D. Mitchell, and Y. Péresse, "Computing finite semigroups," *Journal of Symbolic Computation*, vol. 92, pp. 110–155, 2019.

[4] T. Godin, I. Klimann, and M. Picantin, "On torsion-free semigroups generated by invertible reversible Mealy automata," in *International Conference on Language and Automata Theory and Applications*, pp. 328–339, Springer, Berlin, Germany, 2015.

[5] M. Hall, *The Theory of Groups*, Courier Dover Publications, Mineola, NY, USA, 2018.

[6] I. N. Herstein, *Topics in Algebra*, John Wiley and Sons, Hoboken, NJ, USA, 2006.

[7] J. M. Howie, *Fundamentals of Semigroup Theory*, Clarendon Oxford, Oxford, UK, 1995.

[8] Y. Zhu, "Cayley-symmetric semigroups," *Bulletin of the Korean Mathematical Society*, vol. 52, no. 2, pp. 409–419, 2015.

[9] J. Liu, J. Huang, and A. Chen, "Semigroup generations of unbounded block operator matrices based on the space decomposition," *Operators and Matrices*, vol. 14, no. 2, pp. 295–304, 2020.

[10] F. Smarandache, *A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Probability, and Statistics*, American Research Press, Rehoboth, DE, USA, 2000.

[11] V. Çetkiny, B. P. Varolz, and H. Aygün, "On neutrosophic submodules of a module," *Hacettepe Journal of Mathematics and Statistics*, vol. 46, no. 4, pp. 791–799, 2017.

[12] V. I. Kandasamy and F. Smarandache, "Semi-idempotents in neutrosophic rings," *Mathematics*, vol. 7, no. 6, p. 507, 2019.

[13] V. I. Kandasamy and F. Smarandache, "Neutrosophic triplets in neutrosophic rings," *Mathematics*, vol. 7, no. 6, p. 563, 2019.

[14] W. B. Kandasamy, V. I. Kandasamy, and F. Smarandache, "Neutrosophic quadruple vector spaces and their properties," *Mathematics*, vol. 7, p. 758, 2019.

[15] A. Saha and S. Broumi, "New operators on interval valued neutrosophic sets," *Neutrosophic Sets System*, vol. 28, p. 10, 2019.

[16] F. Smarandache and M. Ali, "Neutrosophic triplet group," *Neural Computing and Applications*, vol. 29, no. 7, pp. 595–601, 2018.

[17] S. Omran and A. Elrawy, "Continuous and bounded operators on neutrosophic normed spaces," *Neutrosophic Sets and Systems*, vol. 46, pp. 276–289, 2021.

[18] R. Sahin and M. Karabacak, "A novel similarity measure for single-valued neutrosophic sets and their applications in medical diagnosis, taxonomy, and clustering analysis," in *Optimization Theory Based on Neutrosophic and Plithogenic Sets*, pp. 315–341, Elsevier, Amsterdam, Netherlands, 2020.

[19] A. Jain, B. Pal Nandi, C. Gupta, and D. K. Tayal, "Large-sized document-level sentiment analysis using neutrosophic set and particle swarm optimization," *Soft Computing*, vol. 24, no. 1, pp. 3–15, 2020.

[20] B. P. Varol, V. Cetkin, and H. Aygun, "A new view on neutrosophic matrix," *Journal of Hyperstructures*, vol. 8, no. 1, pp. 48–57, 2019.

[21] F. Karaaslan and K. Hayat, "Some new operations on single-valued neutrosophic matrices and their applications in multi-criteria group decision making," *Applied Intelligence*, vol. 48, no. 12, pp. 4594–4614, 2018.

[22] M. Khan, S. Anis, K. Bibi, S. Iqbal, and F. Smarandache, "Complex neutrosophic soft matrices framework: an application in signal processing," *Journal of Mathematics*, vol. 2021, Article ID 8887824, 10 pages, 2021.

[23] W. B. Vasantha, I. Kandasamy, and F. Smarandache, "Neutrosophic components semigroups and multiset neutrosophic components semigroups," *Symmetry*, vol. 12, p. 818, 2020.