

We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

5,400

Open access books available

133,000

International authors and editors

165M

Downloads

Our authors are among the

154

Countries delivered to

TOP 1%

most cited scientists

12.2%

Contributors from top 500 universities



WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com



More Functions Associated with Neutrosophic $gs\alpha^*$ - Closed Sets in Neutrosophic Topological Spaces

P. Anbarasi Rodrigo and S. Maheswari

Abstract

The concept of neutrosophic continuous function was very first introduced by A.A. Salama et al. The main aim of this paper is to introduce a new concept of Neutrosophic continuous function namely Strongly Neutrosophic $gs\alpha^*$ - continuous functions, Perfectly Neutrosophic $gs\alpha^*$ - continuous functions and Totally Neutrosophic $gs\alpha^*$ - continuous functions in Neutrosophic topological spaces. These concepts are derived from strongly generalized neutrosophic continuous function and perfectly generalized neutrosophic continuous function. Several interesting properties and characterizations are derived and compared with already existing neutrosophic functions.

Keywords: Neutrosophic $gs\alpha^*$ - closed set, Neutrosophic $gs\alpha^*$ - open set, Strongly Neutrosophic $gs\alpha^*$ - continuous function, Perfectly Neutrosophic $gs\alpha^*$ - continuous function, Totally Neutrosophic $gs\alpha^*$ - continuous function

1. Introduction

The concept of Neutrosophic set theory was introduced by F. Smarandache [1] and it comes from two concept, one is intuitionistic fuzzy sets introduced by K. Atanassov's [2] and the other is fuzzy sets introduced by L.A. Zadeh's [3]. It includes three components, truth, indeterminacy and false membership function. R. Dhavaseelan and S. Jafari [4] has discussed about the concept of strongly generalized neutrosophic continuous function. Further he also introduced the topic of perfectly generalized neutrosophic continuous function. The real life application of neutrosophic topology is applied in Information Systems, Applied Mathematics etc.

In this paper, we introduce some new concepts related to Neutrosophic $gs\alpha^*$ - continuous function namely Strongly Neutrosophic $gs\alpha^*$ - continuous function, Perfectly Neutrosophic $gs\alpha^*$ - continuous function, Totally Neutrosophic $gs\alpha^*$ - continuous function.

2. Preliminaries

Definition 2.1: [5] Let \mathbb{P} be a non-empty fixed set. A Neutrosophic set H on the universe \mathbb{P} is defined as $H = \{ \langle p, (t_H(p), i_H(p), f_H(p)) \rangle : p \in \mathbb{P} \}$ where $t_H(p), i_H(p), f_H(p)$ represent the degree of membership function $t_H(p)$, the degree of indeterminacy $i_H(p)$ and the degree of non-membership function $f_H(p)$ respectively for each element $p \in \mathbb{P}$ to the set H . Also, $t_H, i_H, f_H : \mathbb{P} \rightarrow]-0, 1^+]$ and -0

$\leq t_H(\mathcal{P}) + i_H(\mathcal{P}) + f_H(\mathcal{P}) \leq 3^+$. Set of all Neutrosophic set over \mathbb{P} is denoted by $N_{eu}(\mathbb{P})$.

Definition 2.2: [8] Let \mathbb{P} be a non-empty set.

$\mathcal{A} = \{ \langle \mathcal{P}, (t_{\mathcal{A}}(\mathcal{P}), i_{\mathcal{A}}(\mathcal{P}), f_{\mathcal{A}}(\mathcal{P})) \rangle : \mathcal{P} \in \mathbb{P} \}$ and $\mathcal{B} = \{ \langle \mathcal{P}, (t_{\mathcal{B}}(\mathcal{P}), i_{\mathcal{B}}(\mathcal{P}), f_{\mathcal{B}}(\mathcal{P})) \rangle : \mathcal{P} \in \mathbb{P} \}$ are neutrosophic sets, then

- i. $\mathcal{A} \subseteq \mathcal{B}$ if $t_{\mathcal{A}}(\mathcal{P}) \leq t_{\mathcal{B}}(\mathcal{P}), i_{\mathcal{A}}(\mathcal{P}) \leq i_{\mathcal{B}}(\mathcal{P}), f_{\mathcal{A}}(\mathcal{P}) \geq f_{\mathcal{B}}(\mathcal{P})$ for all $\mathcal{P} \in \mathbb{P}$.
- ii. Intersection of two neutrosophic set \mathcal{A} and \mathcal{B} is defined as $\mathcal{A} \cap \mathcal{B} = \{ \langle \mathcal{P}, (\min(t_{\mathcal{A}}(\mathcal{P}), t_{\mathcal{B}}(\mathcal{P})), \min(i_{\mathcal{A}}(\mathcal{P}), i_{\mathcal{B}}(\mathcal{P})), \max(f_{\mathcal{A}}(\mathcal{P}), f_{\mathcal{B}}(\mathcal{P}))) \rangle : \mathcal{P} \in \mathbb{P} \}$.
- iii. Union of two neutrosophic set \mathcal{A} and \mathcal{B} is defined as $\mathcal{A} \cup \mathcal{B} = \{ \langle \mathcal{P}, (\max(t_{\mathcal{A}}(\mathcal{P}), t_{\mathcal{B}}(\mathcal{P})), \max(i_{\mathcal{A}}(\mathcal{P}), i_{\mathcal{B}}(\mathcal{P})), \min(f_{\mathcal{A}}(\mathcal{P}), f_{\mathcal{B}}(\mathcal{P}))) \rangle : \mathcal{P} \in \mathbb{P} \}$.
- iv. $\mathcal{A}^c = \{ \langle \mathcal{P}, (f_{\mathcal{A}}(\mathcal{P}), 1 - i_{\mathcal{A}}(\mathcal{P}), t_{\mathcal{A}}(\mathcal{P})) \rangle : \mathcal{P} \in \mathbb{P} \}$.
- v. $0_{N_{eu}} = \{ \langle \mathcal{P}, (0, 0, 1) \rangle : \mathcal{P} \in \mathbb{P} \}$ and $1_{N_{eu}} = \{ \langle \mathcal{P}, (1, 1, 0) \rangle : \mathcal{P} \in \mathbb{P} \}$.

Definition 2.3: [5] A neutrosophic topology ($N_{eu}T$) on a non-empty set \mathbb{P} is a family $\tau_{N_{eu}}$ of neutrosophic sets in \mathbb{P} satisfying the following axioms,

- i. $0_{N_{eu}}, 1_{N_{eu}} \in \tau_{N_{eu}}$.
- ii. $\mathcal{A}_1 \cap \mathcal{A}_2 \in \tau_{N_{eu}}$ for any $\mathcal{A}_1, \mathcal{A}_2 \in \tau_{N_{eu}}$.
- iii. $\bigcup \mathcal{A}_i \in \tau_{N_{eu}}$ for every family $\{ \mathcal{A}_i / i \in \Omega \} \subseteq \tau_{N_{eu}}$.

In this case, the ordered pair $(\mathbb{P}, \tau_{N_{eu}})$ or simply \mathbb{P} is called a neutrosophic topological space ($N_{eu}TS$). The elements of $\tau_{N_{eu}}$ is neutrosophic open set ($N_{eu} - OS$) and $\tau_{N_{eu}}^c$ is neutrosophic closed set ($N_{eu} - CS$).

Definition 2.4: [6] A neutrosophic set \mathcal{A} in a $N_{eu}TS$ $(\mathbb{P}, \tau_{N_{eu}})$ is called a neutrosophic generalized semi alpha star closed set ($N_{eu}gs\alpha^* - CS$) if $N_{eu}\alpha - int(N_{eu}\alpha - cl(\mathcal{A})) \subseteq N_{eu} - int(\mathcal{G})$, whenever $\mathcal{A} \subseteq \mathcal{G}$ and \mathcal{G} is $N_{eu}\alpha^* -$ open set.

Definition 2.5: [7] A neutrosophic topological space $(\mathbb{P}, \tau_{N_{eu}})$ is called a $N_{eu}gs\alpha^* - T_{1/2}$ space if every $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$ is a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.

Definition 2.6: A neutrosophic function $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ is said to be

1. neutrosophic continuous [8] if the inverse image of each $N_{eu} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$ is a $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.
2. $N_{eu}gs\alpha^* -$ continuous [7] if the inverse image of each neutrosophic closed set in $(\mathbb{Q}, \sigma_{N_{eu}})$ is a $N_{eu}gs\alpha^* -$ closed set in $(\mathbb{P}, \tau_{N_{eu}})$.
3. $N_{eu}gs\alpha^* -$ irresolute map [7] if the inverse image of each $N_{eu}gs\alpha^* -$ closed set in $(\mathbb{Q}, \sigma_{N_{eu}})$ is a $N_{eu}gs\alpha^* -$ closed set in $(\mathbb{P}, \tau_{N_{eu}})$.
4. strongly neutrosophic continuous [4] if the inverse image of each neutrosophic set in $(\mathbb{Q}, \sigma_{N_{eu}})$ is both $N_{eu} - OS$ and $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.
5. perfectly neutrosophic continuous [4] if the inverse image of each $N_{eu} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$ is both $N_{eu} - OS$ and $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.

Definition 2.7: [9] Let $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}\}$ is a neutrosophic topological space over \mathbb{P} . Then $(\mathbb{P}, \tau_{N_{eu}})$ is called neutrosophic discrete topological space.

Definition 2.8: A neutrosophic topological space $(\mathbb{P}, \tau_{N_{eu}})$ is called a neutrosophic clopen set (N_{eu} -clopen set) if it is both N_{eu} -OS and N_{eu} -CS in $(\mathbb{P}, \tau_{N_{eu}})$.

3. Strongly neutrosophic $gs\alpha^*$ -continuous function

Definition 3.1: A neutrosophic function $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ is said to be strongly $N_{eu}gs\alpha^*$ -continuous if the inverse image of every $N_{eu}gs\alpha^*$ -CS in $(\mathbb{Q}, \sigma_{N_{eu}})$ is a N_{eu} -CS in $(\mathbb{P}, \tau_{N_{eu}})$. (ie) $f^{-1}(A)$ is a N_{eu} -CS in $(\mathbb{P}, \tau_{N_{eu}})$ for every $N_{eu}gs\alpha^*$ -CS A in $(\mathbb{Q}, \sigma_{N_{eu}})$.

Theorem 3.2: Every strongly $N_{eu}gs\alpha^*$ -continuous is neutrosophic continuous, but not conversely.

Proof:

Let $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be any neutrosophic function. Let A be any N_{eu} -CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since every N_{eu} -CS is $N_{eu}gs\alpha^*$ -CS, then A is $N_{eu}gs\alpha^*$ -CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since f is strongly $N_{eu}gs\alpha^*$ -continuous, then $f^{-1}(A)$ is N_{eu} -CS in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, f is neutrosophic continuous.

Example 3.3: Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, A\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, B\}$ are N_{eu} TS on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$ respectively. Also $A = \{\langle p, (0.6, 0.4, 0.4) \rangle\}$ and $B = \{\langle q, (0.4, 0.6, 0.2) \rangle\}$ are $N_{eu}(\mathbb{P})$ and $N_{eu}(\mathbb{Q})$. Define a map $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ by $f(p) = q + 0.2$. Let $B^c = \{\langle q, (0.2, 0.4, 0.4) \rangle\}$ be a N_{eu} -CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $f^{-1}(B^c) = \{\langle p, (0.4, 0.6, 0.6) \rangle\}$. Now, $N_{eu} - cl(f^{-1}(B^c)) = A^c \cap 1_{N_{eu}} = A^c = f^{-1}(B^c) \Rightarrow f^{-1}(B^c)$ is N_{eu} -CS in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, f is neutrosophic continuous, but f is not strongly $N_{eu}gs\alpha^*$ -continuous. Let $C = \{\langle q, (0.1, 0.2, 0.8) \rangle\}$ be a $N_{eu}gs\alpha^*$ -CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $f^{-1}(C) = \{\langle p, (0.3, 0.4, 1) \rangle\}$. Now $N_{eu} - cl(f^{-1}(C)) = A^c \cap 1_{N_{eu}} = A^c \neq f^{-1}(C) \Rightarrow f^{-1}(C)$ is not N_{eu} -CS in $(\mathbb{P}, \tau_{N_{eu}})$.

Theorem 3.4: Let $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be strongly $N_{eu}gs\alpha^*$ -continuous iff the inverse image of every $N_{eu}gs\alpha^*$ -OS in $(\mathbb{Q}, \sigma_{N_{eu}})$ is N_{eu} -OS in $(\mathbb{P}, \tau_{N_{eu}})$.

Proof:

Assume that f is strongly $N_{eu}gs\alpha^*$ -continuous function. Let A be any $N_{eu}gs\alpha^*$ -OS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then A^c is $N_{eu}gs\alpha^*$ -CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since f is strongly $N_{eu}gs\alpha^*$ -continuous, then $f^{-1}(A^c)$ is N_{eu} -CS in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow (f^{-1}(A))^c$ is N_{eu} -CS in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f^{-1}(A)$ is N_{eu} -OS in $(\mathbb{P}, \tau_{N_{eu}})$. Conversely, Let A be any $N_{eu}gs\alpha^*$ -CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then A^c is $N_{eu}gs\alpha^*$ -OS in $(\mathbb{Q}, \sigma_{N_{eu}})$. By hypothesis, $f^{-1}(A^c)$ is N_{eu} -OS in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow (f^{-1}(A))^c$ is N_{eu} -OS in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f^{-1}(A)$ is N_{eu} -CS in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, f is strongly $N_{eu}gs\alpha^*$ -continuous.

Theorem 3.5: Every strongly $N_{eu}gs\alpha^*$ -continuous is $N_{eu}gs\alpha^*$ -continuous, but not conversely.

Proof:

Let $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be any neutrosophic function. Let A be any N_{eu} -CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then A is $N_{eu}gs\alpha^*$ -CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since f is strongly $N_{eu}gs\alpha^*$ -continuous, then $f^{-1}(A)$ is a N_{eu} -CS in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f^{-1}(A)$ is $N_{eu}gs\alpha^*$ -CS in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, f is $N_{eu}gs\alpha^*$ -continuous.

Example 3.6: Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, A\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, B\}$ are N_{eu} TS on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$ respectively. Also $A = \{\langle p, (0.4, 0.5, 0.7) \rangle\}$ and $B = \{\langle q, (0.6, 0.8, 0.4) \rangle\}$ are $N_{eu}(\mathbb{P})$ and $N_{eu}(\mathbb{Q})$.

Define a map $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ by $f(\mathbf{p}) = \mathbf{q}$. Let $\mathbf{B}^c = \{\langle \mathbf{q}, (0.4, 0.2, 0.6) \rangle\}$ be a $N_{eu} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $f^{-1}(\mathbf{B}^c) = \{\langle \mathbf{p}, (0.4, 0.2, 0.6) \rangle\}$. $N_{eu}\alpha^* - OS = N_{eu}\alpha - OS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbf{A}\}$ and $N_{eu}\alpha - CS = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbf{A}^c\}$. $N_{eu}\alpha - cl(f^{-1}(\mathbf{B}^c)) = \mathbf{A}^c \cap 1_{N_{eu}} = \mathbf{A}^c$. Now, $N_{eu}\alpha - int(N_{eu}\alpha - cl(f^{-1}(\mathbf{B}^c))) = \mathbf{A} \subseteq N_{eu} - int(1_{N_{eu}}) = 1_{N_{eu}}$, whenever $f^{-1}(\mathbf{B}^c) \subseteq 1_{N_{eu}} \Rightarrow f^{-1}(\mathbf{B}^c)$ is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, f is $N_{eu}gs\alpha^* -$ continuous. But f is not strongly $N_{eu}gs\alpha^* -$ continuous. Let $\mathcal{C} = \{\langle \mathbf{q}, (0.3, 0.1, 0.7) \rangle\}$ be a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $f^{-1}(\mathcal{C}) = \{\langle \mathbf{p}, (0.3, 0.1, 0.7) \rangle\}$. Now $N_{eu} - cl(f^{-1}(\mathcal{C})) = \mathbf{A}^c \cap 1_{N_{eu}} = \mathbf{A}^c \neq f^{-1}(\mathcal{C}) \Rightarrow f^{-1}(\mathcal{C})$ is not $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.

Theorem 3.7: Every strongly neutrosophic continuous is strongly $N_{eu}gs\alpha^* -$ continuous, but not conversely.

Proof:

Let $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be any neutrosophic function. Let \mathbf{A} be any $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since f is strongly neutrosophic continuous, then $f^{-1}(\mathbf{A})$ is both $N_{eu} - OS$ and $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f^{-1}(\mathbf{A})$ is $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Hence, f is strongly $N_{eu}gs\alpha^* -$ continuous.

Example 3.8: Let $\mathbb{P} = \{\mathbf{p}\}$ and $\mathbb{Q} = \{\mathbf{q}\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbf{A}, \mathcal{C}\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, \mathbf{B}\}$ are $N_{eu}TS$ on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$ respectively. Also $\mathbf{A} = \{\langle \mathbf{p}, (0.4, 0.6, 0.2) \rangle\}$, $\mathcal{C} = \{\langle \mathbf{p}, ([0.4, 1], [0.6, 1], [0, 0.2]) \rangle\}$ and $\mathbf{B} = \{\langle \mathbf{q}, (0.4, 0.6, 0.2) \rangle\}$ are $N_{eu}(\mathbb{P})$ and $N_{eu}(\mathbb{Q})$. Define a map $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ by $f(\mathbf{p}) = \mathbf{q}$. Let $\mathcal{T} = \{\langle \mathbf{q}, ([0, 0.2], [0, 0.4], [0.4, 1]) \rangle\}$ be a $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $f^{-1}(\mathcal{T}) = \{\langle \mathbf{p}, ([0, 0.2], [0, 0.4], [0.4, 1]) \rangle\}$. Now $N_{eu} - cl(f^{-1}(\mathcal{T})) = \mathbf{A}^c \cap \mathcal{C}^c \cap 1_{N_{eu}} = \mathcal{C}^c = f^{-1}(\mathcal{T})$. Therefore, f is strongly $N_{eu}gs\alpha^* -$ continuous. But f is not strongly neutrosophic continuous. Let $\mathbf{E} = \{\langle \mathbf{q}, (0.4, 0.6, 0.2) \rangle\}$ be a neutrosophic set in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $f^{-1}(\mathbf{E}) = \{\langle \mathbf{p}, (0.4, 0.6, 0.2) \rangle\}$. Now $N_{eu} - int(f^{-1}(\mathbf{E})) = 0_{N_{eu}} \cup \mathbf{A} = \mathbf{A} = f^{-1}(\mathbf{E}) \Rightarrow f^{-1}(\mathbf{E})$ is $N_{eu} - OS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Also $N_{eu} - cl(f^{-1}(\mathbf{E})) = 1_{N_{eu}} \neq f^{-1}(\mathbf{E}) \Rightarrow f^{-1}(\mathbf{E})$ is not $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, $f^{-1}(\mathbf{E})$ is not both $N_{eu} - OS$ and $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.

Remark 3.9: Every strongly neutrosophic continuous is $N_{eu}gs\alpha^* -$ continuous, but not conversely. (by Theorem 3.5 & 3.7).

Theorem 3.10: Let $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be neutrosophic function and $(\mathbb{Q}, \sigma_{N_{eu}})$ be $N_{eu}gs\alpha^* - T_{1/2}$ space. Then the following are equivalent.

1. f is strongly $N_{eu}gs\alpha^* -$ continuous.
2. f is neutrosophic continuous.

Proof:

1. \Rightarrow (2), Proof follows from theorem 3.2.
2. \Rightarrow (1), Let \mathbf{A} be any $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since $(\mathbb{Q}, \sigma_{N_{eu}})$ is $N_{eu}gs\alpha^* - T_{1/2}$ space, then \mathbf{A} is $N_{eu} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since f is neutrosophic continuous, then $f^{-1}(\mathbf{A})$ is $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, f is strongly $N_{eu}gs\alpha^* -$ continuous.

Theorem 3.11: Let $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be $N_{eu}gs\alpha^* -$ continuous. Both $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$ are $N_{eu}gs\alpha^* - T_{1/2}$ space, then f is strongly $N_{eu}gs\alpha^* -$ continuous.

Proof:

Let A be any $N_{eu}gs\alpha^*$ - CS in $(Q, \sigma_{N_{eu}})$. Since $(Q, \sigma_{N_{eu}})$ is $N_{eu}gs\alpha^*$ - $T_{1/2}$ space, then A is N_{eu} - CS in $(Q, \sigma_{N_{eu}})$. Since f is $N_{eu}gs\alpha^*$ - continuous, then $f^{-1}(A)$ is $N_{eu}gs\alpha^*$ - CS in $(P, \tau_{N_{eu}})$. Since $(P, \tau_{N_{eu}})$ is $N_{eu}gs\alpha^*$ - $T_{1/2}$ space, then $f^{-1}(A)$ is N_{eu} - CS in $(P, \tau_{N_{eu}})$. Therefore, f is strongly $N_{eu}gs\alpha^*$ - continuous.

Theorem 3.12: Let $f : (P, \tau_{N_{eu}}) \rightarrow (Q, \sigma_{N_{eu}})$ be strongly $N_{eu}gs\alpha^*$ - continuous, then f is $N_{eu}gs\alpha^*$ - irresolute.

Proof:

Let A be any $N_{eu}gs\alpha^*$ - CS in $(Q, \sigma_{N_{eu}})$. Since f is strongly $N_{eu}gs\alpha^*$ - continuous, then $f^{-1}(A)$ is N_{eu} - CS in $(P, \tau_{N_{eu}}) \Rightarrow f^{-1}(A)$ is $N_{eu}gs\alpha^*$ - CS in $(P, \tau_{N_{eu}})$. Hence, f is $N_{eu}gs\alpha^*$ - irresolute.

Theorem 3.13: Let $f : (P, \tau_{N_{eu}}) \rightarrow (Q, \sigma_{N_{eu}})$ be $N_{eu}gs\alpha^*$ - irresolute and $(P, \tau_{N_{eu}})$ be $N_{eu}gs\alpha^*$ - $T_{1/2}$ space, then f is strongly $N_{eu}gs\alpha^*$ - continuous.

Proof:

Let A be any $N_{eu}gs\alpha^*$ - CS in $(Q, \sigma_{N_{eu}})$. Since f is $N_{eu}gs\alpha^*$ - irresolute, then $f^{-1}(A)$ is $N_{eu}gs\alpha^*$ - CS in $(P, \tau_{N_{eu}})$. Since $(P, \tau_{N_{eu}})$ is $N_{eu}gs\alpha^*$ - $T_{1/2}$ space, then $f^{-1}(A)$ is N_{eu} - CS in $(P, \tau_{N_{eu}})$. Therefore, f is strongly $N_{eu}gs\alpha^*$ - continuous.

Theorem 3.14: Let $f : (P, \tau_{N_{eu}}) \rightarrow (Q, \sigma_{N_{eu}})$ and $g : (Q, \sigma_{N_{eu}}) \rightarrow (R, \gamma_{N_{eu}})$ be strongly $N_{eu}gs\alpha^*$ - continuous, then $gof : (P, \tau_{N_{eu}}) \rightarrow (R, \gamma_{N_{eu}})$ is strongly $N_{eu}gs\alpha^*$ - continuous.

Proof:

Let A be any $N_{eu}gs\alpha^*$ - CS in $(R, \gamma_{N_{eu}})$. Since g is strongly $N_{eu}gs\alpha^*$ - continuous, then $g^{-1}(A)$ is N_{eu} - CS in $(Q, \sigma_{N_{eu}}) \Rightarrow g^{-1}(A)$ is $N_{eu}gs\alpha^*$ - CS in $(Q, \sigma_{N_{eu}})$. Since f is strongly $N_{eu}gs\alpha^*$ - continuous, then $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is N_{eu} - CS in $(P, \tau_{N_{eu}})$. Therefore, gof is strongly $N_{eu}gs\alpha^*$ - continuous.

Theorem 3.15: Let $f : (P, \tau_{N_{eu}}) \rightarrow (Q, \sigma_{N_{eu}})$ be strongly $N_{eu}gs\alpha^*$ - continuous and $g : (Q, \sigma_{N_{eu}}) \rightarrow (R, \gamma_{N_{eu}})$ be $N_{eu}gs\alpha^*$ - continuous, then $gof : (P, \tau_{N_{eu}}) \rightarrow (R, \gamma_{N_{eu}})$ is neutrosophic continuous.

Proof:

Let A be any N_{eu} - CS in $(R, \gamma_{N_{eu}})$. Since g is $N_{eu}gs\alpha^*$ - continuous, then $g^{-1}(A)$ is $N_{eu}gs\alpha^*$ - CS in $(Q, \sigma_{N_{eu}})$. Since f is strongly $N_{eu}gs\alpha^*$ - continuous, then $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is N_{eu} - CS in $(P, \tau_{N_{eu}})$. Therefore, gof is neutrosophic continuous.

Theorem 3.16: Let $f : (P, \tau_{N_{eu}}) \rightarrow (Q, \sigma_{N_{eu}})$ be strongly $N_{eu}gs\alpha^*$ - continuous and $g : (Q, \sigma_{N_{eu}}) \rightarrow (R, \gamma_{N_{eu}})$ be $N_{eu}gs\alpha^*$ - irresolute, then $gof : (P, \tau_{N_{eu}}) \rightarrow (R, \gamma_{N_{eu}})$ is strongly $N_{eu}gs\alpha^*$ - continuous.

Proof:

Let A be any $N_{eu}gs\alpha^*$ - CS in $(R, \gamma_{N_{eu}})$. Since g is $N_{eu}gs\alpha^*$ - irresolute, then $g^{-1}(A)$ is $N_{eu}gs\alpha^*$ - CS in $(Q, \sigma_{N_{eu}})$. Since f is strongly $N_{eu}gs\alpha^*$ - continuous, then $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is N_{eu} - CS in $(P, \tau_{N_{eu}})$. Therefore, gof is strongly $N_{eu}gs\alpha^*$ - continuous.

Theorem 3.17: Let $f : (P, \tau_{N_{eu}}) \rightarrow (Q, \sigma_{N_{eu}})$ be $N_{eu}gs\alpha^*$ - continuous and $g : (Q, \sigma_{N_{eu}}) \rightarrow (R, \gamma_{N_{eu}})$ be strongly $N_{eu}gs\alpha^*$ - continuous, then $gof : (P, \tau_{N_{eu}}) \rightarrow (R, \gamma_{N_{eu}})$ is $N_{eu}gs\alpha^*$ - irresolute.

Proof:

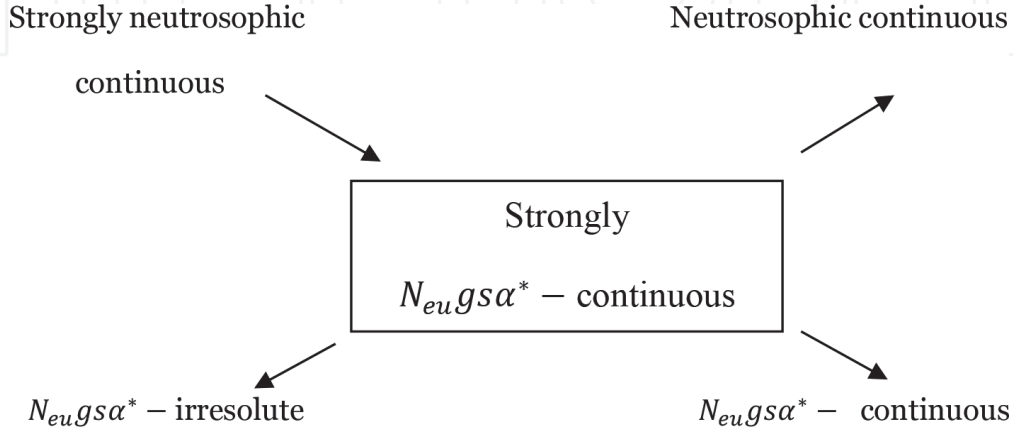
Let A be any $N_{eu}gs\alpha^*$ - CS in $(R, \gamma_{N_{eu}})$. Since g is strongly $N_{eu}gs\alpha^*$ - continuous, then $g^{-1}(A)$ is N_{eu} - CS in $(Q, \sigma_{N_{eu}})$. Since f is $N_{eu}gs\alpha^*$ - continuous, then $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is $N_{eu}gs\alpha^*$ - CS in $(P, \tau_{N_{eu}})$. Hence, gof is $N_{eu}gs\alpha^*$ - irresolute.

Theorem 3.18: Let $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be neutrosophic continuous and $g : (\mathbb{Q}, \sigma_{N_{eu}}) \rightarrow (\mathbb{R}, \gamma_{N_{eu}})$ be strongly $N_{eu}gs\alpha^*$ – continuous, then $gof : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{R}, \gamma_{N_{eu}})$ is strongly $N_{eu}gs\alpha^*$ – continuous.

Proof:

Let A be any $N_{eu}gs\alpha^*$ – CS in $(\mathbb{R}, \gamma_{N_{eu}})$. Since g is strongly $N_{eu}gs\alpha^*$ – continuous, then $g^{-1}(A)$ is N_{eu} – CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since f is neutrosophic continuous, then $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is N_{eu} – CS in $(\mathbb{P}, \tau_{N_{eu}})$. Hence, gof is strongly $N_{eu}gs\alpha^*$ – continuous.

Inter-relationship 3.19:



4. Perfectly neutrosophic $gs\alpha^*$ -continuous function

Definition 4.1: A neutrosophic function $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ is said to be perfectly $N_{eu}gs\alpha^*$ – continuous if the inverse image of every $N_{eu}gs\alpha^*$ – CS in $(\mathbb{Q}, \sigma_{N_{eu}})$ is both N_{eu} – OS and N_{eu} – CS (ie, N_{eu} – clopen set) in $(\mathbb{P}, \tau_{N_{eu}})$.

Theorem 4.2: Every perfectly $N_{eu}gs\alpha^*$ – continuous is strongly $N_{eu}gs\alpha^*$ – continuous, but not conversely.

Proof:

Let $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be any neutrosophic function. Let A be any $N_{eu}gs\alpha^*$ – CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since f is perfectly $N_{eu}gs\alpha^*$ – continuous, then $f^{-1}(A)$ is both N_{eu} – OS and N_{eu} – CS in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f^{-1}(A)$ is N_{eu} – CS in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, f is strongly $N_{eu}gs\alpha^*$ – continuous.

Example 4.3: Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, A, C\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, B\}$ are N_{eu} TS on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$ respectively. Also $A = \{\langle p, (0.7, 0.8, 0.3) \rangle\}$, $C = \{\langle p, ([0.7, 1], [0.8, 1], [0, 0.3]) \rangle\}$ and $B = \{\langle q, (0.7, 0.8, 0.3) \rangle\}$ are $N_{eu}(\mathbb{P})$ and $N_{eu}(\mathbb{Q})$. Define a map $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ by $f(p) = q$. Let $X = \{\langle q, ([0, 0.3], [0, 0.2], [0.7, 1]) \rangle\}$ be a $N_{eu}gs\alpha^*$ – CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $f^{-1}(X) = \{\langle p, ([0, 0.3], [0, 0.2], [0.7, 1]) \rangle\}$. Now $N_{eu} - cl(f^{-1}(X)) = A^c \cap C^c \cap 1_{N_{eu}} = C^c = f^{-1}(X)$. Therefore, f is strongly $N_{eu}gs\alpha^*$ – continuous. But f is not perfectly $N_{eu}gs\alpha^*$ – continuous, because $f^{-1}(X)$ is not both N_{eu} – OS and N_{eu} – CS in $(\mathbb{P}, \tau_{N_{eu}})$. Since, $N_{eu} - int(f^{-1}(X)) = 0_{N_{eu}} \neq f^{-1}(X) \Rightarrow f^{-1}(X)$ is not N_{eu} – CS in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, $f^{-1}(X)$ is not both N_{eu} – OS and N_{eu} – CS in $(\mathbb{P}, \tau_{N_{eu}})$.

Theorem 4.4: Every perfectly $N_{eu}gs\alpha^*$ – continuous is perfectly neutrosophic continuous, but not conversely.

Proof:

Let $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be any neutrosophic function. Let A be any $N_{eu} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then A is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since f is perfectly $N_{eu}gs\alpha^* -$ continuous, then $f^{-1}(A)$ is both $N_{eu} - OS$ and $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, f is perfectly neutrosophic continuous.

Example 4.5: Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, A, C, E\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, B\}$ are $N_{eu}TS$ on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$ respectively. Also $A = \{\langle p, (0.4, 0.2, 0.6) \rangle\}$, $C = \{\langle p, (0.6, 0.8, 0.4) \rangle\}$, $E = \{\langle p, ([0, 0.4], [0, 0.2], [0.6, 1]) \rangle\}$ and $B = \{\langle q, (0.6, 0.8, 0.4) \rangle\}$ are $N_{eu}(\mathbb{P})$ and $N_{eu}(\mathbb{Q})$. Define a map $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ by $f(p) = q$. Let $B^c = \{\langle q, (0.4, 0.2, 0.6) \rangle\}$ be a $N_{eu} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $f^{-1}(B^c) = \{\langle p, (0.4, 0.2, 0.6) \rangle\}$. Now $N_{eu} - cl(f^{-1}(B^c)) = A^c \cap C^c \cap E^c \cap 1_{N_{eu}} = C^c = f^{-1}(B^c)$. Also, $N_{eu} - int(f^{-1}(B^c)) = A \cup E \cup 0_{N_{eu}} = A = f^{-1}(B^c) \Rightarrow f^{-1}(B^c)$ is both $N_{eu} - OS$ and $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, f is perfectly neutrosophic continuous. But f is not perfectly $N_{eu}gs\alpha^* -$ continuous. Let $X = \{\langle q, ([0, 0.4], [0, 0.2], [0.6, 1]) \rangle\}$ be $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $f^{-1}(X) = \{\langle p, ([0, 0.4], [0, 0.2], [0.6, 1]) \rangle\}$. Since, $N_{eu} - int(f^{-1}(X)) = E \cup 0_{N_{eu}} = E = f^{-1}(X) \Rightarrow f^{-1}(X)$ is $N_{eu} - OS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Also, $N_{eu} - cl(f^{-1}(X)) = A^c \cap C^c \cap E^c \cap 1_{N_{eu}} = C^c \neq f^{-1}(X) \Rightarrow f^{-1}(X)$ is not $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, $f^{-1}(X)$ is not both $N_{eu} - OS$ and $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.

Theorem 4.6: Let $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be perfectly $N_{eu}gs\alpha^* -$ continuous iff the inverse image of every $N_{eu}gs\alpha^* - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$ is both $N_{eu} - OS$ and $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.

Proof:

Assume that f is perfectly $N_{eu}gs\alpha^* -$ continuous function. Let A be any $N_{eu}gs\alpha^* - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then A^c is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since f is perfectly $N_{eu}gs\alpha^* -$ continuous, then $f^{-1}(A^c)$ is both $N_{eu} - OS$ and $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow (f^{-1}(A))^c$ is both $N_{eu} - OS$ and $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f^{-1}(A)$ is both $N_{eu} - OS$ and $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Conversely, Let A be any $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then A^c is $N_{eu}gs\alpha^* - OS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. By hypothesis, $f^{-1}(A^c)$ is both $N_{eu} - OS$ and $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow (f^{-1}(A))^c$ is both $N_{eu} - OS$ and $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f^{-1}(A)$ is both $N_{eu} - OS$ and $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, f is perfectly $N_{eu}gs\alpha^* -$ continuous.

Theorem 4.7: Let $(\mathbb{P}, \tau_{N_{eu}})$ be a neutrosophic discrete topological space and $(\mathbb{Q}, \sigma_{N_{eu}})$ be any neutrosophic topological space. Let $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be a neutrosophic function, then the following statements are true.

1. f is strongly $N_{eu}gs\alpha^* -$ continuous.
2. f is perfectly $N_{eu}gs\alpha^* -$ continuous.

Proof:

1. \Rightarrow (2), Let A be any $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since f is strongly $N_{eu}gs\alpha^* -$ continuous, then $f^{-1}(A)$ is $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Since $(\mathbb{P}, \tau_{N_{eu}})$ is neutrosophic discrete topological space, then $f^{-1}(A)$ is $N_{eu} - OS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f^{-1}(A)$ is both $N_{eu} - OS$ and $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, f is perfectly $N_{eu}gs\alpha^* -$ continuous.
2. \Rightarrow (1), Let A be any $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since f is perfectly $N_{eu}gs\alpha^* -$ continuous, then $f^{-1}(A)$ is both $N_{eu} - OS$ and $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f^{-1}(A)$ is $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, f is strongly $N_{eu}gs\alpha^* -$ continuous.

Theorem 4.8: Let $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be perfectly neutrosophic continuous and $(\mathbb{Q}, \sigma_{N_{eu}})$ be $N_{eu}gs\alpha^* - T_{1/2}$ space, then f is perfectly $N_{eu}gs\alpha^* -$ continuous.

Proof:

Let A be any $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since $(\mathbb{Q}, \sigma_{N_{eu}})$ is $N_{eu}gs\alpha^* - T_{1/2}$ space, then A is $N_{eu} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since f is perfectly neutrosophic continuous, then $f^{-1}(A)$ is both $N_{eu} - OS$ and $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, f is perfectly $N_{eu}gs\alpha^* -$ continuous.

Theorem 4.9: Let $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ and $g : (\mathbb{Q}, \sigma_{N_{eu}}) \rightarrow (\mathbb{R}, \gamma_{N_{eu}})$ be perfectly $N_{eu}gs\alpha^* -$ continuous, then $gof : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{R}, \gamma_{N_{eu}})$ is perfectly $N_{eu}gs\alpha^* -$ continuous.

Proof:

Let A be any $N_{eu}gs\alpha^* - CS$ in $(\mathbb{R}, \gamma_{N_{eu}})$. Since g is perfectly $N_{eu}gs\alpha^* -$ continuous, then $g^{-1}(A)$ is both $N_{eu} - OS$ and $N_{eu} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow g^{-1}(A)$ is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since f is perfectly $N_{eu}gs\alpha^* -$ continuous, then $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is both $N_{eu} - OS$ and $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, gof is perfectly $N_{eu}gs\alpha^* -$ continuous.

Theorem 4.10: Let $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be neutrosophic continuous and $g : (\mathbb{Q}, \sigma_{N_{eu}}) \rightarrow (\mathbb{R}, \gamma_{N_{eu}})$ be perfectly $N_{eu}gs\alpha^* -$ continuous, then $gof : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{R}, \gamma_{N_{eu}})$ is strongly $N_{eu}gs\alpha^* -$ continuous.

Proof:

Let A be any $N_{eu}gs\alpha^* - CS$ in $(\mathbb{R}, \gamma_{N_{eu}})$. Since g is perfectly $N_{eu}gs\alpha^* -$ continuous, then $g^{-1}(A)$ is both $N_{eu} - OS$ and $N_{eu} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since f is neutrosophic continuous, then $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, gof is strongly $N_{eu}gs\alpha^* -$ continuous.

Theorem 4.11: Let $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be perfectly $N_{eu}gs\alpha^* -$ continuous and $g : (\mathbb{Q}, \sigma_{N_{eu}}) \rightarrow (\mathbb{R}, \gamma_{N_{eu}})$ be strongly $N_{eu}gs\alpha^* -$ continuous, then $gof : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{R}, \gamma_{N_{eu}})$ is perfectly $N_{eu}gs\alpha^* -$ continuous.

Proof:

Let A be any $N_{eu}gs\alpha^* - CS$ in $(\mathbb{R}, \gamma_{N_{eu}})$. Since g is strongly $N_{eu}gs\alpha^* -$ continuous, then $g^{-1}(A)$ is $N_{eu} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}}) \Rightarrow g^{-1}(A)$ is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since f is perfectly $N_{eu}gs\alpha^* -$ continuous, then $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is both $N_{eu} - OS$ and $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, gof is perfectly $N_{eu}gs\alpha^* -$ continuous.

5. Totally neutrosophic $gs\alpha^* -$ continuous function

Definition 5.1: A neutrosophic function $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ is said to be totally $N_{eu}gs\alpha^* -$ continuous if the inverse image of every $N_{eu} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$ is both $N_{eu}gs\alpha^* - OS$ and $N_{eu}gs\alpha^* - CS$ (ie, $N_{eu}gs\alpha^* -$ clopen set) in $(\mathbb{P}, \tau_{N_{eu}})$.

Definition 5.2: A neutrosophic topological space $(\mathbb{P}, \tau_{N_{eu}})$ is called a $N_{eu}gs\alpha^* -$ clopen set ($N_{eu}gs\alpha^* -$ clopen set) if it is both $N_{eu}gs\alpha^* - OS$ and $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.

Example 5.3: Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, A\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, B\}$ are N_{eu} TS on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$ respectively. Also $A = \{\langle p, (0.4, 0.5, 0.7) \rangle\}$ and $B = \{\langle q, (0.2, 0.7, 0.8) \rangle\}$ are $N_{eu}(\mathbb{P})$ and $N_{eu}(\mathbb{Q})$. Define a map $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ by $f(p) = q$. Let $B^c = \{\langle q, (0.8, 0.3, 0.2) \rangle\}$ be a $N_{eu} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $f^{-1}(B^c) = \{\langle p, (0.8, 0.3, 0.2) \rangle\}$. $N_{eu}\alpha - OS = N_{eu}\alpha - OS = \{0_{N_{eu}}, 1_{N_{eu}}, A\}$ and $N_{eu}\alpha - CS = \{0_{N_{eu}}, 1_{N_{eu}}, A^c\}$. $N_{eu}\alpha - cl(f^{-1}(B^c)) = 1_{N_{eu}}$. Now, $N_{eu}\alpha - int(N_{eu}\alpha - cl(f^{-1}(B^c))) = 1_{N_{eu}} \subseteq N_{eu} - int(1_{N_{eu}}) = 1_{N_{eu}}$, whenever $f^{-1}(B^c) \subseteq 1_{N_{eu}} \Rightarrow f^{-1}(B^c)$ is $N_{eu}gs\alpha^* - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.

Also, $N_{eu}\alpha\text{-int} (f^{-1}(B^c)) = 0_{N_{eu}}$. Now, $N_{eu}\alpha\text{-cl} (N_{eu}\alpha\text{-int} (f^{-1}(B^c))) = 0_{N_{eu}} \supseteq N_{eu} - cl(0_{N_{eu}}) = 0_{N_{eu}}$, whenever $f^{-1}(B^c) \supseteq 0_{N_{eu}} \Rightarrow f^{-1}(B^c)$ is $N_{eu}gs\alpha^*$ - OS in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, f is totally $N_{eu}gs\alpha^*$ - continuous.

Theorem 5.4: Every perfectly $N_{eu}gs\alpha^*$ - continuous is totally $N_{eu}gs\alpha^*$ - continuous, but not conversely.

Proof:

Let $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be any neutrosophic function. Let A be any N_{eu} - CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then A is $N_{eu}gs\alpha^*$ - CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since f is perfectly $N_{eu}gs\alpha^*$ - continuous, then $f^{-1}(A)$ is both N_{eu} - OS and N_{eu} - CS in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f^{-1}(A)$ is both $N_{eu}gs\alpha^*$ - OS and $N_{eu}gs\alpha^*$ - CS in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, f is totally $N_{eu}gs\alpha^*$ - continuous.

Example 5.5: Let $\mathbb{P} = \{\mathbf{p}\}$ and $\mathbb{Q} = \{\mathbf{q}\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, A\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, B\}$ are N_{eu} TS on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$ respectively. Also $A = \{\langle \mathbf{p}, (0.2, 0.4, 0.6) \rangle\}$ and $B = \{\langle \mathbf{q}, (0.6, 0.8, 0.4) \rangle\}$ are $N_{eu}(\mathbb{P})$ and $N_{eu}(\mathbb{Q})$. Define a map $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ by $f(\mathbf{p}) = \mathbf{q}$. Let $B^c = \{\langle \mathbf{q}, (0.4, 0.2, 0.6) \rangle\}$ be a N_{eu} - CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $f^{-1}(B^c) = \{\langle \mathbf{p}, (0.4, 0.2, 0.6) \rangle\}$. $N_{eu}\alpha^*$ - OS = $N_{eu}\alpha$ - OS = $\{0_{N_{eu}}, 1_{N_{eu}}, A\}$ and $N_{eu}\alpha$ - CS = $\{0_{N_{eu}}, 1_{N_{eu}}, A^c\}$. $N_{eu}\alpha\text{-cl} (f^{-1}(B^c)) = A^c \cap 1_{N_{eu}} = A^c$. Now, $N_{eu}\alpha\text{-int} (N_{eu}\alpha\text{-cl} (f^{-1}(B^c))) = A \cup 0_{N_{eu}} = A \subseteq N_{eu} - int(1_{N_{eu}}) = 1_{N_{eu}}$, whenever $f^{-1}(B^c) \subseteq 1_{N_{eu}} \Rightarrow f^{-1}(B^c)$ is $N_{eu}gs\alpha^*$ - CS in $(\mathbb{P}, \tau_{N_{eu}})$. Also, $N_{eu}\alpha\text{-int} (f^{-1}(B^c)) = 0_{N_{eu}}$. Now, $N_{eu}\alpha\text{-cl} (N_{eu}\alpha\text{-int} (f^{-1}(B^c))) = 0_{N_{eu}} \supseteq N_{eu} - cl(0_{N_{eu}}) = 0_{N_{eu}}$, whenever $f^{-1}(B^c) \supseteq 0_{N_{eu}} \Rightarrow f^{-1}(B^c)$ is $N_{eu}gs\alpha^*$ - OS in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, f is totally $N_{eu}gs\alpha^*$ - continuous. But f is not perfectly $N_{eu}gs\alpha^*$ - continuous. Let $X = \{\langle \mathbf{q}, (0.3, 0.1, 0.8) \rangle\}$ be $N_{eu}gs\alpha^*$ - CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $f^{-1}(X) = \{\langle \mathbf{p}, (0.3, 0.1, 0.8) \rangle\}$. Now, $N_{eu}\text{-int} (f^{-1}(X)) = 0_{N_{eu}} \neq f^{-1}(X) \Rightarrow f^{-1}(X)$ is not N_{eu} - OS in $(\mathbb{P}, \tau_{N_{eu}})$. Also, $N_{eu}\text{-cl} (f^{-1}(X)) = A^c \neq f^{-1}(X) \Rightarrow f^{-1}(X)$ is not N_{eu} - CS in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, $f^{-1}(X)$ is not both N_{eu} - OS and N_{eu} - CS in $(\mathbb{P}, \tau_{N_{eu}})$.

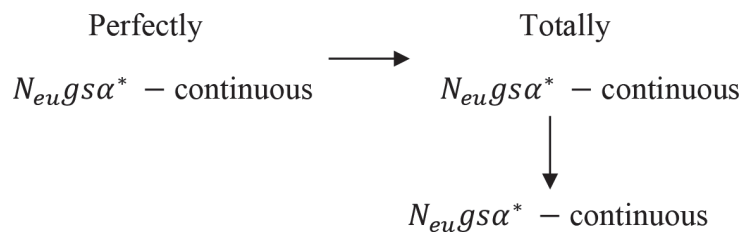
Theorem 5.6: Every totally $N_{eu}gs\alpha^*$ - continuous is $N_{eu}gs\alpha^*$ - continuous.

Proof:

Let $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be any neutrosophic function. Let A be any N_{eu} - CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since f is totally $N_{eu}gs\alpha^*$ - continuous, then $f^{-1}(A)$ is both $N_{eu}gs\alpha^*$ - OS and $N_{eu}gs\alpha^*$ - CS in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f^{-1}(A)$ is $N_{eu}gs\alpha^*$ - CS in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, f is $N_{eu}gs\alpha^*$ - continuous.

Example 5.7: Let $\mathbb{P} = \{\mathbf{p}\}$ and $\mathbb{Q} = \{\mathbf{q}\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, A\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, B\}$ are N_{eu} TS on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$ respectively. Also $A = \{\langle \mathbf{p}, (0.7, 0.6, 0.5) \rangle\}$ and $B = \{\langle \mathbf{q}, (0.7, 0.8, 0.3) \rangle\}$ are $N_{eu}(\mathbb{P})$ and $N_{eu}(\mathbb{Q})$. Define a map $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ by $f(\mathbf{p}) = \mathbf{q}$. Let $B^c = \{\langle \mathbf{q}, (0.3, 0.2, 0.7) \rangle\}$ be a N_{eu} - CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $f^{-1}(B^c) = \{\langle \mathbf{p}, (0.3, 0.2, 0.7) \rangle\}$. $N_{eu}\alpha^*$ - OS = $N_{eu}\alpha$ - OS = $\{0_{N_{eu}}, 1_{N_{eu}}, A, D\}$ and $N_{eu}\alpha$ - CS = $\{0_{N_{eu}}, 1_{N_{eu}}, A^c, E\}$, where $D = \{\langle \mathbf{p}, ([0.7, 1], [0.6, 1], [0, 0.5]) \rangle\}$, $E = \{\langle \mathbf{p}, ([0, 0.5], [0, 0.4], [0.7, 1]) \rangle\}$. $N_{eu}\alpha\text{-cl} (f^{-1}(B^c)) = A^c \cap F \cap 1_{N_{eu}} = F$, where $F = \{\langle \mathbf{p}, ([0.3, 0.5], [0.2, 0.4], 0.7) \rangle\}$. Now, $N_{eu}\alpha\text{-int} (N_{eu}\alpha\text{-cl} (f^{-1}(B^c))) = 0_{N_{eu}} \subseteq N_{eu} - int(A), N_{eu} - int(D), N_{eu} - int(1_{N_{eu}}) = A, 1_{N_{eu}}$, whenever $f^{-1}(B^c) \subseteq A, 1_{N_{eu}} \Rightarrow f^{-1}(B^c)$ is $N_{eu}gs\alpha^*$ - CS in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, f is $N_{eu}gs\alpha^*$ - continuous. But f is not totally $N_{eu}gs\alpha^*$ - continuous, because $f^{-1}(B^c)$ is not $N_{eu}gs\alpha^*$ - OS in $(\mathbb{P}, \tau_{N_{eu}})$. Since $N_{eu}\alpha\text{-cl} (N_{eu}\alpha\text{-int} (f^{-1}(B^c))) = 0_{N_{eu}} \not\supseteq N_{eu} - cl(J) = A^c$, whenever $f^{-1}(B^c) \supseteq J$, where $J = \{\langle \mathbf{p}, ([0, 0.3], [0, 0.2], [0.7, 1]) \rangle\} \Rightarrow f^{-1}(B^c)$ is not $N_{eu}gs\alpha^*$ - OS in $(\mathbb{P}, \tau_{N_{eu}})$.

Inter-relationship 5.8:



Theorem 5.9: Let $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be totally $N_{eu}gs\alpha^*$ – continuous and $(\mathbb{Q}, \sigma_{N_{eu}})$ be $N_{eu}gs\alpha^* - T_{1/2}$ space, then f is $N_{eu}gs\alpha^* -$ irresolute.

Proof:

Let \mathcal{A} be any $N_{eu}gs\alpha^* -$ CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since $(\mathbb{Q}, \sigma_{N_{eu}})$ is $N_{eu}gs\alpha^* - T_{1/2}$ space, then \mathcal{A} is $N_{eu} -$ CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since f is totally $N_{eu}gs\alpha^* -$ continuous, then $f^{-1}(\mathcal{A})$ is both $N_{eu}gs\alpha^* -$ OS and $N_{eu}gs\alpha^* -$ CS in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f^{-1}(\mathcal{A})$ is $N_{eu}gs\alpha^* -$ CS in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, f is $N_{eu}gs\alpha^* -$ irresolute.

Theorem 5.10: Let $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ and $g : (\mathbb{Q}, \sigma_{N_{eu}}) \rightarrow (\mathbb{R}, \gamma_{N_{eu}})$ be totally $N_{eu}gs\alpha^* -$ continuous and $(\mathbb{Q}, \sigma_{N_{eu}})$ be $N_{eu}gs\alpha^* - T_{1/2}$ space, then $gof : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{R}, \gamma_{N_{eu}})$ is totally $N_{eu}gs\alpha^* -$ continuous.

Proof:

Let \mathcal{A} be any $N_{eu} -$ CS in $(\mathbb{R}, \gamma_{N_{eu}})$. Since g is totally $N_{eu}gs\alpha^* -$ continuous, then $g^{-1}(\mathcal{A})$ is both $N_{eu}gs\alpha^* -$ OS and $N_{eu}gs\alpha^* -$ CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since $(\mathbb{Q}, \sigma_{N_{eu}})$ is $N_{eu}gs\alpha^* - T_{1/2}$ space, then $g^{-1}(\mathcal{A})$ is $N_{eu} -$ CS in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since f is totally $N_{eu}gs\alpha^* -$ continuous, then $f^{-1}(g^{-1}(\mathcal{A})) = (gof)^{-1}(\mathcal{A})$ is both $N_{eu}gs\alpha^* -$ OS and $N_{eu}gs\alpha^* -$ CS in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, gof is totally $N_{eu}gs\alpha^* -$ continuous.

Author details


P. Anbarasi Rodrigo¹ and S. Maheswari^{2*}

¹ Assistant Professor, Department of Mathematics, St. Mary's College (Autonomous), Thoothukudi, Affiliated by Manonmaniam Sundaranar University, Tirunelveli, India

² Research Scholar, Register number : 20212212092003, St. Mary's College (Autonomous), Thoothukudi, Affiliated by Manonmaniam Sundaranar University, Tirunelveli, India

*Address all correspondence to: mahma1295@gmail.com

IntechOpen

© 2021 The Author(s). Licensee IntechOpen. This chapter is distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/3.0>), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited. 

References

- [1] Floretin Smarandache, Neutrosophic Sets & Systems, University of New Mexico, vol.13, pp.90-95, 2016.
 Set: A Generalization of Intuitionistic Fuzzy Set, Jorunal of Defense Resources Management, 2010,107-116.
- [2] Atanassov, K., Intuitionistic Fuzzy Sets, Fuzzy Sets And Systems, pp. 87-94, 1986.
- [3] Zadeh, L.A., Fuzzy Sets, Inform And Control, Vol.8, pp.338-353, 1965.
- [4] Dhavaseelan, R., & Jafari, S., Generalized Neutrosophic Closed Sets, New trends in Neutrosophic Theory And Applications, Vol.2, pp.261-273, Jan 2017.
- [5] Sreeja, D., & Sarankumar, T., Generalized Alpha Closed Sets in Neutrosophic Topological Spaces, JASC, Journal of Applied Science & Computations, ISSN:1076-5131, Vol.5, Issue 11, pp.1816-1823, Nov-2018.
- [6] Anbarasi Rodrigo, P., Maheswari, S., Neutrosophic Generalized Semi Alpha Star Closed Sets in Neutrosophic Topological Spaces, Paper presented in International Conference on Mathematics, Statistics, Computers And Information Sciences, 2021.
- [7] Anbarasi Rodrigo, P., Maheswari, S., Functions Related to Neutrosophic $gs\alpha^*$ – Closed Sets in Neutrosophic Topological Spaces, Paper presented in 24th FAI International Conference on Global Trends of Data Analytics in Business Management, Social Sciences, Medical Sciences and Decision Making 24th FAI-ICDBSMD 2021.
- [8] Blessie Rebecca, S., Francina Shalini, A., Neutrosophic Generalized Regular Contra Continuity in Neutrosophic Topological Spaces, Research in Advent Technology, Vol.7, No.2, E-ISSN: 2321-9637, pp.761-765, Feb 2019.
- [9] Serkan Karatas, Cemil Kuru, Neutrosophic Topology, Neutrosophic