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Review Article

On Different Types of Single-Valued Neutrosophic Covering Rough Set with Application in Decision-Making

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This paper aims to propose the notion of Type-1 single-valued neutrosophic complementary β-neighborhood (briefly, Type-1 SVN complementary β-neighborhood) and use it to introduce a novel class of 1-single-valued neutrosophic β-covering rough set (briefly, 1-SVN β-CRS). Then, we will merge the 1-SVN β-neighborhood and 1-SVN complementary β-neighborhood to create new two models of 1-SVN β-CRS. Furthermore, we will discuss the relationships between the present work and Wang and Zhang's work. For further study on Type-2 Wang and Zhang's models, we will define the 2-SVN complementary β-neighborhood and use it to present a novel class of 2-SVN β-CRS. Also, we combine the 2-SVN β-neighborhood and 2-SVN complementary β-neighborhood to investigate the new two models of 2-SVN β-CRS. Lately, we will demonstrate two illustrative examples as real problems to show the differences between two of our approaches and Wang and Zhang's approach.

1. Introduction

In 1982, the world-known new notion called rough sets (briefly, RSs) dealt with uncertain data on the hand of Pawlak [1, 2]. This notion helped researchers in several areas of research to develop these areas through RS, for instance, there are many published papers (see [3–19]). The famed generalization of RS is covering rough sets (briefly, CRSs). The CRSs were studied by many specialists from different views which made an evolution in many fields such as computer science, mathematics, and chemistry. Some of the relevant studies helped scholars to solve many life problems [20–33]. Consequently, in 1990, the notions of fuzzy rough sets (briefly, FRSs) and rough fuzzy sets (briefly, RFSs) are defined by Dubois and Prade [34] from the merging between the CRS and the fuzzy sets (briefly, FSs) which appeared by Zadeh [35]. From this point of view, the new kinds of covering fuzzy rough sets (briefly, CFRSs) through the fuzzy β-neighborhoods were called fuzzy β covering rough sets (briefly, FβCRSs) (see [36]). To complete this study, Yang et al. [37, 38] defined several basic notions of fuzzy complementary β-neighborhoods, fuzzy β-minimal description, and fuzzy β-maximal description to establish new classes of FβCRS.

The notion of single-valued neutrosophic sets (briefly, SVNS) was developed by Wang et al. [39]. SVNS is a natural extension of the intuitionistic fuzzy set (briefly, IFS) [40]. Smarandache [41] investigated a new set called neutrosophic set as a generalization of mathematical tools (i.e., fuzzy set [35], interval-valued fuzzy set [42], IFS [40], and interval-valued intuitionistic fuzzy set [43]). In 2015, Mondal and Pramanik [44] demonstrated a new terminology called rough neutrosophic set. By using SVN relation, Yang et al. [45] introduced the SVN rough set model, and based on the notion of Type-1 SVN β-neighborhoods, Wang and Zhange [46] proposed two models of Type-1 SVNβ-covering rough sets (briefly, SVNβ-CRS). Furthermore, they presented a new kind of SVNβ-CRS called Type-2 SVNβ-CRS utilizing Type-2 SVN β-neighborhoods in [47]. The notions of neutrosophic soft rough sets and its generalizations are presented in [48–53].

By the above discussion and extend the other work (see [46, 47]) in SVNβ-CRS. We will generalize these methods in [46, 47] by boosting the lower approximation and minimizing the upper approximation, which is a big challenge to every author. Consequently, the motivation of this paper is to improve this area is obtained by introducing the notion of 1-SVN complementary β-neighborhood (resp., 2-SVN complementary β-neighborhood) to build novel classes of 1-SVNβ-CRS (resp., 2-SVNβ-CRS). And, by joining 1-SVN β-neighborhoods (resp., 2-SVN β-neighborhoods) and 1-SVN complementary β-neighborhood (resp., 2-SVN complementary β-neighborhood), we obtain two new SVN β-neighborhoods which establish two new models of 1-SVNβ-CRS (resp., 2-SVNβ-CRS). Also, we discuss the properties of the two proposed covering methods. Finally, we apply our work (i.e., two proposed methods) to solve decision-making problems.

The organization of this article is as follows. In Section 2, we give a basic notion about the presented study. Section 3 establishes the definition of 1-SVN complementary β-neighborhood, and hence, a new model of 1-SVNβ-CRS is proposed. Also, by merging between the 1-SVN β-neighborhoods and its complementary, we set up two other models of 1-SVNβ-CRS. Thus, the relevant characteristics are also studied. Section 4 constructs the notion of 2-SVN complementary β-neighborhood, and thus, a new model of 2-SVNβ-CRS is proposed. By merging between the 2-SVN β-neighborhoods and its complementary, we also set up two other models of 2-SVN β-CRS. Then, the relevant properties are also discussed. The decision-making approaches to the two methods are mentioned in Sections 3 and 4 are investigated in Section 5. Also, in this section, we compare between our approach and Wang and Zhang's approach. Section 6 shows the overall benefits of our study.

2. Preliminaries

In this section, we review some basic terminologies about the subject of this study.

Definition 1 (Cf. [26]). Assume that Ω is a universe and $\widetilde{\Gamma}$ is a family of subsets of Ω. If no element in $\widetilde{\Gamma}$ is empty and $\Omega = \cup_{\widetilde{C}\in\widetilde{\Gamma}}\widetilde{C}$, then $\widetilde{\Gamma}$ is called a covering of Ω, and $(\Omega, \widetilde{\Gamma})$ is called a covering approximation space (briefly, CAS).

Definition 2 (Cf. [54, 55]). Assume that Ω is a universe. We say $\widetilde{\Gamma} = \{\widetilde{C}_1, \widetilde{C}_2, \ldots, \widetilde{C}_m\}$, with $\widetilde{C}_i \in \mathscr{F}(\Omega)\,(i = 1, 2, \ldots, m)$, a fuzzy covering (briefly, FC) of Ω if $(\cup_{i=1}^{m}\widetilde{C}_i)(x) = 1$, for each $x \in \Omega$.

The notion of fuzzy β-covering was discovered by Ma [36] $(0 < \beta \leq 1)$. This notion is considered as a generalization of FC. If $\widetilde{\Gamma} = \{\widetilde{C}_1, \widetilde{C}_2, \ldots, \widetilde{C}_m\}$, with $\widetilde{C}_i \in \mathscr{F}(\Omega)\,(i = 1, 2, \ldots, m)$, then $(\Omega, \widetilde{\Gamma})$ is called a fuzzy β-covering approximation space (briefly, FβCAS).

Definition 3 (Cf. [54, 55]). Assume that Ω is not an empty set. For each $x \in \Omega$, define the SVN set $\mathscr{A} \subseteq \Omega$ as the following formula:

$$\mathscr{A} = \{\langle x, \mathscr{T}_{\mathscr{A}}(x), \mathscr{I}_{\mathscr{A}}(x), \mathscr{F}_{\mathscr{A}}(x)\rangle\}. \tag{1}$$

where $\mathscr{T}_{\mathscr{A}}\colon \Omega \longrightarrow [0,1]$ is the degree of truth membership of the element x to $\mathscr{A}$, $\mathscr{I}_{\mathscr{A}}\colon \Omega \longrightarrow [0,1]$ is the degree of indeterminacy membership of the element x to $\mathscr{A}$, and $\mathscr{F}_{\mathscr{A}}\colon \Omega \longrightarrow [0,1]$ is the degree of falsity membership. These variables satisfy $0 \leq \mathscr{T}_{\mathscr{A}} + \mathscr{I}_{\mathscr{A}} + \mathscr{F}_{\mathscr{A}} \leq 3$.

In 2018, Wang et al. [39] established the notion of SVN β-covering approximation space, and then, Wang and Zhang [46, 47] used this notion to create two types of the covering method as in the following definition.

Definition 4 (Cf. [46, 47]). Let Ω be a universe and SVN (Ω) be the SVN power set of Ω. For a SVN number $\beta = (a, b, c)$, we call $\widetilde{\Gamma} = \{\widetilde{C}_1, \widetilde{C}_2, \ldots, \widetilde{C}_m\}$, with $\widetilde{C}_i \in \mathrm{SVN}(\Omega)$ $(i = 1, 2, \ldots, m)$, a Type-1 SVN β-covering (a Type-2 SVN β-covering) of Ω if $\widetilde{C}_i(x) \geq \beta$ $(\widetilde{C}_i(x) \succcurlyeq \beta)$, for each $x \in \Omega$. Moreover, $(\Omega, \widetilde{\Gamma})$ is called a Type-1 SVN β-covering approximation space (a Type-2 SVN β-covering approximation space) (briefly, 1-SVN β CAS (2-SVNβCAS)).

If $\mathscr{A} = \langle a_1, b_1, c_1\rangle$ and $\mathscr{B} = \langle a_2, b_2, c_2\rangle$ are two SVN numbers, then

(i) $\mathscr{A} \leq \mathscr{B} \Leftrightarrow a_1 \leq a_2, b_1 \geq b_2, c_1 \geq c_2$
(ii) $\mathscr{A} \geq \mathscr{B} \Leftrightarrow a_1 \geq a_2, b_1 \leq b_2, c_1 \leq c_2$
(iii) $\mathscr{A} \prec \mathscr{B} \Leftrightarrow a_1 \leq a_2, b_1 \leq b_2, c_1 \geq c_2$
(iv) $\mathscr{A} \succcurlyeq \mathscr{B} \Leftrightarrow a_1 \geq a_2, b_1 \geq b_2, c_1 \leq c_2$

Here, $\forall \mathscr{A}, \mathscr{B} \in \mathrm{SVN}(\Omega)$, and we have the following relation, union, and intersection operations.

For Type-1,

(1) $\mathscr{A} \subseteq \mathscr{B} \Leftrightarrow \mathscr{T}_{\mathscr{A}} \leq \mathscr{T}_{\mathscr{B}}, \mathscr{I}_{\mathscr{B}} \leq \mathscr{I}_{\mathscr{A}}, \mathscr{F}_{\mathscr{A}} \leq \mathscr{F}_{\mathscr{B}}\ \forall x \in \Omega$
(2) $\mathscr{A} \cap \mathscr{B} = \{\langle x, \mathscr{T}_{\mathscr{A}} \wedge \mathscr{T}_{\mathscr{B}}, \mathscr{I}_{\mathscr{A}} \vee \mathscr{I}_{\mathscr{B}}, \mathscr{F}_{\mathscr{A}} \vee \mathscr{F}_{\mathscr{B}}\rangle\}$
(3) $\mathscr{A} \cup \mathscr{B} = \{\langle x, \mathscr{T}_{\mathscr{A}} \vee \mathscr{T}_{\mathscr{B}}, \mathscr{I}_{\mathscr{A}} \wedge \mathscr{I}_{\mathscr{B}}, \mathscr{F}_{\mathscr{A}} \wedge \mathscr{F}_{\mathscr{B}}\rangle\}$

For Type-2,

(1) $\mathscr{A} \sqsubseteq \mathscr{B} \Leftrightarrow \mathscr{T}_{\mathscr{A}} \leq \mathscr{T}_{\mathscr{B}}, \mathscr{I}_{\mathscr{B}} \leq \mathscr{I}_{\mathscr{A}}, \mathscr{F}_{\mathscr{A}} \leq \mathscr{F}_{\mathscr{B}}\ \forall x \in \Omega$
(2) $\mathscr{A} \sqcap \mathscr{B} = \{\langle x, \mathscr{T}_{\mathscr{A}} \wedge \mathscr{T}_{\mathscr{B}}, \mathscr{I}_{\mathscr{A}} \wedge \mathscr{I}_{\mathscr{B}}, \mathscr{F}_{\mathscr{A}} \vee \mathscr{F}_{\mathscr{B}}\rangle\}$
(3) $\mathscr{A} \sqcup \mathscr{B} = \{\langle x, \mathscr{T}_{\mathscr{A}} \vee \mathscr{T}_{\mathscr{B}}, \mathscr{I}_{\mathscr{A}} \vee \mathscr{I}_{\mathscr{B}}, \mathscr{F}_{\mathscr{A}} \wedge \mathscr{F}_{\mathscr{B}}\rangle\}$

Definition 5 (Cf. [46, 47]). Let $(\Omega, \widetilde{\Gamma})$ be a 1-SVN β CAS with $\widetilde{\Gamma} = \{\widetilde{C}_1, \widetilde{C}_2, \ldots, \widetilde{C}_m\}$ for some $\beta = \langle a, b, c\rangle$. Then, for each $x \in \Omega$, define the Type-1 SVN β-neighborhood (the Type-2 SVN β-neighborhood) of x as follows:

$$\begin{aligned} {}_1\widetilde{\mathcal{N}}_x^{\beta} &= \cap\left\{\widetilde{C}_i \in \widetilde{\Gamma}\colon \widetilde{C}_i(x) \geq \beta\right\} \\ &= \cap\left\{\widetilde{C}_i \in \widetilde{\Gamma}\colon \mathcal{T}_{\widetilde{C}_i} \geq a, \mathcal{I}_{\widetilde{C}_i} \leq b, \mathcal{F}_{\widetilde{C}_i} \leq c\right\}, \\ {}_2\widetilde{\mathcal{N}}_x^{\beta} &= \sqcap\left\{\widetilde{C}_i \in \widetilde{\Gamma}\colon \widetilde{C}_i(x) \succcurlyeq \beta\right\} \\ &= \sqcap\left\{\widetilde{C}_i \in \widetilde{\Gamma}\colon \mathcal{T}_{\widetilde{C}_i} \geq a, \mathcal{I}_{\widetilde{C}_i} \geq b, \mathcal{F}_{\widetilde{C}_i} \leq c\right\}. \end{aligned} \quad (2)$$

Definition 6 (Cf. [46, 47]). Let $(\Omega, \widetilde{\Gamma})$ be a 1-SVNβCAS (resp., 2-SVNβCAS) for some $\beta = \langle a, b, c\rangle$. For each $x \in \Omega$ and $\mathscr{A} \in \mathrm{SVN}(\Omega)$, define the first type of Type-1 SVN lower approximation (1-1-SVNLA) $\mathscr{L}_1^1(\mathscr{A})$ (resp., the first type of Type-2 SVN lower approximation (1-2-SVNLA) $\mathscr{L}_1^2(\mathscr{A})$) and the first type of Type-1 SVN upper approximation (1-1-SVNUA) $\mathscr{U}_1^1(\mathscr{A})$ (resp., the first type of Type-2 SVN upper approximation (1-2-SVNUA) $\mathscr{U}_1^2(\mathscr{A})$) as follows:

$$\begin{aligned} \mathscr{L}_1^1(\mathscr{A}) &= \left\{\langle x, \wedge_{y\in\Omega}\left(\mathcal{F}_{{}_1\widetilde{\mathcal{N}}_x^{\beta}}(y) \vee \mathcal{T}_{\mathscr{A}}(y)\right), \wedge_{y\in\Omega}\left(\left(1 - \mathcal{I}_{{}_1\widetilde{\mathcal{N}}_x^{\beta}}(y)\right) \vee \mathcal{I}_{\mathscr{A}}(y)\right), \vee_{y\in\Omega}\left(\mathcal{T}_{{}_1\widetilde{\mathcal{N}}_x^{\beta}}(y) \wedge \mathcal{F}_{\mathscr{A}}(y)\right)\rangle\right\}, \\ \mathscr{U}_1^1(\mathscr{A}) &= \left\{\langle x, \vee_{y\in\Omega}\left(\mathcal{T}_{{}_1\widetilde{\mathcal{N}}_x^{\beta}}(y) \wedge \mathcal{T}_{\mathscr{A}}(y)\right), \vee_{y\in\Omega}\left(\mathcal{I}_{{}_1\widetilde{\mathcal{N}}_x^{\beta}}(y) \wedge \mathcal{I}_{\mathscr{A}}(y)\right), \wedge_{y\in\Omega}\left(\mathcal{F}_{{}_1\widetilde{\mathcal{N}}_x^{\beta}}(y) \vee \mathcal{F}_{\mathscr{A}}(y)\right)\rangle\right\}, \\ \mathscr{L}_1^2(\mathscr{A}) &= \left\{\langle x, \wedge_{y\in\Omega}\left(\mathcal{F}_{{}_2\widetilde{\mathcal{N}}_x^{\beta}}(y) \vee \mathcal{T}_{\mathscr{A}}(y)\right), \wedge_{y\in\Omega}\left(\left(1 - \mathcal{I}_{{}_2\widetilde{\mathcal{N}}_x^{\beta}}(y)\right) \vee \mathcal{I}_{\mathscr{A}}(y)\right), \vee_{y\in\Omega}\left(\mathcal{T}_{{}_2\widetilde{\mathcal{N}}_x^{\beta}}(y) \wedge \mathcal{F}_{\mathscr{A}}(y)\right)\rangle\right\}, \\ \mathscr{U}_1^2(\mathscr{A}) &= \left\{\langle x, \vee_{y\in\Omega}\left(\mathcal{T}_{{}_2\widetilde{\mathcal{N}}_x^{\beta}}(y) \wedge \mathcal{T}_{\mathscr{A}}(y)\right), \vee_{y\in\Omega}\left(\mathcal{I}_{{}_2\widetilde{N}_x^{\beta}}(y) \wedge \mathcal{I}_{\mathscr{A}}(y)\right), \wedge_{y\in\Omega}\left(\mathcal{F}_{{}_2\widetilde{N}_x^{\beta}}(y) \vee \mathcal{F}_{\mathscr{A}}(y)\right)\rangle\right\}. \end{aligned} \quad (3)$$

If $\mathscr{L}_1^1(\mathscr{A})$ (resp., $\mathscr{L}_1^2(\mathscr{A})$) $\neq \mathscr{U}_1^1(\mathscr{A})$ (resp., $\mathscr{U}_1^2(\mathscr{A})$), then $\mathscr{A}$ is called the first type of Type-1 SVN β-covering rough sets (resp., the first type of Type-2 SVN β-covering rough sets) (briefly, 1-1-SVNβCRSs (resp., 1-2-SVNβCRSs)).

3. Type-1 SVN Complementary β-Neighborhood and Three New Kinds of Type-1 SVNβ-CRS

We will propose the concept of a type-1 SVN complementary β-neighborhood and three new kinds of Type-1 SVNβ-CRS and introduce several definitions, propositions, and examples as indicated below.

Definition 7. Let $(\Omega, \widetilde{\Gamma})$ be a 1-SVNβCAS with $\widetilde{\Gamma} = \left\{\widetilde{C}_1, \widetilde{C}_2, \ldots, \widetilde{C}_m\right\}$, for some $\beta = \langle a, b, c\rangle$. Then, for each $x \in \Omega$, define the type-1 SVN complementary β-neighborhood of x as follows:

$$ {}_1\widetilde{\mathscr{M}}_x^{\beta}(y) = {}_1\widetilde{\mathcal{N}}_y^{\beta}(x), \quad \forall y \in \Omega. \quad (4)$$

Example 1. Let $(\Omega, \widetilde{\Gamma})$ be a 1-SVNβCAS, $\Omega = \{x_1, x_2, x_3, x_4, x_5\}$ and $\widetilde{\Gamma} = \left\{\widetilde{C}_1, \widetilde{C}_2, \widetilde{C}_3, \widetilde{C}_4\right\}$, where $\beta = \langle 0.5, 0.3, 0.8\rangle$ are summarized in Table 1.

In Table 1, ${}_1\widetilde{\mathcal{N}}_{x_1}^{\beta} = \widetilde{C}_1 \cap \widetilde{C}_2$, ${}_1\widetilde{\mathcal{N}}_{x_2}^{\beta} = \widetilde{C}_1 \cap \widetilde{C}_2 \cap \widetilde{C}_4$, ${}_1\widetilde{\mathcal{N}}_{x_3}^{\beta} = \widetilde{C}_3 \cap \widetilde{C}_4$, ${}_1\widetilde{\mathcal{N}}_{x_4}^{\beta} = \widetilde{C}_1 \cap \widetilde{C}_4$, and ${}_1\widetilde{\mathcal{N}}_{x_5}^{\beta} = \widetilde{C}_2 \cap \widetilde{C}_3 \cap \widetilde{C}_4$.

Table 2 contains the results of type-1 SVN β-neighborhood.

Thus, we can obtain the values of type-1 SVN complementary β-neighborhood as in Table 3.

Hence, we can merge ${}_1\widetilde{\mathcal{N}}_x^{\beta}$ and ${}_1\widetilde{\mathscr{M}}_x^{\beta}$ and compute ${}_1\widetilde{\mathcal{N}}_x^{\beta} \cap {}_1\widetilde{\mathscr{M}}_x^{\beta}$ as Table 4.

Also, we can compute ${}_1\widetilde{\mathcal{N}}_x^{\beta} \cup {}_1\widetilde{\mathscr{M}}_x^{\beta}$ as set in Table 5.

Proposition 1. *Let $(\Omega, \widetilde{\Gamma})$ be a 1-SVNβCAS, for some $\beta = \langle a, b, c\rangle$ and for each $x, y, z \in \Omega$. Then, the following statements hold:*

(1) ${}_1\widetilde{\mathscr{M}}_x^{\beta}(x) \geq \beta$
(2) If ${}_1\widetilde{\mathscr{M}}_x^{\beta}(y) \geq \beta$ and ${}_1\widetilde{\mathscr{M}}_y^{\beta}(z) \geq \beta$, then ${}_1\widetilde{\mathscr{M}}_x^{\beta}(z) \geq \beta$
(3) $0 < \beta_1 \leq \beta_2 \leq \beta$, then ${}_1\widetilde{\mathscr{M}}_x^{\beta_1} \subseteq {}_1\widetilde{\mathscr{M}}_x^{\beta_2}$

Proof

(1) It follows directly from Definitions 5 and 7.
(2) Since ${}_1\widetilde{\mathscr{M}}_x^{\beta}(y) \geq \beta$, then ${}_1\widetilde{\mathcal{N}}_y^{\beta}(x) \geq \beta$. If $\widetilde{C}_i(x) \geq \beta$, then $\widetilde{C}_i(y) \geq \beta$, and since ${}_1\widetilde{\mathscr{M}}_y^{\beta}(z) \geq \beta$, then ${}_1\widetilde{\mathcal{N}}_z^{\beta}(y) \geq \beta$. If $\widetilde{C}_i(y) \geq \beta$, then $\widetilde{C}_i(z) \geq \beta$. Therefore, ${}_1\widetilde{\mathscr{M}}_x^{\beta}(z) \geq \beta$.
(3) For each $x \in \Omega, 0 < \beta_1 \leq \beta_2 \leq \beta$, then $\cap\left\{\widetilde{C}_i \in \widetilde{\Gamma}\colon \widetilde{C}_i(x) \geq \beta_1\right\} \supseteq \cap\left\{\widetilde{C}_i \in \widetilde{\Gamma}\colon \widetilde{C}_i(x) \geq \beta_2\right\}$. Thus, by Definition 7, we have ${}_1\widetilde{\mathscr{M}}_x^{\beta_1} \subseteq {}_1\widetilde{\mathscr{M}}_x^{\beta_2}$. □

Proposition 2. *Let $(\Omega, \widetilde{\Gamma})$ be a 1-SVNβCAS, for some $\beta = \langle a, b, c\rangle$. For each $x, y \in \Omega$,*

$$ {}_1\widetilde{\mathscr{M}}_x^{\beta}(y) \geq \beta \Leftrightarrow {}_1\widetilde{\mathscr{M}}_y^{\beta} \subseteq {}_1\widetilde{\mathscr{M}}_x^{\beta}. \quad (5)$$

Proof. Let ${}_1\widetilde{\mathscr{M}}_x^{\beta}(y) \geq \beta$, $\mathcal{T}_{{}_1\widetilde{\mathscr{M}}_x^{\beta}}(y) = \mathcal{T} \cap \widetilde{C}_i(y) = \wedge\ \mathcal{T}_{\widetilde{C}_i}(y) \geq a$, $\mathcal{I}_{{}_1\widetilde{\mathscr{M}}_x^{\beta}}(y) = \mathcal{I} \cap \widetilde{C}_i(y) = \vee \mathcal{I}_{\widetilde{C}_i}(y) \leq b$, and $\mathcal{F}_{{}_1\widetilde{\mathscr{M}}_x^{\beta}}(y) = \mathcal{F} \cap \widetilde{C}_i(y) = \vee \mathcal{F}_{\widetilde{C}_i}(y) \leq c$. Then, $\left\{\widetilde{C}_i \in \widetilde{\Gamma}\colon \mathcal{T}_{\widetilde{C}_i}(x) \geq a, \mathcal{I}_{\widetilde{C}_i}(x) \leq b, \mathcal{F}_{\widetilde{C}_i}(x) \leq c\right\} \subseteq \left\{\widetilde{C}_i \in \widetilde{\Gamma}\colon \mathcal{T}_{\widetilde{C}_i}(y) \geq a, \mathcal{I}_{\widetilde{C}_i}(y) \leq\right.$

Table 1: $(\Omega, \widetilde{\Gamma})$.

	$\widetilde{C}_1$	$\widetilde{C}_2$	$\widetilde{C}_3$	$\widetilde{C}_4$
x_1	$\langle 0.7, 0.2, 0.5\rangle$	$\langle 0.6, 0.2, 0.4\rangle$	$\langle 0.4, 0.1, 0.5\rangle$	$\langle 0.1, 0.5, 0.6\rangle$
x_2	$\langle 0.5, 0.3, 0.2\rangle$	$\langle 0.5, 0.2, 0.8\rangle$	$\langle 0.4, 0.5, 0.4\rangle$	$\langle 0.6, 0.1, 0.7\rangle$
x_3	$\langle 0.4, 0.5, 0.2\rangle$	$\langle 0.2, 0.3, 0.6\rangle$	$\langle 0.5, 0.2, 0.4\rangle$	$\langle 0.6, 0.3, 0.4\rangle$
x_4	$\langle 0.6, 0.1, 0.7\rangle$	$\langle 0.4, 0.5, 0.7\rangle$	$\langle 0.3, 0.6, 0.5\rangle$	$\langle 0.5, 0.3, 0.2\rangle$
x_5	$\langle 0.3, 0.2, 0.6\rangle$	$\langle 0.7, 0.3, 0.5\rangle$	$\langle 0.6, 0.3, 0.5\rangle$	$\langle 0.8, 0.1, 0.2\rangle$

Table 2: ${}_1\widetilde{\mathcal{N}}^{\beta}_{x_i}, \forall i = 1, 2, \ldots, 5$.

	x_1	x_2	x_3	x_4	x_5
${}_1\widetilde{\mathcal{N}}^{\beta}_{x_1}$	$\langle 0.6, 0.2, 0.5\rangle$	$\langle 0.5, 0.3, 0.8\rangle$	$\langle 0.2, 0.5, 0.6\rangle$	$\langle 0.4, 0.5, 0.7\rangle$	$\langle 0.3, 0.3, 0.6\rangle$
${}_1\widetilde{\mathcal{N}}^{\beta}_{x_2}$	$\langle 0.1, 0.5, 0.6\rangle$	$\langle 0.5, 0.3, 0.8\rangle$	$\langle 0.2, 0.5, 0.6\rangle$	$\langle 0.4, 0.5, 0.7\rangle$	$\langle 0.3, 0.3, 0.6\rangle$
${}_1\widetilde{\mathcal{N}}^{\beta}_{x_3}$	$\langle 0.1, 0.5, 0.6\rangle$	$\langle 0.4, 0.5, 0.7\rangle$	$\langle 0.5, 0.3, 0.4\rangle$	$\langle 0.3, 0.6, 0.5\rangle$	$\langle 0.6, 0.3, 0.5\rangle$
${}_1\widetilde{\mathcal{N}}^{\beta}_{x_4}$	$\langle 0.1, 0.5, 0.6\rangle$	$\langle 0.5, 0.3, 0.7\rangle$	$\langle 0.4, 0.5, 0.4\rangle$	$\langle 0.5, 0.3, 0.7\rangle$	$\langle 0.3, 0.2, 0.6\rangle$
${}_1\widetilde{\mathcal{N}}^{\beta}_{x_5}$	$\langle 0.1, 0.5, 0.6\rangle$	$\langle 0.4, 0.5, 0.8\rangle$	$\langle 0.2, 0.3, 0.6\rangle$	$\langle 0.3, 0.6, 0.7\rangle$	$\langle 0.6, 0.3, 0.5\rangle$

Table 3: ${}_1\widetilde{\mathcal{M}}^{\beta}_{x_i}, \forall i = 1, 2, \ldots, 5$.

	x_1	x_2	x_3	x_4	x_5
${}_1\widetilde{\mathcal{M}}^{\beta}_{x_1}$	$\langle 0.5, 0.8, 0.6\rangle$	$\langle 0.8, 0.7, 0.5\rangle$	$\langle 0.6, 0.5, 0.2\rangle$	$\langle 0.7, 0.5, 0.4\rangle$	$\langle 0.6, 0.7, 0.3\rangle$
${}_1\widetilde{\mathcal{M}}^{\beta}_{x_2}$	$\langle 0.6, 0.5, 0.1\rangle$	$\langle 0.8, 0.7, 0.5\rangle$	$\langle 0.6, 0.5, 0.2\rangle$	$\langle 0.7, 0.5, 0.4\rangle$	$\langle 0.6, 0.7, 0.3\rangle$
${}_1\widetilde{\mathcal{M}}^{\beta}_{x_3}$	$\langle 0.6, 0.5, 0.1\rangle$	$\langle 0.7, 0.5, 0.4\rangle$	$\langle 0.4, 0.7, 0.5\rangle$	$\langle 0.5, 0.4, 0.3\rangle$	$\langle 0.5, 0.7, 0.6\rangle$
${}_1\widetilde{\mathcal{M}}^{\beta}_{x_4}$	$\langle 0.6, 0.5, 0.1\rangle$	$\langle 0.7, 0.7, 0.5\rangle$	$\langle 0.4, 0.5, 0.4\rangle$	$\langle 0.7, 0.7, 0.5\rangle$	$\langle 0.6, 0.8, 0.3\rangle$
${}_1\widetilde{\mathcal{M}}^{\beta}_{x_5}$	$\langle 0.6, 0.5, 0.1\rangle$	$\langle 0.8, 0.5, 0.4\rangle$	$\langle 0.6, 0.7, 0.2\rangle$	$\langle 0.7, 0.4, 0.3\rangle$	$\langle 0.5, 0.7, 0.6\rangle$

Table 4: ${}_1\widetilde{\mathcal{N}}^{\beta}_{x_i} \cap {}_1\widetilde{\mathcal{M}}^{\beta}_{x_i}, \forall i = 1, 2, \ldots, 5$.

	x_1	x_2	x_3	x_4	x_5
${}_1\widetilde{\mathcal{N}}^{\beta}_{x_1} \cap {}_1\widetilde{\mathcal{M}}^{\beta}_{x_1}$	$\langle 0.5, 0.8, 0.6\rangle$	$\langle 0.5, 0.7, 0.8\rangle$	$\langle 0.2, 0.5, 0.6\rangle$	$\langle 0.4, 0.5, 0.7\rangle$	$\langle 0.3, 0.7, 0.6\rangle$
${}_1\widetilde{\mathcal{N}}^{\beta}_{x_2} \cap {}_1\widetilde{\mathcal{M}}^{\beta}_{x_2}$	$\langle 0.1, 0.5, 0.6\rangle$	$\langle 0.5, 0.7, 0.8\rangle$	$\langle 0.2, 0.5, 0.6\rangle$	$\langle 0.4, 0.5, 0.7\rangle$	$\langle 0.3, 0.7, 0.6\rangle$
${}_1\widetilde{\mathcal{N}}^{\beta}_{x_3} \cap {}_1\widetilde{\mathcal{M}}^{\beta}_{x_3}$	$\langle 0.1, 0.5, 0.6\rangle$	$\langle 0.4, 0.5, 0.7\rangle$	$\langle 0.4, 0.7, 0.5\rangle$	$\langle 0.3, 0.6, 0.5\rangle$	$\langle 0.5, 0.7, 0.6\rangle$
${}_1\widetilde{\mathcal{N}}^{\beta}_{x_4} \cap {}_1\widetilde{\mathcal{M}}^{\beta}_{x_4}$	$\langle 0.1, 0.5, 0.6\rangle$	$\langle 0.5, 0.7, 0.7\rangle$	$\langle 0.4, 0.5, 0.4\rangle$	$\langle 0.5, 0.7, 0.7\rangle$	$\langle 0.3, 0.8, 0.6\rangle$
${}_1\widetilde{\mathcal{N}}^{\beta}_{x_5} \cap {}_1\widetilde{\mathcal{M}}^{\beta}_{x_5}$	$\langle 0.1, 0.5, 0.6\rangle$	$\langle 0.4, 0.5, 0.8\rangle$	$\langle 0.2, 0.7, 0.6\rangle$	$\langle 0.3, 0.6, 0.7\rangle$	$\langle 0.5, 0.7, 0.6\rangle$

Table 5: ${}_1\widetilde{\mathcal{N}}^{\beta}_{x_i} \cup {}_1\widetilde{\mathcal{M}}^{\beta}_{x_i}, \forall i = 1, 2, \ldots, 5$.

	x_1	x_2	x_3	x_4	x_5
${}_1\widetilde{\mathcal{N}}^{\beta}_{x_1} \cup {}_1\widetilde{\mathcal{M}}^{\beta}_{x_1}$	$\langle 0.6, 0.2, 0.5\rangle$	$\langle 0.8, 0.3, 0.5\rangle$	$\langle 0.6, 0.5, 0.2\rangle$	$\langle 0.7, 0.5, 0.4\rangle$	$\langle 0.6, 0.3, 0.3\rangle$
${}_1\widetilde{\mathcal{N}}^{\beta}_{x_2} \cup {}_1\widetilde{\mathcal{M}}^{\beta}_{x_2}$	$\langle 0.6, 0.5, 0.1\rangle$	$\langle 0.8, 0.3, 0.5\rangle$	$\langle 0.6, 0.5, 0.2\rangle$	$\langle 0.7, 0.5, 0.4\rangle$	$\langle 0.6, 0.3, 0.3\rangle$
${}_1\widetilde{\mathcal{N}}^{\beta}_{x_3} \cup {}_1\widetilde{\mathcal{M}}^{\beta}_{x_3}$	$\langle 0.6, 0.5, 0.1\rangle$	$\langle 0.7, 0.5, 0.4\rangle$	$\langle 0.5, 0.3, 0.4\rangle$	$\langle 0.5, 0.4, 0.3\rangle$	$\langle 0.6, 0.3, 0.5\rangle$
${}_1\widetilde{\mathcal{N}}^{\beta}_{x_4} \cup {}_1\widetilde{\mathcal{M}}^{\beta}_{x_4}$	$\langle 0.6, 0.5, 0.1\rangle$	$\langle 0.7, 0.3, 0.5\rangle$	$\langle 0.4, 0.5, 0.4\rangle$	$\langle 0.7, 0.3, 0.5\rangle$	$\langle 0.6, 0.2, 0.3\rangle$
${}_1\widetilde{\mathcal{N}}^{\beta}_{x_5} \cup {}_1\widetilde{\mathcal{M}}^{\beta}_{x_5}$	$\langle 0.6, 0.5, 0.1\rangle$	$\langle 0.8, 0.5, 0.4\rangle$	$\langle 0.6, 0.3, 0.2\rangle$	$\langle 0.7, 0.4, 0.3\rangle$	$\langle 0.6, 0.3, 0.5\rangle$

$b, \mathcal{F}_{\widetilde{C}_i}(y) \le c\}$. Thus, ${}_1\widetilde{\mathcal{M}}^{\beta}_y \subseteq {}_1\widetilde{\mathcal{M}}^{\beta}_x$. On the contrary, let ${}_1\widetilde{\mathcal{M}}^{\beta}_y \subseteq {}_1\widetilde{\mathcal{M}}^{\beta}_x$. Then, $\mathcal{T}_{{}_1\widetilde{\mathcal{M}}^{\beta}_x}(y) \ge \mathcal{T}_{{}_1\widetilde{\mathcal{M}}^{\beta}_y}(y) \ge a, \mathcal{I}_{{}_1\widetilde{\mathcal{M}}^{\beta}_x}(y) \le \mathcal{I}_{\widetilde{\mathcal{M}}^{\beta}_y}(y) \le b$, and $\mathcal{F}_{{}_1\widetilde{\mathcal{M}}^{\beta}_x}(y) \le \mathcal{F}_{{}_1\widetilde{\mathcal{M}}^{\beta}_y}(y) \le c$. Hence, ${}_1\widetilde{\mathcal{M}}^{\beta}_x(y) \ge \beta$. □

Now, we present the three new types of 1-1-SVNβCRSs based on Definitions 5 and 7 as indicated below.

Definition 8. Consider $(\Omega, \widetilde{\Gamma})$ is a 1-SVNβCAS with $\widetilde{\Gamma} = \{\widetilde{C}_1, \widetilde{C}_2, \ldots, \widetilde{C}_m\}$, for some $\beta = \langle a, b, c\rangle$. For each $x \in \Omega$ and $\mathcal{A} \in \text{SVN}(\Omega)$, then we have the following paradigms:

Paradigm 1: the second type of Type-1 SVN lower approximation (2-1-SVNLA) $\mathcal{L}^1_2(\mathcal{A})$ and the second type of Type-1 SVN upper approximation (2-1-SVNUA) $\mathcal{U}^1_2(\mathcal{A})$ are as follows:

$$\begin{aligned}
\mathcal{L}^1_2(\mathcal{A}) &= \left\{\langle x, \wedge_{y\in\Omega}\left(\mathcal{F}_{{}_1\widetilde{\mathcal{M}}^{\beta}_x}(y) \vee \mathcal{T}_{\mathcal{A}}(y)\right), \wedge_{y\in\Omega}\left(\left(1 - \mathcal{I}_{{}_1\widetilde{\mathcal{M}}^{\beta}_x}(y)\right) \vee \mathcal{I}_{\mathcal{A}}(y)\right), \vee_{y\in\Omega}\left(\mathcal{T}_{{}_1\widetilde{\mathcal{M}}^{\beta}_x}(y) \wedge \mathcal{F}_{\mathcal{A}}(y)\right)\rangle\right\},\\
\mathcal{U}^1_2(\mathcal{A}) &= \left\{\langle x, \vee_{y\in\Omega}\left(\mathcal{T}_{{}_1\widetilde{\mathcal{M}}^{\beta}_x}(y) \wedge \mathcal{T}_{\mathcal{A}}(y)\right), \vee_{y\in\Omega}\left(\mathcal{I}_{{}_1\widetilde{\mathcal{M}}^{\beta}_x}(y) \wedge \mathcal{I}_{\mathcal{A}}(y)\right), \wedge_{y\in\Omega}\left(\mathcal{F}_{{}_1\widetilde{\mathcal{M}}^{\beta}_x}(y) \vee \mathcal{F}_{\mathcal{A}}(y)\right)\rangle\right\}.
\end{aligned} \tag{6}$$

If $\mathcal{L}^1_2(\mathcal{A}) \ne \mathcal{U}^1_2(\mathcal{A})$, then $\mathcal{A}$ is called the second type of Type-1 SVN β-covering rough sets (briefly, 2-1-SVNβCRSs).

Paradigm 2: the third type of Type-1 SVN lower approximation (3-1-SVNLA) $\mathcal{L}^1_3(\mathcal{A})$ and the third type of Type-1 SVN upper approximation (3-1-SVNUA) $\mathcal{U}^1_3(\mathcal{A})$ are introduced as follows:

$$\begin{aligned}
\mathcal{L}^1_3(\mathcal{A}) &= \left\{\langle x, \wedge_{y\in\Omega}\left(\mathcal{F}_{{}_1\widetilde{\mathcal{N}}^{\beta}_x \cap {}_1\widetilde{\mathcal{M}}^{\beta}_x}(y) \vee \mathcal{T}_{\mathcal{A}}(y)\right), \wedge_{y\in\Omega}\left(\left(1 - \mathcal{I}_{{}_1\widetilde{\mathcal{N}}^{\beta}_x \cap {}_1\widetilde{\mathcal{M}}^{\beta}_x}(y)\right) \vee \mathcal{I}_{\mathcal{A}}(y)\right), \vee_{y\in\Omega}\left(\mathcal{T}_{{}_1\widetilde{\mathcal{N}}^{\beta}_x \cap {}_1\widetilde{\mathcal{M}}^{\beta}_x}(y) \wedge \mathcal{F}_{\mathcal{A}}(y)\right)\rangle\right\},\\
\mathcal{U}^1_3(\mathcal{A}) &= \left\{\langle x, \vee_{y\in\Omega}\left(\mathcal{T}_{{}_1\widetilde{\mathcal{N}}^{\beta}_x \cap {}_1\widetilde{\mathcal{M}}^{\beta}_x}(y) \wedge \mathcal{T}_{\mathcal{A}}(y)\right), \vee_{y\in\Omega}\left(\mathcal{I}_{{}_1\widetilde{\mathcal{N}}^{\beta}_x \cap {}_1\widetilde{\mathcal{M}}^{\beta}_x}(y) \wedge \mathcal{I}_{\mathcal{A}}(y)\right), \wedge_{y\in\Omega}\left(\mathcal{F}_{{}_1\widetilde{\mathcal{N}}^{\beta}_x \cap {}_1\widetilde{\mathcal{M}}^{\beta}_x}(y) \vee \mathcal{F}_{\mathcal{A}}(y)\right)\rangle\right\}.
\end{aligned} \tag{7}$$

If $\mathcal{L}^1_3(\mathcal{A}) \ne \mathcal{U}^1_3(\mathcal{A})$, then $\mathcal{A}$ is called the third type of Type-1 SVN β-covering rough sets (briefly, 3-1-SVNβCRSs).

Paradigm 3: the fourth type of Type-1 SVN lower approximation (4-1-SVNLA) $\mathcal{L}^1_4(\mathcal{A})$ and the fourth type of Type-1 SVN upper approximation (4-1-SVNUA) $\mathcal{U}^1_4(\mathcal{A})$ are proposed as follows:

$$\begin{aligned}
\mathcal{L}^1_4(\mathcal{A}) &= \left\{\langle x, \wedge_{y\in\Omega}\left(\mathcal{F}_{{}_1\widetilde{\mathcal{N}}^{\beta}_x \cup {}_1\widetilde{\mathcal{M}}^{\beta}_x}(y) \vee \mathcal{T}_{\mathcal{A}}(y)\right), \wedge_{y\in\Omega}\left(\left(1 - \mathcal{I}_{{}_1\widetilde{\mathcal{N}}^{\beta}_x \cup {}_1\widetilde{\mathcal{M}}^{\beta}_x}(y)\right) \vee \mathcal{I}_{\mathcal{A}}(y)\right), \vee_{y\in\Omega}\left(\mathcal{T}_{{}_1\widetilde{\mathcal{N}}^{\beta}_x \cup {}_1\widetilde{\mathcal{M}}^{\beta}_x}(y) \wedge \mathcal{F}_{\mathcal{A}}(y)\right)\rangle\right\},\\
\mathcal{U}^1_4(\mathcal{A}) &= \left\{\langle x, \vee_{y\in\Omega}\left(\mathcal{T}_{{}_1\widetilde{\mathcal{N}}^{\beta}_x \cup {}_1\widetilde{\mathcal{M}}^{\beta}_x}(y) \wedge \mathcal{T}_{\mathcal{A}}(y)\right), \vee_{y\in\Omega}\left(\mathcal{I}_{{}_1\widetilde{\mathcal{N}}^{\beta}_x \cup {}_1\widetilde{\mathcal{M}}^{\beta}_x}(y) \wedge \mathcal{I}_{\mathcal{A}}(y)\right), \wedge_{y\in\Omega}\left(\mathcal{F}_{{}_1\widetilde{\mathcal{N}}^{\beta}_x \cup {}_1\widetilde{\mathcal{M}}^{\beta}_x}(y) \vee \mathcal{F}_{\mathcal{A}}(y)\right)\rangle\right\}.
\end{aligned} \tag{8}$$

If $\mathcal{L}^1_4(\mathcal{A}) \ne \mathcal{U}^1_4(\mathcal{A})$, then $\mathcal{A}$ is called the fourth type of Type-1 SVN β-covering rough sets (briefly, 4-1-SVNβCRSs).

Example 2. Consider Example 1 if $\beta = \langle 0.5, 0.3, 0.8\rangle$ and $\mathcal{A} = ((0.5, 0.3, 0.6)/x_1) + ((0.4, 0.5, 0.1)/x_2) + ((0.3, 0.2,$

$0.6)/x_3) + ((0.5, 0.3, 0.4)/x_4) + ((0.7, 0.2, 0.3)/x_5)$; then, we have the following results:

$$\begin{aligned}
\mathscr{L}_1^1(\mathscr{A}) &= \{\langle x_1, 0.6, 0.5, 0.5\rangle, \langle x_2, 0.6, 0.5, 0.4\rangle, \langle x_3, 0.4, 0.4, 0.5\rangle, \langle x_4, 0.4, 0.5, 0.4\rangle, \langle x_5, 0.6, 0.4, 0.3\rangle\},\\
\mathscr{U}_1^1(\mathscr{A}) &= \{\langle x_1, 0.6, 0.3, 0.5\rangle, \langle x_2, 0.4, 0.3, 0.6\rangle, \langle x_3, 0.6, 0.5, 0.5\rangle, \langle x_4, 0.5, 0.3, 0.6\rangle, \langle x_5, 0.6, 0.5, 0.5\rangle\},\\
\mathscr{L}_2^1(\mathscr{A}) &= \{\langle x_1, 0.3, 0.3, 0.6\rangle, \langle x_2, 0.3, 0.3, 0.6\rangle, \langle x_3, 0.4, 0.3, 0.5\rangle, \langle x_4, 0.4, 0.2, 0.4\rangle, \langle x_5, 0.3, 0.3, 0.6\rangle\},\\
\mathscr{U}_2^1(\mathscr{A}) &= \{\langle x_1, 0.6, 0.5, 0.3\rangle, \langle x_2, 0.6, 0.5, 0.3\rangle, \langle x_3, 0.6, 0.5, 0.4\rangle, \langle x_4, 0.6, 0.5, 0.3\rangle, \langle x_5, 0.6, 0.5, 0.4\rangle\},\\
\mathscr{L}_3^1(\mathscr{A}) &= \{\langle x_1, 0.6, 0.3, 0.5\rangle, \langle x_2, 0.6, 0.3, 0.4\rangle, \langle x_3, 0.5, 0.3, 0.4\rangle, \langle x_4, 0.4, 0.2, 0.3\rangle, \langle x_5, 0.6, 0.3, 0.3\rangle\},\\
\mathscr{U}_3^1(\mathscr{A}) &= \{\langle x_1, 0.5, 0.5, 0.6\rangle, \langle x_2, 0.4, 0.5, 0.6\rangle, \langle x_3, 0.5, 0.5, 0.5\rangle, \langle x_4, 0.5, 0.5, 0.6\rangle, \langle x_5, 0.5, 0.5, 0.6\rangle\},\\
\mathscr{L}_4^1(\mathscr{A}) &= \{\langle x_1, 0.3, 0.5, 0.6\rangle, \langle x_2, 0.3, 0.5, 0.6\rangle, \langle x_3, 0.4, 0.5, 0.5\rangle, \langle x_4, 0.4, 0.5, 0.5\rangle, \langle x_5, 0.3, 0.5, 0.6\rangle\},\\
\mathscr{U}_4^1(\mathscr{A}) &= \{\langle x_1, 0.6, 0.3, 0.3\rangle, \langle x_2, 0.6, 0.3, 0.3\rangle, \langle x_3, 0.6, 0.5, 0.4\rangle, \langle x_4, 0.6, 0.3, 0.3\rangle, \langle x_5, 0.6, 0.5, 0.4\rangle\}.
\end{aligned} \tag{9}$$

Next, we will present Proposition 3 for the 2-1-SVNβCRS model; also, it satisfies in case of the 3-1-SVNβCRS and the 4-1-SVNβCRS models.

Proposition 3. *Let $(\Omega, \widetilde{\Gamma})$ be a 1-SVNβCAS, for some $\beta = \langle a, b, c\rangle$. For each $x, y, z \in \Omega$ and $\mathscr{A}, \mathscr{B} \in SVN(\Omega)$. Then, the following statements hold:*

(1) *(SVNL1) $\mathscr{L}_2^1(\mathscr{A}^c) = (\mathscr{U}_2^1(\mathscr{A}))^c$.*
   (SVNU1) $\mathscr{U}_2^1(\mathscr{A}^c) = (\mathscr{L}_2^1(\mathscr{A}))^c$.
(2) *If $\mathscr{A} \subseteq \mathscr{B}$, then*
   (SVNL2) $\mathscr{L}_2^1(\mathscr{A}) \subseteq \mathscr{L}_2^1(\mathscr{B})$.
   (SVNU2) $\mathscr{U}_2^1(\mathscr{A}) \subseteq \mathscr{U}_2^1(\mathscr{B})$.
(3) *(SVNL3) $\mathscr{L}_2^1(\mathscr{A} \cap \mathscr{B}) = \mathscr{L}_2^1(\mathscr{A}) \cap \mathscr{L}_2^1(\mathscr{B})$.*
   (SVNU3) $\mathscr{U}_2^1(\mathscr{A} \cap \mathscr{B}) \subseteq \mathscr{U}_2^1(\mathscr{A}) \cap \mathscr{U}_2^1(\mathscr{B})$.
(4) *(SVNL4) $\mathscr{L}_2^1(\mathscr{A} \cup \mathscr{B}) \supseteq \mathscr{L}_2^1(\mathscr{A}) \cup \mathscr{L}_2^1(\mathscr{B})$.*
   (SVNU4) $\mathscr{U}_2^1(\mathscr{A} \cap \mathscr{B}) = \mathscr{U}_2^1(\mathscr{A}) \cap \mathscr{U}_2^1(\mathscr{B})$.

Proof. We shall only prove (SVNL1), (SVNL2), (SVNL3), and (SVNL4).

(SVNL1):

$$\begin{aligned}
\mathscr{L}_2^1(\mathscr{A}^c) &= \left\{\langle x, \wedge_{y\in\Omega}\left(\mathscr{F}_{{}_1\widetilde{\mathscr{M}}_x^\beta}(y) \vee \mathscr{T}_{\mathscr{A}^c}(y)\right), \wedge_{y\in\Omega}\left(\left(1 - \mathscr{I}_{{}_1\widetilde{\mathscr{M}}_x^\beta}(y)\right) \vee \mathscr{I}_{\mathscr{A}^c}(y)\right), \vee_{y\in\Omega}\left(\mathscr{T}_{{}_1\widetilde{\mathscr{M}}_x^\beta}(y) \wedge \mathscr{F}_{\mathscr{A}^c}(y)\right)\rangle\right\}\\
&= \left\{\langle x, \wedge_{y\in\Omega}\left(\mathscr{F}_{{}_1\widetilde{\mathscr{M}}_x^\beta}(y) \vee \mathscr{F}_{\mathscr{A}}(y)\right), \wedge_{y\in\Omega}\left(\left(1 - \mathscr{I}_{{}_1\widetilde{\mathscr{M}}_x^\beta}(y)\right) \vee (1 - \mathscr{I}_{\mathscr{A}}(y))\right), \vee_{y\in\Omega}\left(\mathscr{T}_{{}_1\widetilde{\mathscr{M}}_x^\beta}(y) \wedge \mathscr{T}_{\mathscr{A}}(y)\right)\rangle\right\}\\
&= \left(\mathscr{U}_2^1(\mathscr{A})\right)^c.
\end{aligned} \tag{10}$$

(SVNL2): let $\mathscr{A}, \mathscr{B} \in \mathrm{SVN}(\Omega)$ such that $\mathscr{A} \subseteq \mathscr{B}$ (i.e., $\mathscr{T}_{\mathscr{A}} \leq \mathscr{T}_{\mathscr{B}}, \mathscr{I}_{\mathscr{B}} \leq \mathscr{I}_{\mathscr{A}}$ and $\mathscr{F}_{\mathscr{B}} \leq \mathscr{F}_{\mathscr{A}}$) and $x \in \Omega$. Then, we get the following result:

$$\begin{aligned}
\mathscr{L}_2^1(\mathscr{A})(x) &= \left\{\langle x, \wedge_{y\in\Omega}\left(\mathscr{F}_{{}_1\widetilde{\mathscr{M}}_x^\beta}(y) \vee \mathscr{T}_{\mathscr{A}}(y)\right), \wedge_{y\in\Omega}\left(\left(1 - \mathscr{I}_{{}_1\widetilde{\mathscr{M}}_x^\beta}(y)\right) \vee \mathscr{I}_{\mathscr{A}}(y)\right), \vee_{y\in\Omega}\left(\mathscr{T}_{{}_1\widetilde{\mathscr{M}}_x^\beta}(y) \wedge \mathscr{F}_{\mathscr{A}}(y)\right)\rangle\right\},\\
\mathscr{T}_{\mathscr{L}_2^1(\mathscr{A})} &= \wedge_{y\in\Omega}\left(\mathscr{F}_{{}_1\widetilde{\mathscr{M}}_x^\beta}(y) \vee \mathscr{T}_{\mathscr{A}}(y)\right) \leq \wedge_{y\in\Omega}\left(\mathscr{F}_{{}_1\widetilde{\mathscr{M}}_x^\beta}(y) \vee \mathscr{T}_{\mathscr{B}}(y)\right) = \mathscr{T}_{\mathscr{L}_2^1(\mathscr{B})},\\
\mathscr{I}_{\mathscr{L}_2^1(\mathscr{A})} &= \wedge_{y\in\Omega}\left(\left(1 - \mathscr{I}_{{}_1\widetilde{\mathscr{M}}_x^\beta}(y)\right) \vee \mathscr{I}_{\mathscr{A}}(y)\right) \geq \wedge_{y\in\Omega}\left(\left(1 - \mathscr{I}_{{}_1\widetilde{\mathscr{M}}_x^\beta}(y)\right) \vee \mathscr{I}_{\mathscr{B}}(y)\right) = \mathscr{I}_{\mathscr{L}_2^1(\mathscr{B})},\\
\mathscr{F}_{\mathscr{L}_2^1(\mathscr{A})} &= \vee_{y\in\Omega}\left(\mathscr{T}_{{}_1\widetilde{\mathscr{M}}_x^\beta}(y) \wedge \mathscr{F}_{\mathscr{A}}(y)\right) \geq \vee_{y\in\Omega}\left(\mathscr{T}_{{}_1\widetilde{\mathscr{M}}_x^\beta}(y) \wedge \mathscr{F}_{\mathscr{B}}(y)\right) = \mathscr{F}_{\mathscr{L}_2^1(\mathscr{B})}.
\end{aligned} \tag{11}$$

Therefore, $\mathscr{L}_2^1(\mathscr{A}) \subseteq \mathscr{L}_2^1(\mathscr{B})$.

(SVNL3): if $x \in \Omega$, then we have

$$\begin{aligned}
\mathscr{L}_2^1(\mathscr{A})(x) &= \left\{\langle x, \wedge_{y\in\Omega}\left(\mathscr{F}_{{}_1\widetilde{\mathscr{M}}_x^\beta}(y) \vee \mathscr{T}_{\mathscr{A}\cap\mathscr{B}}(y)\right), \wedge_{y\in\Omega}\left(\left(1-\mathscr{I}_{{}_1\widetilde{\mathscr{M}}_x^\beta}(y)\right) \vee \mathscr{I}_{\mathscr{A}\cap\mathscr{B}}(y)\right), \vee_{y\in\Omega}\left(\mathscr{T}_{{}_1\widetilde{\mathscr{M}}_x^\beta}(y) \wedge \mathscr{F}_{\mathscr{A}\cap\mathscr{B}}(y)\right)\rangle\right\} \\
&= \left\{\langle x, \wedge_{y\in\Omega}\left(\mathscr{F}_{{}_1\widetilde{\mathscr{M}}_x^\beta}(y) \vee (\mathscr{T}_{\mathscr{A}}(y) \wedge \mathscr{T}_{\mathscr{B}}(y))\right), \wedge_{y\in\Omega}\left(\left(1-\mathscr{I}_{{}_1\widetilde{\mathscr{M}}_x^\beta}(y)\right) \vee (\mathscr{I}_{\mathscr{A}}(y) \vee \mathscr{I}_{\mathscr{B}}(y))\right), \vee_{y\in\Omega}\left(\mathscr{T}_{{}_1\widetilde{\mathscr{M}}_x^\beta}(y) \wedge (\mathscr{F}_{\mathscr{A}}(y) \vee \mathscr{F}_{\mathscr{B}}(y))\right)\rangle\right\} \\
&= \left\{\langle x, \wedge_{y\in\Omega}\left(\left(\mathscr{F}_{{}_1\widetilde{\mathscr{M}}_x^\beta}(y) \vee \mathscr{T}_{\mathscr{A}}(y)\right) \wedge \left(\mathscr{F}_{{}_1\widetilde{\mathscr{M}}_x^\beta}(y) \vee \mathscr{T}_{\mathscr{B}}(y)\right)\right), \wedge_{y\in\Omega}\left(\left(\left(1-\mathscr{I}_{{}_1\widetilde{\mathscr{M}}_x^\beta}(y)\right) \vee \mathscr{I}_{\mathscr{A}}(y)\right) \vee \left(\left(1-\mathscr{I}_{{}_1\widetilde{\mathscr{M}}_x^\beta}(y)\right) \vee \mathscr{I}_{\mathscr{B}}(y)\right)\right),\right. \\
&\qquad \left. \vee_{y\in\Omega}\left(\left(\mathscr{T}_{{}_1\widetilde{\mathscr{M}}_x^\beta}(y) \wedge \mathscr{F}_{\mathscr{A}}(y)\right) \vee \left(\mathscr{T}_{{}_1\widetilde{\mathscr{M}}_x^\beta}(y) \wedge \mathscr{F}_{\mathscr{B}}(y)\right)\right)\rangle\right\} \\
&= \mathscr{L}_2^1(\mathscr{A}) \cap \mathscr{L}_2^1(\mathscr{B}).
\end{aligned} \tag{12}$$

(SVNL4): since $\mathscr{A}\cup\mathscr{B}\supseteq\mathscr{A}$, then, by (SVNL2), we have $\mathscr{L}_2^1(\mathscr{A}\cup\mathscr{B})\supseteq\mathscr{L}_2^1(\mathscr{A})$. Similarly, $\mathscr{A}\cup\mathscr{B}\supseteq\mathscr{B}$; then, by (SVNL2), we have $\mathscr{L}_2^1(\mathscr{A}\cup\mathscr{B})\supseteq\mathscr{L}_2^1(\mathscr{B})$. Thus, $\mathscr{L}_2^1(\mathscr{A}\cup\mathscr{B})\supseteq\mathscr{L}_2^1(\mathscr{A})\cup\mathscr{L}_2^1(\mathscr{B})$. □

Now, we proceed to explain some relationships among these models.

Proposition 4. *Let $(\Omega,\widetilde{\Gamma})$ be a 1-SVNβCAS and $\mathscr{A}\in SVN(\Omega)$. Then, we have the following properties:*

(1) $\mathscr{L}_4^1(\mathscr{A})\leq\mathscr{L}_2^1(\mathscr{A})\leq\mathscr{L}_3^1(\mathscr{A})$
(2) $\mathscr{L}_4^1(\mathscr{A})\leq\mathscr{L}_1^1(\mathscr{A})\leq\mathscr{L}_3^1(\mathscr{A})$
(3) $\mathscr{U}_3^1(\mathscr{A})\leq\mathscr{U}_2^1(\mathscr{A})\leq\mathscr{L}_4^1(\mathscr{A})$
(4) $\mathscr{U}_3^1(\mathscr{A})\leq\mathscr{U}_1^1(\mathscr{A})\leq\mathscr{U}_4^1(\mathscr{A})$

Proof. The proof is clear from Definition 8. □

Proposition 5. *Let $(\Omega,\widetilde{\Gamma})$ be a 1-SVNβCAS and $\mathscr{A}\in SVN(\Omega)$. Then, we have the following properties:*

(1) $\mathscr{L}_3^1(\mathscr{A})\geq\mathscr{L}_1^1(\mathscr{A})\cup\mathscr{L}_2^1(\mathscr{A})$
(2) $\mathscr{U}_3^1(\mathscr{A})\leq\mathscr{U}_1^1(\mathscr{A})\cap\mathscr{U}_2^1(\mathscr{A})$
(3) $\mathscr{L}_4^1(\mathscr{A})\leq\mathscr{L}_1^1(\mathscr{A})\cap\mathscr{L}_2^1(\mathscr{A})$
(4) $\mathscr{U}_4^1(\mathscr{A})\geq\mathscr{U}_1^1(\mathscr{A})\cup\mathscr{U}_2^1(\mathscr{A})$

Proof (straightforward) □

4. Type-2 SVN Complementary β-Neighborhood and Three New Kinds of Type-2 SVNβ-CRS

Definition 9. Let $(\Omega,\widetilde{\Gamma})$ be a 2-SVNβCAS with $\widetilde{\Gamma}=\{\widetilde{C}_1,\widetilde{C}_2,\ldots,\widetilde{C}_m\}$, for some $\beta=\langle a,b,c\rangle$. Then, for each $x\in\Omega$, define the type-2 SVN complementary β-neighborhood of x as follows:

$$ {}_2\widetilde{\mathscr{M}}_x^\beta(y) = {}_2\widetilde{\mathscr{N}}_y^\beta(x), \quad \forall y\in\Omega. \tag{13}$$

Example 3. Consider Example 1, $\beta=\langle 0.5,0.1,0.8\rangle$ and $\mathscr{A}=((0.6,0.3,\ 0.5)/x_1)+((0.4,0.5,\ 0.1)/x_2)+((0.3,0.2,0.6)/x_3)+((0.5,0.3,0.4)/x_4)+((0.7,0.2,0.3)/x_5)$. Then, the values of type-2 SVN β-neighborhood are seen in Table 6:

$$\begin{aligned}
{}_2\widetilde{\mathscr{N}}_{x_1}^\beta &= \widetilde{C}_1\sqcap\widetilde{C}_2,\\
{}_2\widetilde{\mathscr{N}}_{x_2}^\beta &= \widetilde{C}_1\sqcap\widetilde{C}_2\sqcap\widetilde{C}_4,\\
{}_2\widetilde{\mathscr{N}}_{x_3}^\beta &= \widetilde{C}_3\sqcap\widetilde{C}_4,\\
{}_2\widetilde{\mathscr{N}}_{x_4}^\beta &= \widetilde{C}_1\sqcap\widetilde{C}_4,\\
{}_2\widetilde{\mathscr{N}}_{x_5}^\beta &= \widetilde{C}_2\sqcap\widetilde{C}_3\sqcap\widetilde{C}_4.
\end{aligned} \tag{14}$$

Also, we compute type-2 SVN complementary β-neighborhood as in Table 7.

Thus, we can merge ${}_2\widetilde{\mathscr{M}}_x^\beta$ and ${}_2\widetilde{\mathscr{M}}_x^\beta$ and calculate ${}_2\widetilde{\mathscr{N}}_x^\beta\sqcap{}_2\widetilde{\mathscr{M}}_x^\beta$ as Table 8.

Furthermore, we can calculate ${}_2\widetilde{\mathscr{N}}_x^\beta\sqcup{}_2\widetilde{\mathscr{M}}_x^\beta$, as set in Table 9.

Proposition 6. *Let $(\Omega,\widetilde{\Gamma})$ be a 2-SVNβCAS, for some $\beta=\langle a,b,c\rangle$ and for each $x,y,z\in\Omega$. Then, the following statements hold:*

(1) ${}_2\widetilde{\mathscr{M}}_x^\beta(x)\succcurlyeq\beta$
(2) If ${}_2\widetilde{\mathscr{M}}_x^\beta(y)\succcurlyeq\beta$ and ${}_2\widetilde{\mathscr{M}}_y^\beta(z)\succcurlyeq\beta$, then ${}_2\widetilde{\mathscr{M}}_x^\beta(z)\succcurlyeq\beta$
(3) $0\prec\beta_1\preccurlyeq\beta_2\preccurlyeq\beta$, then ${}_2\widetilde{\mathscr{M}}_x^{\beta_1}\sqsubseteq{}_2\widetilde{\mathscr{M}}_x^{\beta_2}$

Proof

(1) follows directly from Definitions 5 and 9.

(2) Since ${}_2\widetilde{\mathscr{M}}_x^\beta(y)\succcurlyeq\beta$, then ${}_2\widetilde{\mathscr{N}}_y^\beta(x)\succcurlyeq\beta$. If $\widetilde{C}_i(x)\succcurlyeq\beta$, then $\widetilde{C}_i(y)\succcurlyeq\beta$, and since ${}_2\widetilde{\mathscr{M}}_y^\beta(z)\succcurlyeq\beta$, then ${}_2\widetilde{\mathscr{N}}_z^\beta(y)\succcurlyeq\beta$. If $\widetilde{C}_i(y)\succcurlyeq\beta$, then $\widetilde{C}_i(z)\succcurlyeq\beta$. Therefore, ${}_2\widetilde{\mathscr{M}}_x^\beta(z)\succcurlyeq\beta$.

(3) For each $x\in\Omega, 0\prec\beta_1\prec\beta_2\prec\beta$, then $\sqcap\{\widetilde{C}_i\in\widetilde{\Gamma}: \widetilde{C}_i(x)\succcurlyeq\beta_1\}\sqsupseteq\sqcap\{\widetilde{C}_i\in\widetilde{\Gamma}: \widetilde{C}_i(x)\succcurlyeq\beta_2\}$. Thus, by Definition 9, we have ${}_2\widetilde{\mathscr{M}}_x^{\beta_1}\sqsubseteq{}_2\widetilde{\mathscr{M}}_x^{\beta_2}$. □

Table 6: ${}_2\widetilde{\mathscr{N}}^{\beta}_{x_i}, \forall i = 1, 2, \ldots, 5$.

	x_1	x_2	x_3	x_4	x_5
${}_2\widetilde{\mathscr{N}}^{\beta}_{x_1}$	$\langle 0.6, 0.2, 0.5\rangle$	$\langle 0.5, 0.2, 0.8\rangle$	$\langle 0.2, 0.3, 0.6\rangle$	$\langle 0.4, 0.1, 0.7\rangle$	$\langle 0.3, 0.2, 0.6\rangle$
${}_2\widetilde{\mathscr{N}}^{\beta}_{x_2}$	$\langle 0.1, 0.2, 0.6\rangle$	$\langle 0.5, 0.1, 0.8\rangle$	$\langle 0.2, 0.3, 0.6\rangle$	$\langle 0.4, 0.1, 0.7\rangle$	$\langle 0.3, 0.1, 0.6\rangle$
${}_2\widetilde{\mathscr{N}}^{\beta}_{x_3}$	$\langle 0.1, 0.1, 0.6\rangle$	$\langle 0.4, 0.1, 0.7\rangle$	$\langle 0.5, 0.2, 0.4\rangle$	$\langle 0.3, 0.3, 0.5\rangle$	$\langle 0.6, 0.1, 0.5\rangle$
${}_2\widetilde{\mathscr{N}}^{\beta}_{x_4}$	$\langle 0.1, 0.2, 0.6\rangle$	$\langle 0.5, 0.1, 0.7\rangle$	$\langle 0.4, 0.3, 0.4\rangle$	$\langle 0.5, 0.1, 0.7\rangle$	$\langle 0.3, 0.1, 0.6\rangle$
${}_2\widetilde{\mathscr{N}}^{\beta}_{x_5}$	$\langle 0.1, 0.1, 0.6\rangle$	$\langle 0.4, 0.1, 0.8\rangle$	$\langle 0.2, 0.2, 0.6\rangle$	$\langle 0.3, 0.3, 0.7\rangle$	$\langle 0.6, 0.1, 0.5\rangle$

Table 7: ${}_2\widetilde{\mathscr{M}}^{\beta}_{x_i}, \forall i = 1, 2, \ldots, 5$.

	x_1	x_2	x_3	x_4	x_5
${}_2\widetilde{\mathscr{M}}^{\beta}_{x_1}$	$\langle 0.5, 0.8, 0.6\rangle$	$\langle 0.8, 0.8, 0.5\rangle$	$\langle 0.6, 0.7, 0.2\rangle$	$\langle 0.7, 0.9, 0.4\rangle$	$\langle 0.6, 0.8, 0.3\rangle$
${}_2\widetilde{\mathscr{M}}^{\beta}_{x_2}$	$\langle 0.6, 0.8, 0.1\rangle$	$\langle 0.8, 0.9, 0.5\rangle$	$\langle 0.6, 0.7, 0.2\rangle$	$\langle 0.7, 0.9, 0.4\rangle$	$\langle 0.6, 0.9, 0.3\rangle$
${}_2\widetilde{\mathscr{M}}^{\beta}_{x_3}$	$\langle 0.6, 0.9, 0.1\rangle$	$\langle 0.7, 0.9, 0.4\rangle$	$\langle 0.4, 0.8, 0.5\rangle$	$\langle 0.5, 0.7, 0.3\rangle$	$\langle 0.5, 0.9, 0.6\rangle$
${}_2\widetilde{\mathscr{M}}^{\beta}_{x_4}$	$\langle 0.6, 0.8, 0.1\rangle$	$\langle 0.7, 0.9, 0.5\rangle$	$\langle 0.4, 0.7, 0.4\rangle$	$\langle 0.7, 0.9, 0.5\rangle$	$\langle 0.6, 0.9, 0.3\rangle$
${}_2\widetilde{\mathscr{M}}^{\beta}_{x_5}$	$\langle 0.6, 0.9, 0.1\rangle$	$\langle 0.8, 0.9, 0.4\rangle$	$\langle 0.6, 0.8, 0.2\rangle$	$\langle 0.7, 0.7, 0.3\rangle$	$\langle 0.5, 0.9, 0.6\rangle$

Table 8: ${}_2\widetilde{\mathscr{N}}^{\beta}_{x_i} \sqcap {}_2\widetilde{\mathscr{M}}^{\beta}_{x_i}, \forall i = 1, 2, \ldots, 5$.

	x_1	x_2	x_3	x_4	x_5
${}_2\widetilde{\mathscr{N}}^{\beta}_{x_1} \sqcap {}_2\widetilde{\mathscr{M}}^{\beta}_{x_1}$	$\langle 0.5, 0.2, 0.6\rangle$	$\langle 0.5, 0.2, 0.8\rangle$	$\langle 0.2, 0.3, 0.6\rangle$	$\langle 0.4, 0.1, 0.7\rangle$	$\langle 0.3, 0.2, 0.6\rangle$
${}_2\widetilde{\mathscr{N}}^{\beta}_{x_2} \sqcap {}_2\widetilde{\mathscr{M}}^{\beta}_{x_2}$	$\langle 0.1, 0.2, 0.6\rangle$	$\langle 0.5, 0.1, 0.8\rangle$	$\langle 0.2, 0.3, 0.6\rangle$	$\langle 0.4, 0.1, 0.7\rangle$	$\langle 0.3, 0.1, 0.6\rangle$
${}_2\widetilde{\mathscr{N}}^{\beta}_{x_3} \sqcap {}_2\widetilde{\mathscr{M}}^{\beta}_{x_3}$	$\langle 0.1, 0.1, 0.6\rangle$	$\langle 0.4, 0.1, 0.7\rangle$	$\langle 0.4, 0.2, 0.5\rangle$	$\langle 0.3, 0.3, 0.5\rangle$	$\langle 0.5, 0.1, 0.6\rangle$
${}_2\widetilde{\mathscr{N}}^{\beta}_{x_4} \sqcap {}_2\widetilde{\mathscr{M}}^{\beta}_{x_4}$	$\langle 0.1, 0.2, 0.6\rangle$	$\langle 0.5, 0.1, 0.7\rangle$	$\langle 0.4, 0.3, 0.4\rangle$	$\langle 0.5, 0.1, 0.7\rangle$	$\langle 0.3, 0.1, 0.6\rangle$
${}_2\widetilde{\mathscr{N}}^{\beta}_{x_5} \sqcap {}_2\widetilde{\mathscr{M}}^{\beta}_{x_5}$	$\langle 0.1, 0.1, 0.6\rangle$	$\langle 0.4, 0.1, 0.8\rangle$	$\langle 0.2, 0.2, 0.6\rangle$	$\langle 0.3, 0.3, 0.7\rangle$	$\langle 0.5, 0.1, 0.6\rangle$

Table 9: ${}_2\widetilde{\mathscr{N}}^{\beta}_{x_i} \sqcup {}_2\widetilde{\mathscr{M}}^{\beta}_{x_i}, \forall i = 1, 2, \ldots, 5$.

	x_1	x_2	x_3	x_4	x_5
${}_2\widetilde{\mathscr{N}}^{\beta}_{x_1} \sqcup {}_2\widetilde{\mathscr{M}}^{\beta}_{x_1}$	$\langle 0.6, 0.8, 0.5\rangle$	$\langle 0.8, 0.8, 0.5\rangle$	$\langle 0.6, 0.7, 0.2\rangle$	$\langle 0.7, 0.9, 0.4\rangle$	$\langle 0.6, 0.8, 0.3\rangle$
${}_2\widetilde{\mathscr{N}}^{\beta}_{x_2} \sqcup {}_2\widetilde{\mathscr{M}}^{\beta}_{x_2}$	$\langle 0.6, 0.8, 0.1\rangle$	$\langle 0.8, 0.9, 0.5\rangle$	$\langle 0.6, 0.7, 0.2\rangle$	$\langle 0.7, 0.9, 0.4\rangle$	$\langle 0.6, 0.9, 0.3\rangle$
${}_2\widetilde{\mathscr{N}}^{\beta}_{x_3} \sqcup {}_2\widetilde{\mathscr{M}}^{\beta}_{x_3}$	$\langle 0.6, 0.9, 0.1\rangle$	$\langle 0.7, 0.9, 0.4\rangle$	$\langle 0.5, 0.8, 0.4\rangle$	$\langle 0.5, 0.7, 0.3\rangle$	$\langle 0.6, 0.9, 0.5\rangle$
${}_2\widetilde{\mathscr{N}}^{\beta}_{x_4} \sqcup {}_2\widetilde{\mathscr{M}}^{\beta}_{x_4}$	$\langle 0.6, 0.8, 0.1\rangle$	$\langle 0.7, 0.9, 0.5\rangle$	$\langle 0.4, 0.7, 0.4\rangle$	$\langle 0.7, 0.9, 0.5\rangle$	$\langle 0.6, 0.9, 0.3\rangle$
${}_2\widetilde{\mathscr{N}}^{\beta}_{x_5} \sqcup {}_2\widetilde{\mathscr{M}}^{\beta}_{x_5}$	$\langle 0.6, 0.9, 0.1\rangle$	$\langle 0.8, 0.9, 0.4\rangle$	$\langle 0.6, 0.8, 0.2\rangle$	$\langle 0.7, 0.7, 0.3\rangle$	$\langle 0.6, 0.9, 0.5\rangle$

Proposition 7. *Let $(\Omega, \widetilde{\Gamma})$ be a 1-SVNβCAS, for some $\beta = \langle a, b, c\rangle$. For each $x, y \in \Omega$, ${}_2\widetilde{\mathscr{M}}^{\beta}_{x}(y) \succcurlyeq \beta \Leftrightarrow {}_2\widetilde{\mathscr{M}}^{\beta}_{y} \sqsubseteq {}_2\widetilde{\mathscr{M}}^{\beta}_{x}$.*

Proof. Let ${}_2\widetilde{\mathscr{M}}^{\beta}_{x}(y) \succcurlyeq \beta$, $\mathscr{T}_{{}_2\widetilde{\mathscr{M}}^{\beta}_{x}}(y) = \mathscr{T} \sqcap \widetilde{C}_i(y) = \wedge \mathscr{T}_{\widetilde{C}_i}(y) \geq a$, $\mathscr{I}_{{}_2\widetilde{\mathscr{M}}^{\beta}_{x}}(y) = \mathscr{I} \sqcap \widetilde{C}_i(y) = \vee \mathscr{I}_{\widetilde{C}_i}(y) \geq b$, and $\mathscr{F}_{{}_2\widetilde{\mathscr{M}}^{\beta}_{x}}(y) = \mathscr{F} \sqcap \widetilde{C}_i(y) = \vee \mathscr{F}_{\widetilde{C}_i}(y) \leq c$. Then, $\{\widetilde{C}_i \in \widetilde{\Gamma}\colon \mathscr{T}_{\widetilde{C}_i}(x) \geq a, \mathscr{I}_{\widetilde{C}_i}(x) \geq b, \mathscr{F}_{\widetilde{C}_i}(x) \leq c\} \subseteq \{\widetilde{C}_i \in \widetilde{\Gamma}\colon \mathscr{T}_{\widetilde{C}_i}(y) \geq a, \mathscr{I}_{\widetilde{C}_i}(y) \geq b, \mathscr{F}_{\widetilde{C}_i}(y) \leq c\}$ Thus, ${}_2\widetilde{\mathscr{M}}^{\beta}_{y} \sqsubseteq {}_2\widetilde{\mathscr{M}}^{\beta}_{x}$. On the contrary, let ${}_2\widetilde{\mathscr{M}}^{\beta}_{y} \sqsubseteq {}_2\widetilde{\mathscr{M}}^{\beta}_{x}$. Then, $\mathscr{T}_{{}_2\widetilde{\mathscr{M}}^{\beta}_{x}}(y) \geq \mathscr{T}_{{}_2\widetilde{\mathscr{M}}^{\beta}_{y}}(y) \geq a$, $\mathscr{I}_{{}_2\widetilde{\mathscr{M}}^{\beta}_{x}}(y) \geq \mathscr{I}_{{}_2\widetilde{\mathscr{M}}^{\beta}_{y}}(y) \geq b$, and $\mathscr{F}_{{}_2\widetilde{\mathscr{M}}^{\beta}_{x}}(y) \leq \mathscr{F}_{{}_2\widetilde{\mathscr{M}}^{\beta}_{y}}(y) \leq c$. Hence, ${}_2\widetilde{\mathscr{M}}^{\beta}_{x}(y) \succcurlyeq \beta$. □

Here, we construct three new types of 2-1-SVNβCRSs based on Definitions 5 and 9 as seen below.

Definition 10. Consider $(\Omega, \widetilde{\Gamma})$ be a 2-SVNβCAS with $\widetilde{\Gamma} = \{\widetilde{C}_1, \widetilde{C}_2, \ldots, \widetilde{C}_m\}$ for some $\beta = \langle a, b, c\rangle$. For each $x \in \Omega$ and $\mathscr{A} \in \mathrm{SVN}(\Omega)$, then we have the following paradigms.

Paradigm 1: The second type of Type-2 SVN lower approximation (2-2-SVNLA) $\mathscr{L}^2_2(\mathscr{A})$ and the second type of Type-2 SVN upper approximation (2-2-SVNUA) $\mathscr{U}_2^2(\mathscr{A})$ are found as follows:

$$
\begin{aligned}
\mathscr{L}_2^2(\mathscr{A}) &= \left\{\langle x, \wedge_{y\in\Omega}\left(\mathscr{F}_{{}_2\widetilde{\mathscr{M}}_x^{\beta}}(y)\vee\mathscr{T}_{\mathscr{A}}(y)\right), \wedge_{y\in\Omega}\left(\left(1-\mathscr{I}_{{}_2\widetilde{\mathscr{M}}_x^{\beta}}(y)\right)\vee\mathscr{I}_{\mathscr{A}}(y)\right), \vee_{y\in\Omega}\left(\mathscr{T}_{{}_2\widetilde{\mathscr{M}}_x^{\beta}}(y)\wedge\mathscr{F}_{\mathscr{A}}(y)\right)\rangle\right\},\\
\mathscr{U}_2^2(\mathscr{A}) &= \left\{\langle x, \vee_{y\in\Omega}\left(\mathscr{T}_{{}_2\widetilde{\mathscr{M}}_x^{\beta}}(y)\wedge\mathscr{T}_{\mathscr{A}}(y)\right), \vee_{y\in\Omega}\left(\mathscr{I}_{{}_2\widetilde{\mathscr{M}}_x^{\beta}}(y)\wedge\mathscr{I}_{\mathscr{A}}(y)\right), \wedge_{y\in\Omega}\left(\mathscr{F}_{{}_2\widetilde{\mathscr{M}}_x^{\beta}}(y)\vee\mathscr{F}_{\mathscr{A}}(y)\right)\rangle\right\}.
\end{aligned}
\tag{15}
$$

If $\mathscr{L}_2^2(\mathscr{A})\neq\mathscr{U}_2^2(\mathscr{A})$, then $\mathscr{A}$ is called the second type of Type-2 SVN β-covering rough sets (briefly, 2-2-SVNβCRSs)

Paradigm 2: the third type of Type-2 SVN lower approximation (3-2-SVNLA) $\mathscr{L}_3^2(\mathscr{A})$ and the third type of Type-2 SVN upper approximation (3-2-SVNUA) $\mathscr{U}_3^2(\mathscr{A})$ are introduced as follows:

$$
\begin{aligned}
\mathscr{L}_3^2(\mathscr{A}) &= \left\{\langle x, \wedge_{y\in\Omega}\left(\mathscr{F}_{{}_2\widetilde{\mathscr{N}}_x^{\beta}\sqcap_2\widetilde{\mathscr{M}}_x^{\beta}}(y)\vee\mathscr{T}_{\mathscr{A}}(y)\right), \wedge_{y\in\Omega}\left(\left(1-\mathscr{I}_{{}_2\widetilde{\mathscr{N}}_x^{\beta}\sqcap_1\widetilde{\mathscr{M}}_x^{\beta}}(y)\right)\vee\mathscr{I}_{\mathscr{A}}(y)\right), \vee_{y\in\Omega}\left(\mathscr{T}_{{}_2\widetilde{\mathscr{N}}_x^{\beta}\sqcap_2\widetilde{\mathscr{M}}_x^{\beta}}(y)\wedge\mathscr{F}_{\mathscr{A}}(y)\right)\rangle\right\},\\
\mathscr{U}_3^2(\mathscr{A}) &= \left\{\langle x, \vee_{y\in\Omega}\left(\mathscr{T}_{{}_2\widetilde{\mathscr{N}}_x^{\beta}\sqcap_2\widetilde{\mathscr{M}}_x^{\beta}}(y)\wedge\mathscr{T}_{\mathscr{A}}(y)\right), \vee_{y\in\Omega}\left(\mathscr{I}_{{}_2\widetilde{\mathscr{N}}_x^{\beta}\sqcap_2\widetilde{\mathscr{M}}_x^{\beta}}(y)\wedge\mathscr{I}_{\mathscr{A}}(y)\right), \wedge_{y\in\Omega}\left(\mathscr{F}_{2\widetilde{\mathscr{N}}_x^{\beta}\sqcap_2\widetilde{\mathscr{M}}_x^{\beta}}(y)\vee\mathscr{F}_{\mathscr{A}}(y)\right)\rangle\right\}.
\end{aligned}
\tag{16}
$$

If $\mathscr{L}_3^2(\mathscr{A})\neq\mathscr{U}_3^2(\mathscr{A})$, then $\mathscr{A}$ is called the third type of Type-2 SVN β-covering rough sets (briefly, 3-2-SVNβCRSs).

Paradigm 3: the fourth type of Type-2 SVN lower approximation (4-2-SVNLA) $\mathscr{L}_4^2(\mathscr{A})$ and the fourth type of Type-2 SVN upper approximation (4-2-SVNUA) $\mathscr{U}_4^2(\mathscr{A})$ are proposed as follows:

$$
\begin{aligned}
\mathscr{L}_4^2(\mathscr{A}) &= \left\{\langle x, \wedge_{y\in\Omega}\left(\mathscr{F}_{{}_2\widetilde{\mathscr{N}}x^{\beta}\sqcup_2\widetilde{\mathscr{M}}_x^{\beta}}(y)\vee\mathscr{T}_{\mathscr{A}}(y)\right), \wedge_{y\in\Omega}\left(\left(1-\mathscr{I}_{{}_2\widetilde{\mathscr{N}}x^{\beta}\sqcup_2\widetilde{\mathscr{M}}_x^{\beta}}(y)\right)\vee\mathscr{I}_{\mathscr{A}}(y)\right), \vee_{y\in\Omega}\left(\mathscr{T}_{{}_2\widetilde{\mathscr{N}}x^{\beta}\sqcup_2\widetilde{\mathscr{M}}_x^{\beta}}(y)\wedge\mathscr{F}_{\mathscr{A}}(y)\right)\rangle\right\},\\
\mathscr{U}_4^2(\mathscr{A}) &= \left\{\langle x, \vee_{y\in\Omega}\left(\mathscr{T}_{{}_2\widetilde{\mathscr{N}}x^{\beta}\sqcup_2\widetilde{\mathscr{M}}_x^{\beta}}(y)\wedge\mathscr{T}_{\mathscr{A}}(y)\right), \vee_{y\in\Omega}\left(\mathscr{I}_{{}_2\widetilde{\mathscr{N}}x^{\beta}\sqcup_1\widetilde{\mathscr{M}}_x^{\beta}}(y)\wedge\mathscr{I}_{\mathscr{A}}(y)\right), \wedge_{y\in\Omega}\left(\mathscr{F}_{{}_2\widetilde{\mathscr{N}}x^{\beta}\sqcup_2\widetilde{\mathscr{M}}_x^{\beta}}(y)\vee\mathscr{F}_{\mathscr{A}}(y)\right)\rangle\right\}.
\end{aligned}
\tag{17}
$$

If $\mathscr{L}_4^2(\mathscr{A})\neq\mathscr{U}_4^2(\mathscr{A})$, then $\mathscr{A}$ is called the fourth type of Type-2 SVN β-covering rough sets (briefly, 4-2-SVNβCRSs).

Example 4. Consider Example 1 if $\beta=\langle 0.5, 0.1, 0.8\rangle$ and $\mathscr{A}=((0.6,0.3,0.5)/x_1)+((0.4,0.5,0.1)/x_2)+((0.3,0.2,0.6)/x_3)+((0.5,0.3,0.4)/x_4)+((0.7,0.2,0.3)/x_5)$; then, we have the following results:

$$
\begin{aligned}
\mathscr{L}_1^2(\mathscr{A}) &= \{\langle x_1, 0.6, 0.7, 0.5\rangle, \langle x_2, 0.6, 0.7, 0.4\rangle, \langle x_3, 0.4, 0.7, 0.5\rangle, \langle x_4, 0.4, 0.7, 0.4\rangle, \langle x_5, 0.6, 0.7, 0.3\rangle\},\\
\mathscr{U}_1^2(\mathscr{A}) &= \{\langle x_1, 0.6, 0.2, 0.5\rangle, \langle x_2, 0.4, 0.2, 0.6\rangle, \langle x_3, 0.6, 0.3, 0.5\rangle, \langle x_4, 0.5, 0.2, 0.6\rangle, \langle x_5, 0.6, 0.3, 0.5\rangle\},\\
\mathscr{L}_2^2(\mathscr{A}) &= \{\langle x_1, 0.3, 0.2, 0.6\rangle, \langle x_2, 0.3, 0.2, 0.6\rangle, \langle x_3, 0.4, 0.2, 0.5\rangle, \langle x_4, 0.4, 0.2, 0.4\rangle, \langle x_5, 0.3, 0.2, 0.6\rangle\},\\
\mathscr{U}_2^2(\mathscr{A}) &= \{\langle x_1, 0.6, 0.5, 0.3\rangle, \langle x_2, 0.6, 0.5, 0.3\rangle, \langle x_3, 0.6, 0.5, 0.4\rangle, \langle x_4, 0.6, 0.5, 0.3\rangle, \langle x_5, 0.6, 0.5, 0.4\rangle\},\\
\mathscr{L}_3^2(\mathscr{A}) &= \{\langle x_1, 0.6, 0.7, 0.5\rangle, \langle x_2, 0.6, 0.7, 0.4\rangle, \langle x_3, 0.5, 0.7, 0.4\rangle, \langle x_4, 0.4, 0.7, 0.3\rangle, \langle x_5, 0.6, 0.7, 0.3\rangle\},\\
\mathscr{U}_3^2(\mathscr{A}) &= \{\langle x_1, 0.5, 0.2, 0.6\rangle, \langle x_2, 0.4, 0.2, 0.6\rangle, \langle x_3, 0.5, 0.3, 0.5\rangle, \langle x_4, 0.5, 0.2, 0.6\rangle, \langle x_5, 0.5, 0.3, 0.6\rangle\},\\
\mathscr{L}_4^2(\mathscr{A}) &= \{\langle x_1, 0.3, 0.2, 0.6\rangle, \langle x_2, 0.3, 0.2, 0.6\rangle, \langle x_3, 0.4, 0.2, 0.5\rangle, \langle x_4, 0.4, 0.2, 0.5\rangle, \langle x_5, 0.3, 0.2, 0.6\rangle\},\\
\mathscr{U}_4^2(\mathscr{A}) &= \{\langle x_1, 0.6, 0.5, 0.3\rangle, \langle x_2, 0.6, 0.5, 0.3\rangle, \langle x_3, 0.6, 0.5, 0.4\rangle, \langle x_4, 0.6, 0.5, 0.3\rangle, \langle x_5, 0.6, 0.5, 0.4\rangle\}.
\end{aligned}
\tag{18}
$$

In the following, we will propose Proposition 8 for the 2-2-SVNβCRS model; also, it fulfills in case of the 3-2-SVNβCRS and the 4-2-SVNβCRS models.

Proposition 8. *Let* $(\Omega, \widetilde{\Gamma})$ *be a 1-SVNβCAS, for some* $\beta = \langle a, b, c\rangle$. *For each* $x, y, z \in \Omega$ *and* $\mathscr{A}, \mathscr{B} \in SVN(\Omega)$, *then the following statements hold:*

(1) (SVNL1): $\mathscr{L}_2^2(\mathscr{A}^c) = (\mathscr{U}_2^2(\mathscr{A}))^c$.
 (SVNU1): $\mathscr{U}_2^2(\mathscr{A}^c) = (\mathscr{L}_2^2(\mathscr{A}))^c$.

(2) If $\mathscr{A} \subseteq \mathscr{B}$, then
 (SVNL2): $\mathscr{L}_2^2(\mathscr{A}) \subseteq \mathscr{L}_2^2(\mathscr{B})$.
 (SVNU2): $\mathscr{U}_2^2(\mathscr{A}) \subseteq \mathscr{U}_2^2(\mathscr{B})$.

(3) (SVNL3): $\mathscr{L}_2^2(\mathscr{A} \cap \mathscr{B}) = \mathscr{L}_2^2(\mathscr{A}) \cap \mathscr{L}_2^2(\mathscr{B})$.
 (SVNU3): $\mathscr{U}_2^2(\mathscr{A} \cap \mathscr{B}) \subseteq \mathscr{U}_2^2(\mathscr{A}) \cap \mathscr{U}_2^2(\mathscr{B})$.

(4) (SVNL4): $\mathscr{L}_2^2(\mathscr{A} \cup \mathscr{B}) \supseteq \mathscr{L}_2^2(\mathscr{A}) \cup \mathscr{L}_2^2(\mathscr{B})$.
 (SVNU4): $\mathscr{U}_2^2(\mathscr{A} \cap \mathscr{B}) = \mathscr{U}_2^2(\mathscr{A}) \cap \mathscr{U}_2^1(\mathscr{B})$.

(5) (SVNL5): $\mathscr{L}_2^2(\Omega) = \Omega$.
 (SVNU5): $\mathscr{U}_2^2(\varnothing) = \varnothing$.

Proof. We shall only prove (SVNL1), (SVNL2), (SVNL3), and (SVNL4).

(SVNL1):

$$\begin{aligned}
\mathscr{L}_2^2(\mathscr{A}^c) &= \left\{\langle x, \wedge_{y\in\Omega}\left(\mathscr{F}_{2\cdot\widetilde{\mathscr{M}}_x^\beta}(y) \vee \mathscr{T}_{\mathscr{A}^c}(y)\right), \wedge_{y\in\Omega}\left(\left(1-\mathscr{I}_{2\cdot\widetilde{\mathscr{M}}_x^\beta}(y)\right)\vee \mathscr{I}_{\mathscr{A}^c}(y)\right), \vee_{y\in\Omega}\left(\mathscr{T}_{2\cdot\widetilde{\mathscr{M}}_x^\beta}(y)\wedge \mathscr{F}_{\mathscr{A}^c}(y)\right)\rangle\right\}\\
&= \left\{\langle x, \wedge_{y\in\Omega}\left(\mathscr{F}_{2\cdot\widetilde{\mathscr{M}}_x^\beta}(y) \vee \mathscr{F}_{\mathscr{A}}(y)\right), \wedge_{y\in\Omega}\left(\left(1-\mathscr{I}_{2\cdot\widetilde{\mathscr{M}}_x^\beta}(y)\right)\vee (1-\mathscr{I}_{\mathscr{A}}(y))\right), \vee_{y\in\Omega}\left(\mathscr{T}_{2\cdot\widetilde{\mathscr{M}}_x^\beta}(y)\wedge \mathscr{T}_{\mathscr{A}}(y)\right)\rangle\right\}\\
&= \left(\mathscr{U}_2^2(\mathscr{A})\right)^c.
\end{aligned} \tag{19}$$

(SVNL2): let $\mathscr{A}, \mathscr{B} \in \mathrm{SVN}(\Omega)$ such that $\mathscr{A} \sqsubseteq \mathscr{B}$ (i.e., $\mathscr{T}_{\mathscr{A}} \leq \mathscr{T}_{\mathscr{B}}, \mathscr{I}_{\mathscr{B}} \leq \mathscr{I}_{\mathscr{A}}$ and $\mathscr{F}_{\mathscr{B}} \leq \mathscr{F}_{\mathscr{A}}$) and $x \in \Omega$. Then, we get the following result:

$$\begin{aligned}
&\mathscr{L}_2^2(\mathscr{A})(x) = \left\{\langle x, \wedge_{y\in\Omega}\left(\mathscr{F}_{2\cdot\widetilde{\mathscr{M}}_x^\beta}(y) \vee \mathscr{T}_{\mathscr{A}}(y)\right), \wedge_{y\in\Omega}\left(\left(1-\mathscr{I}_{2\cdot\widetilde{\mathscr{M}}_x^\beta}(y)\right)\vee \mathscr{I}_{\mathscr{A}}(y)\right), \vee_{y\in\Omega}\left(\mathscr{T}_{2\cdot\widetilde{\mathscr{M}}_x^\beta}(y)\wedge \mathscr{F}_{\mathscr{A}}(y)\right)\rangle\right\},\\
&\mathscr{T}_{\mathscr{L}_2^2(\mathscr{A})} = \wedge_{y\in\Omega}\left(\mathscr{F}_{2\cdot\widetilde{\mathscr{M}}_x^\beta}(y) \vee \mathscr{T}_{\mathscr{A}}(y)\right) \leq \wedge_{y\in\Omega}\left(\mathscr{F}_{2\cdot\widetilde{\mathscr{M}}_x^\beta}(y) \vee \mathscr{T}_{\mathscr{B}}(y)\right) = \mathscr{T}_{\mathscr{L}_2^2(\mathscr{B})},\\
&\mathscr{I}_{\mathscr{L}_2^2(\mathscr{A})} = \wedge_{y\in\Omega}\left(\left(1-\mathscr{I}_{2\cdot\widetilde{\mathscr{M}}_x^\beta}(y)\right)\vee \mathscr{I}_{\mathscr{A}}(y)\right) \leq \wedge_{y\in\Omega}\left(\left(1-\mathscr{I}_{2\cdot\widetilde{\mathscr{M}}_x^\beta}(y)\right)\vee \mathscr{I}_{\mathscr{B}}(y)\right) = \mathscr{I}_{\mathscr{L}_2^2(\mathscr{B})},\\
&\mathscr{F}_{\mathscr{L}_2^2(\mathscr{A})} = \vee_{y\in\Omega}\left(\mathscr{T}_{2\cdot\widetilde{\mathscr{M}}_x^\beta}(y)\wedge \mathscr{F}_{\mathscr{A}}(y)\right) \geq \vee_{y\in\Omega}\left(\mathscr{T}_{2\cdot\widetilde{\mathscr{M}}_x^\beta}(y)\wedge \mathscr{F}_{\mathscr{B}}(y)\right) = \mathscr{F}_{\mathscr{L}_2^2(\mathscr{B})}.
\end{aligned} \tag{20}$$

Therefore, $\mathscr{L}_2^2(\mathscr{A}) \sqsubseteq \mathscr{L}_2^2(\mathscr{B})$.

(SVNL3): If $x \in \Omega$, then we have

$$\begin{aligned}
\mathscr{L}_2^2(\mathscr{A})(x) &= \left\{\langle x, \wedge_{y\in\Omega}\left(\mathscr{F}_{2\cdot\widetilde{\mathscr{M}}_x^\beta}(y) \vee \mathscr{T}_{\mathscr{A}\cap\mathscr{B}}(y)\right), \wedge_{y\in\Omega}\left(\left(1-\mathscr{I}_{2\cdot\widetilde{\mathscr{M}}_x^\beta}(y)\right)\vee \mathscr{I}_{\mathscr{A}\cap\mathscr{B}}(y)\right), \vee_{y\in\Omega}\left(\mathscr{T}_{2\cdot\widetilde{\mathscr{M}}_x^\beta}(y)\wedge \mathscr{F}_{\mathscr{A}\cap\mathscr{B}}(y)\right)\rangle\right\}\\
&= \left\{\langle x, \wedge_{y\in\Omega}\left(\mathscr{F}_{2\cdot\widetilde{\mathscr{M}}_x^\beta}(y) \vee (\mathscr{T}_{\mathscr{A}}(y)\wedge\mathscr{T}_{\mathscr{B}}(y))\right), \wedge_{y\in\Omega}\left(\left(1-\mathscr{I}_{2\cdot\widetilde{\mathscr{M}}_x^\beta}(y)\right)\vee (\mathscr{I}_{\mathscr{A}}(y)\wedge\mathscr{I}_{\mathscr{B}}(y))\right), \vee_{y\in\Omega}\left(\mathscr{T}_{2\cdot\widetilde{\mathscr{M}}_x^\beta}(y)\wedge (\mathscr{F}_{\mathscr{A}}(y)\vee\mathscr{F}_{\mathscr{B}}(y))\right)\rangle\right\}\\
&= \left\{\langle x, \wedge_{y\in\Omega}\left(\left(\mathscr{F}_{2\cdot\widetilde{\mathscr{M}}_x^\beta}(y) \vee \mathscr{T}_{\mathscr{A}}(y)\right)\wedge\left(\mathscr{F}_{2\cdot\widetilde{\mathscr{M}}_x^\beta}(y) \vee \mathscr{T}_{\mathscr{B}}(y)\right)\right), \wedge_{y\in\Omega}\left(\left(\left(1-\mathscr{I}_{2\cdot\widetilde{\mathscr{M}}_x^\beta}(y)\right)\vee \mathscr{I}_{\mathscr{A}}(y)\right)\wedge\left(\left(1-\mathscr{I}_{2\cdot\widetilde{\mathscr{M}}_x^\beta}(y)\right)\vee \mathscr{I}_{\mathscr{B}}(y)\right)\right),\right.\\
&\left.\vee_{y\in\Omega}\left(\left(\mathscr{T}_{2\cdot\widetilde{\mathscr{M}}_x^\beta}(y)\wedge (\mathscr{F}_{\mathscr{A}}(y)\right)\vee\left(\mathscr{T}_{2\cdot\widetilde{\mathscr{M}}_x^\beta}(y)\wedge \mathscr{F}_{\mathscr{B}}(y)\right)\right)\rangle\rangle\right\}\\
&= \mathscr{L}_2^2(\mathscr{A}) \sqcap \mathscr{L}_2^2(\mathscr{B}).
\end{aligned} \tag{21}$$

(SVNL4): since $\mathscr{A} \sqcup \mathscr{B} \sqsupseteq \mathscr{A}$, then by SVNL2 we have $\mathscr{L}_2^2(\mathscr{A} \sqcup \mathscr{B}) \sqsupseteq \mathscr{L}_2^2(\mathscr{A})$. Similarly, $\mathscr{A} \sqcup \mathscr{B} \sqsupseteq \mathscr{B}$; then, by SVNL2, we have $\mathscr{L}_2^2(\mathscr{A} \sqcup \mathscr{B}) \sqsupseteq \mathscr{L}_2^2(\mathscr{B})$. Thus, $\mathscr{L}_2^2(\mathscr{A} \sqcup \mathscr{B}) \sqsupseteq \mathscr{L}_2^2(\mathscr{A}) \sqcup \mathscr{L}_2^2(\mathscr{B})$.

(SVNL5): since SVN universe is $\Omega = \langle x, 1, 1, 0\rangle$ and SVN empty set is $\varnothing = \langle x, 0, 0, 1\rangle$, then we have

$$
\begin{aligned}
\mathscr{L}_2^2(\Omega) &= \left\{\langle x, \wedge_{y\in\Omega}\left(\mathscr{F}_{{}_2\widetilde{\mathscr{M}}_x^\beta}(y)\vee\mathscr{T}_\Omega(y)\right), \wedge_{y\in\Omega}\left(\left(1-\mathscr{I}_{{}_2\widetilde{\mathscr{M}}_x^\beta}(y)\right)\vee\mathscr{I}_\Omega(y)\right), \vee_{y\in\Omega}\left(\mathscr{T}_{{}_2\widetilde{\mathscr{M}}_x^\beta}(y)\wedge\mathscr{F}_\Omega(y)\right)\rangle\right\} \\
&= \left\{\langle x, \wedge_{y\in\Omega}\left(\mathscr{F}_{{}_2\widetilde{\mathscr{M}}_x^\beta}(y)\vee 1\right), \wedge_{y\in\Omega}\left(\left(1-\mathscr{I}_{{}_2\widetilde{\mathscr{M}}_x^\beta}(y)\right)\vee 1\right), \vee_{y\in\Omega}\left(\mathscr{T}_{{}_2\widetilde{\mathscr{M}}_x^\beta}(y)\wedge 0\right)\rangle\right\} \\
&= \{\langle x, 1, 1, 0\rangle\} \\
&= \Omega.
\end{aligned}
\tag{22}
$$

□

In the following, we give some relationships among these models.

Proposition 9. *Let $(\Omega, \widetilde{\Gamma})$ be a 2-SVNβCAS and $\mathscr{A} \in SVN(\Omega)$. Then, we have the following properties:*

(1) $\mathscr{L}_4^2(\mathscr{A}) \preccurlyeq \mathscr{L}_2^2(\mathscr{A}) \preccurlyeq \mathscr{L}_3^2(\mathscr{A})$
(2) $\mathscr{L}_4^2(\mathscr{A}) \preccurlyeq \mathscr{L}_1^2(\mathscr{A}) \preccurlyeq \mathscr{L}_3^2(\mathscr{A})$
(3) $\mathscr{U}_3^2(\mathscr{A}) \preccurlyeq \mathscr{U}_2^2(\mathscr{A}) \preccurlyeq \mathscr{U}_4^2(\mathscr{A})$
(4) $\mathscr{U}_3^2(\mathscr{A}) \preccurlyeq \mathscr{U}_1^2(\mathscr{A}) \preccurlyeq \mathscr{U}_4^2(\mathscr{A})$

Proof. The proof is clear from Definition 10. □

Proposition 10. *Let $(\Omega, \widetilde{\Gamma})$ be a 2-SVNβCAS and $\mathscr{A} \in SVN(\Omega)$. Then, we have the following properties:*

(1) $\mathscr{L}_3^2(\mathscr{A}) \succcurlyeq \mathscr{L}_1^2(\mathscr{A}) \sqcup \mathscr{L}_2^2(\mathscr{A})$
(2) $\mathscr{U}_3^2(\mathscr{A}) \preccurlyeq \mathscr{U}_1^2(\mathscr{A}) \sqcap \mathscr{U}_2^2(\mathscr{A})$
(3) $\mathscr{L}_4^2(\mathscr{A}) \succcurlyeq \mathscr{L}_1^2(\mathscr{A}) \sqcap \mathscr{L}_2^2(\mathscr{A})$
(4) $\mathscr{U}_4^2(\mathscr{A}) \preccurlyeq \mathscr{U}_1^2(\mathscr{A}) \sqcup \mathscr{U}_2^2(\mathscr{A})$

Proof (clear). □

5. Decision-Making Approach to DM Based on SVNβCRSs

5.1. Description and Process

5.1.1. Method I. Assume that $\Omega = \{x_r : r = 1, \ldots, k\}$ is the set of alternatives (patients), m is main attributes (symptoms) (e.g., cough and fever) $V = \{y_i : i = 1, 2, \ldots, m\}$ of A disease, $\widetilde{\mathscr{C}}_i(x_r) = \langle \mathscr{T}_{\widetilde{\mathscr{C}}_i}(x_r), \mathscr{I}_{\widetilde{\mathscr{C}}_i}(x_r), \mathscr{F}_{\widetilde{\mathscr{C}}_i}(x_r)\rangle$ indicates the symptom value for each patient which is known by a doctor D, for some $\beta = \langle a, b, c\rangle$, and $(\Omega, \widetilde{\Gamma})$ is a Type-1 SVN β-CRS, where $\mathscr{T}_{\widetilde{\mathscr{C}}_i}(x_r) \in [0, 1]$ (i.e., the degree that doctor D confirms the patient x_r has symptom y_i), $\mathscr{I}_{\widetilde{\mathscr{C}}_i}(x_r) \in [0, 1]$ (i.e., the degree that doctor D is not sure if the patient x_r has symptom y_i), $\mathscr{F}_{\widetilde{\mathscr{C}}_i}(x_r) \in [0, 1]$ (i.e., the degree that doctor D confirms the patient x_r does not have any symptom y_i), and $0 \leq \mathscr{T}_{\widetilde{\mathscr{C}}_i}(x_r) + \mathscr{I}_{\widetilde{\mathscr{C}}_i}(x_r) + \mathscr{F}_{\widetilde{\mathscr{C}}_i}(x_r) \leq 3$. According to the presented covering methods, we propose a decision-making algorithm to obtain the result by the following steps:

Step 1: consider, for each $x_r \in \Omega$, there is at least one $y_i \in V$ such that the symptom value $\mathscr{C}_i$ for patient x_r is not less than β, where β is a critical value.

Step 2: consider $\mathscr{A}(x_r) = \langle d, e, f\rangle$ is the evaluation by a decision maker D, where d is a possible degree, e is an indeterminacy degree, and f is an impossible degree of A disease.

Step 3: based on this information, use Definition 8 and 3-1-SVNβCRSs model to calculate the lower and upper approximation of $\mathscr{A}$.

Step 4: calculate $\mathfrak{R}_{\mathscr{A}}$ by the following equation:

$$
\mathfrak{R}_{\mathscr{A}} = \mathscr{U}_3^1(\mathscr{A}) \oplus \mathscr{L}_3^1(\mathscr{A}), \tag{23}
$$

where $\mathscr{A}\oplus\mathscr{B} = \{\langle x, \mathscr{T}_{\mathscr{A}}(x) + \mathscr{T}_{\mathscr{B}}(x) - \mathscr{T}_{\mathscr{A}}(x)^* \mathscr{T}_{\mathscr{B}}(x), \mathscr{I}_{\mathscr{A}}(x)^* \mathscr{I}_{\mathscr{A}B}(x), \mathscr{F}_{\mathscr{A}}(x)^* \mathscr{F}_{\mathscr{B}}(x)\rangle : x \in \Omega\}$.

Step 5: calculate the decision method by the following formula:

$$
\mathscr{S}(x) = \frac{\mathscr{T}_{\mathfrak{R}_{\mathscr{A}}}(x)}{\sqrt{\left(\mathscr{T}_{\mathfrak{R}_{\mathscr{A}}}(x)\right)^2 + \left(\mathscr{I}_{\mathfrak{R}_{\mathscr{A}}}(x)\right)^2 + \left(\mathscr{F}_{\mathfrak{R}_{\mathscr{A}}}(x)\right)^2}}, \tag{24}
$$

hence, ranking the alternatives.

Based on these steps, we give an algorithm to solve the decision-making problems based on Definition 8. The steps corresponding to it are summarized in Algorithm 1.

5.1.2. Method II. Suppose that $\Omega = \{x_r : r = 1, \ldots, k\}$ is the set of alternatives (papers), m is main attributes (symptoms) (e.g., spot and steak) $V = \{y_i : i = 1, 2, \ldots, m\}$ of A paper trouble, $\widetilde{\mathscr{C}}_i(x_r) = \langle \mathscr{T}_{\widetilde{\mathscr{C}}_i}(x_r), \mathscr{I}_{\widetilde{\mathscr{C}}_i}(x_r), \mathscr{F}_{\widetilde{\mathscr{C}}_i}(x_r)\rangle$ indicates the symptom value for each paper which known by an investigator I, for some $\beta = \langle a, b, c\rangle$, and $(\Omega, \widetilde{\Gamma})$ is a Type-2 SVN β-CRS, where $\mathscr{T}_{\widetilde{\mathscr{C}}_i}(x_r) \in [0, 1]$ (i.e., the degree that the investigator I asserts the paper x_r has symptom y_i), $\mathscr{I}_{\widetilde{\mathscr{C}}_i}(x_r) \in [0, 1]$ (i.e., the degree that the investigator I is not sure whether the paper x_r has symptom y_i), $\mathscr{F}_{\widetilde{\mathscr{C}}_i}(x_r) \in [0, 1]$ (i.e., the degree that investigator I affirms paper x_r does not have any symptom y_i), and $0 \leq \mathscr{T}_{\widetilde{\mathscr{C}}_i}(x_r) + \mathscr{I}_{\widetilde{\mathscr{C}}_i}(x_r) +$

Input: SVN decision information system $(\Omega, \widetilde{\Gamma}, \beta, \mathscr{A})$.
Output: Decision-making.
(1) Enter $\mathscr{A}, \beta$ and Ω.
(2) From Definition 5, compute the 1-SVN β-neighborhood ${}_1\widetilde{\mathscr{N}}_x^\beta$.
(3) From **Step 2** and by Definition 7, compute 1-SVN complementary β-neighborhood ${}_1\widetilde{\mathscr{M}}_x^\beta$.
(4) From **Steps 2 and 3** and by Definition 8, compute 3-1-SVNβCRSs $\mathscr{L}_3^1(\mathscr{A})$ and $\mathscr{U}_3^1(\mathscr{A})$
(5) Compute $\mathfrak{R}_{\mathscr{A}}$.
(6) Compute the cosine similarity measure $\mathcal{S}(x)$.
(7) Obtain the decision.

Algorithm 1: Algorithm for a 1-SVNβCRSs to make a decision.

Input: SVN decision information system $(\Omega, \widetilde{\Gamma}, \beta, \mathscr{A})$.
Output: Decision-making.
(1) Enter $\mathscr{A}, \beta$, and Ω.
(2) From Definition 5, compute the 2-SVN β-neighborhood ${}_2\widetilde{\mathscr{N}}_x^\beta$.
(3) From **Step 2** and by Definition 9, compute the 2-SVN complementary β-neighborhood ${}_2\widetilde{\mathscr{M}}_x^\beta$.
(4) From **Steps 2 and 3** and by Definition 10, compute 3-2-SVNβCRSs $\mathscr{L}_3^2(\mathscr{A})$ and $\mathscr{U}_3^2(\mathscr{A})$
(5) Compute $\mathfrak{R}_{\mathscr{A}}$.
(6) Compute the cosine similarity measure $\mathcal{S}(x)$.
(7) Obtain the decision.

Algorithm 2: Algorithm for 2-SVNβCRSs to make a decision.

$\mathscr{F}_{\widetilde{\mathscr{C}}_i}(x_r) \leq 3$. According to the presented covering methods, we propose a decision-making algorithm to obtain the result by the following steps:

Step 1: consider, for each $x_r \in \Omega$, there is at least one $y_i \in V$ such that the symptom value $\mathscr{C}_i$ for paper x_r is not less than β (i.e., $\widetilde{C}_i(x) \succcurlyeq \beta$), where β is a critical value.

Step 2: consider $\mathscr{A}(x_r) = \langle d, e, f \rangle$ is the evaluation by a decision maker I, where d is a possible degree, e is an indeterminacy degree, and f is an impossible degree of A disease.

Step 3: based on this information, use Definition 10 and 3-2-SVNβCRSs model to calculate the lower and upper approximation of $\mathscr{A}$.

Step 4: calculate $\mathfrak{R}_{\mathscr{A}}$ by the following equation:

$$\mathfrak{R}_{\mathscr{A}} = \mathscr{U}_3^2(\mathscr{A}) \oplus \mathscr{L}_3^2(\mathscr{A}), \quad (25)$$

where $\mathscr{A} \oplus \mathscr{B} = \{\langle x, \mathscr{T}_{\mathscr{A}}(x) + \mathscr{T}_{\mathscr{B}}(x) - \mathscr{T}_{\mathscr{A}}(x)^* \mathscr{T}_{\mathscr{B}}(x), \mathscr{I}_{\mathscr{A}}(x)^* \mathscr{I}_{\mathscr{B}}(x), \mathscr{F}_{\mathscr{A}}(x)^* \mathscr{F}_{\mathscr{B}}(x) \rangle : x \in \Omega\}$.

Step 5: calculate the decision method by the following formula.

$$\mathcal{S}(x) = \frac{\mathscr{T}_{\mathfrak{R}_{\mathscr{A}}}(x)}{\sqrt{\left(\mathscr{T}_{\mathfrak{R}_{\mathscr{A}}}(x)\right)^2 + \left(\mathscr{I}_{\mathfrak{R}_{\mathscr{A}}}(x)\right)^2 + \left(\mathscr{F}_{\mathfrak{R}_{\mathscr{A}}}(x)\right)^2}}, \quad (26)$$

hence, ranking the alternatives.

Based on these steps, we give an algorithm to solve the decision-making problems based on Definition 10. The steps corresponding to it are summarized in Algorithm 2.

5.2. Numerical Example

Example 5. Diseased people form a set $\Omega = \{x_1, x_2, x_3, x_4, x_5\}$ and their relevant symptoms are collected by the attribute set $V = \{\text{cough}(y_1), \text{fever}(y_2), \text{sore}(y_3), \text{headache}(y_4)\}$ for A disease. Here, the following steps of the algorithm described are implemented.

Step 1: under the attribute set, doctor D estimates each patient and presents its decisions with suitable values which are summarized in Table 1.

Step 2: consider $\beta = \langle 0.5, 0.3, 0.8 \rangle$ is a critical and $\widetilde{\Gamma} = \{\widetilde{C}_1, \widetilde{C}_2, \widetilde{C}_3, \widetilde{C}_4\}$ is a Type-1 SVNβCRS. Then, we compute the Type-1 SVN β-neighborhood ${}_1\widetilde{\mathscr{N}}_x^\beta$ and the Type-1 SVN complementary β-neighborhood ${}_1\widetilde{\mathscr{M}}_x^\beta$, as shown in Tables 2 and 3.

Consider $\mathscr{A} = ((0.6, 0.3, 0.5)/x_1) + ((0.4, 0.5, 0.1)/x_2) + ((0.3, 0.2, 0.6)/x_3) + ((0.5, 0.3, 0.4)/x_4) + ((0.7, 0.2, 0.3)/x_5)$.

Step 3: by Definition 8 and 3-1-SVNβRSs model, we have the following results:

$$\mathscr{L}_3^1(\mathscr{A}) = \{\langle x_1, 0.6, 0.3, 0.5\rangle, \langle x_2, 0.6, 0.3, 0.4\rangle, \langle x_3, 0.5, 0.3, 0.4\rangle, \langle x_4, 0.4, 0.2, 0.3\rangle, \langle x_5, 0.6, 0.3, 0.3\rangle\},$$
$$\mathscr{U}_3^1(\mathscr{A}) = \{\langle x_1, 0.5, 0.5, 0.6\rangle, \langle x_2, 0.4, 0.5, 0.6\rangle, \langle x_3, 0.5, 0.5, 0.5\rangle, \langle x_4, 0.5, 0.5, 0.6\rangle, \langle x_5, 0.5, 0.5, 0.6\rangle\}. \quad (27)$$

Step 4: compute $\mathfrak{R}_{\mathscr{A}}$ as follows:

$$\mathfrak{R}_{\mathscr{A}} = \mathscr{L}_3^1(\mathscr{A}) \oplus \mathscr{U}_3^1(\mathscr{A}) = \{\langle x_1, 0.8, 0.15, 0.3\rangle, \langle x_2, 0.76, 0.15, 0.24\rangle, \langle x_3, 0.75, 0.15, 0.2\rangle, \langle x_4, 0.7, 0.1, 0.18\rangle, \langle x_5, 0.8, 0.15, 0.18\rangle\}. \quad (28)$$

Step 5: according to the above information, we get $\mathscr{S}(x)$ as follows:

$$\mathscr{S}(x_1) = 0.923,$$
$$\mathscr{S}(x_2) = 0.938,$$
$$\mathscr{S}(x_3) = 0.949, \quad (29)$$
$$\mathscr{S}(x_4) = 0.959,$$
$$\mathscr{S}(x_5) = 0.964,$$

and hence, we get the ranking order as

$$\mathscr{S}(x_5) > \mathscr{S}(x_4) > \mathscr{S}(x_3) > \mathscr{S}(x_2) > \mathscr{S}(x_1). \quad (30)$$

So, by the above computation, the verdict of the decision maker D is x_5.

Example 6. Let $\Omega = \{x_1, x_2, x_3, x_4, x_5\}$ be the set of papers, and their relevant symptoms are collected by the attribute set $V = \{\text{spot}(y_1), \text{steak}(y_2), \text{crater}(y_3), \text{fracture}(y_4)\}$ for A paper error. Here, the following steps of the algorithm described are implemented.

Step 1: under the attribute set, investigator I estimates each paper and presents its decisions with suitable values which are summarized in Table 1.

Step 2: consider $\beta = \langle 0.5, 0.1, 0.8\rangle$ is a critical and $\widetilde{\Gamma} = \{\widetilde{C}_1, \widetilde{C}_2, \widetilde{C}_3, \widetilde{C}_4\}$ is a 2-SVNβCRS. Then, we compute the Type-2 SVN β-neighborhood ${}_2\widetilde{\mathscr{N}}_x^\beta$ and the Type-2 SVN complementary β-neighborhood ${}_2\widetilde{\mathscr{M}}_x^\beta$, as shown in Tables 6 and 7. Consider $\mathscr{A} = ((0.6, 0.3, 0.5)/x_1) + ((0.4, 0.5, 0.1)/x_2) + ((0.3, 0.2, 0.6)/x_3) + ((0.5, 0.3, 0.4)/x_4) + ((0.7, 0.2, 0.3)/x_5)$.

Step 3: by Definition 10 and 3-2-SVNβRSs model, we have the following results:

$$\mathscr{L}_3^2(\mathscr{A}) = \{\langle x_1, 0.6, 0.7, 0.5\rangle, \langle x_2, 0.6, 0.7, 0.4\rangle, \langle x_3, 0.5, 0.7, 0.4\rangle, \langle x_4, 0.4, 0.7, 0.3\rangle, \langle x_5, 0.6, 0.7, 0.3\rangle\},$$
$$\mathscr{U}_3^2(\mathscr{A}) = \{\langle x_1, 0.5, 0.2, 0.6\rangle, \langle x_2, 0.4, 0.2, 0.6\rangle, \langle x_3, 0.5, 0.3, 0.5\rangle, \langle x_4, 0.5, 0.2, 0.6\rangle, \langle x_5, 0.5, 0.3, 0.6\rangle\}. \quad (31)$$

Step 4: compute $\mathfrak{R}_{\mathscr{A}}$ as follows:

$$\mathfrak{R}_{\mathscr{A}} = \mathscr{L}_3^2(\mathscr{A}) \oplus \mathscr{U}_3^2(\mathscr{A}) = \{\langle x_1, 0.8, 0.14, 0.3\rangle, \langle x_2, 0.76, 0.14, 0.24\rangle, \langle x_3, 0.75, 0.21, 0.2\rangle, \langle x_4, 0.7, 0.14, 0.18\rangle, \langle x_5, 0.8, 0.21, 0.18\rangle\}. \quad (32)$$

Step 5: according to above information, we get $\mathscr{S}(x)$ as follows:

$$\mathscr{S}(x_1) = 0.924,$$
$$\mathscr{S}(x_2) = 0.939,$$
$$\mathscr{S}(x_3) = 0.933, \quad (33)$$
$$\mathscr{S}(x_4) = 0.951,$$
$$\mathscr{S}(x_5) = 0.945,$$

and hence, we get the ranking order as

$$\mathscr{S}(x_4) > \mathscr{S}(x_5) > \mathscr{S}(x_2) > \mathscr{S}(x_3) > \mathscr{S}(x_1). \quad (34)$$

So, by the above calculations, the verdict of the decision maker I is x_4.

5.3. Comparative Analysis. The major purpose of our presented work is eligible to raise the lower approximation and reduce the upper approximation of the previous study by Wang and Zhang's methods [46, 47], as visible in Examples 2 and 4. To clarify the comparisons between Wang and Zhang's methods [46, 47] and our methods, the sorting outcomes of these decision-making models are listed in Table 10 for 1-SVNβCAS and Table 11 for 2-SVNβCAS.

Table 10: Sorting outcomes for 1-SVNβCAS.

Different methods	Obtain a decision
Wang and Zhang's model [46]	$x_5 \succ x_1 \succ x_2 \succ x_3 \succ x_4$
Our model	$x_5 \succ x_4 \succ x_3 \succ x_2 \succ x_1$

Table 11: Sorting outcomes for 2-SVNβCAS.

Different methods	Obtain a decision
Wang and Zhang's model [47]	$x_5 \succ x_1 \succ x_2 \succ x_4 \succ x_3$
Our model	$x_4 \succ x_5 \succ x_2 \succ x_3 \succ x_1$

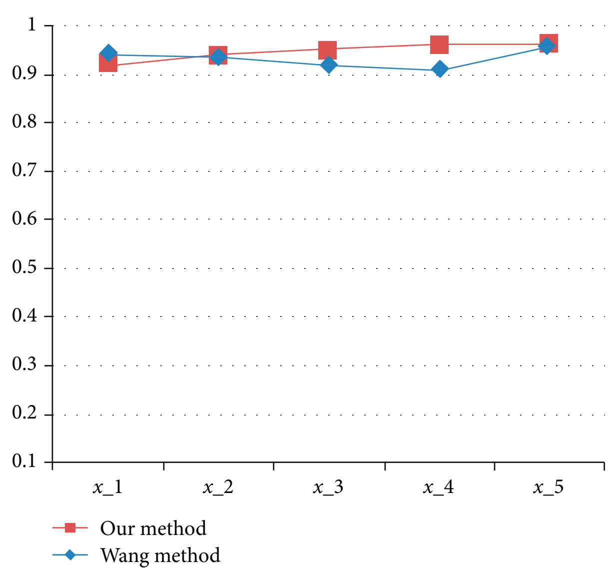

Figure 1: The representations of the results by using our model and Wang and Zhang's model [46].

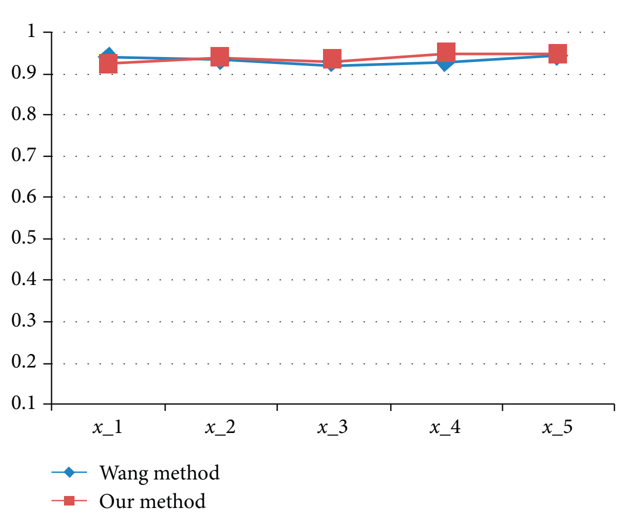

Figure 2: The representations of the results by using our model and Wang and Zhang's model [47].

An easy way to explain these outcomes, see Figures 1 and 2 which simplify the comparisons between our presented method and the previous one.

Figure 1 explained the differences between the outcomes using our model (3-1-SVNβCAS) and the last one (1-1-SVNβCRSs). Furthermore, Figure 2 illustrated the comparisons between the values through our model (3-2-SVNβCAS) and the previous one (1-2-SVNβCRSs). Thus, there are slight differences among these distinct methods, and these variations made our model better than others.

6. Conclusion

This work is extended to Wang and Zhang's studies in [46, 47]. We presented the definitions of 1-SVN complementary β-neighborhoods and 2-SVN complementary β-neighborhoods. We use them to set up new models of 1-SVN β-CRS and 2-SVN β-CRS, respectively. Moreover, by merging the Type-1 neighborhoods (resp., Type-2 neighborhoods) and Type-1 complementary neighborhoods (resp., Type-2 complementary neighborhoods), we obtain two new types of Type-1 neighborhoods and Type-2 neighborhoods, respectively. Thus, two new classes of 1-SVN β-CRS and 2-SVN β-CRS are investigated. To explain the differences between these new and older types of covering methods, see Examples 2 and 4. For more clarification about them, see Figures 1 and 2. There are some issues in these two covering methods:

(1) If $\beta = (0.5, 0.1, 0.8)$ in Example 2, then $\widetilde{\Gamma}$ is not 1-SVNβCRSs, but it is applicable in 2-SVNβCRSs

(2) If $\beta = (0.5, 0.3, 0.8)$ in Example 4, then $\widetilde{\Gamma}$ is not 2-SVNβCRSs, but it is applicable in 1-SVNβCRSs

In short, the two methods are considered complementary to each other, which means if there are some failures in 1-SVNβCRSs, the 2-SVNβCRSs is working instead and vice versa.

In the future, we can extend the results of this study as a combination between 1-SVN (or 2-SVN) complementary β-neighborhoods and published papers (see [50–55]). In addition, one may investigate further based on 1-SVN (or 2-SVN) complementary β-neighborhoods with some links to topology as in [26, 48]. Finally, there are many areas (for example, several comparative of this proposed method) which can be presented by researchers in the next paper.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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