Properties of Group Neutro-Topological Space

Article · June 2022

DOI: 10.54216/JNFS.030104

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Abstract

Unlike traditional algebraic structures, where all operations are well-defined and all axioms are completely true, NeutroAlgebras and AntiAlgebras allow operations to be partially well-defined and axioms to be partially true or fully outer-defined, and axioms to be completely false. These NeutroAlgebras and AntiAlgebras represent a new research subject based on real-world examples. Since an empty set is not a subgroup of a group by observing this, the article leads to learning group neutro-topological space. We introduced the notion of a group neutro-topological space and investigated its properties.

Keywords: Topological Space; Neutrosophic Set; NeutroAlgebras; Neutro-Topological Space; Group Neutro-Topological Space.

1. Introduction

Florentin Smarandache [1] defined the idea of neutrosophic logic and the concept of neutrosophic set in 1998. After that, the concepts of the neutrosophic set have been applied in many branches of sciences and technology. Salama and Alblowi [5] introduced the concept of neutrosophic topological space. Narmada et al. [6] discussed separation axioms in ordered neutrosophic bitopological space. Mwchahary and Basumatary [8] did their work in neutrosophic bitopological space. After defining the neutrosophic group, Sumathi and Arokiarani [4] defined the concept of the topological group structure of the neutrosophic set. Also, Sumathi and Arockiarani [7] defined the fuzzy neutrosophic group.

In recent years, there has been a surge in academic interest in neutrosophic set theory. Florentin Smarandache first defined the idea of neutro-structures and anti-structures [2, 3]. An algebraic structure space is divided into three regions: A, the set of elements that satisfy the conditions of the algebraic structure, the truth region; Neutro-A, the set of elements that do not meet the conditions of the algebraic structure, the uncertainty region and the anti-A, the set of elements that do not meet the conditions of the algebraic structure, the inaccuracy region. Without using neutrosophic sets and neutrosophic numbers, the structure of neutrosophic logic has been transferred to the structure of classical algebras. Şahin et al. [10] studied Neutro-Topological Space and Anti-Topological Space. Smarandache [9] studied neutroalgebra as a generalization of partial algebra. Many researchers [11-19] studied neutroalgebra. In this study, group neutro-topological space and its properties are discussed.

Doi : https://doi.org/10.54216/JNFS.030104

Received: March 02, 2022 Accepted: May 27, 2022

2. Preliminaries

Definition 2.1: [9] The Neutro-sophistication of the Law

(i) Let X be a non-empty set and $*$ be binary operation. For some elements $(a,b) \in (X,X), (a*b) \in X$(degree of well defined (T)) and for other elements $(x,y),(p,q) \in (X,X)$; $[x*y$ is indeterminate (degree of indeterminacy (I)), or $p*q \notin X$ (degree of outer-defined (F)], where (T,I,F) is different from (1,0,0) that represents the Classical Law, and from (0,0,1) that represents the AntiLaw.

(ii) In neutro-algebra, the classical well-defined for $*$ binary operation is divided into three regions: degree of well-defined (T), degree of indeterminacy (I), and degree of outer-defined (F) similar to neutrosophic set and neutrosophic logic.

Definition 2.2: [10]

Let X be the non-empty set and τ be a collection of subsets of X. Then τ is said to be topology on X, and (X,τ) is said to be a topological space, if it satisfies the following conditions:

(i) $\emptyset, X \in \tau$.

(ii) $A \cap B \in \tau$ for any $A, B \in \tau$.

(iii) $\cup A_i \in \tau$ for any arbitrary family $\{A_i : i \in \Lambda\} \subseteq \tau$.

Definition 2.3: [10]

Let X be the non-empty set and τ be a collection of subsets of X. Then τ is said to be a neutro-topology on X, and the pair (X,τ) is said to be a neutro-topological space, if at least one of the following conditions hold good:

(i) $[(\emptyset_N \in \tau, X_N \notin \tau)\ or\ (X_N \in \tau, \emptyset_N \notin \tau)]$ or $[\emptyset_N, X_N \in_{\sim} \tau]$.

(ii) For any n elements $a_1, a_2, \dots, a_n \in \tau, \bigcap_{i=1}^{n} a_i \in \tau$ [degree of truth T] and for other n elements $b_1, b_2, \dots, b_n \in \tau, p_1, p_2, \dots, p_n \in (\bigcap_{i=1}^{n} b_i \notin \tau)$[degree of falsehood F]; [or $(\bigcap_{i=1}^{n} p_i$ is indeterminate (degree of indeterminacy$I$)], where n is finite; where(T,I,F) is different from (1,0,0) that represents the Classical Axiom, and from (0,0,1) that represents the AntiAxiom.

(iii) For any n elements $a_1, a_2, \dots, a_n \in \tau, \bigcup_{i=1} a_i \in \tau$[degree of truth T] and for other n elements $b_1, b_2, \dots, b_n \in \tau, p_1, p_2, \dots, p_n \in \tau$; $[(\bigcup_{i=I} b_i \notin \tau)$[degree of falsehood F] or $(\bigcup_{i=I} p_i$is indeterminate (degree of indeterminacy I)], where n is finite; where (T,I,F) is different from (1,0,0) that represents the Classical Axiom, and from (0,0,1) that represents the AntiAxiom.)].

Remark 2.1:[10]

The symbol "$\in_{\sim}$" will be used for situations where it is an unclear appurtenance (not sure if an element belongs or not to a set). For example, if it is not certain whether "a" is a member of the set P, then it is denoted by a $\in_{\sim} P$.

Theorem 2.1: [10]

Let (X,τ) be a classical topological space. Then $(X, \tau - \emptyset)$ is a neutro-topological space.

Theorem 2.2: [10]

Let (X,τ) be a classical topological space. Then $(X, \tau - X)$ is a neutro-topological space.

3. Main Results

Definition 3.1

Let G be a group. Then the collection(τ) of subgroups of G is said to be a neutro-topological space on G, and G with neutro-topological space (τ) is said to be a group neutro-topological space and is denoted by (G,τ). They are neutro-topological space and, respectively, neutro-topology (and not classical ones) since the first axiom is only partially true because the empty set does not belong to the neutro-topological space nor to τ.

Example 3.1

Let $G = \{1, -1, i, -i\}$ then $(G, \circ)$ be a group and $\tau = \{G, \{1\}, \{1, -1\}\}$ be a collection of subgroups of G. Then it is clear that $G \in \tau, \emptyset \notin \tau$.

Doi : https://doi.org/10.54216/JNFS.030104
Received: March 02, 2022 Accepted: May 27, 2022

Then it can be verified that (G,τ) satisfies the conditions of the group neutro-topological space.

Proposition 3.1

Let G be a group; then, we can form a group neutro-topological space (G,τ), where τ is the collection of subgroups of G.

Proof:
Clearly, $G \in \tau$ since $G \subseteq G$ and $\emptyset \notin \tau$.
Let $P, Q \in \tau$ be any two elements.
Then it is clear that P and Q are subgroups of G.
We have to show that $P \cap Q$ is a subgroup of G.
Let $x \in P \cap Q \Rightarrow x \in P \text{ and } x \in Q$
$\Rightarrow x^{-1} \in P \text{ and } x^{-1} \in Q$
$\Rightarrow x^{-1} \in P \cap Q$
Thus, $P \cap Q \in \tau$.
Let, $\{P_i : i \in \Lambda\} \subseteq \tau$, where P_i's are subgroups of G.
Case -1 If one subgroup is contained in other i.e., $P_i \subseteq P_i \subseteq P_i \subseteq \ldots \subseteq P_i$.
Let, $x \in \cup P_i \Rightarrow \exists \text{ an element } x_i \in P_i$, for some $i \in \Lambda$.
$\Rightarrow x_i^{-1} \in P_i$, for some i
$\Rightarrow x_i \in \cup P_i$
Thus, $\cup_{i\in\Lambda} P_i \in \tau$.
Case-2 If one subgroup is not contained in another subgroup, then clearly $\cup_{i\in\Lambda} P_i$ is not a group, and $\cup_{i\in\Lambda} P_i \notin \tau$.
Thus, it is clear that (G,τ) satisfies the conditions {i, ii, iii} in definition 2.3.
Hence τ is a neutro-topology on G and (G,τ) is a group neutro-topological space.

Theorem 3.1

A set $\mathcal{A}$ of the group neutro-topological space(G,τ) is τ-closed if and only if $\mathcal{A}^c$ is τ-open.

Proof:
Let $\mathcal{A}$ be a τ-open, then
$x \in \mathcal{A} \Rightarrow x^{-1} \in \mathcal{A}$
and let $x \in \mathcal{A}^c \Rightarrow x \notin \mathcal{A}$
$\Rightarrow x^{-1} \notin \mathcal{A}$
$\Rightarrow x^{-1} \in \mathcal{A}^c$
Thus, $x \in \mathcal{A}^c \Rightarrow x^{-1} \in \mathcal{A}^c$
Hence, $\mathcal{A}^c$ is τ-open set i.e., $\mathcal{A}$ is τ-closed set.
The converse part can be established similarly.

Theorem 3.2

Every subgroup of group G is τ-open set in group neutro-topological space.

Proof:
Let $\mathcal{A}$ be a subgroup of a group G.
To prove that $\mathcal{A} \in \tau$.
$x \in \mathcal{A} \Rightarrow x^{-1} \in \mathcal{A}$
Hence, $\mathcal{A} \in \tau$.

Remark 3.1

The converse of the above Theorem is not true in general, i.e., a subgroup is a τ-open set need not be subgroup of G. This follows from the following example.

Example 3.2

Let G be a group and τ be a collection of proper subgroups of G with $\emptyset$.
Then clearly, (τ, G) is group neutro-topological space.

Doi : https://doi.org/10.54216/JNFS.030104
Received: March 02, 2022 Accepted: May 27, 2022

Now, $\emptyset$ is an open set of τ which is not a subgroup of G.

Proposition 3.2

Let $\mathcal{A}$ be a subgroup of G, then $\forall\, x, y \in \mathcal{A}$; we have $xy \in \mathcal{A}$.

Proposition 3.3

Let (G, τ) be a group neutro-topological space on the group G and $\mathcal{A}, \mathcal{B} \in \tau$ such that $x \in \mathcal{A}, y \in \mathcal{B} \Rightarrow xy \in \mathcal{A}\,\mathcal{B}$.

Theorem 3.3

Let $\mathcal{A}$ be a τ-open set in group neutro-topological space(G, τ). Then $\mathcal{A}$ is subgroup of G if and only if $\forall\, x, y^{-1} \in \mathcal{A} \Rightarrow xy^{-1} \in \mathcal{A}$.

Proof: Let $\mathcal{A}$ be any subgroup of the group G.
Let $\mathcal{A} \in \tau$, then $x \in \mathcal{A} \Rightarrow x^{-1} \in \mathcal{A}$.
So, $\mathcal{A} \in \tau$ and is also subgroup of G, then the result follows by the Proposition 3.2.
Conversely, for any $x, y \in \mathcal{A} \Rightarrow xy^{-1} \in \mathcal{A}$

$$\Rightarrow x(y^{-1})^{-1} \in \mathcal{A}$$

$$\Rightarrow xy \in \mathcal{A}.$$

Hence, $\mathcal{A}$ is subgroup of G.

Theorem 3.4

If $\mathcal{A}$ and $\mathcal{B}$ are τ-open sets in G, then $\mathcal{A}\,\mathcal{B}$ is also τ-open set provided. G is an abelian group.

Proof:
Let$x \in \mathcal{A}$ and $y \in \mathcal{B}$, then by the Proposition 3.3, we have $xy \in \mathcal{A}\,\mathcal{B}$.
Since $\mathcal{A}, \mathcal{B} \in \tau$, so $x^{-1} \in \mathcal{A}$ and $y^{-1} \in \mathcal{B}$.
Again, by the Proposition 3.3, we have

$$(x^{-1}y^{-1}) \in \mathcal{A}\,\mathcal{B} \Rightarrow (yx)^{-1} \in \mathcal{A}\,\mathcal{B}$$

$$\Rightarrow (xy)^{-1} \in \mathcal{A}\,\mathcal{B} \ [\because G \text{ is abelian}]$$

Thus, $xy \in \mathcal{A}\,\mathcal{B} \Rightarrow (xy)^{-1} \in \mathcal{A}\,\mathcal{B}$

$$\Rightarrow \mathcal{A}\,\mathcal{B} \in \tau.$$

6. Conclusion

In this article, following the definition of neutro-topological space, group neutro-topological space is investigated. The definition of group neutro-topological space has been introduced. Some properties of the group neutro-topological space are investigated, and some of their properties have been established. It is expected that the work done will help in further investigation of the group neutro-topological space.

Funding: "This research received no external funding."

Conflicts of Interest: “The authors declare no conflict of interest.”

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Received: March 02, 2022 Accepted: May 27, 2022

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Doi : https://doi.org/10.54216/JNFS.030104
Received: March 02, 2022 Accepted: May 27, 2022