

Generalized Parametric Fuzzy Divergence Model Based on Single Valued Neutrosophic Sets

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ABSTRACT

In diverse real-life decision-making problems, the divergence models establish informative measures in conducting impreciseness and improbability surrounded by various factors of the decision-making process. This eagerness encourages one to instigate new fuzzy divergence models in convincing flexibility in assorted disciplines. The present communication is an accurate footstep in the direction of shaping a single-valued generalized neutrosophic parametric fuzzy divergence model and complete examination of its wide-ranging properties.

Keywords: Fuzzy divergence, Truth-membership function, Probability spaces, Single valued neutrosophic set.

INTRODUCTION

The distance models supply a crucial liability because of their applications in the direction of assortment fields and one of the core concerns is to discover such a well-fitting distance model in the probability spaces. Such a model is a dynamic tool for the pronouncement of innumerable solutions concerning multidisciplinary optimization problems. It has been a matter of fact that the basic essential and practical well-known divergence model in the probability spaces is owed to Kullback and Leibler [11] with the quantitative structure having legitimate properties. After this divergence model in probability spaces, numerous new divergence models have been produced by a variety of researchers. Some of these are:

$$D_{\lambda}(P:Q) = \frac{1}{\lambda} \sum_{i=1}^n (1 + \lambda p_i) \ln \frac{1 + \lambda p_i}{1 + \lambda q_i}, \lambda > 0 \quad (1.1)$$

which is Ferreri's [7] model.

$$D(P:Q) = \frac{1}{\alpha - 1} \ln \left(\sum_{i=1}^n p_i^{\alpha+\beta-1} q_i^{1-\alpha} \bigg/ \sum_{i=1}^n p_i^{\beta} \right) \quad (1.2)$$

which are Kapur [9] divergence models.

$$D_{\alpha}(P:Q) = \frac{1}{2^{\beta}-1} \left[\left(\sum_{i=1}^n p_i^{\alpha} q_i^{1-\alpha} \right)^{\frac{\beta-1}{\alpha-1}} - 1 \right], \alpha, \beta > 0, \alpha \neq 1, \beta \neq 1. \quad (1.3)$$

which is Sharma and Mittal's [19] model.

Lin [12] observed that Kullback and Leibler's [11] model is indeterminate if $q_i = 0$ and $p_i \neq 0$ and to overcome this problem, produced a modified description of Kullback and Leibler's [11] divergence. Parkash and Kumar [16] created an analogous divergence model in the subsequent look:

$$D(P;Q) = \sum_{i=1}^n p_i \ln \frac{p_i}{\left(\frac{\sqrt{p_i} + \sqrt{q_i}}{2} \right)^2} \quad (1.4)$$

Stimulated by the idea of weighted information wrought by Guiasu [8], Kapur [9] created quite a lot of suitable weighted divergence models, some of which are:

$$D_1(P;Q;W) = \sum_{i=1}^n w_i \left[\frac{p_i - p_i \ln p_i - q_i + q_i \ln q_i}{\ln q_i} + p_i - q_i \right] \quad (1.5)$$

Motivated by the technique employed by Kapur [9], Parkash, Kumar, and Kakkar [17] produced the subsequent weighted divergence model:

$$D_{\alpha}(P,Q;W) = \frac{1}{2^{1-\alpha}-1} \sum_{i=1}^n w_i \left[(\alpha-1) p_i - p_i^{\alpha} q_i^{1-\alpha} + \alpha p_i^{\frac{1}{\alpha}} q_i^{1-\frac{1}{\alpha}} + 2(1-\alpha) q_i \right], \alpha > 1 \quad (1.6)$$

Employing the above configuration, quite a lot of complementary weighted divergence models can be developed to deliver their applications.

On the erstwhile, Zadeh's [22] unbelievable theory of fuzzy sets extended the possibility of research and this novelty gave internment to divergence models in a fuzzy environment. Bhandari and Pal [3] recommended a divergence model for such situations whereas Couso, Jains, and Montes [4] twisted fuzzy divergence models to accept the prevalent situations. Some recent developments regarding the divergence models in the fuzzy environment have been made by Kapur [10], Parkash, Sharma, and Kumar [18], Markechová, Mosapour, and Ebrahimzadeh [13], Mishra et. al. [14], etc.

It is supplementary that the above hypothesis was additionally promoted with the perception of intuitionistic fuzzy sets (IFSs) originated by Atanassov [2]. Originally projected by Smarandache [20], neutrosophy is a stem of philosophy analyzing the ancestor, environment, and purview of neutralities, along with their exchanges with dissimilar approximate spectra. The quantified unambiguity in the neutrosophic set plays a critical responsibility in numerous applications. The intuitionistic fuzzy sets (IFSs) think regarding together types of memberships and are capable of controlling the partial information. A single-valued neutrosophic set (SVNS) delivers an episode of a neutrosophic set fashioned to make

available supplementary likelihood to signify uncertainty in the real world for applying in decision-making. Some pioneers to develop models on neutrosophic sets include Das et al. [5], Wei and Zhang [21], Pamucar and Bozanic [15], Abobala et al. [1], Zulfarnain et al. [23], Farooq, Saqlain and Rehman [6], etc.

The present communication consists of investigations and proposals of a new generalized parametric fuzzy divergence model based on SVNPs and has graceful properties to augment the employability of the model. In the sequel; we shape the model with certain well desirable properties indispensable for the development of our model.

2. GENERALIZED PARAMETRIC FUZZY DIVERGENCE MODEL BASED ON SVNPs

With the allocation of all standard notations of truth-membership function, indeterminacy-membership function, and falsity-membership function, let G and H be two fuzzy sets having these values as $T_G(x_d), I_G(x_d), F_G(x_d)$ and $T_H(x_d), I_H(x_d), F_H(x_d)$ respectively.

We now recommend the subsequent parametric fuzzy divergence model:

$$J_{r\{\gamma\}}^s(G, H) = \frac{r}{n(2-s)} \sum_{i=1}^n \left[T_G^{\frac{r}{2-s}}(x_d) \log \frac{T_G^{\frac{r}{2-s}}(x_d)}{\gamma T_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_H^{\frac{r}{2-s}}(x_d)} + \left(1 - T_G^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - T_G^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma T_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_H^{\frac{r}{2-s}}(x_d) \right)} \right. \\ \left. + I_G^{\frac{r}{2-s}}(x_d) \log \frac{I_G^{\frac{r}{2-s}}(x_d)}{\gamma I_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_H^{\frac{r}{2-s}}(x_d)} + \left(1 - I_G^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - I_G^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma I_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_H^{\frac{r}{2-s}}(x_d) \right)} \right. \\ \left. + F_G^{\frac{r}{2-s}}(x_d) \log \frac{F_G^{\frac{r}{2-s}}(x_d)}{\gamma F_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_H^{\frac{r}{2-s}}(x_d)} + \left(1 - F_G^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - F_G^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma F_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_H^{\frac{r}{2-s}}(x_d) \right)} \right];$$

$$0 \leq \gamma \leq 1; r, s > 0, s \neq 2$$

It may be noted from the definition of $J_{r\{\gamma\}}^s(G, H)$ is not a symmetric measure, so to imbue the measure with symmetry, a generalized symmetric fuzzy divergence measure based on neutrosophic sets can be defined as follows

Definition 2.1. A generalized symmetric fuzzy divergence model for two SVNPs G and H based on r, s and γ is given as

$$J_{r\{\gamma\}}^s(G, H) = J_{r\{\gamma\}}^s(G, H) + J_{r\{\gamma\}}^s(H, G)$$

From the definition of $D_E^{\hat{\alpha}}(\hat{A}, \hat{B})$, It has been observed that

$$(i) \quad D_E^{\hat{\alpha}}(\hat{A}, \hat{B}) \geq 0$$

- (ii) $D_{\hat{E}}^{\hat{\alpha}}(\hat{A}, \hat{B}) = 0$ if and only if $\mu_{\hat{A}}(x_i) = \mu_{\hat{B}}(x_i) \& \nu_{\hat{A}}(x_i) = \nu_{\hat{B}}(x_i)$
- (iii) $D_{\hat{E}}^{\hat{\alpha}}(\hat{A}, \hat{B}) \neq D_{\hat{E}}^{\hat{\alpha}}(\hat{B}, \hat{A})$ but $K_{\hat{E}}^{\hat{\alpha}}(\hat{A}, \hat{B}) = D_{\hat{E}}^{\hat{\alpha}}(\hat{A}, \hat{B}) + D_{\hat{E}}^{\hat{\alpha}}(\hat{B}, \hat{A})$ is symmetric.

We have confirmed that it satisfies every one advantageous possession of being a fuzzy divergence, that's why we assert that the above appearance represents an indisputable model of fuzzy divergence.

Property2.1. If G and H be the two IFSs defined on universal set X , such that they satisfy for any $x_i \in X$ either $G \subseteq H$ -or $G \supseteq H$, then

$$J_{r\{\gamma\}}^s(G \cup H; G \cap H) = J_{r\{\gamma\}}^s(G; H).$$

Proof: It is clear that

$$J_{r\{\gamma\}}^s(G \cup H; G \cap H) = J_{r\{\gamma\}}^s(G \cup H | G \cap H) + J_{r\{\gamma\}}^s(G \cap H | G \cup H)$$

$$\text{Now, } J_{r\{\gamma\}}^s(G \cup H | G \cap H)$$

$$= \frac{r}{n(2-s)} \sum_{i=1}^n \left[\begin{aligned} & T_{G \cup H}^{\frac{r}{2-s}}(x_d) \log \frac{T_{G \cup H}^{\frac{r}{2-s}}(x_d)}{\gamma T_{G \cup H}^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_{G \cap H}^{\frac{r}{2-s}}(x_d)} + \left(1 - T_{G \cup H}^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - T_{G \cup H}^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma T_{G \cup H}^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_{G \cap H}^{\frac{r}{2-s}}(x_d) \right)} \\ & + I_{G \cup H}^{\frac{r}{2-s}}(x_d) \log \frac{I_{G \cup H}^{\frac{r}{2-s}}(x_d)}{\gamma I_{G \cup H}^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_{G \cap H}^{\frac{r}{2-s}}(x_d)} + \left(1 - I_{G \cup H}^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - I_{G \cup H}^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma I_{G \cup H}^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_{G \cap H}^{\frac{r}{2-s}}(x_d) \right)} \\ & + F_{G \cup H}^{\frac{r}{2-s}}(x_d) \log \frac{F_{G \cup H}^{\frac{r}{2-s}}(x_d)}{\gamma F_{G \cup H}^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_{G \cap H}^{\frac{r}{2-s}}(x_d)} + \left(1 - F_{G \cup H}^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - F_{G \cup H}^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma F_{G \cup H}^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_{G \cap H}^{\frac{r}{2-s}}(x_d) \right)} \end{aligned} \right]$$

$$= \frac{r}{n(2-s)} \sum_{x \in X_1} \left[\begin{aligned} & T_H^{\frac{r}{2-s}}(x_d) \log \frac{T_H^{\frac{r}{2-s}}(x_d)}{\gamma T_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_G^{\frac{r}{2-s}}(x_d)} + \left(1 - T_H^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - T_H^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma T_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_G^{\frac{r}{2-s}}(x_d) \right)} \\ & + I_H^{\frac{r}{2-s}}(x_d) \log \frac{I_H^{\frac{r}{2-s}}(x_d)}{\gamma I_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_G^{\frac{r}{2-s}}(x_d)} + \left(1 - I_H^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - I_H^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma I_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_G^{\frac{r}{2-s}}(x_d) \right)} \\ & + F_H^{\frac{r}{2-s}}(x_d) \log \frac{F_H^{\frac{r}{2-s}}(x_d)}{\gamma F_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_G^{\frac{r}{2-s}}(x_d)} + \left(1 - F_H^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - F_H^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma F_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_G^{\frac{r}{2-s}}(x_d) \right)} \end{aligned} \right]$$

$$\begin{aligned}
 & \left[T_G^{\frac{r}{2-s}}(x_d) \log \frac{T_G^{\frac{r}{2-s}}(x_d)}{\gamma T_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_H^{\frac{r}{2-s}}(x_d)} + \left(1 - T_G^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - T_G^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma T_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_H^{\frac{r}{2-s}}(x_d) \right)} \right. \\
 & + I_G^{\frac{r}{2-s}}(x_d) \log \frac{I_G^{\frac{r}{2-s}}(x_d)}{\gamma I_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_H^{\frac{r}{2-s}}(x_d)} + \left(1 - I_G^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - I_G^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma I_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_H^{\frac{r}{2-s}}(x_d) \right)} \\
 & \left. + F_G^{\frac{r}{2-s}}(x_d) \log \frac{F_G^{\frac{r}{2-s}}(x_d)}{\gamma F_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_H^{\frac{r}{2-s}}(x_d)} + \left(1 - F_G^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - F_G^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma F_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_H^{\frac{r}{2-s}}(x_d) \right)} \right] \\
 & + \frac{r}{n(2-s)} \sum_{x \in X_2}
 \end{aligned}$$

$$J_{r\{\gamma\}}^s(G \cap H | G \cup H)$$

$$\begin{aligned}
 & \left[T_{G \cap H}^{\frac{r}{2-s}}(x_d) \log \frac{T_{G \cap H}^{\frac{r}{2-s}}(x_d)}{\gamma T_{G \cap H}^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_{G \cup H}^{\frac{r}{2-s}}(x_d)} + \left(1 - T_{G \cap H}^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - T_{G \cap H}^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma T_{G \cap H}^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_{G \cup H}^{\frac{r}{2-s}}(x_d) \right)} \right. \\
 & + I_{G \cap H}^{\frac{r}{2-s}}(x_d) \log \frac{I_{G \cap H}^{\frac{r}{2-s}}(x_d)}{\gamma I_{G \cap H}^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_{G \cup H}^{\frac{r}{2-s}}(x_d)} + \left(1 - I_{G \cap H}^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - I_{G \cap H}^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma I_{G \cap H}^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_{G \cup H}^{\frac{r}{2-s}}(x_d) \right)} \\
 & \left. + F_{G \cap H}^{\frac{r}{2-s}}(x_d) \log \frac{F_{G \cap H}^{\frac{r}{2-s}}(x_d)}{\gamma F_{G \cap H}^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_{G \cup H}^{\frac{r}{2-s}}(x_d)} + \left(1 - F_{G \cap H}^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - F_{G \cap H}^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma F_{G \cap H}^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_{G \cup H}^{\frac{r}{2-s}}(x_d) \right)} \right] \\
 & = \frac{r}{n(2-s)} \sum_{i=1}^n
 \end{aligned}$$

$$\begin{aligned}
 & \left[T_G^{\frac{r}{2-s}}(x_d) \log \frac{T_G^{\frac{r}{2-s}}(x_d)}{\gamma T_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_H^{\frac{r}{2-s}}(x_d)} + \left(1 - T_G^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - T_G^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma T_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_H^{\frac{r}{2-s}}(x_d) \right)} \right. \\
 & + I_G^{\frac{r}{2-s}}(x_d) \log \frac{I_G^{\frac{r}{2-s}}(x_d)}{\gamma I_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_H^{\frac{r}{2-s}}(x_d)} + \left(1 - I_G^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - I_G^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma I_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_H^{\frac{r}{2-s}}(x_d) \right)} \\
 & \left. + F_G^{\frac{r}{2-s}}(x_d) \log \frac{F_G^{\frac{r}{2-s}}(x_d)}{\gamma F_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_H^{\frac{r}{2-s}}(x_d)} + \left(1 - F_G^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - F_G^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma F_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_H^{\frac{r}{2-s}}(x_d) \right)} \right] \\
 & = \frac{r}{n(2-s)} \sum_{x \in X_1}
 \end{aligned}$$

$$\begin{aligned}
& + \frac{r}{n(2-s)} \sum_{x \in X_2} \left[\begin{aligned} & T_H^{\frac{r}{2-s}}(x_d) \log \frac{T_H^{\frac{r}{2-s}}(x_d)}{\gamma T_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_G^{\frac{r}{2-s}}(x_d)} + \left(1 - T_H^{\frac{r}{2-s}}(x_d)\right) \log \frac{1 - T_H^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma T_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_G^{\frac{r}{2-s}}(x_d)\right)} \\ & + I_H^{\frac{r}{2-s}}(x_d) \log \frac{I_H^{\frac{r}{2-s}}(x_d)}{\gamma I_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_G^{\frac{r}{2-s}}(x_d)} + \left(1 - I_H^{\frac{r}{2-s}}(x_d)\right) \log \frac{1 - I_H^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma I_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_G^{\frac{r}{2-s}}(x_d)\right)} \\ & + F_H^{\frac{r}{2-s}}(x_d) \log \frac{F_H^{\frac{r}{2-s}}(x_d)}{\gamma F_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_G^{\frac{r}{2-s}}(x_d)} + \left(1 - F_H^{\frac{r}{2-s}}(x_d)\right) \log \frac{1 - F_H^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma F_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_G^{\frac{r}{2-s}}(x_d)\right)} \end{aligned} \right]
\end{aligned}$$

By adding the above equations, we get

$$\begin{aligned}
J_{r\{\gamma\}}^s(G \cup H | G \cap H) + J_{r\{\gamma\}}^s(G \cap H | G \cup H) &= J_{r\{\gamma\}}^s(G | H) + J_{r\{\gamma\}}^s(H | G) \\
&= J_{r\{\gamma\}}^s(G; H)
\end{aligned}$$

Hence the result holds.

Property 2.2. For any two SVNSSs G and H , we have

- (a) $J_{r\{\gamma\}}^s(G; G \cup H) = J_{r\{\gamma\}}^s(H; G \cap H)$,
- (b) $J_{r\{\gamma\}}^s(G; G \cap H) = J_{r\{\gamma\}}^s(H; G \cup H)$,
- (c) $J_{r\{\gamma\}}^s(G; G \cup H) + J_{r\{\gamma\}}^s(G; G \cap H) = J_{r\{\gamma\}}^s(G; H)$,
- (d) $J_{r\{\gamma\}}^s(H; G \cup H) + J_{r\{\gamma\}}^s(H; G \cap H) = J_{r\{\gamma\}}^s(G; H)$.

Proof: We prove only (a). All other parts (b), (c), and (d)) can be proved similarly.

$$J_{r\{\gamma\}}^s(G | G \cup H) + J_{r\{\gamma\}}^s(G \cap H | G) = J_{r\{\gamma\}}^s(H | G \cap H) + J_{r\{\gamma\}}^s(G \cap H | H).$$

From the definition of divergence, let $J_{r\{\gamma\}}^s(G | G \cup H)$

$$\begin{aligned}
& = \frac{r}{n(2-s)} \sum_{i=1}^n \left[\begin{aligned} & T_G^{\frac{r}{2-s}}(x_d) \log \frac{T_G^{\frac{r}{2-s}}(x_d)}{\gamma T_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_{G \cup H}^{\frac{r}{2-s}}(x_d)} + \left(1 - T_G^{\frac{r}{2-s}}(x_d)\right) \log \frac{1 - T_G^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma T_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_{G \cup H}^{\frac{r}{2-s}}(x_d)\right)} \\ & + I_G^{\frac{r}{2-s}}(x_d) \log \frac{I_G^{\frac{r}{2-s}}(x_d)}{\gamma I_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_{G \cup H}^{\frac{r}{2-s}}(x_d)} + \left(1 - I_G^{\frac{r}{2-s}}(x_d)\right) \log \frac{1 - I_G^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma I_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_{G \cup H}^{\frac{r}{2-s}}(x_d)\right)} \\ & + F_G^{\frac{r}{2-s}}(x_d) \log \frac{F_G^{\frac{r}{2-s}}(x_d)}{\gamma F_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_{G \cup H}^{\frac{r}{2-s}}(x_d)} + \left(1 - F_G^{\frac{r}{2-s}}(x_d)\right) \log \frac{1 - F_G^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma F_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_{G \cup H}^{\frac{r}{2-s}}(x_d)\right)} \end{aligned} \right]
\end{aligned}$$

$$\begin{aligned}
& \left[T_G^{\frac{r}{2-s}}(x_d) \log \frac{T_G^{\frac{r}{2-s}}(x_d)}{\gamma T_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_H^{\frac{r}{2-s}}(x_d)} + \left(1 - T_G^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - T_G^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma T_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_H^{\frac{r}{2-s}}(x_d) \right)} \right. \\
& + I_G^{\frac{r}{2-s}}(x_d) \log \frac{I_G^{\frac{r}{2-s}}(x_d)}{\gamma I_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_H^{\frac{r}{2-s}}(x_d)} + \left(1 - I_G^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - I_G^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma I_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_H^{\frac{r}{2-s}}(x_d) \right)} \\
& \left. + F_G^{\frac{r}{2-s}}(x_d) \log \frac{F_G^{\frac{r}{2-s}}(x_d)}{\gamma F_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_H^{\frac{r}{2-s}}(x_d)} + \left(1 - F_G^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - F_G^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma F_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_H^{\frac{r}{2-s}}(x_d) \right)} \right] \\
& + \frac{r}{n(2-s)} \sum_{x \in X_2} \left[T_G^{\frac{r}{2-s}}(x_d) \log \frac{T_G^{\frac{r}{2-s}}(x_d)}{\gamma T_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_G^{\frac{r}{2-s}}(x_d)} + \left(1 - T_G^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - T_G^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma T_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_G^{\frac{r}{2-s}}(x_d) \right)} \right. \\
& + I_G^{\frac{r}{2-s}}(x_d) \log \frac{I_G^{\frac{r}{2-s}}(x_d)}{\gamma I_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_G^{\frac{r}{2-s}}(x_d)} + \left(1 - I_G^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - I_G^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma I_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_G^{\frac{r}{2-s}}(x_d) \right)} \\
& \left. + F_G^{\frac{r}{2-s}}(x_d) \log \frac{F_G^{\frac{r}{2-s}}(x_d)}{\gamma F_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_G^{\frac{r}{2-s}}(x_d)} + \left(1 - F_G^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - F_G^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma F_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_G^{\frac{r}{2-s}}(x_d) \right)} \right] \\
& = \frac{r}{n(2-s)} \sum_{x \in X_1} \left[T_G^{\frac{r}{2-s}}(x_d) \log \frac{T_G^{\frac{r}{2-s}}(x_d)}{\gamma T_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_H^{\frac{r}{2-s}}(x_d)} + \left(1 - T_G^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - T_G^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma T_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_H^{\frac{r}{2-s}}(x_d) \right)} \right. \\
& + I_G^{\frac{r}{2-s}}(x_d) \log \frac{I_G^{\frac{r}{2-s}}(x_d)}{\gamma I_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_H^{\frac{r}{2-s}}(x_d)} + \left(1 - I_G^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - I_G^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma I_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_H^{\frac{r}{2-s}}(x_d) \right)} \\
& \left. + F_G^{\frac{r}{2-s}}(x_d) \log \frac{F_G^{\frac{r}{2-s}}(x_d)}{\gamma F_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_H^{\frac{r}{2-s}}(x_d)} + \left(1 - F_G^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - F_G^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma F_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_H^{\frac{r}{2-s}}(x_d) \right)} \right]
\end{aligned}$$

and $J_{r\{\gamma\}}^s(G \cup H|G)$

$$\begin{aligned}
& \left[T_{G \cup H}^{\frac{r}{2-s}}(x_d) \log \frac{T_{G \cup H}^{\frac{r}{2-s}}(x_d)}{\gamma T_{G \cup H}^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_G^{\frac{r}{2-s}}(x_d)} + \left(1 - T_{G \cup H}^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - T_{G \cup H}^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma T_{G \cup H}^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_G^{\frac{r}{2-s}}(x_d) \right)} \right. \\
& + I_{G \cup H}^{\frac{r}{2-s}}(x_d) \log \frac{I_{G \cup H}^{\frac{r}{2-s}}(x_d)}{\gamma I_{G \cup H}^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_G^{\frac{r}{2-s}}(x_d)} + \left(1 - I_{G \cup H}^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - I_{G \cup H}^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma I_{G \cup H}^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_G^{\frac{r}{2-s}}(x_d) \right)} \\
& \left. + F_{G \cup H}^{\frac{r}{2-s}}(x_d) \log \frac{F_{G \cup H}^{\frac{r}{2-s}}(x_d)}{\gamma F_{G \cup H}^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_G^{\frac{r}{2-s}}(x_d)} + \left(1 - F_{G \cup H}^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - F_{G \cup H}^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma F_{G \cup H}^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_G^{\frac{r}{2-s}}(x_d) \right)} \right] \\
& = \frac{r}{n(2-s)} \sum_{i=1}^n \left[T_H^{\frac{r}{2-s}}(x_d) \log \frac{T_H^{\frac{r}{2-s}}(x_d)}{\gamma T_H^{\frac{r}{2-s}}(x_i) + (1-\gamma) T_G^{\frac{r}{2-s}}(x_d)} + \left(1 - T_H^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - T_H^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma T_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_G^{\frac{r}{2-s}}(x_d) \right)} \right. \\
& + I_H^{\frac{r}{2-s}}(x_d) \log \frac{I_H^{\frac{r}{2-s}}(x_d)}{\gamma I_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_G^{\frac{r}{2-s}}(x_d)} + \left(1 - I_H^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - I_H^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma I_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_G^{\frac{r}{2-s}}(x_d) \right)} \\
& \left. + F_H^{\frac{r}{2-s}}(x_d) \log \frac{F_H^{\frac{r}{2-s}}(x_d)}{\gamma F_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_G^{\frac{r}{2-s}}(x_d)} + \left(1 - F_H^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - F_H^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma F_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_G^{\frac{r}{2-s}}(x_d) \right)} \right] \\
& = \frac{r}{n(2-s)} \sum_{x \in X_1} \left[T_G^{\frac{r}{2-s}}(x_d) \log \frac{T_G^{\frac{r}{2-s}}(x_d)}{\gamma T_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_G^{\frac{r}{2-s}}(x_d)} + \left(1 - T_G^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - T_G^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma T_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_G^{\frac{r}{2-s}}(x_d) \right)} \right. \\
& + I_G^{\frac{r}{2-s}}(x_d) \log \frac{I_G^{\frac{r}{2-s}}(x_d)}{\gamma I_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_G^{\frac{r}{2-s}}(x_d)} + \left(1 - I_G^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - I_G^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma I_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_G^{\frac{r}{2-s}}(x_d) \right)} \\
& \left. + F_G^{\frac{r}{2-s}}(x_d) \log \frac{F_G^{\frac{r}{2-s}}(x_d)}{\gamma F_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_G^{\frac{r}{2-s}}(x_d)} + \left(1 - F_G^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - F_G^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma F_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_G^{\frac{r}{2-s}}(x_d) \right)} \right]
\end{aligned}$$

$$\begin{aligned}
 & \left[T_H^{\frac{r}{2-s}}(x_d) \log \frac{T_H^{\frac{r}{2-s}}(x_d)}{\gamma T_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_G^{\frac{\hat{\alpha}}{2-\hat{\beta}}}(x_d)} + \left(1 - T_H^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - T_H^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma T_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_G^{\frac{\hat{\alpha}}{2-\hat{\beta}}}(x_d) \right)} \right. \\
 & + I_H^{\frac{r}{2-s}}(x_d) \log \frac{I_H^{\frac{r}{2-s}}(x_d)}{\gamma I_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_G^{\frac{\hat{\alpha}}{2-\hat{\beta}}}(x_d)} + \left(1 - I_H^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - I_H^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma I_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_G^{\frac{\hat{\alpha}}{2-\hat{\beta}}}(x_d) \right)} \\
 & \left. + F_H^{\frac{r}{2-s}}(x_d) \log \frac{F_H^{\frac{r}{2-s}}(x_d)}{\gamma F_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_G^{\frac{\hat{\alpha}}{2-\hat{\beta}}}(x_d)} + \left(1 - F_H^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - F_H^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma F_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_G^{\frac{\hat{\alpha}}{2-\hat{\beta}}}(x_d) \right)} \right] \\
 & = \frac{r}{n(2-s)} \sum_{x \in X_1}
 \end{aligned}$$

Similarly, $J_{r\{\gamma\}}^s(H|G \cap H)$

$$\begin{aligned}
 & \left[T_G^{\frac{r}{2-s}}(x_d) \log \frac{T_G^{\frac{r}{2-s}}(x_d)}{\gamma T_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_H^{\frac{r}{2-s}}(x_d)} + \left(1 - T_G^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - T_G^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma T_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_H^{\frac{r}{2-s}}(x_d) \right)} \right. \\
 & + I_G^{\frac{r}{2-s}}(x_d) \log \frac{I_G^{\frac{r}{2-s}}(x_d)}{\gamma I_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_H^{\frac{r}{2-s}}(x_d)} + \left(1 - I_G^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - I_G^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma I_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_H^{\frac{r}{2-s}}(x_d) \right)} \\
 & \left. + F_G^{\frac{r}{2-s}}(x_d) \log \frac{F_G^{\frac{r}{2-s}}(x_d)}{\gamma F_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_H^{\frac{r}{2-s}}(x_d)} + \left(1 - F_G^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - F_G^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma F_G^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_H^{\frac{r}{2-s}}(x_d) \right)} \right] \\
 & = \frac{r}{n(2-s)} \sum_{x \in X_2}
 \end{aligned}$$

and,

$J_{r\{\gamma\}}^s(G \cap H|H)$

$$\begin{aligned}
 & \left[T_H^{\frac{r}{2-s}}(x_d) \log \frac{T_H^{\frac{r}{2-s}}(x_d)}{\gamma T_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_G^{\frac{r}{2-s}}(x_d)} + \left(1 - T_H^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - T_H^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma T_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) T_G^{\frac{r}{2-s}}(x_d) \right)} \right. \\
 & + I_H^{\frac{r}{2-s}}(x_d) \log \frac{I_H^{\frac{r}{2-s}}(x_d)}{\gamma I_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_G^{\frac{r}{2-s}}(x_d)} + \left(1 - I_H^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - I_H^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma I_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) I_G^{\frac{r}{2-s}}(x_d) \right)} \\
 & \left. + F_H^{\frac{r}{2-s}}(x_d) \log \frac{F_H^{\frac{r}{2-s}}(x_d)}{\gamma F_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_G^{\frac{r}{2-s}}(x_d)} + \left(1 - F_H^{\frac{r}{2-s}}(x_d) \right) \log \frac{1 - F_H^{\frac{r}{2-s}}(x_d)}{1 - \left(\gamma F_H^{\frac{r}{2-s}}(x_d) + (1-\gamma) F_G^{\frac{r}{2-s}}(x_d) \right)} \right] \\
 & = \frac{r}{n(2-s)} \sum_{x \in X_1}
 \end{aligned}$$

By using equation (2.1), we get the result.

CONCLUDING REMARKS

In the dynamic invented story of divergence models, one gets inspired to summarize pioneering models to persuade flexibility in divergent disciplines. A new-fangled fuzzy divergence model on SVN_Ss with valuable properties has been investigated in the present communication. Such numerous innovative fuzzy divergence models can be produced for the continuance of research in these disciplines.

REFERENCES

- [1] Abobala, M., Hatip, A., Olgun, N., Broumi, S., Salama, A. and Khaled, H.E. (2021). The algebraic creativity in the neutrosophic square matrices. *Neutrosophic Sets and Systems*, 40, 1-11.
- [2] Atanassov, K.T.(1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20, 87-96.
- [3] Bhandari, D., Pal, N.R. (1993). Some new information measure for fuzzy sets. *Information Science*, 67(3), 209-228.
- [4] Couso, I., Jains, V., Montes, S. (2000). Fuzzy divergence measures. *Acta Univ. M. Belii. Ser. Math.*, 8, 21-26.
- [5] Das, S., Roy, B.K., Kar, M.B., Kar, S. and Pamucar D. (2020). Neutrosophic fuzzy set and its application in decision making. *Journal of Ambient Intelligence and Humanized Computing* .<https://doi.org/10.1007/s12652-020-01808-3>.
- [6] Farooq, M. U., Saqlain, M. and Rehman, Z. (2021). The selection of LASER as surgical instrument in medical using neutrosophic soft set with generalized fuzzy TOPSIS, WSM and WPM along with MATLAB coding. *Neutrosophic Sets and Systems*, 40,29-44.
- [7] Ferreri, C. (1980). Hypoentropy and related heterogeneity divergence measures. *Statistica* 40, 155-168.
- [8] Guiasu, S. (1971). Weighted entropy. *Reports on Mathematical Physics*, 2, 165-171.
- [9] Kapur, J.N. (1995). Measures of Information and their Applications, Wiley Eastern, New York.
- [10] Kapur, J.N.(1997). Measures of fuzzy information. *Mathematical Sciences Trust Society*, Delhi.
- [11]Kullback, S. and Leibler, R.A. (1951). On information and sufficiency. *The Annals Mathematical Statistics*, 22(1), 79–86.
- [12]Lin, J.(1991). Divergence measure based on Shannon entropy. *IEEE Transactions on Information Theory*, 37(1),145-151.
- [13]Markechová, D.,Mosapour, B. and Ebrahimzadeh, A. (2018).R-norm entropy and R-norm divergence in fuzzy probability spaces.*Entropy*,20(4),Paper No. 272, 18 pp.
- [14]Mishra, A.R., Rani, P., Pardasani, K.R., Mardani, A., Stevic, Z. and Pamucar, D. (2020). A novel entropy and divergence measures with multi-criteria service quality assessment using interval-valued intuitionistic fuzzy TODIM method. *Soft Computing*, <https://doi.org/10.1007/s00500-019-04627-7>.

- [15] Pamucar D. and Bozanic, D. (2019). "Selection of a location for the development of multimodal logistics centre: application of single-valued neutrosophic MABAC model". *Operational Research in Engineering Sciences: Theory and Applications*, 2(2), 55–71.
- [16] Parkash, O. and Kumar, R. (2016): New parametric and non –parametric measures of cross entropy, *Canadian Journal of Pure and Applied Sciences*, 10(2), 3921-3925.
- [17] Parkash, O., Kumar, R. and Kakkar, P. (2017). Two new weighted divergence measures in probability spaces. *International Journal of Pure and Applied Mathematics* 116(19), 643-653.
- [18] Parkash, O., Sharma, P.K., Kumar, S. (2006). Two new measures of fuzzy divergence and their properties. *SQU Journal for Science*, 11, 69-77.
- [19] Sharma, B.D. and Mittal, D.P.(1977). New Non-additive measures of relative information. *Journal of Combinatorics, Information and System Science*, 2, 122-133.
- [20] Smarandache, F. (1999). A unifying field in logics neutrosophy: neutrosophic probability, set and logic, American Research Press, Rehoboth.
- [21] Wei, G. and Zhang, Z. (2019). Some single-valued neutrosophic Bonferroni power aggregation operators in multiple attribute decision making. *Journal of Ambient Intelligence and Humanized Computing*, 10, 863–882.
- [22] Zadeh, L.A. (1965). Fuzzy sets. *Information and Control*, 8(3), 338-353.
- [23] Zulqarnain, R. M., Xin, X. L., Ali, B., Broumi, S., Abdal, S. and Ahmad, M. I. (2021). Decision-making approach based on correlation coefficient with its properties under interval-valued neutrosophic hyper soft set environment. *Neutrosophic Sets and Systems*, 40, 12-28.