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# Properties of Detour Central and Detour Boundary Vertices in Neutrosophic Graphs

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## Abstract

In this paper, the idea of neutrosophic detour boundary vertices and neutrosophic detour center vertices in neutrosophic graph is introduced. Here the concept of neutrosophic detour eccentricity, neutrosophic detour radius, neutrosophic detour diameter, neutrosophic detour neighbourhood vertex are studied. Several interesting properties related to of eccentric nodes, peripheral nodes, and central nodes in neutrosophic graphs are established.

**Keywords:** Neutrosophic detour eccentricity; Neutrosophic detour centre; Neutrosophic detour self-centered graph; Neutrosophic detour boundary vertex; Neutrosophic detour boundary graph

## 1. Introduction

The concept of “Fuzzy sets” was introduced by L. Zadeh [6] in 1965, by introducing the concept of degree of true membership. It was the first successful attempt towards incorporating non – probabilistic uncertainty. Since then in many applications involving uncertainty, fuzzy sets and fuzzy logic are applied. Atanassov in 1986, further generalized the fuzzy sets and introduced “Intuitionistic Fuzzy Sets (IFS)”, by adding the concept of degree of false membership to the fuzzy sets ( $\langle T_A(x), F_A(x) \rangle$ ). In adding to it, there was an restriction that the sum of true membership and false membership is less than or equal to one. Atanassov also introduced the concept of “Intuitionistic Fuzzy Graphs” and “Intuitionistic Fuzzy Relation”. Although fuzzy set theory is very successful in handling uncertainties, it couldn't handle all sorts of uncertainties such as problems involving incomplete information.

Recently a new theory which is known as Neutrosophic logic and sets has been introduced. Florentin Smarandache [4], in 1995, introduced “Neutrosophic Logic” by adding the concept of degree of indeterminate membership to the Intuitionistic fuzzy set. A neutrosophic is a set in which every element has a degree of truth, indeterminacy and falsity values  $\langle T_A(x), I_A(x), F_A(x) \rangle$ . NSs are generalized form of IFS, as there are no limitations between the degree of truth, degree of indeterminacy and degree of falsity. All these degrees lie between  $[0, 1]$  such that  $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ . Neutrosophy as a newly-born science is a branch of philosophy that studies the origin, nature and scope of neutralities.

The concept of detour hasn't ever been introduced on any kind of NGs. The concept of fuzzy detour  $\mu$ -distance was introduced by Nagoor Gani and Umamaheswari [2,3]. In this paper, a new idea of distance called Neutrosophic Detour distance is introduced. Neutrosophic detour boundary vertices, NDC vertices and their properties with necessary examples are studied. Note that in this paper connected neutrosophic graph and spanning neutrosophic graph is denoted by CNG and SNG respectively.

## 2. Preliminaries

### Definition 2.1 [5]

Let  $Y$  be a given non – empty set. A Neutrosophic set (NS)  $A$  in  $Y$  is characterized by a true membership function  $T_A$  an indeterminate membership function  $I_A$  and a false membership function  $F_A$  which are defined as

$T_A, I_A, F_A : Y \rightarrow [0, 1]$  such that  $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ , for every  $x$  in  $Y$ .

### Definition 2.2 [5]

Let  $G^* = (V, E)$  be a Graph. A Neutrosophic graph (NG) is a pair  $G = (A, B)$  of  $G^*$ , where

$A : V \rightarrow [0, 1]$  is a NS in  $V$  and  $B : E \rightarrow [0, 1]$  is a NS in  $E$  such that

$$T_B(xy) \leq \{T_A(x) \wedge T_A(y)\}$$

$$I_B(xy) \leq \{I_A(x) \wedge I_A(y)\}$$

$$F_B(xy) \geq \{F_A(x) \vee F_A(y)\} \quad \forall \{x, y\} \in V$$

### Definition 2.3 [5]

Let  $G = (A, B)$  be a NG on a given set  $Y$ .  $G$  is called a CNG if the following conditions are satisfied.

$$T_B(x, y) = \{T_A(x) \wedge T_A(y)\}$$

$$I_B(x, y) = \{I_A(x) \wedge I_A(y)\}$$

$$F_B(x, y) = \{F_A(x) \vee F_A(y)\} \quad \forall \{x, y\} \in Y$$

### Definition 2.4 [5]

A NG  $G_I = (A_I, B_I)$  is called a N subgraph of  $G = (A, B)$  if

$$i. A_I \subseteq A \quad \forall x \in Y_I \subseteq Y$$

$$ii. B_I = B \quad \forall (x, y) \in E_I = E.$$

### Definition 2.5 [5]

Let  $G = (A, B)$ ,  $G_I = (A_I, B_I)$  be NG on set  $Y$ . Then  $G$  is called the SNG of  $G$  if

$$i. A_I = A \quad \forall x \in Y_I = Y$$

$$ii. B_I \subseteq B \quad \forall (x, y) \in E_I \subseteq E.$$

### Definition 2.6 [5]

Let  $G = (A, B)$  be a NG on  $Y$  and  $y_0, y_n$  be two given vertices such that  $n \in \mathbb{N}$ . Then,

a) A distinct sequence of vertices  $P : y_0, y_1, \dots, y_n$  in  $G$  is called a  $T$ -path of length  $n$  from  $y_0$  to  $y_n$ , if  $T_B(y_i, y_{i+1}) > 0$ , for  $i = 0, 1, \dots, n-1$ .

•  $T(P)_G = \min \{T_B(y_i, y_{i+1})\}$  is called the *strength* of this  $T$ -path with respect to path  $P$ .

b) A distinct sequence of vertices  $P : y_0, y_1, \dots, y_n$  in  $G$  is called a  $I$ -path of length  $n$  from  $y_0$  to  $y_n$ , if  $I_B(y_i, y_{i+1}) > 0$ , for  $i = 0, 1, \dots, n-1$ .

•  $I(P)_G = \min \{I_B(y_i, y_{i+1})\}$  is called the *strength* of this  $I$ -path with respect to path  $P$ .

- c) A distinct sequence of vertices  $P : y_0, y_1, \dots, y_n$  in  $G$  is called a  $F$ -path of length  $n$  from  $y_0$  to  $y_n$ , if  $F_B(y_i, y_{i+1}) > 0$ , for  $i = 0, 1, \dots, n-1$ .
- $F(P)_G = \min \{F_B(y_i, y_{i+1})\}$  is called the *strength* of this  $F$ -path with respect to path  $P$ .

### 3. A view on Neutrosophic Detour Distance

**Definition 3.1:** The length of a path  $P : x_0, \dots, x_n$  is  $l(P) = \langle T_l(P), I_l(P), F_l(P) \rangle$

where,  $T_l(P) = \sum_{i=1}^n \frac{1}{T_B(v_i, v_{i+1})}$

$$I_l(P) = \sum_{i=1}^n \frac{1}{I_B(v_i, v_{i+1})}$$

$$F_l(P) = \sum_{i=1}^n \frac{1}{F_B(v_i, v_{i+1})}$$

**Definition 3.2:** Let  $G = (A, B)$  be a CNG. The Neutrosophic detour distance (NDD)  $D_\Delta(x, y) = \langle T_\Delta(x, y), I_\Delta(x, y), F_\Delta(x, y) \rangle$  between vertices  $x$  and  $y$  is defined to be the maximum length of any  $x$ - $y$  path.

Similarly, the N distance  $D_\delta(x, y) = \langle T_\delta(x, y), I_\delta(x, y), F_\delta(x, y) \rangle$  is the minimum length of any  $x$ - $y$  path.

**Definition 3.3:** Let  $G = (A, B)$  be a NG. The strength of connectedness between two vertices  $x$  and  $y$  is defined as

$$T^c(x, y) = \max \{T_\Delta(x, y)\}$$

$$I^c(x, y) = \max \{I_\Delta(x, y)\} \text{ and}$$

$$F^c(x, y) = \max \{F_\Delta(x, y)\} \quad \forall \{x, y\} \in A_I$$

for each,  $\langle T_\Delta(x, y), I_\Delta(x, y), F_\Delta(x, y) \rangle > 0$

**Definition 3.4:** A NG  $G = (A, B)$  is a CNG if  $\langle T^c(x, y), I^c(x, y), F^c(x, y) \rangle > 0$ ,  $\forall \{x, y\} \in Y$ .

**Definition 3.5:** A ND eccentricity of a vertex  $v$  of a NG  $G = (A, B)$  is the maximum ND distance from  $v$  to any vertex of  $G$  i.e.,  $e_\Delta(v) = \langle T_e(v), I_e(v), F_e(v) \rangle$  where,  $T_e(v) = \max \{T_\Delta(x, y)\}$ ,  $I_e(v) = \max \{I_\Delta(x, y)\}$  and  $F_e(v) = \max \{F_\Delta(x, y)\}$ , for any vertex  $x$  of  $G$ .

**Definition 3.6:** A ND radius  $r_\Delta(G) = \langle T_r, I_r, F_r \rangle$  of  $G$  is the minimum ND eccentricity among the vertices of  $G$ .

Similarly, the N radius  $r_\delta(G)$  of  $G$  is the minimum N eccentricity among the vertices of  $G$ .

**Definition 3.8:** A ND diameter  $d_\Delta(G) = \langle T_d, I_d, F_d \rangle$  of  $G$  is the maximum ND eccentricity among the vertices of  $G$ .

Similarly, the N diameter  $d_\delta(G)$  of  $G$  is the maximum N eccentricity among the vertices of  $G$ .

### 4. View on Neutrosophic Detour Boundary (NDB) Vertices:

**Definition 4.1:** Let  $G = (A, B)$  be a non-trivial CNG. For a vertex  $y \in Y$ ,

$$T_\Delta^-(y) = \min \{T_\Delta(x, y) : x \in Y - \{y\}\}$$

$$I_\Delta^-(y) = \min \{I_\Delta(x, y) : x \in Y - \{y\}\} \text{ and}$$

$$F_\Delta^-(y) = \min \{F_\Delta(x, y) : x \in Y - \{y\}\}$$

where  $\langle T_\Delta^-(y), I_\Delta^-(y), F_\Delta^-(y) \rangle = D_\Delta^-(y)$

A vertex  $x (\neq y)$  is called a ND neighbor of  $y$  if one of the following condition satisfied.

$$T_\Delta(x, y) = T_\Delta^-(y)$$

$$I_\Delta(x, y) = I_\Delta^-(y) \text{ and}$$

$$F_\Delta(x, y) = F_\Delta^-(y)$$

The collection of ND neighbourhood of  $y$  is denoted by  $N_\Delta(y)$ .

**Definition 4.2:** A vertex  $y$  in a CNG  $G$  is said to be a NDB vertex of a vertex  $x$ , if any of the two conditions is satisfied.

$$\begin{aligned} T_{\Delta}(x, w) &\leq T_{\Delta}(x, y) \\ I_{\Delta}(x, w) &\leq I_{\Delta}(x, y) \text{ and} \\ F_{\Delta}(x, w) &\leq F_{\Delta}(x, y) \quad \forall w \in Y \end{aligned}$$

**Definition 4.3:** A vertex is said to be a cut vertex of a graph  $G : (A, B)$  if removal of it reduces the strength of connectedness between some pair of vertices

**Definition 4.4:** A NG  $G$  is said to be a block if the graph is a CNG and has no cut vertex. Block is denoted by  $B$ .

**Example: 4.1**

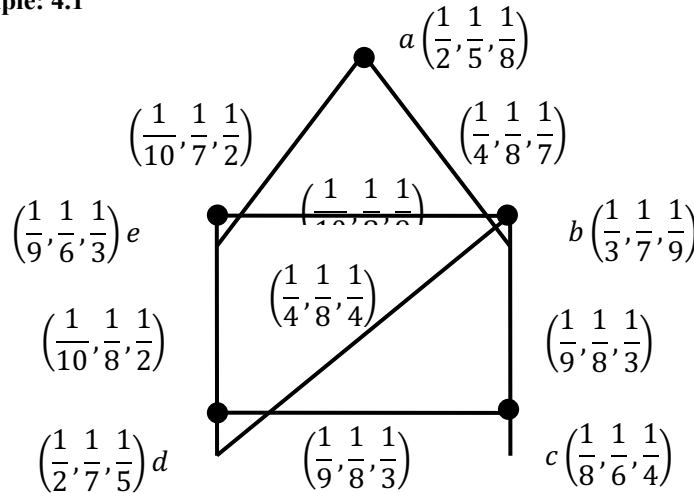


Figure 1 : NDB vertices

**ND Distance**

$$\begin{aligned} D_{\Delta}(a, b) &= \langle 38, 31, 10 \rangle, D_{\Delta}(a, c) = \langle 33, 31, 11 \rangle \\ D_{\Delta}(a, e) &= \langle 32, 32, 15 \rangle, D_{\Delta}(a, d) = \langle 38, 31, 13 \rangle \\ D_{\Delta}(b, e) &= \langle 28, 24, 9 \rangle, D_{\Delta}(b, d) = \langle 20, 23, 11 \rangle \\ D_{\Delta}(b, c) &= \langle 33, 31, 14 \rangle, D_{\Delta}(e, d) = \langle 32, 31, 15 \rangle \\ D_{\Delta}(e, c) &= \langle 27, 23, 13 \rangle, D_{\Delta}(d, c) = \langle 33, 30, 15 \rangle \end{aligned}$$

Table 1 Eccentricity, Radius and Diameter for Figure 1 example

Vertex	$e_{\Delta}(a)$	$r_{\Delta}(a)$	$d_{\Delta}(a)$
a	$\langle 38,32,15 \rangle$	$\langle 32,31,14 \rangle$	$\langle 38,32,15 \rangle$
b	$\langle 38,31,14 \rangle$		
c	$\langle 33,31,15 \rangle$		
d	$\langle 38,31,15 \rangle$		
e	$\langle 32,32,15 \rangle$		

**ND Neighborhood**

$$D_{\Delta}^{-(a)} = \langle 32,31,10 \rangle, T_{\Delta}(a,e) = T_{\Delta}^{-(a)} = 31, I_{\Delta}^{-(a)} = I_{\Delta}(a,b) = I_{\Delta}(a,c) = I_{\Delta}(a,d) = 31.$$

Therefore,  $e, b, c, d$  are the ND neighbourhood vertices of  $a$ .

$$D_{\Delta}^{-(b)} = \langle 20,23,9 \rangle, T_{\Delta}^{-(b)} = T_{\Delta}(b,d) = 20, F_{\Delta}^{-(b)} = F_{\Delta}(b,e) = 9.$$

Therefore,  $e$  and  $d$  are the N neighbourhood vertices of  $a$ .

$$D_{\Delta}^{-(c)} = \langle 27,23,11 \rangle, T_{\Delta}^{-(c)} = T_{\Delta}(e,c) = 27, F_{\Delta}^{-(c)} = F_{\Delta}(a,c) = 11.$$

Therefore,  $e$  and  $a$  are the N neighbourhood vertices of  $a$ .

$$D_{\Delta}^{-(d)} = \langle 20,23,11 \rangle = D_{\Delta}(b,d).$$

Therefore,  $b$  is N neighbourhood vertex of  $d$ .

$$D_{\Delta}^{-(e)} = \langle 27,23,9 \rangle, T_{\Delta}^{-(e)} = T_{\Delta}(e,c) = 27, F_{\Delta}^{-(e)} = F_{\Delta}(b,e) = 9.$$

Therefore,  $b$  and  $c$  are the N neighbourhood of  $e$ .

**Neutrosophic detour boundary(NDB) vertices**

$T_{\Delta}(a, c) \leq T_{\Delta}(a, b)$  and  $I_{\Delta}(a, c) = I_{\Delta}(a, b)$ . Therefore  $b$  is the NDB vertex of  $a$ .  
 $T_{\Delta}(a, b) = T_{\Delta}(a, d)$  and  $I_{\Delta}(a, b) = I_{\Delta}(a, d)$ . Therefore  $d$  is the NDB vertex of  $a$ .  
 $T_{\Delta}(b, e) = T_{\Delta}(b, a)$  and  $I_{\Delta}(b, e) = I_{\Delta}(b, a)$ . Therefore  $a$  is the NDB vertex of  $b$ .  
 $T_{\Delta}(b, d) \leq T_{\Delta}(b, a)$  and  $I_{\Delta}(b, d) \leq I_{\Delta}(b, a)$ . Therefore  $a$  is the NDB vertex of  $b$ .  
 $T_{\Delta}(c, a) = T_{\Delta}(c, b)$  and  $I_{\Delta}(c, a) = I_{\Delta}(c, b)$ . Therefore  $b$  is the NDB vertex of  $a$ .  
 $T_{\Delta}(c, e) \leq T_{\Delta}(c, a)$  and  $I_{\Delta}(c, e) \leq I_{\Delta}(c, a)$ . Therefore  $a$  is the NDB vertex of  $c$ .  
 $T_{\Delta}(d, e) \leq T_{\Delta}(d, a)$  and  $I_{\Delta}(d, e) = I_{\Delta}(d, a)$ . Therefore  $a$  is the NDB vertex of  $e$ .

$T_{\Delta}(d, b) \leq T_{\Delta}(d, a)$  and  $I_{\Delta}(d, b) \leq I_{\Delta}(d, a)$ . Therefore  $a$  is the NDB vertex of  $d$ .

$T_{\Delta}(e, c) \leq T_{\Delta}(e, b)$  and  $I_{\Delta}(e, c) \leq I_{\Delta}(e, b)$ . Therefore  $b$  is the NDB vertex of  $e$ .

$T_{\Delta}(e, d) \leq T_{\Delta}(e, a)$  and  $I_{\Delta}(e, d) \leq I_{\Delta}(e, a)$ . Therefore  $a$  is the NDB vertex of  $e$ .

**Proposition 4.1:** Let  $x$  and  $y$  be the two vertices of a CNG  $G$ ,

$$0 \leq T_{\delta}(x, y) \leq T_{\Delta}(x, y) < \infty$$

$$0 \leq I_{\delta}(x, y) \leq I_{\Delta}(x, y) < \infty$$

$$0 \leq F_{\delta}(x, y) \leq F_{\Delta}(x, y) < \infty.$$

**Proof:** It is obvious that the distance  $D_{\delta}(x, y) \geq 0$  and is a metric. The ND distance is the maximum length of any  $x$ - $y$  path. Therefore,

$$T_{\Delta}(x, y) \geq T_{\delta}(x, y)$$

$$I_{\Delta}(x, y) \geq I_{\delta}(x, y)$$

$$F_{\Delta}(x, y) \geq F_{\delta}(x, y) \text{ and also } D_{\Delta}(x, y) < \infty$$

Hence proved.

**Proposition 4.2:** Let  $x$  and  $y$  be the two vertices of a CNG  $G$ ,  $D_{\Delta}(x, y) = 0$  if and only if  $D_{\delta}(x, y) = 0$  if and only if

$$x = y.$$

**Proof:**

Let  $G$  be a CNG and  $D_{\delta}(x, y)$  is a metric. Therefore by 2.6,  $D_{\delta}(x, y) = 0$  if and only if  $x = y$ . Also since  $D_{\Delta}(x, y)$  is metric, therefore we know,  $D_{\Delta}(x, y) = 0$  if and only if  $x = y$ . By the proposition 3.1.1, we have  $D_{\Delta}(x, y) = 0$  if and only if  $D_{\delta}(x, y) = 0$  if and only if  $x = y$ .

**Proposition 4.3:** For every CNG  $G$  which  $r_{\delta}(G) = a$  and  $r_{\Delta}(G) = b$ , where  $a, b$  are any two real numbers, then

$$1 \leq a \leq b < \infty.$$

**Proof:**

Let  $G: (A, B)$  be a CNG such that  $r_{\delta}(G) = a$ .

Now,  $a = r_{\delta}(G) = \min \{e_{\delta}(y) : y \in Y\}$

$$= \min \{\max D_{\delta}(x, y) : x \in G\}$$

$$\leq \min \{\max D_{\Delta}(x, y) : x \in G\}$$

$$= \min \{e_{\Delta}(y)\}$$

$$= r_{\Delta}(G) = b$$

Since for every  $x, y \in G$ ,  $D_{\delta}(x, y) \subseteq D_{\Delta}(x, y)$ . Obviously,  $a \geq 1$ . Therefore,  $1 \leq a \leq b < \infty$ .

**Proposition 4.5:** A CNG  $G$  for every vertex  $y$ ,  $d_{\Delta}(G) - e_{\Delta}(G) \geq k$ , where  $k$  is arbitrary non – negative real number.

**Proof:**

Let  $G: (A, B)$  be a CNG and  $y$  be any vertex of  $G$ .

By 3.7 and 3.8,  $T_r(G) \leq T_e(y) \leq T_d(G)$

$$I_r(G) \leq I_e(y) \leq I_d(G)$$

and  $F_r(G) \leq F_e(y) \leq F_d(G)$ . is true for any vertex  $y$  of  $G$ .

Considering the right side inequality, we can have,

$$T_d(G) - T_e(y) \geq k_T$$

$$I_d(G) - I_e(y) \geq k_I$$

$$F_d(G) - F_e(y) \geq k_F$$

where  $< k_T, k_I, k_F > = k$  and where  $k \geq 0$ .

### 5. View on Neutrosophic Detour Central (NDC) Vertices

**Definition 5.1:** A vertex  $x$  is a NDC node of  $G$  if,

$$T_e(x) = T_r(x), I_e(x) = I_r(x) \text{ and } F_e(x) = F_r(x)$$

The collection of NDC node is denoted by  $C_\Delta(G)$ .

**Definition 5.2:** Let  $G : (A, B)$  be a CNG. A N subgraph induced by  $C_\Delta(G)$  is denoted by  $[C_\Delta(G)] = H : (A_I, B_I)$  is called centre of  $G$ . A CNG  $G$  is self centered if each node is a central node.

**Definition 5.3:** A vertex  $x$  is called a peripheral vertex if,  $e_\Delta(x) = d_\Delta(G)$ .

$$T_e(x) = T_d(x)$$

$$I_e(x) = I_d(x)$$

and

$$F_e(x) = F_d(x)$$

The collection of ND peripheral vertex is denoted by  $C_\Delta(G)$ .

**Example**

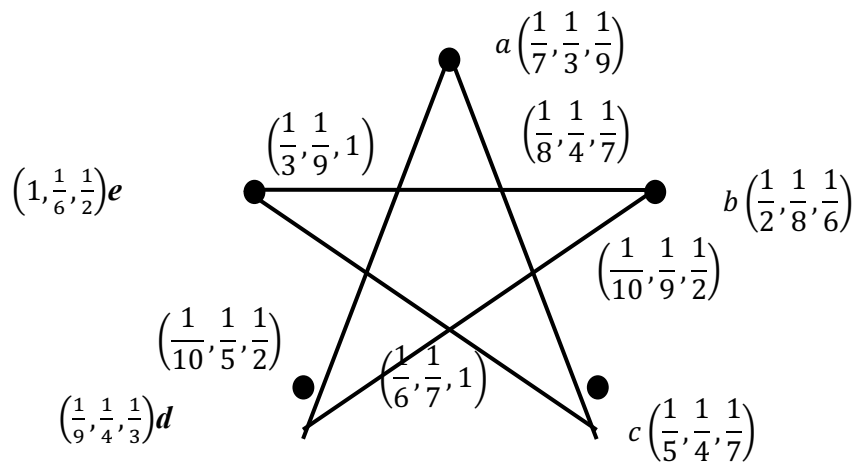


Figure 2: NDC Vertices



Table 2 Eccentricity, Radius and Diameter for Figure 2 example

Vertex	$e_{\Delta}(a)$	$r_{\Delta}(a)$	$d_{\Delta}(a)$
a	$\langle 29,30,11 \rangle$	$\langle 27,25,11 \rangle$	$\langle 34,30,12 \rangle$
b	$\langle 31,25,12 \rangle$		
c	$\langle 34,30,12 \rangle$		
d	$\langle 27,25,11 \rangle$		
e	$\langle 31,25,11 \rangle$		

In the above example, the ND center vertex is  $d$ ,

since  $e_{\Delta}(d) = r_{\Delta}(G)$  and the N detour peripheral vertex is  $c$ , since  $e_{\Delta}(c) = d_{\Delta}(G)$ .

**Proposition 5.1:**  $C_{\Delta}(G)$  of every CNG  $G$  lies in a Single block of  $G$ .

**Proof:**

Let us prove this theorem by contradiction. Suppose that, a CNG  $G$  whose ND centre does not lie in a single block of  $G$ . Let  $B_1$  and  $B_2$  be any two blocks containing the vertices of  $C_{\Delta}(G)$ . All vertices which are not

N cut vertex either belong to  $B_1$  or to  $B_2$ . Let  $y$  be a N cut vertex of  $G$  which lie outside the blocks  $B_1$  and  $B_2$ . We know that  $G$  is a CNG. Therefore the blocks  $B_1$  and  $B_2$  can only be connected through N cut vertices.

Let  $x \in B_1$  such that,

$$\langle T_e(x), I_e(x), F_e(x) \rangle = \langle T_r(G), I_r(G), F_r(G) \rangle.$$

Let  $w \in B_2$  be a NDC vertex of  $G$  such that

$$\langle T_e(y), I_e(y), F_e(y) \rangle = \langle T_{\Delta}(y, w), I_{\Delta}(y, w), F_{\Delta}(y, w) \rangle.$$

$$\text{i.e., } T_e(x) = T_e(w)$$

$$I_e(x) = I_e(w)$$

$$\text{and } F_e(x) = F_e(w).$$

Let  $P_1, P_2$  and  $P_3$  be the paths  $x-y, y-w$  and  $x-w$  respectively. The ND distance  $P_1$  and  $P_2$  together form the ND distance of  $P_3$  in  $G$ . Since  $y$  lies in the path  $x-w$  ( $P_3$ ).

Then,

$$T_e(w) \geq T_{\Delta}(w, x) > T_{\Delta}(w, y) = T_e(y)$$

$$I_e(w) \geq I_{\Delta}(w, x) > I_{\Delta}(w, y) = I_e(y) \text{ and}$$

$$F_e(w) \geq F_{\Delta}(w, x) > F_{\Delta}(w, y) = F_e(y)$$

Contradicting to the fact that  $w$  is a NDC vertex of  $G$ .

**Proposition 5.2:** Let  $G$  be a CNG with N cut vertices and let  $y$  be a vertex of  $G$ . If  $e_{\Delta}(G) = r_{\Delta}(G)$ , then  $y$  is not a N cut vertex of  $G$ .

**Proof:**

$e_{\Delta}(G) = r_{\Delta}(G)$  implies that  $y$  is a NDC node of  $G$ , that is,  $y \in C_{\Delta}(G)$ . Since the NDC vertices of  $G$  lie in a single block of  $G$  and this block does not contain any N cut node of  $G$ ,  $y$  cannot be a N cut node of  $G$ .

## 6. Conclusions

An application of the detour distance that is closely related with real-life situations such as power and water delivery over a large area. Etc. Power/water demand can be classified over a specific time period, such as daily, seasonal, sporadic, and so on. If we consider a specific city, demand in different locations may fluctuate depending on consumption. Demands can change on a daily, monthly, or annual basis. Typically, we prefer to distribute electricity and water supplies based on demand across various sections of the country efficiently and without significant power/water loss. A power plant requires an additional electric power supply to meet demand for a set amount of time without failure. However, station officials do not have enough time to call all other power plants. At the same time, officials were required to take quick action in coordination with all of the nearby power plants in order to provide additional power. We may use the idea of  $N$  detour distances in the NG to tackle this key problem, which is a better and more effective method for analysing interdeterminancy information. Consider all power plants and water treatment plants within a city. The stations are represented by vertices, while the transmission lines between them are represented by edges. Every edge has a membership value, interdeterminancy membership, and false membership value connected with it. This method can be applied to a large network. Furthermore, consider each detour distance between all pairs of vertices within a network, and determine all detour bases from there. This detour basis will be critical in connecting and sharing facilities amongst networks. In algebraic structures and fuzzy models, the notion of neutrosophy is employed as the framework. In this paper, the idea of NDB vertices and NDC vertices in NG are introduced. Moreover, the concept of ND eccentricity, ND radius, ND diameter, ND neighborhood vertex are studied. Several interesting properties related to of eccentric nodes, peripheral nodes, a NDC nodes in NG are established with the necessary examples.

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