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Confidence Neutrosophic Number Linear Programming Methods Based on Probability Distributions and Their Applications in Production Planning Problems

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A neutrosophic number linear programming (NN-LP) method reveals a useful tool for solving optimal production planning problems in an indeterminate environment. The optimal feasible solutions of the decision variables and the objective function in the NN-LP method are obtained only depending on the subjectively specified uncertain range of neutrosophic numbers without considering some distribution and confidence level of product sample data. Due to the lack of some probability distribution and confidence level/interval in indeterminate situations, existing indeterminate optimization methods are difficult to guarantee the reliability and credibility of the optimal interval feasible solutions. To solve these problems, this study proposes confidence neutrosophic number linear programming methods based on some probability distributions to objectively determine the confidence level/interval of a sample dataset/multivalued set in the NN-LP problems and strengthen the reliability and credibility of the optimal interval feasible solutions. Therefore, this article first introduces a transformation method from product sample datasets (multivalued sets) with normal and log-normal distributions to confidence neutrosophic numbers (CNNs) (confidence intervals) from a probabilistic point of view. Then, CNN linear programming (CNN-LP) models and solution methods are proposed according to the normal and log-normal distributions and confidence levels of product sample datasets to obtain the optimal interval feasible solutions. Furthermore, the proposed CNN-LP methods are applied to two real cases in a manufacturing company of Shaoxing City in China to perform the production planning problems under normal and log-normal distributions and 95% CNN/confidence interval of the product sample datasets. Compared with existing related linear programming methods, the proposed CNN-LP methods can make the optimal interval feasible solutions more reasonable and credible/reliable than the existing linear programming methods and overcome the defects of the existing linear programming methods in indeterminate situations.

1. Introduction

In the fierce market competition, optimal management and decision-making have become more and more important and necessary in the current production and operation of enterprises. Thus, the production planning problem (PPP) of an enterprise is a critical issue in production management. Then, many optimization/planning methods have been proposed to solve actual PPPs [1–6]. Then, these PPPs usually contain indeterminate and incomplete information of their objective functions and/or constraints in real production environments. Therefore, many uncertain optimization/planning methods [7–14] have been proposed and widely used in engineering management and decision-making to solve uncertain optimization/planning problems in actual problems. However, most uncertain optimization/planning methods used interval numbers or fuzzy numbers to express uncertain parameters in objective functions and constraints and then transformed the objective functions and constraints into deterministic planning problems in

order to yield the optimal exact/crisp solutions in the solution process of uncertain problems [7–14]. Furthermore, some researchers [15–17] also proposed trapezoidal or triangular neutrosophic number programming methods to solve single-objective or multiobjective linear programming problems in indeterminate environments. Certain researchers [18] further proposed a nonlinear programming model to perform matrix games in the setting of single-valued neutrosophic numbers. However, these optimal crisp feasible solutions of decision variables and objective functions obtained in uncertain optimization/planning problems are not really meaningful complete solutions but are actually special solutions in uncertain problems. In fact, the planning problem in the uncertain environment should obtain the uncertain optimal solutions of the objective function and the decision variables. Therefore, it is necessary for us to find a suitable optimization method that can obtain the uncertain optimal solutions in uncertain PPPs.

As a pioneer of neutrosophic theory, Smarandache [19–21] proposed a neutrosophic number (NN) from a symbolic point of view, whose expression is $y = b + \lambda I$ with a certain part b and an uncertain part λI. It can be seen that NN can express an uncertain value/changeable interval number corresponding to an uncertain value/range of $[I^L, I^U]$ in real life.

After that, NNs have been widely used to deal with uncertain problems. For example, Ye [22] proposed a method to solve group decision-making problems with uncertain information under the neutrosophic number environment, where alternatives are ranked in view of the possibility ranking approach of NNs as deneutralization and the best one is finally selected. Then, Ye [23] first proposed some operations of NNs from an algebraic point of view and developed NN linear programming (NN-LP) models that included the NN objective function and NN constraint equations and their solution methods in indeterminate situations. In nonlinear programming problems, Ye et al. [24] introduced the concepts of NN nonlinear functions, NN constraint inequalities, and NN nonlinear programming models, gave optimal interval solutions of the NN nonlinear programming models with various constraint problems, and then verified the efficiency of the proposed methods by numerical examples. Furthermore, Tu et al. [25] introduced NN optimization models and solution methods based on the Matlab built-in function "fmincon()" and NN operations and then applied them to actual production problems to obtain optimal interval solutions under an uncertain environment. Furthermore, Pramanik and Dey [26] presented a NN multilevel linear programming approach to perform linear programming problems. Banerjee and Pramanik [27] introduced a NN-LP model to solve single-objective linear programming problems. Then, they [28] further presented another NN-LP model to solve multiobjective linear programming problems. Pramanik and Dey [29] also introduced a bilevel NN-LP method for linear programming problems. Khalifa [30] proposed a NN optimization approach to perform the multiobjective assignment problems. Aslam [31] introduced an exponential distribution sampling plan method

based on the neutrosophic interval statistical method to solve the neutrosophic nonlinear optimization problem of sampling plans by analyzing the certain part and the uncertain part.

As mentioned above, the existing NN-LP models/methods have been improved or extended to solve the NN single- and multiobjective linear programming problems with uncertain information. However, there are still shortcomings in the optimization methods and applications mentioned above. For example, when we treat NNs as changeable intervals in optimization models, the choice of the uncertain range/interval of NNs will depend on subjectively given values rather than objectively given values, which lack some distribution and confidence level of product sample datasets from a probabilistic point of view. Therefore, the existing indeterminate optimization methods are difficult to guarantee the reliability and rationality of the optimal interval feasible solutions due to the lack of some probability distribution and confidence level/interval in indeterminate situations.

From a probabilistic point of view, a confidence interval reveals the probability that a parameter falls between a pair of values around the mean. Then, a confidence level of $1-\alpha$ for a level α in probability and statistics reflects that a $(1-\alpha)\%$ confidence interval will contain $(1-\alpha)\%$ of all probability values and $\alpha\%$ of all probability values will be outside the confidence interval. Based on the motivation of the confidence level/interval and the existing NN-LP methods, this study proposes confidence neutrosophic number linear programming (CNN-LP) methods based on some probability distributions to objectively determine the confidence level/interval of a sample dataset/multivalued set in the NN-LP problems and overcome the defects of the existing NN-LP methods. From a probabilistic point of view, this article first presents a transformation method from multivalued sets (product sample datasets) with normal and log-normal distributions (two typical distributions in PPPs) to confidence neutrosophic numbers (CNNs) (i.e., confidence intervals). Then, CNN-LP models and solution methods are proposed according to normal and log-normal distributions and confidence levels of product sample datasets to obtain optimal interval feasible solutions. Furthermore, the proposed CNN-LP methods are applied to two real cases to solve PPPs under the normal and log-normal distributions and 95% CNN/confidence interval of product sample datasets. Compared with existing related linear programming methods, the proposed CNN-LP methods can make the optimal interval feasible solutions more reasonable and credible/reliable than the existing linear programming methods and overcome the defects of the existing linear programming methods in indeterminate situations.

In general, this study reveals the following obvious contributions:

(1) The proposed transformation method from a sample dataset/multivalued set to CNN can provide a more reasonable and credible information expression than the existing NN/interval number expression from a probabilistic point of view.

(2) The confidence levels/CNNs of product sample datasets are integrated into the NN-LP models to ensure the reliability and creditability of the programming results in the situation of indeterminacy.

(3) The proposed CNN-LP models can flexibly meet the requirements of optimal solutions in actual programming applications depending on some distributions and confidence levels.

The rest of the article is composed of these parts. Section 2 introduces NN operations and NN-LP methods as preliminaries. Section 3 first presents a transformation method from a sample dataset/multivalued set to CNN from a probabilistic point of view and then develops CNN-LP models with normal and log-normal distributions and confidence levels and their solution methods in indeterminate environments. In Section 4, the proposed CNN-LP methods are applied to two cases of PPPs with normal and log-normal distributions and confidence levels of 0.95 related to the product sample datasets to indicate their applications and efficiency. Section 5 compares the proposed CNN-LP methods with existing related linear programming methods to prove the effectivity and feasibility of the proposed models and solution methods, and the comparative results show that the proposed CNN-LP methods are more reasonable and credible than the existing methods. Section 6 gives the conclusions and further research directions of this study.

2. Preliminaries of NNs and NN-LP Methods

In terms of the concept of NNs presented by Smarandache [19–21], NN consists of its determinate part b and its indeterminate part λI, then its mathematical expression is $y = b + \lambda I$ for $b, \lambda \in R$ (all real numbers) with indeterminacy $I \in [I^L, I^U]$. Therefore, NN can express an indeterminate value/range, which depends on the value/range of I.

Two NNs are set as $y_1 = b_1 + \lambda_1 I$ and $y_2 = b_2 + \lambda_2 I$ for $I \in [I^L, I^U]$. Then, Ye [23] defined the algebraic operations as follows:

(1) $y_1 + y_2 = b_1 + b_2 + (\lambda_1 + \lambda_2)I = [b_1 + b_2 + \lambda_1 I^L + \lambda_2 I^L, b_1 + b_2 + \lambda_1 I^U + \lambda_2 I^U]$;

(2) $y_1 - y_2 = b_1 - b_2 + (\lambda_1 - \lambda_2)I = [b_1 - b_2 + \lambda_1 I^L - \lambda_2 I^L, b_1 - b_2 + \lambda_1 I^U - \lambda_2 I^U]$;

$$y_1 \times y_2 = b_1 b_2 + (b_1 \lambda_2 + b_2 \lambda_1)I + b_1 b_2 I^2 =$$

$$(3) \left[\min \begin{pmatrix} (b_1 + \lambda_1 I^L)(b_2 + \lambda_2 I^L), \\ (b_1 + \lambda_1 I^L)(b_2 + \lambda_2 I^U), \\ (b_1 + \lambda_1 I^U)(b_2 + \lambda_2 I^L), \\ (b_1 + \lambda_1 I^U)(b_2 + \lambda_2 I^U) \end{pmatrix}, \max \begin{pmatrix} (b_1 + \lambda_1 I^L)(b_2 + \lambda_2 I^L), \\ (b_1 + \lambda_1 I^L)(b_2 + \lambda_2 I^U), \\ (b_1 + \lambda_1 I^U)(b_2 + \lambda_2 I^L), \\ (b_1 + \lambda_1 I^U)(b_2 + \lambda_2 I^U) \end{pmatrix} \right];$$

(4) $(y_1/y_2) = ((b_1 + \lambda_1 I)/(b_2 + \lambda_2 I)) = ([b_1 + \lambda_1 I^L, b_1 + \lambda_1 I^U]/[b_1 + \lambda_1 I^L, b_1 + \lambda_1 I^U]) = [\min((b_1 + \lambda_1 I^L)/(b_2 + \lambda_2 I^U), (b_1 + \lambda_1 I^L)/(b_2 + \lambda_2 I^L), (b_1 + \lambda_1 I^U)/(b_2 + \lambda_2 I^U), (b_1 + \lambda_1 I^U)/(b_2 + \lambda_2 I^L)), \max((b_1 + \lambda_1 I^L)/(b_2 + \lambda_2 I^U), (b_1 + \lambda_1 I^L)/(b_2 + \lambda_2 I^L), (b_1 + \lambda_1 I^U)/(b_2 + \lambda_2 I^U), (b_1 + \lambda_1 I^U)/(b_2 + \lambda_2 I^L))]$

In linear programming problems, the mathematical programming model usually contains three elements: decision variables, objective functions, and constraint conditions. In the traditional mathematical programming model, the coefficients and variables in the objective function and constraint equations are usually regarded as certain/exact values. In an uncertain environment, Ye [23] proposed a NN-LP model with general constraint conditions:

$$\text{Max } F(X, I) = \sum_{j=1}^{n} y_{cj} x_j$$

$$\text{s.t.} \quad \sum_{j=1}^{n} y_{ij} x_j \le y_i, \quad i = 1, 2, \ldots, q \quad (1)$$

$$x_j \ge 0, j = 1, 2, \ldots, n,$$

$$x_j, y_{ij}, y_{cj}, y_i \in Y,$$

where $X = (x_1, x_2, \ldots, x_n)$ is a vector of n decision variables; I is indeterminacy; Y is all NNs and x_j, y_{cj}, y_{ij}, $y_i \in Y$.

To solve the NN-LP model, Ye [23] introduced the NN simplex algorithm to produce a group of optimal interval solutions of PPP based on subjectively given/assumed ranges of indeterminacy $I \in [I^L, I^U]$.

3. CNN-LP Methods with Normal and Log-Normal Distributions

3.1. Confidence Neutrosophic Numbers/Confidence Intervals. Since the NN $y = b + \lambda I$ for $I \in [I^L, I^U]$ can be treated as any interval value depending on a range of I, it can be denoted as $y = [b + \lambda I^L, b + \lambda I^U]$ for $y \in Y$. From a probabilistic point of view, a possible value of the random variable y usually falls within $[b + \lambda I^L, b + \lambda I^U]$, which depends on some distribution of y. Assuming that the distribution function of y is p(y), then the probability of y in $[b + \lambda I^L, b + \lambda I^U]$ is yielded by P(y):

$$P(y) = p(b + \lambda I^L \le y \le b + \lambda I^U) = \int_{b+\lambda I^L}^{b+\lambda I^U} p(y) dy. \quad (2)$$

In view of the probability of y, we can use the concept of a confidence interval under some confidence level to guarantee that the sample dataset of y falls within a certain confidence interval (e.g., 95% confidence interval for a level $\alpha = 0.05$) under a certain distribution condition. According to the confidence level/interval concept, we can propose the definition of CNN.

Definition 1. Set $A = \{a_1, a_2, \ldots, a_n\}$ as a sample dataset (a multivalued set) with the normal distribution in a NN-LP

problem. Regarding the confidence level of $1-\alpha$, the CNN/ confidence interval y is given by the following equation:

$$y = \left[b + \lambda I^L, b + \lambda I^U \right] = \left[b - \frac{\delta}{\sqrt{n}} t_{\alpha/2}, b + \frac{\delta}{\sqrt{n}} t_{\alpha/2} \right], \qquad (3)$$

$$y^L = b + \lambda I^L = b - \frac{\delta}{\sqrt{n}} t_{\alpha/2} \le y \le b + t_{\alpha/2} \frac{\delta}{\sqrt{n}} = b + \lambda I^U = y^U, \qquad (4)$$

where $\lambda = \delta/\sqrt{n}$ is an indeterminate parameter, $[I^L, I^U] = [-t_{\alpha/2}, t_{\alpha/2}]$ is the indeterminate range of I, and the mean b and the standard deviation δ of the sample dataset A can be obtained by the following formulae:

$$b = \frac{1}{n} \sum_{s=1}^{n} a_s, \qquad (5)$$

$$\delta = \sqrt{S_A^2} = \sqrt{\frac{1}{n-1} \sum_{s=1}^{n} (a_s - b)^2}. \qquad (6)$$

Remark 1. Reference [32]. A confidence level of $1-\alpha$ will be established at $(1-\alpha)\%$ (calculated statistic based on the sample), and there is a $(1-\alpha)\%$ chance of being correct for the entire population within the established confidence level.

Remark 2. Reference [32]. $t_{\alpha/2}$ is a critical value to be used for confidence interval calculation. Then, its value usually takes $t_{\alpha/2} = 1.645$, 1.96, and 2.576 corresponding to 90%, 95%, and 99% confidence intervals for levels $\alpha = 0.1$, 0.05, and 0.01, which are by far the most widely used, especially the 95% confidence interval for a level $\alpha = 0.05$ is the most commonly used.

Definition 2. Set $A = \{a_1, a_2, \ldots, a_n\}$ as a sample dataset (a multivalued set) with the log-normal distribution in a NN-LP problem. Regarding the confidence level of $1-\alpha$, the CNN (confidence interval) is given by the following equation:

$$e^y = \left[e^{b-(\delta/\sqrt{n})t_{\alpha/2}}, e^{b+(\delta/\sqrt{n})t_{\alpha/2}} \right] = \left[e^{y^L}, e^{y^U} \right], \qquad (7)$$

where $y = [b + \lambda I^L, b + \lambda I^U] = [b - (\delta/\sqrt{n})t_{\alpha/2}, b + (\delta/\sqrt{n})t_{\alpha/2}]$ for $[I^L, I^U] = [-t_{\alpha/2}, t_{\alpha/2}]$ and $\lambda = \delta/\sqrt{n}$. Then, the mean b and the standard deviation δ of the sample dataset A can be obtained by the following formulae:

$$b = \frac{1}{n} \sum_{s=1}^{n} \ln a_s, \qquad (8)$$

$$\delta = \sqrt{S_A^2} = \sqrt{\frac{1}{n-1} \sum_{s=1}^{n} (\ln a_s - b)^2}. \qquad (9)$$

3.2. CNN-LP Methods with Normal and Log-Normal Distributions. In terms of CNNs corresponding to the normal and log-normal distributions and the confidence levels of $1-\alpha$, this part proposes the CNN-LP models and solution methods in indeterminate linear programming problems.

First, we establish the CNN-LP model and the solution method with normal distribution and the confidence level of $1-\alpha$ of all sample datasets (multivalued sets) in PPP.

CNN-LP Type I. Regarding the normal distribution and the confidence level of $1-\alpha$ of product sample datasets, we propose the following CNN-LP model:

$$\text{Max} \, F_1(X, I) = y_{c1}x_1 + y_{c2}x_2 + \cdots + y_{cn}x_n$$

$$\text{s.t.} \quad y_{i1}x_1 + y_{i2}x_2 + \cdots + y_{in}x_n \le y_i, \quad i = 1, 2, \ldots, q$$

$$x_j \ge 0, \quad j = 1, 2, \ldots, n \qquad (10)$$

$$x_j, y_{ij}, y_i, y_{cj} \in Y,$$

where y_{cj}, y_{ij}, y_i ($j = 1, 2, \ldots, n$; $i = 1, 2, \ldots, q$) are CNNs (confidence intervals), which are yielded by equations (3)–(6) for all sample datasets (multivalued sets) in PPP.

In view of the above CNN-LP model, optimal interval feasible solutions of the decision variables $x_j \in X$ ($j = 1, 2, \ldots, n$) and the CNN objective function $F_1(X, I)$ can be obtained by the lower bound and upper bound of CNNs in the CNN-LP model. Therefore, the solution method can use the built-in function "linprog()" in Matlab to obtain optimal interval feasible solutions.

Second, we establish the CNN-LP model and solution method with the log-normal distribution and the confidence level of $1-\alpha$ of all sample datasets (multivalued sets) in PPP.

CNN-LP Type II. Regarding the log-normal distribution and the confidence level of $1-\alpha$ of product sample datasets (multivalued sets), we propose the following CNN-LP model:

$$\text{Max}\, F_2\,(X, I) = e^{y_{c1}}x_1 + e^{y_{c2}}x_2 + \cdots + e^{y_{cn}}x_n$$

$$\text{s.t.}\quad e^{y_{i1}}x_1 + e^{y_{i2}}x_2 + \cdots + e^{y_{in}}x_n \le e^{y_i},\quad i = 1, 2, \ldots, q$$

$$x_j \ge 0,\quad j = 1, 2, \ldots, n \tag{11}$$

$$x_j, y_{ij}, y_i, y_{cj} \in Y,$$

where y_{cj}, y_{ij}, y_i ($j = 1, 2, \ldots, n$; $i = 1, 2, \ldots, q$) are CNNs (confidence intervals), which are yielded by equations (7)–(9) for all sample datasets (multivalued sets) in PPP.

In view of the above CNN-LP model, optimal interval feasible solutions of the decision variables $x_j \in X$ ($j = 1, 2, \ldots, n$) and the CNN objective function $F_2(X, I)$ can be given by the lower bound and upper bound of CNNs in the CNN-LP model. Thus, the solution method can use the built-in function "linprog()" in Matlab to obtain optimal interval feasible solutions.

In CNN-LP problems, the CNN-LP methods of the two CNN-LP types are composed of the following steps:

Step 1. Using the "histfit()" function in MATLAB, we can give the fitting curves of the sample datasets in a PPP problem and determine some distributions.

Step 2. Using equations (3)–(6) or equations (7)–(9) with the confidence level of $1−\alpha$ (the commonly used level $\alpha = 0.05$), we can get CNNs.

Step 3. On the basis of CNNs, we can establish the CNN-LP model under the normal or log-normal distribution condition (the common distributions).

Step 4. Regarding the lower and upper bounds of all CNNs in the NN-LP model, we can use the built-in function "linprog()" in MATLAB to get the optimal NN/interval feasible solutions of the CNN-LP model.

Step 5. End.

4. Actual Production Planning Problems

This section applies the proposed two CNN-LP types to two typical cases of PPPs adopted from a manufacturing company of Shaoxing City in China. To facilitate modeling of PPPs, it is necessary to define the following notations and assumptions in PPPs.

4.1. Notations. The notations in Table 1 are defined to establish the CNN-LP models for Cases 1 and 2.

4.2. Assumptions. The production planning models are built-in terms of the following assumptions:

  (i) Raw materials can sufficiently satisfy the production of products.

  (ii) Fluctuations in worker productivity are not taken into account.

  (iii) The maximum profit that can be achieved is only considered under the maximum production capacity of the equipment, without considering the relationship with customer needs.

  (iv) Startup time between machines/devices is ignored.

4.3. Case 1 with the Normal Distribution. In the production process of Case 1, a manufacturing company of Shaoxing City in China wants to produce two kinds of products, which are denoted by the product C and the product D. Then, they need to beprocessed by the machines I, II, and III. The sample datasets obtained in the production process of Case 1 are listed in Tables 2–4.

Regarding this actual case, CNN-LP type 1 can be applied to this PPPand then its CNN-LP methodis composed of the following steps:

  Step 1. To determine some distribution of the sample datasets of the C and *D* products in Case 1, we take the sample dataset C_1 in Table 2 as a calculation example. Using the "histfit()" function in MATLAB, the fitting curve of the sample dataset C_1 reflects a normal distribution, which is shown in Figure 1.

  Then, the mean and standard deviation of the sample dataset C1 can be calculated by equations (5) and (6):

  (i) $b_{C1} = 1/20 \sum_{s=1}^{20} c_{1s} = 9.22$;

  (ii) $\delta_{C1} = \sqrt{S_{C_1}^2} = \sqrt{1/(20-1)\sum_{s=1}^{20}(c_{1s} - b_{C1})^2} = 3.07$.

  Step 2. Using equation (3) with the confidence level of $1−\alpha = 0.95$ (the commonly used level $\alpha = 0.05$), we can get the following CNN:

$$y_{C1} = \left[b_{C1} + \lambda_{C1}I^L, b_{C1} + \lambda_{C1}I^U \right]$$

$$= \left[b_{C1} - \frac{\delta_{C1}}{\sqrt{n}}t_{\alpha/2}, b_{C1} + \frac{\delta_{C1}}{\sqrt{n}}t_{\alpha/2} \right]$$

$$= \left[9.22 - \frac{1.96 \times 3.07}{\sqrt{20}}, 9.22 + \frac{1.96 \times 3.07}{\sqrt{20}} \right] \tag{12}$$

$$= [7.79, 10.66],$$

where $[I^L, I^U] = [−1.96, 1.96]$ for $t_{\alpha/2} = 1.96$ is the indeterminate range of I.

By the similar calculational way, using equations (3)–(6) for the corresponding sample datasets in Tables 2–4, we can obtain all CNNs y_{Cj}, y_{Dj}, y_{Ei}, y_{Pi} ($j = 1, 2, 3, 4, 5$; $i = 1, 2, 3$) in the NN-LP problem. Thus, we give the average values b_{Cj}, b_{Dj}, b_{Ej}, and b_{Pj}, the standard deviations δ_{Cj}, δ_{Dj}, and δ_{Ej}, and δ_{Pj}, and the

TABLE 1: Definitions of notations.

Dataset	Definition
$X = \{x_1, x_2\}$	x_1: product C (piece); x_2: product D (piece)
F(X, I)	Object function of the product profit ($)
$C_1 = \{c_{11}, c_{12}, \ldots, c_{120}\}$	Material cost of the product C ($/piece)
$C_2 = \{c_{21}, c_{22}, \ldots, c_{220}\}$	Selling price of the product C ($/piece)
$C_3 = \{c_{31}, c_{32}, \ldots, c_{320}\}$	Production time of the product C for the machine I (h/piece)
$C_4 = \{c_{41}, c_{42}, \ldots, c_{420}\}$	Production time of the product C for the machine II (h/piece)
$C_5 = \{c_{51}, c_{52}, \ldots, c_{520}\}$	Production time of the product C for the machine III (h/piece)
$D_1 = \{d_{11}, d_{12}, \ldots, d_{120}\}$	Material cost of the product D ($/piece)
$D_2 = \{d_{21}, d_{22}, \ldots, d_{220}\}$	Selling price of the product D ($/piece)
$D_3 = \{d_{31}, d_{32}, \ldots, d_{320}\}$	Production time of the product D for the machine I (h/piece)
$D_4 = \{d_{41}, d_{42}, \ldots, d_{420}\}$	Production time of the product D for the machine II (h/piece)
$D_5 = \{d_{51}, d_{52}, \ldots, d_{520}\}$	Production time of the product D for the machine III (h/piece)
$E_1 = \{e_{11}, e_{12}, \ldots, e_{120}\}$	Validity time of the machine I (h/machine)
$E_2 = \{e_{21}, e_{22}, \ldots, e_{220}\}$	Validity time of the machine II (h/machine)
$E_3 = \{e_{31}, e_{32}, \ldots, e_{320}\}$	Validity time of the machine III (h/machine)
$P_1 = \{p_{11}, p_{12}, \ldots, p_{120}\}$	Processing fee of the machine I ($/hour/machine)
$P_2 = \{p_{21}, p_{22}, \ldots, p_{220}\}$	Processing fee of the machine II ($/hour/machine)
$P_3 = \{p_{31}, p_{32}, \ldots, p_{320}\}$	Processing fee of the machine III ($/hour/machine)

TABLE 2: Sample datasets of the product C in Case 1.

	Sample dataset									
C_1	7.68	9.03	8.82	16.60	10.32	6.49	5.08	11.38	8.41	10.95
	6.51	11.69	3.56	13.70	11.54	9.33	5.52	7.81	9.76	10.31
C_2	30.16	51.00	27.98	28.22	25.71	45.47	29.78	38.76	20.17	29.85
	40.93	23.48	35.45	39.94	51.01	40.77	43.52	31.81	29.79	35.51
C_3	9.92	9.10	11.75	7.73	11.13	13.75	14.17	8.69	11.25	7.55
	11.17	5.82	5.69	7.70	5.41	11.25	10.18	12.47	10.77	11.63
C_4	4.77	6.10	12.43	6.86	11.14	4.68	9.76	6.91	6.19	10.41
	8.35	8.83	8.84	8.54	5.86	5.10	9.31	6.64	12.19	6.36
C_5	5.18	2.83	5.87	4.73	5.06	5.54	6.28	3.79	2.80	4.84
	3.77	5.64	2.22	6.29	5.46	3.33	4.52	6.26	8364	4.47

TABLE 3: Sample datasets of the product D in Case 1.

	Sample dataset									
D_1	18.07	16.97	10.47	13.86	27.15	5.60	16.88	15.25	15.93	13.96
	15.96	12.49	3.84	10.50	15.41	15.55	17.62	27.33	4.08	18.08
D_2	48.89	36.00	28.95	38.57	18.28	59.82	54.52	32.93	28.82	18.57
	18.19	32.27	58.99	27.15	68.52	24.01	78.53	27.54	41.79	57.73
D_3	6.84	8.43	6.67	6.08	9.29	8.08	8.43	6.99	8.35	7.21
	8.48	7.14	8.37	10.72	8.83	7.53	7.76	6.48	8.11	9.06
D_4	10.03	12.83	11.59	8.40	7.20	8.26	7.30	10.99	8.36	8.73
	7.69	9.99	7.62	10.09	7.50	9.42	8.27	10.11	7.90	7.92
D_5	4.23	5.34	5.80	7.88	4.95	6.93	7.75	6.50	8.01	7.95
	10.58	4.89	6.20	8.47	6.77	6.62	5.94	8.75	8.16	8.31

CNNs y_{Cj}, y_{Dj}, y_{Ei}, and y_{Pi} ($j = 1, 2, 3, 4, 5$; $i = 1, 2, 3$) in Table 5.

Step 3. Based on CNNs in Table 5, we can establish the following CNN-LP model:

$$\text{Max } F_1(X, I) = (y_{C2} - y_{C1})x_1 + (y_{D2} - y_{D1})x_2 - (y_{C3}y_{P1} + y_{C4}y_{P2} + y_{C5}y_{P3})x_1$$
$$- (y_{D3}y_{P1} + y_{D4}y_{P2} + y_{D5}y_{P3})x_2$$

TABLE 4: Sample datasets of the machine processing in Case 1.

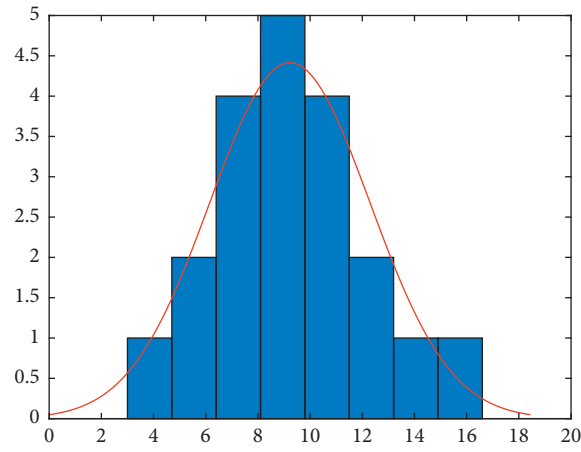|	Sample dataset									
E_1	114405	95865	211154	172704	84946	105394	2025052	128075	199660	200037
	120641	206990	191790	86690	170593	192656	198432	136224	198219	97180
E_2	94854	113150	167409	158263	143842	98001	126749	114818	176946	122914
	163782	186577	176906	157588	105981	203550	105117	122272	140359	160960
E_3	88485	101649	94468	76790	61911	111360	120801	89928	72162	107164
	78518	80483	102182	65836	56692	94731	94412	89522	95947	105316
P_1	0.75	0.76	0.30	0.71	0.58	0.45	0.46	0.86	0.44	0.35
	0.64	1.00	0.61	0.66	0.41	0.93	0.62	0.71	0.41	0.77
P_2	0.92	0.45	0.81	0.66	0.83	0.72	0.74	0.49	0.70	1.03
	1.32	0.45	1.51	1.26	0.85	0.71	0.59	1.05	0.59	0.67
P_3	0.68	0.71	0.33	0.39	0.33	0.41	0.27	0.13	0.39	0.23
	0.13	0.73	0.37	0.46	0.57	0.16	0.46	0.47	0.41	0.42

FIGURE 1: Histogram and fitting curve of the sampled dataset C_1 in Case 1.

TABLE 5: Average values, standard deviations, and CNNs in the NN-LP problem of Case 1.

b_{Cj}	b_{Dj}	b_{Ei}	b_{Pi}	δ_{Cj}	δ_{Dj}	δ_{Ei}	δ_{Pi}	y_{Cj}	y_{Dj}	y_{Ei}	y_{Pi}
9.22	14.7			6.13	15.26			[7.79, 10.66]	[11.87, 17.61]		
34.27	40.00			15.26	17.80			[27.13, 41.41]	[31.67, 48.33]		
9.86	7.94	155690	0.62	2.57	1.11	46986	0.2	[8.65, 11.06]	[7.42, 8.46]	[133700, 177680]	[0.53, 0.71]
7.96	9.01	142000	0.82	2.38	1.56	32103	0.29	[6.85, 9.08]	[8.28, 9.74]	[126980, 157030]	[0.68, 0.95]
4.88	7.00	89418	0.39	1.56	1.57	17027	0.10	[4.17, 5.58]	[6.26, 7.74]	[81449, 97387]	[0.32, 0.48]

$$\text{s.t.} \quad y_{C3}x_1 + y_{D3}x_2 \leq y_{E1}$$

$$y_{C4}x_1 + y_{D4}x_2 \leq y_{E2}$$

$$y_{C5}x_1 + y_{D5}x_2 \leq y_{E3}$$

$$x_k \geq 0, k = 1, 2;$$

$$x_k, y_{Cj}, y_{Dj}, y_{Ei}, y_{Pi} \in Y, \quad j = 1, 2, 3, 4, 5; i = 1, 2, 3. \tag{13}$$

Substituting all CNNs in Table 5 into the above CNN-LP model, we can get the final CNN-LP model:

TABLE 6: Sample datasets of the product C in Case 2.

					Sample dataset					
C_1	9.3	15.87	7.81	16.96	11.45	13.23	9.62	6.50	4.30	9.75
	13.59	7.80	6.89	7.73	5.22	7.05	8.25	9.23	8.74	12.25
C_2	24.29	20.60	57.41	40.79	18.70	22.42	52.95	27.43	51.84	52.01
	25.68	55.33	48.33	18.99	40.03	48.71	51.27	29.49	51.18	20.85
C_3	12.12	8.95	12.33	15.25	6.16	13.58	15.45	10.12	16.84	10.45
	6.88	5.17	9.02	12.36	10.97	11.29	8.39	10.41	6.10	7.94
C_4	8.13	13.12	7.89	11.60	9.57	7.99	4.95	6.41	7.21	10.75
	10.52	8.69	7.82	9.34	6.80	10.90	6.44	5.14	7.70	5.11
C_5	4.41	4.78	4.61	5.38	6.27	4.58	5.74	8.47	6.62	6.41
	3.92	4.26	5.38	4.85	3.07	3.18	6.81	.3099	9.33	2.57

$$
\begin{aligned}
\text{Max } F_1(X, I) = &\ ([27.13, 41.41] - [7.79, 10.66])x_1 + ([31.67, 48.33] - [11.87, 17.61])x_2 \\
& - ([8.65, 11.06] \times [0.53, 0.71] + [6.85, 9.08] \\
& \times [0.68, 0.95] + [4.17, 5.58] \times [0.32, 0.48])x_1 \\
& - ([7.42, 8.46] \times [0.53, 0.71] + [8.28, 9.74] \\
& \times [0.68, 0.95] + [6.26, 7.74] \times [0.32, 0.48])x_2 \\
= &\ [8.76, 11.59]x_1 + [8.23, 11.76]x_2 \\
\text{s.t.} \quad & [8.65, 11.06]x_1 + [7.42, 8.46]x_2 \le [133700, 177680] \\
& [6.85, 9.08]x_1 + [8.28, 9.74]x_2 \le [126980, 157030] \\
& [4.17, 5.58]x_1 + [6.26, 7.74]x_2 \le [81449, 97387] \\
& x_k \ge 0, k = 1, 2 \\
& x_k \in Y.
\end{aligned}
\tag{14}
$$

Step 4. Regarding the lower bound of all CNNs in the NN-LP model, we can use the built-in function "linprog()" in MATLAB and get the optimal solutions:

inf $x_1 = 10023$, inf $x_2 = 6334$, and inf $F_1(X, I) = 139930$.

Regarding the upper bound of all CNNs in the NN-LP model, we can get the optimal solutions:

sup $x_1 = 14359$, sup $x_2 = 2230$, and sup $F_1(X, I) = 192650$.

Therefore, in the case of the normal distribution and confidence level of 0.95, we can obtain the optimal NN/interval feasible solutions of the CNN-LP model as follows:

$x_1 = [10023, 14359]$, $x_2 = [2230, 6334]$, and $F_1(X, I) = [136510, 196890]$.

4.4. Case 2 with the Log-Normal Distribution.

In theproduction process of Case 2, a manufacturing company of Shaoxing City in Chinawants to produce two kinds of products, which are denoted by the product C and the product D. Then, they need to be processed by the machines I, II, and III. The sample datasets obtained in the production process of Case 2 are listed in Tables 6–8.

Regarding this actual case, CNN-LP type II can be applied to this PPP and then its CNN-LP method iscomposed of the following steps:

Step 1. To obtain CNNs from the sample datasets of the C and D products in Case 2, we take the sample dataset C_1 in Table 6 as a calculation example. Using the "histfit()" function in MATLAB, the fitting curve of the sample dataset C_1 is shown in Figure 2, which reflects that the sample dataset C_1 obeys the log-normal distribution.

Using equations (8) and (9), we can get the mean and standard deviation of the sample dataset C_1:

(i) $b_{C1} = 1/20 \sum_{j=1}^{20} \ln c_{1j} = 2.2$;

(ii) $\delta_{C1} = \sqrt{S_{C1}^2} = \sqrt{1/(20-1) \sum_{s=1}^{20} (\ln c_{1s} - b_{C1})^2} = 0.35$.

Step 2. Using equation (7) with the confidence level of $1 - \alpha = 0.95$ ($\alpha = 0.05$ and $t_{\alpha/2} = 1.96$), we can get the following CNN:

$$
\begin{aligned}
e^{y_{C1}} \in &\ \left[e^{b_{C1} - (\delta_{C1}/\sqrt{n})t_{\alpha/2}}, e^{b_{C1} + (\delta_{C1}/\sqrt{n})t_{\alpha/2}} \right] \\
= &\ \left[e^{y_{C1}^L}, e^{y_{C1}^U} \right] = [7.69, 10.70].
\end{aligned}
\tag{15}
$$

Based on the similar calculational way, when equations (7)–(9) are used for the corresponding sample datasets in Table 6–8, we can obtain the average values
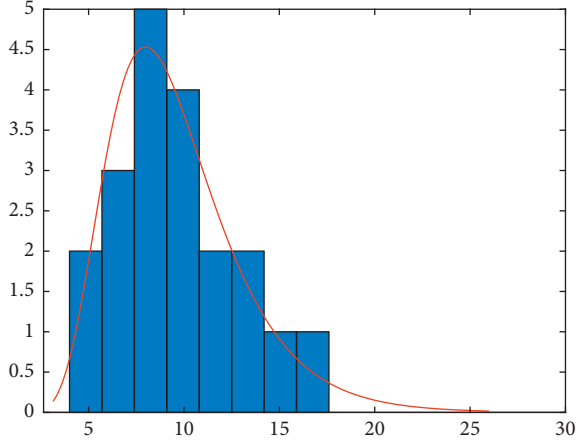
FIGURE 2: Histogram and fitting curve of the sample dataset C_1 in Case 2.

b_{Cj}, b_{Dj}, b_{Ei}, and b_{Pi}, the standard deviations δ_{Cj}, δ_{Dj}, δ_{Ei}, and δ_{Pi}, and the CNNs y_{Cj}, y_{Dj}, y_{Ei}, and y_{Pi} ($j = 1, 2, 3, 4, 5$; $i = 1, 2, 3$) in Case 2 with the log-normal

distribution and the confidence level of $1 - \alpha = 0.95$, as listed in Table 9.

Step 3. In view of the obtained CNNs in the CNN-LP problem, we can establish the following CNN-LP model:

$$
\begin{aligned}
\text{Max } F_2(X, I) = {} & \left(e^{y_{C2}} - e^{y_{C1}}\right)x_1 + \left(e^{y_{D2}} - e^{y_{D1}}\right)x_2 \\
& - \left(e^{y_{C3}}e^{y_{P1}} + e^{y_{C4}}e^{y_{P2}} + e^{y_{C5}}e^{y_{P3}}\right)x_1 \\
& - \left(e^{y_{D3}}e^{y_{P1}} + e^{y_{D4}}e^{y_{P2}} + e^{y_{D5}}e^{y_{P3}}\right)x_2 \\
\text{s.t.} \quad & e^{y_{C3}}x_1 + e^{y_{D3}}x_2 \le e^{y_{E1}} \\
& e^{y_{C4}}x_1 + e^{y_{D4}}x_2 \le e^{y_{E2}} \\
& e^{y_{C5}}x_1 + e^{y_{D5}}x_2 \le e^{y_{E3}} \\
& x_k \ge 0, k = 1, 2 \\
& x_k, y_{Cj}, y_{Dj}, y_{Ei}, y_{Pi} \in Y, \quad j = 1, 2, 3, 4, 5; i = 1, 2, 3.
\end{aligned}
$$

(16)

Substituting all CNNs in Table 9 into the above CNN-LP model, we can get the following final model:

$$
\begin{aligned}
\text{Max } F_2(X, I) = {} & ([28.79, 42.52] - [7.69, 10.70])x_1 + ([31.50, 51.42] - [11.13, 20.49])x_2 \\
& - ([8.58, 11.59] \times [0.52, 0.70] + [7.13, 9.12] \\
& \times [0.67, 0.96] + [4.26, 5.81] \times [0.34, 0.47])x_1 \\
& - ([7.24, 8.85] \times [0.52, 0.70] + [8.08, 10.18] \times [0.67, 0.96] \\
& + [6.30, 7.92] \times [0.34, 0.47])x_2 \\
= {} & [10.41, 12.22]x_1 + [9.05, 11.25]x_2
\end{aligned}
$$

(17)

$$
\begin{aligned}
\text{s.t.} \quad & [8.58, 11.59]x_1 + [7.24, 8.85]x_2 \le [133252, 185350] \\
& [7.13, 9.12]x_1 + [8.08, 10.18]x_2 \le [123007, 161135] \\
& [4.26, 5.81]x_1 + [6.30, 7.92]x_2 \le [79221, 98716] \\
& x_k \ge 0, k = 1, 2 \\
& x_k \in Y.
\end{aligned}
$$

In terms of the lower bound of all CNNs of the CNN-PL model, we can get the optimal feasible solutions:

inf $x_1 = 11457$, inf $x_2 = 4828$, and inf $F_2(X, I) = 162960$.

According to the upper bound of all CNNs in the CNN-LP model, we can get the optimal feasible solutions:

sup $x_1 = 14721$, sup $x_2 = 1665$, and sup $F_2(X, I) = 198620$.

Then, we can obtain the optimal interval feasible solutions of the CNN-LP problem with the log-normal

distribution and the confidence level of $1 - \alpha = 0.95$ as follows:

$x_1 = [11457, 14721]$, $x_2 = [1665, 4828]$, and $F_2(X, I) = [162960, 198620]$.

5. Comparison Analysis

To show the efficiency and rationality of the proposed CNN-LP methods, this section compares the proposed CNN-LP

TABLE 7: Sample datasets of the product D in Case 2.

	Sample dataset									
D_1	22.18	12.11	22.95	35.15	5.74	27.86	36.05	15.46	42.85	16.50
	7.15	4.01	12.30	23.06	18.18	19.26	10.63	16.39	5.62	9.53
D_2	34.68	69.59	23.24	43.67	92.56	35.74	24.14	88.52	3453	52.19
	24.73	47.59	53.00	40.15	48.70	82.50	17.25	45.30	12.13	40.73
D_3	5.95	10.70	10.34	6.74	6.48	9.70	9.90	9.61	8.05	8.52
	8.79	11.83	6.64	6.91	8.05	8.14	6.45	6.20	6.91	7.34
D_4	7.98	8.60	10.20	6.78	9.93	9.24	13.43	8.47	8.67	6.35
	5.96	9.91	8.37	10.50	6.90	16.00	9.36	12.36	8.43	8.60
D_5	10.49	10.96	6.37	6.96	6.40	7.14	5.85	4.79	6.98	5.50
	4.80	11.30	6.71	7.62	8.92	5.02	7.63	7.77	7.15	7.25

TABLE 8: Sample datasets of the machine processing in Case 2.

	Sample dataset									
E_1	107897	65762	279258	216827	192437	180733	235808	184720	86100	198369
	138813	167355	168865	176332	128644	128027	107370	229570	161599	159640
E_2	100675	227526	147926	82318	168843	154091	248403	159812	167417	81119
	128832	173706	128834	176244	110249	144007	131579	116473	144660	135736
E_3	70453	122438	114731	127875	95004	126949	87116	84998	79522	99080
	79721	87148	83010	83843	51960	83376	57696	95852	106401	74284
P_1	0.74	1.30	1.01	0.53	0.42	0.52	0.43	0.89	0.53	0.57
	0.46	0.73	0.45	0.75	0.44	0.65	0.52	0.75	0.48	0.48
P_2	1.24	0.66	1.10	0.53	1.73	0.86	0.79	0.68	0.65	0.66
	0.69	1.34	0.99	0.80	0.53	0.41	1.65	0.75	0.76	0.51
P_3	0.47	0.31	0.36	0.72	0.42	0.53	0.59	0.48	0.42	0.59
	0.27	0.58	0.21	0.27	0.35	0.39	0.24	0.51	0.39	0.29

TABLE 9: Average values, standard deviations, and CNNs in the CNN-LP problem of Case 2.

b_{Cj}	b_{Dj}	b_{Ei}	b_{Pi}	δ_{Cj}	δ_{Dj}	δ_{Ei}	δ_{Pi}	$e^{c_{Di}}$	$e^{y_{Di}}$	$e^{y_{Ei}}$	$e^{y_{Pi}}$
2.20	2.71			0.35	0.66			[7.69, 10.70]	[11.13, 20.49]		
3.56	3.69			0.42	0.53			[28.79, 42.52]	[31.50, 51.42]		
2.30	2.08	11.96	−0.51	0.33	0.20	0.36	0.31	[8.58, 11.59]	[7.24, 8.85]	[133252, 185350]	[0.52, 0.70]
2.08	2.20	11.86	−0.22	0.27	0.24	0.29	0.40	[7.13, 9.12]	[8.08, 10.18]	[123007, 161135]	[0.67, 0.96]
1.60	1.96	11.39	−0.92	0.33	0.36	0.24	0.34	[4.26, 5.81]	[6.30, 7.92]	[79221, 98716]	[0.34, 0.47]

methods with existing traditional linear programming methods through the above actual production planning cases.

5.1. Traditional Linear Programming Methods.
First, we utilize the average values (typical/common values) of the sample datasets in Cases 1 and 2 to construct the traditional

linear programming models with certain values. Using equation (3), all the average values of the sample datasets are listed in Table 10.

Then, we assume that the objective function of the traditional linear programming model is $f(X)$. Based on the traditional linear programming method, we can construct the traditional linear programming model:

$$
\begin{aligned}
\text{Max } f(X) = & (b_{C2} - b_{C1})x_1 + (b_{D2} - b_{D1})x_2 - (b_{C3} \times b_{P1} + b_{C4} \times b_{P2} + b_{C5} \times b_{P3})x_1 \\
& - (b_{D3} \times b_{P1} + b_{D4} \times b_{P2} + b_{D5} \times b_{P3})x_2 \\
\text{s.t. } \quad & b_{C3}x_1 + b_{D3}x_2 \le b_{E1} \\
& b_{C4}x_1 + b_{D4}x_2 \le b_{E2} \\
& b_{C5}x_1 + b_{D5}x_2 \le b_{E3} \\
& x_k \ge 0, k = 1, 2.
\end{aligned}
\tag{18}
$$

TABLE 10: Average values of the sample datasets in Cases 1 and 2.

Case 1				Case 2			
b_{Cj}	b_{Dj}	b_{Ei}	b_{Pi}	b_{Cj}	b_{Dj}	b_{Ei}	b_{Pi}
9.22	14.7			9.59	18.15		
34.27	40.00			37.92	45.55		
9.86	7.94	155690	0.62	10.49	8.16	165710	0.63
7.96	9.01	142000	0.82	8.30	9.30	146420	0.87
4.88	7.00	89418	0.39	5.23	7.28	90573	0.42

Substituting all the average values of the sample datasets for Case 1 in Table 10 into the linear programming model, we obtain the following final model:

$$\text{Max } f_1(X) = (34.27 - 9.22)x_1 + (40.00 - 14.70)x_2 - (9.86 \times 0.62 + 7.96 \times 0.82 + 4.88 \times 0.39)x_1$$
$$- (7.94 \times 0.62 + 9.01 \times 0.82 + 7.00 \times 0.39)x_2$$

$$\text{s.t. } 9.86x_1 + 7.94x_2 \leq 155690$$
$$7.96x_1 + 9.01x_2 \leq 142000$$
$$4.88x_1 + 7.00x_2 \leq 89418$$
$$x_k \geq 0, k = 1, 2.$$

(19)

By the built-in function "linprog()" in MATLAB, we obtain the optimal certain/crisp solutions:
$x_1 = 12589$, $x_2 = 4054$, and $f_1(X) = 170610$.

Substituting all the average values of the sample datasets for Case 2 in Table 10 into the traditional linear programming model, we obtain the following final model:

$$\text{Max } f_2(X) = (37.92 - 9.59)x_1 + (45.55 - 18.15)x_2 - (10.49 \times 0.63 + 8.30 \times 0.87 + 5.23 \times 0.42)x_1$$
$$- (8.16 \times 0.63 + 9.30 \times 0.87 + 7.28 \times 0.42)x_2 = 12.30x_1 + 11.11x_2$$

$$\text{s.t. } 10.49x_1 + 8.16x_2 \leq 165710$$
$$8.30x_1 + 9.30x_2 \leq 146420$$
$$5.23x_1 + 7.28x_2 \leq 90573$$
$$x_k \geq 0, k = 1, 2.$$

(20)

Using the built-in function "linprog()" in MATLAB, we can also give the optimal certain/crisp solutions:
$x_1 = 12933$, $x_2 = 3333$, and $f_2(X) = 184030$.

It is clear that the optimal certain/crisp solutions of the decision variables and objective functions corresponding to Cases 1 and 2 completely fall within the optimal interval feasible solution ranges of the proposed CNN-LP methods, which are shown in Figure 3 and 4. Then, these optimal certain/crisp solutions are only the special solutions of the proposed CNN-LP methods for Cases 1 and 2.

In view of the optimal solutions in Figures 3 and 4, the traditional production planning methods are only the special cases of the proposed CNN-LP methods. Since the traditional production planning methods lose some useful optimal solutions in the optimal interval feasible solutions, they cannot reasonably deal with indeterminate PPPs and also cannot guarantee the credibility/reliability and effectiveness of these optimal solutions from a probabilistic point of view;

while the proposed CNN-LP methods with some distribution and confidence level of product sample datasets can guarantee the credibility/reliability and effectiveness of the optimal interval feasible solutions from a probabilistic point of view. Therefore, the proposed CNN-LP methods are more suitable for actual PPPs than the traditional linear programming methods, since the optimal interval feasible solutions of the former are more flexible and confident/reliable than the optimal exact solutions of the latter in indeterminate situations. Clearly, the proposed CNN-LP methods reveal obvious advantages in indeterminate PPPs.

5.2. General NN-LP Methods. It is known that the general NN-LP methods [23, 25–30] are composed of the NN objective function and the NN constraint equations to reflect the uncertainty in the actual PPPs. Then, the optimal feasible solutions of the decision variables and the objective function
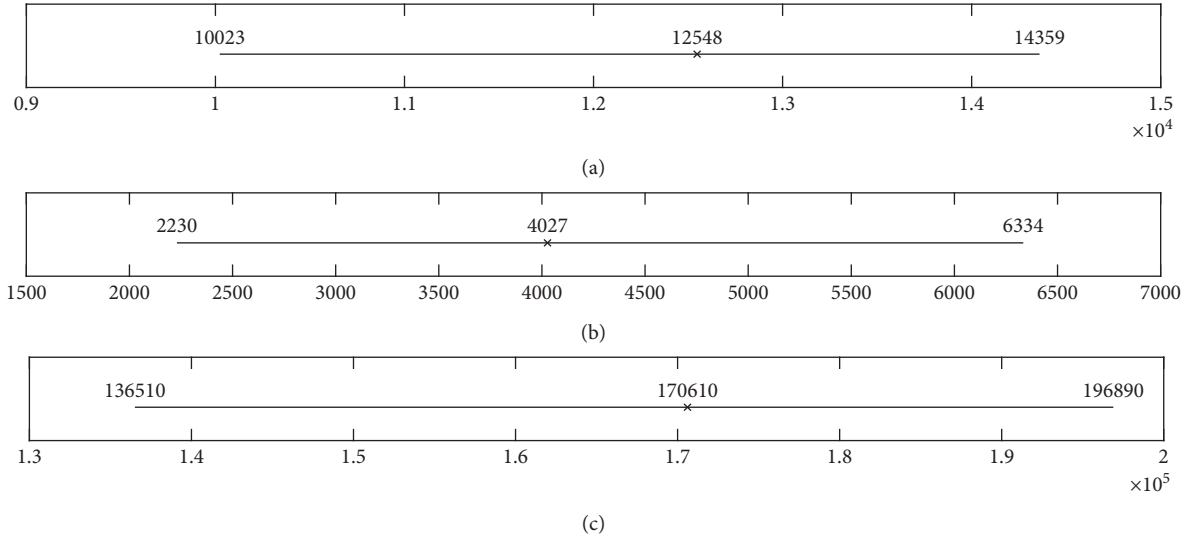
(a)

(b)

(c)

FIGURE 3: Optimal solutions of Case 1. (a) Solutions of x_1 based on the proposed CNN-LP method and the traditional linear programming method; (b) solutions of x_2 based on the proposed CNN-LP method and the traditional linear programming method; (c) solutions of $F_1(X, I)$ and $f_1(X)$ based on the proposed CNN-LP method and the traditional linear programming method.
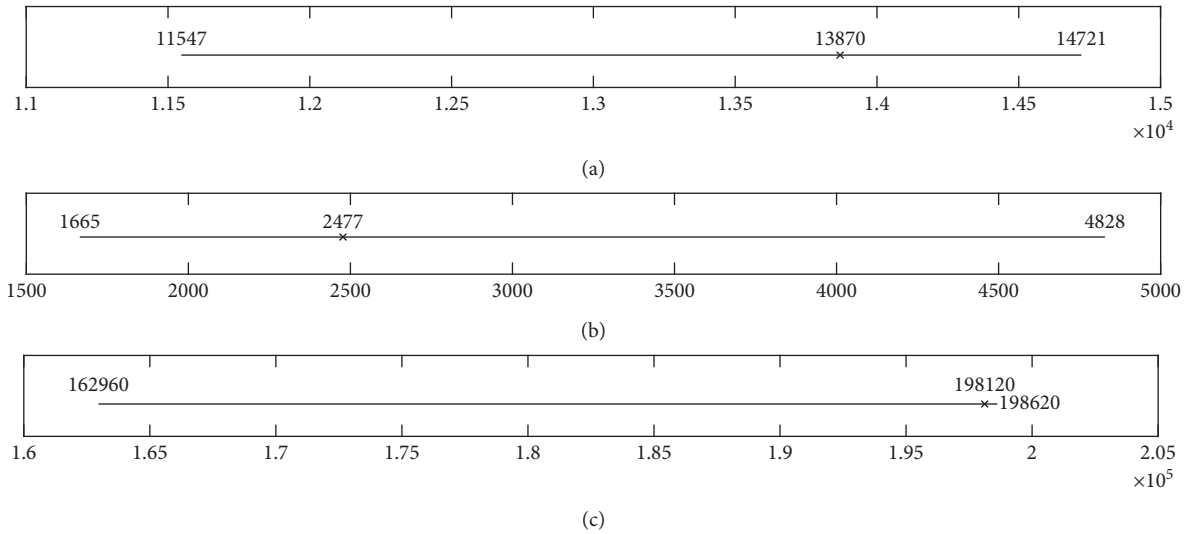

(a)

(b)

(c)

FIGURE 4: Optimal solutions of Case 2. (a) Solutions of x_1 based on the proposed CNN-LP method and the traditional linear programming method; (b) solutions of x_2 based on the proposed CNN-LP method and the traditional linear programming method; (c) solutions of $F_2(X, I)$ and $f_2(X)$ based on the proposed CNN-LP method and the traditional linear programming method.

are obtained only depending on subjectively specified indeterminate values/ranges of NNs. Since the general NN-LP methods [23, 25–30] cannot reflect some distribution and confidence level/interval of sample datasets/multivalued sets from a probabilistic point of view, the optimal feasible solutions cannot ensure their credibility and reliability. The proposed CNN-LP methods are composed of the CNN objective function and the CNN constraint equations corresponding to some distribution and confidence level/interval of product sample datasets to deal with uncertain PPPs. Therefore, the proposed CNN-LP methods can guarantee the credibility/reliability and rationality of their optimal interval feasible solutions from a probabilistic point

of view. In this situation, the proposed CNN-LP methods demonstrate obvious superiority over the general NN-LP methods [23, 25–30] and show their effectiveness and rationality in handling the indeterminate PPPs.

6. Conclusion

In this original study, the presented transformation method from a multivalued set (a product sample dataset) with the normal distribution or log-normal distribution to CNN corresponding to a confidence level of $1-\alpha$ can guarantee the product sample dataset within the CNN/confidence interval from a probabilistic point of view. Then, the

proposed CNN-LP methods can effectively and reasonably handle PPPs in uncertain environments and ensure the credibility and reliability of their optimal interval feasible solutions. Through the built-in function "linprog()" in Matlab in the solution process, the optimal interval feasible solutions of the proposed CNN-LP models can be obtained by the lower and upper bounds of all CNNs in terms of the normal and the log-normal distributions and some confidence level. In actual applications, the proposed CNN-LP methods can perform two production planning cases under the situations of the normal and log-normal distributions and a confidence level of 0.95. Compared with the traditional linear programming methods and the previous NN-LP methods in view of the special cases of Cases 1 and 2, the proposed CNN-LP methods revealed the following main advantages:

(1) The proposed transformation method can reasonably converse product sample datasets into CNNs corresponding to the normal and log-normal distributions and confidence levels of the sample datasets to avoid the specified certain/crisp values or average values only used in traditional optimization methods.

(2) The proposed CNN-LP methods can obtain the optimal interval feasible solutions of the decision variables and objective functions in view of some data distributions and confidence levels to avoid the unreliable/unique crisp special solutions of the traditional linear programming methods.

(3) Compared to previous NN-LP methods, the proposed CNN-LP methods can objectively guarantee the credibility and efficiency of the optimal interval feasible solutions of the decision variables and the objective function based on some probabilistic distributions and confidence levels. Then, the proposed CNN-LP methods can overcome the defects of existing NN-LP methods and reflect a better superiority over existing NN-LP methods in actual PPPs.

However, the proposed CNN-LP methods are only used for linear programming problems with the normal and log-normal distributions in this study, which show their application limitations. In the future work, we shall continue to extend the CNN-LP methods to CNN nonlinear programming problems and then apply them to other fields, such as industrial engineering, mechanical design, and engineering structure design in the situation of different distributions.

Data Availability

All the data are included in the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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