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ON N_m-α-OPEN SETS IN NEUTROSOPHIC MINIMAL STRUCTURE SPACES

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Abstract. In this paper, we introduced the notions of N_m-α-open sets, α-interior and α-closure operators in neutrosophic minimal structures. We investigate some basic propertiesof such notions. Also we introduced the notion of N_m-α-continuous maps and study characterizations of N_m-α-continuous maps by using the α-interior and α-closure operators. We introduced the classes of $N_m lc$-set, $N_m\alpha$lc-sets and study some of its basic properties. Finally, we introduced and studied $N_m lc$-continuous, $N_m\alpha$lc-continuous map, $N_m lc$-irresolute map and $N_m\alpha$lc-irresolute map and investigate some properties of such concepts.

Keywords: Neutrosophic minimal structure spaces, N_m-α-closed, N_m-α-open, $N_m lc$-set, N_m-α-lc-set and N_m-α-continuous.

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1. INTRODUCTION

L. A. Zadeh's [12] Fuzzy set laid the foundation of many theories such as intuitionistic fuzzy set and neutrosophic set, rough sets etc. Later, researchers developed K. T. Atanassov's [4] intuitionistic fuzzy set theory in many fields such as differential equations, topology, computer science and so on. F. Smarandache [10, 11] found that some objects have indeterminacy or neutral other than membership and non-membership. So he coined the notion of neutrosophy. The theories of neutrosophic set have achieved greater success in various areas such as medical diagnosis, database, topology, image processing and decision making problem. While the neutrosophic set is a powerful tool to deal with indeterminate and inconsistent data, the theory of rough set is a powerful mathematical tool to deal with incompleteness. Neutrosophic sets and rough sets are two different topics, none conflicts the other. Valeiru Popa and Noiri [8] introduced the notion of of minimal structure which is a generalization of a topology on a given nonempty set. And they introduced the notion of $\mathcal{M}$-continuous functions as functions defined between minimal structures. M. Karthika et al [7] introduced and studied neutrosophic minimal structure spaces. S. Ganesan [6] introduced and studied N_m-semi open sets. The main objective of this study is to introduce a new hybrid intelligent structure called N_m-α-open sets in neutrosophic minimal structure spaces. The significance of introducing hybrid structures is that the computational techniques, based on any one of these structures alone, will not always yield the best results but a fusion of two or more of them can often give better results. The rest of this paper is organized as follows. Some preliminary concepts required in our work are briefly recalled in section 2. In section 3, the concepts of N_m-α-open, N_m-α-closure, N_m-α-interior, N_m-α-continuous is investigated.

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2. PRELIMINARIES

Definition 2.1. [8] *A subfamily m_x of the power set $\wp(X)$ of a nonempty set X is called a minimal structure (briefly, m-structure) on X if $\emptyset$ in m_x and $X \in m_x$. By (X, m_x), we denote a nonempty set X with a minimal structure m_x on X and call it an m-space. Each member of m_x is said to be m_x-open (or briefly, m-open) and the complement of an m_x-open set is said to be m_x-closed (or briefly, m-closed).*

Definition 2.2. [10, 11] *A neutrosophic set (in short ns) K on a set $X \neq \emptyset$ is defined by $K = \{\prec a, P_K(a), Q_K(a), R_K(a) \succ : a \in X\}$ where $P_K : X \to [0,1]$, $Q_K : X \to [0,1]$ and $R_K : X \to [0,1]$ denotes the membership of an object, indeterminacy and non-membership of an object, for each $a \in X$ to K, respectively and $0 \leq P_K(a) + Q_K(a) + R_K(a) \leq 3$ for each $a \in X$.*

Definition 2.3. [9] *Let $K = \{\prec a, P_K(a), Q_K(a), R_K(a) \succ : a \in X\}$ be a ns. We must introduce the ns $0_\sim$ and $1_\sim$ in X as follows:*
$0_\sim$ may be defined as:

(1) $0_\sim = \{\prec x, 0, 0, 1 \succ : x \in X\}$
(2) $0_\sim = \{\prec x, 0, 1, 1 \succ : x \in X\}$
(3) $0_\sim = \{\prec x, 0, 1, 0 \succ : x \in X\}$
(4) $0_\sim = \{\prec x, 0, 0, 0 \succ : x \in X\}$

$1_\sim$ may be defined as:

(1) $1_\sim = \{\prec x, 1, 0, 0 \succ : x \in X\}$
(2) $1_\sim = \{\prec x, 1, 0, 1 \succ : x \in X\}$
(3) $1_\sim = \{\prec x, 1, 1, 0 \succ : x \in X\}$
(4) $1_\sim = \{\prec x, 1, 1, 1 \succ : x \in X\}$

Proposition 2.4. [9] *For any ns S, then the following conditions are holds:*

(1) $0_\sim \leq S$, $0_\sim \leq 0_\sim$.
(2) $S \leq 1_\sim$, $1_\sim \leq 1_\sim$.

Definition 2.5. [9] *Let $K = \{\prec a, P_K(a), Q_K(a), R_K(a) \succ : a \in X\}$ be a ns.*

(1) *A ns K is an empty set i.e., $K = 0_\sim$ if 0 is membership of an object and 0 is an indeterminacy and 1 is an non-membership of an object respectively. i.e., $0_\sim = \{x, (0, 0, 1) : x \in X\}$*
(2) *A ns K is a universal set i.e., $K = 1_\sim$ if 1 is membership of an object and 1 is an indeterminacy and 0 is an non-membership of an object respectively. $1_\sim = \{x, (1, 1, 0) : x \in X\}$*
(3) $K_1 \cup K_2 = \{a, max\ \{P_{K_1}(a), P_{K_2}(a)\}, max\ \{Q_{K_1}(a), Q_{K_2}(a)\}, min\ \{R_{K_1}(a), R_{K_2}(a)\} : a \in X\}$
(4) $K_1 \cap K_2 = \{a, min\ \{P_{K_1}(a), P_{K_2}(a)\}, min\ \{Q_{K_1}(a), Q_{K_2}(a)\}, max\ \{R_{K_1}(a), R_{K_2}(a)\} : a \in X\}$
(5) $K^c = \{\prec a, R_K(a), 1 - Q_K(a), P_K(a) \succ : a \in X\}$

Definition 2.6. [9] *A neutrosophic topology (nt) in Salama's sense on a nonempty set X is a family τ of ns in X satisfying three axioms:*

(1) *Empty set ($0_\sim$) and universal set ($1_\sim$) are members of τ.*
(2) $K_1 \cap K_2 \in \tau$ *where* $K_1, K_2 \in \tau$.
(3) $\cup K_\delta \in \tau$ *for every* $\{K_\delta : \delta \in \Delta\} \leq \tau$.

Each ns in nt are called neutrosophic open sets. Its complements are called neutrosophic closed sets.

Definition 2.7. [7] *Let the neutrosophic minimal structure space over a universal set X be denoted by N_m. N_m is said to be neutrosophic minimal structure space (in short, nms) over X if it satisfying following the axiom: $0_\sim$, $1_\sim \in N_m$. A family of neutrosophic minimal structure space is denoted by (X, N_{mX}).*
Note that neutrosophic empty set and neutrosophic universal set can form a topology and itis known as neutrosophic minimal structure space.
Each ns in nms is neutrosophic minimal open set. The complement of neutrosophic minimal open set is neutrosophic minimal closed set.

Remark 2.8. [7] *Each ns in nms is neutrosophic minimal open set.*
The complement of neutrosophic minimal open set is neutrosophic minimal closed set.

Definition 2.9. [7] *A is N_m-closed if and only if $N_m cl(A) = A$. Similarly, A is a N_m-open if and only if $N_m int(A) = A$.*

Definition 2.10. [7] *Let N_m be any nms and A be any neutrosophic set. Then*

1. *Every $A \in N_m$ is open and its complement is closed.*
2. *N_m-closure of A = min $\{F : F$ is a neutrosophic minimal closed set and $F \geq A\}$ and it is denoted by $N_m cl(A)$.*
3. *N_m-interior of A = max $\{F : F$ is a neutrosophic minimal open set and $F \leq A\}$ and it is denoted by $N_m int(A)$.*

In general $N_m$int(A) is subset of A and A is a subset of $N_m$cl(A).

Proposition 2.11. [7] *Let R and S are any ns of nms N_m over X. Then*

1. *$N_m^c = \{0, 1, R_i^c\}$ where R_i^c is a complement of ns R_i.*
2. *$X - N_m int(S) = N_m cl(X - S)$.*
3. *$X - N_m cl(S) = N_m int(X - S)$.*
4. *$N_m cl(R^c) = (N_m cl(R))^c = N_m int(R)$.*
5. *N_m closure of an empty set is an empty set and N_m closure of a universal set is a universal set. Similarly, N_m interior of an empty set and universal set respectively an empty and a universal set.*
6. *If S is a subset of R then $N_m cl(S) \leq N_m cl(R)$ and $N_m int(S) \leq N_m int(R)$.*
7. *$N_m cl(N_m cl(R)) = N_m cl(R)$ and $N_m int(N_m int(R)) = N_m int(R)$.*
8. *$N_m cl(R \vee S) = N_m cl(R) \vee N_m cl(S)$.*
9. *$N_m cl(R \wedge S) = N_m cl(R) \wedge N_m cl(S)$.*

Definition 2.12. [7] *Let (X, N_{mX}) be nms.*

1. *Arbitrary union of neutrosophic minimal open sets in (X, N_{mX}) is neutrosophic minimal open. (Union Property).*
2. *Finite intersection of neutrosophic minimal open sets in (X, N_{mX}) is neutrosophic minimal open. (intersection Property).*

Definition 2.13. [7] *A map $f : (X, N_{mX}) \rightarrow (Y, N_{mY})$ is called neutrosophic minimal continuous map if and only if $f^{-1}(V) \in N_{mX}$ whenever $V \in N_{mY}$.*

Definition 2.14. [6] *Let (X, N_{mX}) be a nms and $A \leq X$. A subset A of X is called an N_m-semi-open set if $A \leq N_m cl(N_m int(A))$. The complement of an N_m-semi open set is called an N_m-semi-closed set.*

3. N_m-α-OPEN SETS

Definition 3.1. *Let (X, N_{mX}) be a nms and $A \leq X$. A subset A of X is called an N_m-α-open set if $A \leq N_m int(N_m cl(N_m int(A)))$. The complement of an N_m-α-open set is called an N_m-α-closed set.*

Remark 3.2. *Let (X, $\mathcal{T}$) be a nt and $A \leq X$. A is called an $\mathcal{N}\alpha$-open set* [3] *if $A \leq \mathcal{N}int(\mathcal{N}cl(\mathcal{N}int(A)))$. If the nms N_{mX} is a topology, clearly an N_m-α-open set is $\mathcal{N}\alpha$-open.*

Example 3.3. *Let $X = \{a, b\}$ with $\mathcal{T} = \{0_\sim, A, B, C, D, 1_\sim\}$ and $\mathcal{T}^c = \{1_\sim, F, G, H, I, 0_\sim\}$ where A = $\prec$ (0.5, 0.4, 0.5), (0.5, 0.6, 0.5)$\succ$; B = $\prec$ (0.5, 0.6, 0.5), (0.5, 0.6, 0.6)$\succ$; C = $\prec$ (0.6, 0.6, 0.5), (0.4, 0.4, 0.5)$\succ$; D = $\prec$ (0.5, 0.5, 0.5), (0.5, 0.5, 0.5)$\succ$. F = $\prec$ (0.5, 0.6, 0.5), (0.5, 0.4, 0.5)$\succ$; G = $\prec$ (0.5, 0.4, 0.5), (0.6, 0.4, 0.6)$\succ$; H = $\prec$ (0.5, 0.4, 0.6), (0.5, 0.6, 0.4)$\succ$; I = $\prec$ (0.5, 0.5, 0.5), (0.5, 0.5, 0.5)$\succ$. Now we define the neutrosophic set as follows: V = $\prec$ (0.5, 0.5, 0.5), (0.5, 0.5, 0.5)$\succ$. Let $X = \{a, b\}$ with $N_m = \{0_\sim, E, 1_\sim\}$ and $N_m^c = \{1_\sim, J, 0_\sim\}$ where E = $\prec$ (0.4, 0.3, 0.6), (0.5, 0.4, 0.8)$\succ$; J = $\prec$ (0.6, 0.7, 0.4), (0.8, 0.6, 0.5)$\succ$. We know that $0_\sim = \{\prec x, 0, 0, 1 \succ : x \in X\}$, $1_\sim = \{\prec x, 1, 1, 0 \succ : x \in X\}$ and $0_\sim^c = \{\prec x, 1, 1, 0 \succ : x \in X\}$, $1_\sim^c = \{\prec x, 0, 0, 1 \succ : x \in X\}$. Here, $\mathcal{N}int(V) = D$, $\mathcal{N}cl(\mathcal{N}int(V)) = \mathcal{N}cl(D) = I$, $\mathcal{N}int(\mathcal{N}cl(\mathcal{N}int(V))) = \mathcal{N}int(I) = D$. Therefore, V is a $\mathcal{N}$ α-open but it is not N_m-α-open.*

From Definition of 3.1, obviously the following statement are obtained.

Lemma 3.4. *Let (X, N_{mX}) be a nms. Then*

(1) Every N_m-open set is N_m-α-open.
(2) A is an N_m-α-open set if and only if $A \leq N_m int(N_m cl(N_m int(A)))$.
(3) Every N_m-closed set is N_m-α-closed.
(4) A is an N_m-α-closed set if and only if $N_m cl(N_m int(N_m cl(A))) \leq A$.

Theorem 3.5. *Let (X, N_{mX}) be a nms. Any union of N_m-α-open sets is N_m-α-open.*

Proof. Let A_δ be an N_m-α-open set for $\delta \in \Delta$. From Definition 3.1 and Proposition 2.11(6), it follows $A_\delta \leq N_m int(N_m cl(N_m int(A_\delta))) \leq N_m int(N_m cl(N_m int(\bigcup A_\delta)))$. This implies $\bigcup A_\delta \leq N_m int(N_m cl(N_m int(\bigcup A_\delta)))$. Hence $\bigcup A_\delta$ is an N_m-α-open set. □

Remark 3.6. *Let (X, N_{mX}) be a nms. The intersection of any two N_m-α-open sets may not be N_m-α-open set as shown in the next example.*

Example 3.7. *Let $X = \{a, b\}$ with $N_m = \{0_\sim, P, Q, R, S, 1_\sim\}$ and $N_m^c = \{1_\sim, I, J, K, L, 0_\sim\}$ where P = $\prec$ (0.4, 0.6, 0.5), (0.7, 0.3, 0.5)$\succ$; Q = $\prec$ (0.3, 0.6, 0.8), (0.6, 0.3, 0.5)$\succ$; R = $\prec$ (0.3, 0.7, 0.8), (0.6, 0.5, 0.2)$\succ$; S = $\prec$ (0.4, 0.7, 0.5), (0.6, 0.4, 0.2)$\succ$; I = $\prec$ (0.5, 0.4, 0.4), (0.5, 0.7, 0.7)$\succ$; J = $\prec$ (0.8, 0.4, 0.3), (0.5, 0.7, 0.6)$\succ$; K = $\prec$ (0.8, 0.3, 0.3), (0.2, 0.5, 0.6)$\succ$; L = $\prec$ (0.5, 0.3, 0.4), (0.2, 0.6, 0.6)$\succ$. Now we define the two N_m-α-open sets as follows : D = $\prec$ (0.5, 0.7, 0.5), (0.9, 0.4, 0.5)$\succ$; E = $\prec$ (0.9, 0.8, 0.3), (0.6, 0.4, 0.1)$\succ$. Here*

$N_m int(D) = P$, $N_m cl(N_m int(D)) = N_m cl(P) = 0^c_\sim$, $N_m int(N_m cl(N_m int(D))) = N_m int(0^c_\sim) = 1_\sim$, $N_m int(E) = S$, $N_m cl(N_m int(E)) = N_m cl(S) = 0^c_\sim$, $N_m int(N_m cl(N_m int(E))) = N_m int(0^c_\sim) = 1_\sim$. But $D \wedge E = \prec (0.5, 0.7, 0.5), (0.6, 0.4, 0.5) \succ$ is not a N_m-α-open set in X.

Proposition 3.8. *Let (X, N_{mX}) be a nms. Every N_m-α-open set is N_m-semi-open set.*

Proof. The proof is straightforword from the definitions. □

Example 3.9. *Let $X = \{a, b\}$ with $N_m = \{0_\sim, A, 1_\sim\}$ and $N^c_m = \{1_\sim, B, 0_\sim\}$ where $A = \prec (0.4, 0.3, 0.7), (0.5, 0.4, 0.9) \succ$; $B = \prec (0.7, 0.7, 0.4), (0.9, 0.6, 0.5) \succ$. Now we define the neutrosophic set as follows: $C = \prec (0.5, 0.4, 0.6), (0.5, 0.5, 0.4) \succ$. Here, $N_m int(C) = A$, $N_m cl(N_m int(C)) = N_m cl(A) = B$, $N_m int(N_m cl(N_m int(C))) = N_m int(B) = A$. Therefore, C is a N_m-semi-open but it is not N_m-α-open.*

Definition 3.10. *Let (X, N_{mX}) be a nms. For a subset A of X, the N_m-α-closure of A and the N_m-α-interior of A, denoted by N_m-$\alpha cl(A)$ and N_m-$\alpha int(A)$, respectively, are defined as the following:*

(1) N_m-α-closure of $A = min\ \{F : F$ is N_m-α-closed set and $F \geq A\}$ and it is denoted by N_m-$\alpha cl(A)$.
(2) N_m-α-interior of $A = max\ \{G : G$ is N_m-α-open set and $G \leq A\}$ and it is denoted by N_m-$\alpha int(A)$.

Theorem 3.11. *Let (X, N_{mX}) be a nms and $A \leq X$. Then*

(1) N_m-$\alpha int(A) \leq A$.
(2) If $A \leq B$, then N_m-$\alpha int(A) \leq N_m$-$\alpha int(B)$.
(3) A is N_m-α-open if and only if N_m-$\alpha int(A) = A$.
(4) N_m-$\alpha int(N_m$-$\alpha int(A)) = N_m$-$\alpha int(A)$.
(5) N_m-$\alpha cl(X - A) = X - N_m$-$\alpha int(A)$ and N_m-$\alpha int(X - A) = X - N_m$-$\alpha cl(A)$.

Proof. (1), (2) Obvious.
(3) It follows from Theorem 3.5.
(4) It follows from (3).
(5) For A ≤ X, X − N_m -αint(A) = X − max {U : U ≤ A, U is N_m - α-open } = min { X − U : U ≤ A, U is N_m - α-open } = min { X − U : X − A ≤ X − U, U is N_m-α-open} = N_m-αcl(X − A). Similarly, we have N_m-αint(X − A) = X − N_m-αcl(A). □

Theorem 3.12. *Let (X, N_{mX}) be a nms and $A \leq X$. Then*

(1) $A \leq N_m$-$\alpha cl(A)$.
(2) If $A \leq B$, then N_m-$\alpha cl(A) \leq N_m$-$\alpha cl(B)$.
(3) F is N_m-α-closed if and only if N_m-$\alpha cl(F) = F$.
(4) N_m-$\alpha cl(N_m$-$\alpha cl(A)) = N_m$-$\alpha cl(A)$.

Proof. It is similar to the proof of Theorem 3.11. □

Theorem 3.13. *Let (X, N_{mX}) be a nms and $A \leq X$. Then*

(1) $x \in N_m$-$\alpha cl(A)$ if and only if $A \cap V \neq \emptyset$ for every N_m-α-open set V containing x.
(2) $x \in N_m$-$\alpha int(A)$ if and only if there exists an N_m-α-open set U such that $U \leq A$.

Proof. (1) Suppose there is an N_m-α-open set V containing x such that $A \cap V = \emptyset$. Then $X - V$ is an N_m-α-closed set such that $A \leq X - V$, $x \notin X - V$. This implies $x \notin N_m\text{-}\alpha cl(A)$.

The reverse relation is obvious.

(2) Obvious. □

Lemma 3.14. *Let (X, N_{mX}) be a nms and $A \leq X$. Then*

(1) $N_m cl(N_m int(N_m cl(A))) \leq N_m cl(N_m int(N_m cl(N_m\text{-}\alpha cl(A)))) \leq N_m\text{-}\alpha cl(A)$.
(2) $N_m\text{-}\alpha int(A) \leq N_m int(N_m cl(N_m int(N_m\text{-}\alpha int(A)))) \leq N_m int(N_m cl(N_m int(A)))$.

Proof. (1) For $A \leq X$, by Theorem 3.12, N_m-αcl(A) is an N_m-α-closed set. Hence from Lemma 3.4, we have $N_m\text{cl}(N_m\text{int}(N_m\text{cl}(A))) \leq N_m\text{cl}(N_m\text{int}(N_m\text{cl}(N_m\text{-}\alpha\text{cl}(A)))) \leq N_m\text{-}\alpha\text{cl}(A)$.
(2) It is similar to the proof of (1). □

Definition 3.15. *A map $f : (X, N_{mX}) \to (Y, N_{mY})$ is called N_m-α-continuous map if and only if $f^{-1}(V) \in N_m$-α-open whenever $V \in N_{mY}$.*

Theorem 3.16. *Every neutrosophic minimal continuous is N_m-α-continuous but the conversely.*

Proof. The proof follows from Lemma 3.4 (1). □

Theorem 3.17. *Let $f : X \to Y$ be a map on two nms (X, N_{mX}) and (Y, N_{mY}). Then the following statements are equivalent:*

(1) f is N_m-α-continuous.
(2) $f^{-1}(V)$ is an N_m-α-open set for each N_m-open set V in Y.
(3) $f^{-1}(B)$ is an N_m-α-closed set for each N_m-closed set B in Y.
(4) $f(N_m\text{-}\alpha cl(A)) \leq N_m cl(f(A))$ for $A \leq X$.
(5) $N_m\text{-}\alpha cl(f^{-1}(B)) \leq f^{-1}(N_m cl(B))$ for $B \leq Y$.
(6) $f^{-1}(N_m int(B)) \leq N_m\text{-}\alpha int(f^{-1}(B))$ for $B \leq Y$.

Proof. (1) ⇒ (2) Let V be an N_m-open set in Y and $x \in f^{-1}(V)$. By hypothesis, there exists an N_m-α-open set U_x containing x such that $f(U) \leq V$. This implies $x \in U_x \leq f^{-1}(V)$ for all $x \in f^{-1}(V)$. Hence by Theorem 3.5, $f^{-1}(V)$ is N_m-α-open.
(2) ⇒ (3) Obvious.
(3) ⇒ (4) For $A \leq X$, $f^{-1}(N_m\text{cl}(f(A))) = f^{-1}(\min \{F \leq Y : f(A) \leq F$ and F is N_m-closed$\}) = \min \{f^{-1}(F) \leq X : A \leq f^{-1}(F)$ and F is N_m-α-closed$\} \geq \min \{K \leq X : A \leq K$ and K is N_m-α-closed$\} = N_m\text{-}\alpha\text{cl}(A)$. Hence $f(N_m\text{-}\alpha\text{cl}(A)) \leq N_m\text{cl}(f(A))$.
(4) ⇒ (5) For $A \leq X$, from (4), it follows $f(N_m\text{-}\alpha\text{cl}(f^{-1}(A))) \leq N_m\text{cl}(f(f^{-1}(A))) \leq N_m\text{cl}(A)$. Hence we get (5).
(5) ⇒ (6) For $B \leq Y$, fromm $N_m\text{int}(B) = Y - N_m\text{cl}(Y - B)$ and (5), it follows: $f^{-1}(N_m\text{int}(B)) = f^{-1}(Y - N_m\text{cl}(Y - B)) = X - f^{-1}(N_m\text{cl}(Y - B)) \leq X - N_m\text{-}\alpha\text{cl}(f^{-1}(Y - B)) = N_m\text{-}\alpha\text{int}(f^{-1}(B))$. Hence (6) is obtained.
(6) ⇒ (1) Let $x \in X$ and V an N_m-open set containing f(x). Then from (6) and Proposition 2.11, it follows $x \in f^{-1}(V) = f^{-1}(N_m\text{int}(V)) \leq N_m\text{-}\alpha\text{int}(f^{-1}(V))$. So from Theorem 3.13, we can say that there exists an N_m-α-open set U containing x such that $x \in U \leq f^{-1}(V)$. Hence f is N_m-α-continuous. □

Theorem 3.18. *Let $f : X \to Y$ be a map on two nms (X, N_{mX}) and (Y, N_{mY}). Then the following statements are equivalent:*

(1) f is N_m-α-continuous.

(2) $f^{-1}(V) \leq N_m int(N_m cl(N_m int(f^{-1}(V))))$ for each N_m-open set V in Y.
(3) $N_m cl(N_m int(N_m cl(f^{-1}(F)))) \leq f^{-1}(F)$ for each N_m-closed set F in Y.
(4) $f(N_m cl(N_m int(N_m cl(A)))) \leq N_m cl(f(A))$ for $A \leq X$.
(5) $N_m cl(N_m int(N_m cl(f^{-1}(B)))) \leq f^{-1}(N_m cl(B))$ for $B \leq Y$.
(6) $f^{-1}(N_m int(B)) \leq N_m int(N_m cl(N_m int(f^{-1}(B))))$ for $B \leq Y$.

Proof. (1) ⇔ (2) It follows from Theorem 3.17 and Definition of N_m-α-open sets.
(1) ⇔ (3) It follows from Theorem 3.17 and Lemma 3.4.
(3) ⇒ (4) Let A ≤ X. Then from Theorem 3.17(4) and Lemma 3.14, it follows $N_m cl(N_m int(N_m cl(A))) \leq N_m\text{-}\alpha Cl(A)) \leq f^{-1}(N_m cl(f(A)))$. Hence $f(N_m cl(N_m int(N_m cl(A)))) \leq N_m cl(f(A))$.
(4) ⇒ (5) Obvious.
(5) ⇒ (6) From (5) and Proposition 2.11, it follows: $f^{-1}(N_m int(B)) = f^{-1}(Y - N_m cl(Y - B)) = X - f^{-1}(N_m cl(Y - B)) \leq X - N_m cl(N_m int(N_m cl(f^{-1}(Y - B)))) = N_m int(N_m cl(N_m int(f^{-1}(B))))$. Hence, (6) is obtained.
(6) ⇒ (1) Let V be an N_m-open set in Y. Then by (6) and Proposition 2.11, we have $f^{-1}(V) = f^{-1}(N_m int(V)) \leq N_m int(N_m cl(N_m int(f^{-1}(V))))$. This implies $f^{-1}(V)$ is an N_m-α-open set. Hence by (2), f is N_m-α-continuous. □

Definition 3.19. *A subset A of an nms (X, N_{mX}) is called an N_m-locally closed (briefly, $N_m lc$) sets if $A = S \wedge G$, where S is N_m-open and N is N_m-closed (X, N_{mX}). The class of all N_m-locally closed sets in a nms (X, N_{mX}) is denoted by $N_m LC(X)$.*

Definition 3.20. *A subset A of an nms (X, N_{mX}) is called an N_m-α-locally closed (briefly, $N_m \alpha lc$) sets if $A = S \wedge G$, where S is N_m-α-open and N is N_m-α-closed (X, N_{mX}). The class of all N_m-α-locally closed sets in a nms (X, N_{mX}) is denoted by $N_m \alpha LC(X)$.*

Proposition 3.21. *Every N_m-closed (resp. N_m-open) set is $N_m lc$-set but not conversely.*

Proof. It follows from Definition 3.19. □

Example 3.22. *Let $X = \{a\}$ with $N_m = \{0_\sim, A, 1_\sim\}$ and $N_m^c = \{1_\sim, G, 0_\sim\}$ where $A = \prec (0.5, 0.6, 0.9) \succ$; $G = \prec (0.9, 0.4, 0.5) \succ$. Then the collection of $N_m lc$-sets are $0_\sim \wedge 1_\sim^c = \prec (0, 0, 1) \succ$; $0_\sim \wedge G = \prec (0, 0, 1) \succ$; $0_\sim \wedge 0_\sim^c = \prec (0, 0, 1) \succ$; $A \wedge 1_\sim^c = \prec (0, 0, 1) \succ$; $A \wedge G = \prec (0.5, 0.4, 0.9) \succ$; $A \wedge 0_\sim^c = \prec (0.5, 0.6, 0.9) \succ$; $1_\sim \wedge 1_\sim^c = \prec (0, 0, 1) \succ$; $1_\sim \wedge G = \prec (0.9, 0.4, 0.5) \succ$; $1_\sim \wedge 0_\sim^c = \prec (1, 1, 0) \succ$. Here, G is $N_m lc$-set but it is not N_m-open and A is $N_m lc$-set but it is not N_m-closed.*

Proposition 3.23. *Every N_m-α-closed (resp. N_m-α-open) set is $N_m \alpha lc$-set but not conversely.*

Proof. It follows from Definition 3.20. □

Example 3.24. *Let X and N_m as in the Example 3.7. Then N_m-α-closed set are $D^c = \prec (0.5, 0.3, 0.5), (0.5, 0.6, 0.9) \succ$; $E^c = \prec (0.3, 0.2, 0.9), (0.1, 0.6, 0.6) \succ$. Here, D^c is $N_m \alpha lc$-set but it is not N_m-α-open and D is $N_m \alpha lc$-set but it is not N_m-α-closed.*

Proposition 3.25. *Every $N_m lc$-set is $N_m \alpha lc$-set but not conversely.*

Proof. It follows from Proposition 3.4(1), (3). □

Definition 3.26. *A map f : (X, N_{mX}) → (Y, N_{mY}) is said to be N_m-locally closed-continuous (briefly, N_mLC-continuous) if $f^{-1}(V)$ is N_mLC-set in (X, N_{mX}) for every N_m-open set V of (Y, N_{mY}).*

Definition 3.27. *A map f : (X, N_{mX}) → (Y, N_{mY}) is said to be N_m-α-locally closed-continuous (briefly, $N_m\alpha LC$-continuous) if $f^{-1}(V)$ is $N_m\alpha LC$-set in (X, N_{mX}) for every N_m-open set V of (Y, N_{mY}).*

Theorem 3.28. *Let f : (X, N_{mX}) → (Y, N_{mY}) be a map. Then*

(1) If f is N_m-continuous, then it is N_mLC-continuous.
(2) If f is N_m-continuous, then it is $N_m\alpha LC$-continuous.
(3) If f is N_mLC-continuous, then it is $N_m\alpha LC$-continuous.

Proof. (1) It is an immediate consequence of Proposition 3.21.
(2) It is an immediate consequence of Proposition 3.21 and 3.25.
(3) It is an immediate consequence of Proposition 3.25. □

Definition 3.29. *A map f : (X, N_{mX}) → (Y, N_{mY}) is said to be N_mLC-irresolute (resp. $N_m\alpha LC$-irresolute) if $f^{-1}(V)$is N_mLC-set (resp. $N_m\alpha LC$-set) in (X, N_{mX}) for every N_mLC-set (resp. $N_m\alpha LC$-set) V of (Y, N_{mY}).*

Theorem 3.30. *Let f : (X, N_{mX}) → (Y, N_{mY}) be a map. The*

(1) If f is N_mLC-irresolute then it is N_mLC-continuous.
(2) If f is $N_m\alpha LC$-irresolute then it is $N_m\alpha LC$-continuous.

Proof. (1) Let f : (X, N_{mX}) → (Y, N_{mY}) be a N_mLC-irresolute map. Let V be a N_m-open set of (Y, N_{mY}). Since every N_m-open set is N_mlc-set [by the Proposition 3.21], V is N_mLC-set of (Y, N_{mY}). Since f is N_mLC-irresolute, then $f^{-1}(V)$ is a N_mLC-set of (X, N_{mX}). Therefore f is N_mLC-continuous.

(2) Let f : (X, N_{mX}) → (Y, N_{mY}) be a $N_m\alpha$LC-irresolute map. Let V be a N_m-open set of (Y, N_{mY}). Since every N_m-open set is N_mlc-set and every N_mlc-set is $N_m\alpha$lc-set [by the Proposition 3.21 and Proposition 3.25], V is $N_m\alpha$LC-set of (Y, N_{mY}). Since f is $N_m\alpha$LC-irresolute, then $f^{-1}(V)$ is a $N_m\alpha$LC-set of (X, N_{mX}). Therefore f is N_m-αLC-continuous. □

Theorem 3.31. *Let f : (X, N_{mX}) → (Y, N_{mY}) and g : (Y, N_{mY}) → (Z, N_{mZ}) be any two maps. Then*

(1) g ∘ f is N_mLC-continuous if g is N_m-continuous and f is N_mLC-continuous.
(2) g ∘ f is N_mLC-irresolute if both f and g are N_mLC-irresolute.
(3) g ∘ f is N_mLC-continuous if g is N_mLC-continuous and f is N_mLC-irresolute.

Proof. (1) Since g is a N_m-continuous from (Y, N_{mY}) → (Z, N_{mZ}), for any N_m-open set z as a subset of Z, we get $g^{-1}(z)$ = G is a N_m-open set in (Y, N_{mY}) . As f is a N_mLC-continuous map. We get $(g \circ f)^{-1}(z) = f^{-1}(g^{-1}(z)) = f^{-1}(G)$ = S and S is a N_mLC-set in (X, N_{mX}), since every N_m-open set is is $N_m$lc-set [by the Proposition 3.21]. Hence (g ∘ f) is a N_mLC-continuous map.
(2) Consider two N_mLC-irresolute maps, f : (X, N_{mX}) → (Y, N_{mY}) and g : (Y, N_{mY}) → (Z, N_{mZ}) is a N_mLC-irresolute maps. As g is consider to be a N_mLC-irresolute map, by Definition 3.29, for every N_m-lc-set z ≤ (Z, N_{mZ}), $g^{-1}(z)$ = G is a $N_m$lc-set in (Y, N_{mY}). Again since f is N_mLC-irresolute, $(g \circ f)^{-1}(z) = f^{-1}(g^{-1}(z)) = f^{-1}(G)$ = S and S is a $N_m$lc-set in (X, N_{mX}).

Hence (g ∘ f) is a $N_m LC$-irresolute map.
(3) Let g be a $N_m LC$-continuous map from (Y, N_{mY}) → (Z, N_{mZ}) and z subset of Z be a N_m-open set. Therefore $g^{-1}(z) = G$ is a $N_m$lc-set in (Y, N_{mY}), since every N_m-open set is $N_m$lc-set [by the Proposition 3.21]. Also since f is $N_m LC$-irresolute, we get $(g \circ f)^{-1}(z) = f^{-1}(g^{-1}(z)) = f^{-1}(G) = S$ and S is a $N_m$lc-set in (X, N_{mX}). Hence (g ∘ f) is a $N_m LC$-continuous map. □

Theorem 3.32. *Let f : (X, N_{mX}) → (Y, N_{mY}) and g : (Y, N_{mY}) → (Z, N_{mZ}) be any two maps. Then*

(1) g ∘ f is $N_m\alpha LC$-continuous if g is N_m-continuous and f is $N_m\alpha LC$-continuous.
(2) g ∘ f is $N_m\alpha LC$-irresolute if both f and g are $N_m\alpha LC$-irresolute.
(3) g ∘ f is $N_m\alpha LC$-continuous if g is $N_m\alpha LC$-continuous and f is $N_m\alpha LC$-irresolute.

Proof. (1) Since g is a N_m-continuous from (Y, N_{mY}) → (Z, N_{mZ}), for any N_m-open set z as a subset of Z, we get $g^{-1}(z) = G$ is a N_m-open set in (Y, N_{mY}) . As f is a $N_m\alpha$LC-continuous map. We get $(g \circ f)^{-1}(z) = f^{-1}(g^{-1}(z)) = f^{-1}(G) = S$ and S is a $N_m\alpha$LC-set in (X, N_{mX}), since every N_m-open set is is $N_m$lc-set and every $N_m$lc-set is N_m-αlc-set [by the Propositions 3.21 and 3.25]. Hence (g ∘ f) is a $N_m\alpha$LC-continuous map.
(2) Consider two N_m-αLC-irresolute maps, f : (X, N_{mX}) → (Y, N_{mY}) and g : (Y, N_{mY}) → (Z, N_{mZ}) is a $N_m\alpha$LC-irresolute maps. As g is consider to be a $N_m\alpha$LC-irresolute map, by Definition 3.29, for every $N_m\alpha$LC-set z ≤ (Z, N_{mZ}), $g^{-1}(z) = G$ is a $N_m\alpha$LC-set in (Y, N_{mY}). Again since f is $N_m\alpha$LC-irresolute, $(g \circ f)^{-1}(z) = f^{-1}(g^{-1}(z)) = f^{-1}(G) = S$ and S is a $N_m\alpha$LC-set in (X, N_{mX}). Hence (g ∘ f) is a $N_m\alpha$LC-irresolute map.
(3) Let g be a $N_m\alpha$LC-continuous map from (Y, N_{mY}) → (Z, N_{mZ}) and z subset of Z be a N_m-open set. Therefore $g^{-1}(z) = G$ is a $N_m\alpha$lc-set in (Y, N_{mY}), since every N_m-open set is $N_m$lc-set and every $N_m$lc-set is $N_m\alpha$lc-set [by the Propositions 3.21 and 3.25]. Also since f is $N_m\alpha$LC-irresolute, we get $(g \circ f)^{-1}(z) = f^{-1}(g^{-1}(z)) = f^{-1}(G) = S$ and S is a $N_m\alpha$lc-set in (X, N_{mX}). Hence (g ∘ f) is a $N_m\alpha$LC-continuous map. □

4. CONCLUSION

Neutrosophic set is a general formal framework, which generalizes the concept of classic set, fuzzy set, interval valued fuzzy set, intuitionistic fuzzy set, and interval intuitionistic fuzzy set. Since the world is full of indeterminacy, the neutrosophic minimal structure spaces found its place into contemporary research world. Hence N_m-α-open can also be extended to a neutrosophic spatial region. The results of this study may be help in many researches.

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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