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Neutrosophic θ-Compact Spaces

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Abstract. The idea of neutrosophic θ-Compact along with neutrosophic θ-Almost Compact spaces are developed in neutrosophic topological spaces. We investigate their behavior besides giving characterizations of these spaces.

Keywords: Nneutrosophic θ-open set, neutrosophic strongly θ-continuous, neutrosophic θ-irresolute.

INTRODUCTION

The notion of neutrosophic set was introduced by F. Smarandache [11,12,13] wherein each element has a "degree of membership, degree of non-membership and indeterminacy". These sets were gained remarkable attention in dealing with real life problems that entail uncertainty, impreciseness, vagueness together with indeterminacy. As a consequence, topological spaces have been defined and studied on neutrosophic sets, giving rise to Neutrosophic Topology.

In 2012 [9], neutrosophic topological space (NTS) were introduced. In [10] neutrosophic closed sets along with continuous functions were developed by Salama et al. From this onward tremendous research carried out in neutrosophic topology with its relevance in constrained problems. Many authors demonstrated the sets which are near neutrosophic open sets along with neutrosophic closed sets. In [1] authors initiated neutrosophic semi-open (pre-open besides α-open) functions together with analysed their results. Generalized neutrosophic closed sets were presented in [2]. In [4,5] the idea of neutrosophic generalized α-contra continuous including neutrosophic almost α-contra-continuous mappings are developed and investigated their results. In [7] the authors developed the notion of neutrosophic θ-closure operator and utilizing this, neutrosophic θ-closed set is elucidated. As application of this, neutrosophic θ-continuous, neutrosophic strongly θ-continuous including neutrosophic weakly continuous mappings are characterized concerning the operator defined.

In this paper, we continue the study and present neutrosophic theta-compact and neutrosophic almost theta-compact spaces in NTSs and some their properties are investigated.

PRELIMINARIES

Definition 2.1: [11, 12]: Consider S_1 be a non-empty set. A neutrosophic set (briefly, NS) K is an object so as $\mathrm{K} = \{\langle w, \mu_{\mathrm{K}}(w), \sigma_{\mathrm{K}}(w), \gamma_{\mathrm{K}}(w)\rangle : w \in S_1\}$ however $\mu_{\mathrm{K}}(w), \sigma_{\mathrm{K}}(w)$ with $\gamma_{\mathrm{K}}(w)$ whichever indicates the "degree of membership function (viz $\mu_{\Lambda}(w)$), the degree of indeterminacy (viz $\sigma_K(w)$) as well as the degree of non-membership (viz $\gamma_{\mathrm{K}}(w)$) respectively of each element $x \in S_1$ to the set K".

Note: 2.2[11, 12]:

(i) An $NSK = \{\langle w, \mu_K(w), \sigma_K(w), \Gamma_K(w)\rangle : w \in S_1\}$ can be represented as $\langle \mu_K, \sigma_K, \Gamma_K\rangle$ in $]0^-, 1^+[$ on S_1.

(ii) We indicate $K = \langle \mu_K, \sigma_K, \Gamma_K\rangle$ for the $NSK = \{\langle w, \mu_K(w), \sigma_K(w), \Gamma_K(w)\rangle : w \in S_1\}$.

Definition 2.3: [11, 12]:$0_\aleph = \{\langle q, 0,0,1\rangle : q \in S_1\}$ and $1_\aleph = \{\langle q, 1,1,0\rangle : q \in S_1\}$.

Definition 2.4:[10]: A neutrosophic topology (precisely, $\aleph T)S_1 \neq \emptyset$ is a collection ξ_1 of NSs in S_1satisfies the below conditions:

I. $0_\aleph, 1_\aleph \in \xi_1$,

II. $W_1 \cap W_2 \in T$ being $W_1, W_2 \in \xi_1$,

III. $\cup W_i \in \xi_1$ for arbitrary family $\{W_i | i \in K\} \subseteq \xi_1$.

Here, the (S_1, ξ_1) or S_1 is entitled as NTS along with every NS in ξ_1 is known as neutrosophic open set (briefly, ℵOS) . The complement $\overline{\mathrm{K}}$ of an ℵOS K in S_1 is termed as neutrosophic closed set (ℵCS for short) in S_1.

Definition: 2.5:[10, 11]: Consider K be an NS in an $NTSS_1$. Thereupon
$\aleph int$(K) $=\cup \{M_1 | M_1$is an ℵOS in S_1 with $M_1 \subseteq$ K} is named as neutrosophic interior ($\aleph int$ in short) of K;
$\aleph cl$(K) $=\cap \{M_1 | M_1$ is an ℵCS in S_1 along with $M_1 \supseteq$ K} is named as neutrosophic closure ($\aleph cl$ in short) of K.

Definition: 2.6:[3]: Let S_1 be a nonempty set. Wherever "r, t, s be real standard or non standard subsets" of $]0^-, 1^+[$ thereupon the NS $x_{r,t,s}$ is known as "neutrosophic point(in short, NP)in X " given by$x_{r,t,s}(x_p) = \begin{cases}(r,t,s), & if x = x_p \\ (0,0,1), & if x \neq x_p\end{cases}$ for $x_p \in X$ is named as the support of $x_{r,t,s}$, where r signify the "degree of membership value, t indicates the degree of indeterminacy along with s as the degree of non-membership value" of $x_{r,t,s}$.

Definition 2.7: [7]: A NP $x_{(\alpha,\beta,\gamma)}$ is named as neutrosophicθ -cluster point (ℵθ-cluster point, shortly) of a NS K iff considering respective W in Nϵq-nbd of $x_{(\alpha,\beta,\gamma)}$ with $Ncl(W)\ qK$. The collection of all Nθ-cluster points of K is labelled as neutrosophic θ-closure besides represented as $NCl_\theta(K)$.

An NS K will be Nθ-closed set (NθCS in precise) iff K= $NCl_\theta(K)$. The complement of a NθCS is Nθ-open set (in precise, NθOS).

NEUTROSOPHIC θ-COMPACT SPACES

Definition 3.1: A collection $\{W_j \ / \ j \in J\}$ of NθOSs in NTS S_1 is termed as neutrosophic θ-open cover (ℵTO-cover) of a set R if $R \subseteq \cup \{W_j \ / \ j \in J\}$

Definition 3.2: A NTS S_1 is known as neutrosophic θ-Compact Spaces (ℵTCS) if each ℵTO-cover of S_1 has a countable subcover.

Definition 3.3: A member R of a NTS S_1 is termed as ℵTCS if for each collection $\{W_j \ / \ j \in J\}$ of NθOSs of S_1 in order that $R \subseteq \cup \{W_j \ / \ j \in J\}$, there is a finite subset J_o of J so as $R \subseteq \cup \{W_j \ / \ j \in J\}$.

Remark 3.4: As every NθOS is NOS, so every neutrosophic compact space is ℵTCS.

Theorem 3.5:S_1 is ℵTCS iff each family of Nθ-closed subsets of S_1 with f.i.p. attribute has a non-empty intersection.

Proof: Consider S_1 be a ℵTCS and $R = \{R_j \ / \ j \in J\}$ be any family of Nθ-closed subsets of S_1 including f.i.p. Assume that $\cap \{R_j \ / \ j \in J\} = O_N$. Then $\cup \{R_j^c / \ j \in J\} = 1_N$i.e.$\{R_j^c / \ j \in J\}$ is an ℵTO-cover of S_1. As, S_1 is ℵTCS, there is a finite subset J_o of J such that $\cup \{R_j^c / \ j \in J\} = 1_N$. This inferred that $\cap \{R_j \ / \ j \in J\} = O_N$ which deny the presumption that R has a f.i.p. Thence $\cap \{R_j \ / \ j \in J\} \neq O_N$.

Let $\mathbb{G} = \{G_i \ / \ i \in I\}$ indicate ℵTO-cover of X along with the family $\mathbb{G} = \{G_i^c \ / \ i \in I\}$ of a NθCS. As $\mathbb{G}$ is a cover of S_1, $\cap \{G_i^c \ / \ i \in I_o\} = O_N$. Hence $\mathbb{G}$ doesn't have the f.i.p. i.e. there are bounded number of NθOSs

$\{ G_1, G_2, ------, G_n \}$ in $\mathbb{G}$ such that $\cap \{ G_i^c / i = 1,2, ---n \} = O_N$. This implicit that $\{ G_1, G_2, ------, G_n \}$ is a bounded subcover of X in $\mathbb{G}$. Accordingly, S_1 is ℵTCS.

Theorem 3.6: Consider $(S_1, \mathfrak{I})$ indicate the NTS and $\mathfrak{I}_\theta$ denote N-topology on S_1 generated utilizing the subspace of all NθOSs in S_1. Then $(S_1, \mathfrak{I})$ is ℵTCS if and only if $(S_1, \mathfrak{I}_\theta)$ is N-compact.

Proof: Consider $(S_1, \mathfrak{I}_\theta)$ be N-compact along with $R = \{R_j \ / \ j \ \in \ J \}$ denote the ℵTO-cover of S_1 in $\mathfrak{I}$. As for each $j \in J$, $R_j \in \mathfrak{I}_\theta$, R is an N-open cover of S_1 in $\mathfrak{I}_\theta$. Since $(S_1, \mathfrak{I}_\theta)$ is N-compact, R has a finite subcover of X. Henceforth, $(S_1, \mathfrak{I})$ is ℵTCS.

Assume $(X, \mathfrak{I}_\theta)$ be a ℵTCS and let $R = \{R_j \in \mathfrak{I}_\theta , j \in J \}$ indicate a N-open cover of S_1 in $\mathfrak{I}_\theta$. As, for every $j \in J$, $R_j \in \mathfrak{I}_\theta$, R_j is a NθOS in $(S_1, \mathfrak{I})$. Therefore R, is a NTO-cover of X in $\mathfrak{I}$. Since $(S_1, \mathfrak{I})$ℵTCS, R has finite subcover of S_1. So, $(S_1, \mathfrak{I}_\theta)$ is N-compact.

Theorem 3.7: Let J be a Nθ-closed subset of a ℵTCS S_1. Then J, is also ℵTCS.

Proof: Consider J as a NθCS of S_1 along with $K = \{K \ / \ i \in I \}$ indicate NTO-open cover of J. Since J^c is Nθ-open subset of S_1, $K = \{K_i \ / \ i \in I \} \cup J^c$ is a NTO-cover of S_1. As S_1 is ℵTCS, there is a finite subset I_o of I so as $\cup\{K_i \ / \ i \in I_o \} \cup J^c = 1_N$. Hence J is ℵTCS relative to S_1.

Recall [7] that a function $\mu: (S_1, \mathfrak{I}) \longrightarrow (S_2, \mathfrak{I}^1)$ is known as neutrosophic strongly θ-continuous (in short ℵStr-θ-continuous) if for each point $x_{(\alpha,\beta,\gamma)}$ in S_1 with N-open q-neighborhood W of $\mu(x_{(\alpha,\beta,\gamma)})$, there arises a N-open q-neighborhood Q of $x_{(\alpha,\beta,\gamma)}$ such that $\mu(Cl_\theta\ (Q)) \subseteq W$.

Theorem 3.8: (i) ℵStr-θ-continuous image of ℵTCS is N-compact.

(ii) Consider $(S_1, \mathfrak{I}_1)$ and $(S_2, \mathfrak{I}_2)$ denote NTSs with $\mu: (S_1, \mathfrak{I}_1) \longrightarrow (S_2, \mathfrak{I}_2)$ be a Nθ-irresolute. For a subset K of S_1 is ℵTCS, then $\mu(K)$ is ℵTCS.

Proof: (i) Let $\mu: (S_1, \mathfrak{I}_1) \longrightarrow (S_2, \mathfrak{I}_2)$ denote ℵStr-θ-continuous mapping from ℵTCS S_1 onto NTS Y. Consider $R = \{ R_i / \ i \in I \}$ be a N-open cover of S_2. In view of μ is ℵStr-θ-continuous function, $\mu: (S_1, \mathfrak{I}_\theta) \longrightarrow (S_2, \mathfrak{I}_2)$ is N-continuous function

(ii) Let $M = \{ M_i / \ i \in I \}$ be a NTO-cover of $\mu(K)$ in S_2. As μ is Nθ-irresolute, for each M_i , $\mu^{-1}(M_i)$ is a NθOS. Moreover, $\{\mu^{-1}(M_i) \ / \ i \in I \}$ is NTO-cover of K. As, K is ℵTCS relate to S_1, there prevails a finite subset I_o of I so as $K \subseteq \cup\{\mu^{-1}(M_i) \ / \ i \in I_o \}$. Therefore $\mu(K) \subseteq \cup\{\mu^{-1}(M_i) \ / \ i \in I_o \}$. Hence, $\mu(K)$ is ℵTCS relative to S_2.

Theorem 3.9: LetK_1andK_2be the subsets of NTS S_1. If M is ℵTCS and $K_2$is NθCS in S_1, then $K_1 \cap K_2$ is ℵTCS.

Proof: Let $R = \{ R_i / \ i \in I \}$ be a NTC-open cover of $K_1 \wedge K_2$ in S_1. As, K_2^c is NθOS in S_1, $(\cup\{ R_i / \ i \in I \}) \cup K_2^c$. Since K_1 is ℵTCS, here is a finite member I_o of I in order that $K_1 \subseteq (\cup\{ R_i / \ i \in I \}) \cup K_2^c$. Hence $K_1 \cap K_2 \subseteq (\cup\{ R_i / \ i \in I \})$. So, $K_1 \cap K_2$ is ℵTCS.

Definition 3.10: Consider $(S_1, \mathfrak{I}_1)$ and $(S_2, \mathfrak{I}_2)$ denote NTSs. A mapping $\mu: (S_1, \mathfrak{I}_1) \longrightarrow (S_2, \mathfrak{I}_2)$ is named as neutrosophic strongly θ-open if $\mu(W)$ is NθOS of Y for every NθOS W of X.

Theorem 3.11: Consider$\mu: (S_1, \mathfrak{I}_1) \longrightarrow (S_2, \mathfrak{I}_2)$ be neutrosophic strongly θ-open, bijective and S_2 is ℵTCS, then S_1 is ℵTCS.

Proof: Let $\{J_i : i \in I\}$ be an NTO-cover of S_1 and $\{ \mu(J_i) : i \in I \}$ is NTO-cover of S_2. Since S_2 is ℵTCS, there is a finite subset $I_o$of I so as finite collection $\{ \mu(J_i) : i \in I_o \}$ covers S_2. As $1_{\sim X} = \mu^{-1}(1_{\sim Y}) = \mu^{-1}\mu(\cup_{i \in I_0} J_i) = \cup_{i \in I_0} J_i$ and therefore S_1 is ℵTCS.

NEUTROSOPHIC θ-ALMOST COMPACT SPACES

Definition 4.1: An NTSS_1 is termed as Neutrosophicθ- Almost Compact Space (precisely, ℵTACS) is and only of every family of NTO-cover $\{ \mu_i : i \in I \}$ of S_1, there arises a finite subset I_o of I so as $\cup_{i \in I_o} NCl_\theta(\mu_i) = 1_N$.

Definition 4.2: An NTS S_1 is called as Neutrosophic θ- Regular space (ℵTRS precisely) if for each NθOS K ∈ S_1, $K = \cup\{K_i \in I^X \ / \ K_i \ is$ NθOS, $NCl_\theta \leq$ K}.

Theorem 4.3: Let X be an NTS. If S_1 is ℵTACS and ℵTRS then S_1 is ℵTCS.

Proof: Consider $\{ K_i : i \in I \}$ be an NTO-cover of S_1 such that $\vee_{i\in I} K_i = 1_N$. Since S_1 is ℵTRS, $K_i = \cup\{L_i \in I^X \ / \ L_i \ is$ NθOS, $NCl_\theta \subseteq K_i\}$ for each $i \in I$. Since $1_N = \cup_{i\in I}(\cup_{i\in I} L_i)$ and S_1 is ℵTACS , there arises a finite set I_o of I such that $\cup_{i\in I_o} NCl_\theta(L_i) = 1_N$. But, $NCl_\theta(L_i) \subseteq K_i(NInt_\theta(NCl_\theta(L_i))) \subseteq NCl_\theta(L_i)$. We have $\cup_{i\in I_0} K_i \supseteq \cup_{i\in I_o} NCl_\theta(L_i) = 1_N$ Hence X is ℵTCS.

Theorem 4.4: An NTS S_1 is ℵTACS iff, for each collection $\{W_i : i \in I\}$ of NθOSs having f.i.p, $\cap_{i\in I} NCl_\theta(W_i) \neq 0_\sim$

Proof: Consider a family $\{W_i : i \in I\}$ of NθOSs having f.i.p.. Assume that $\cap_{i\in I} NCl_\theta(W_i) \neq 0_\sim$ and then $\cap_{i\in I} \overline{NCl_\theta(W_i)} = \cup_{i\in I} NInt_\theta(\overline{W_i}) = 1_\sim$. As, S_1 is ℵTACS, there prevails a bounded member I_o of I in order that $\cup_{i\in I_0} NCl_\theta(NInt_\theta(\overline{W_i}) = 1_\sim$ This implies that $\cup_{i\in I_0} NCl_\theta(NInt_\theta(\overline{W_i}) = \cup_{i\in I} NCl_\theta(\overline{NCl_\theta(W_i)} = 1_\sim$. Thus, $\cap_{i\in I_0} NInt_\theta(NCl_\theta(W_i)) = 0_\sim$. But $W_i = NInt_\theta(W_i) \subseteq NInt_\theta(NCl_\theta(W_i))$. This implies that $\cap_{i\in I_0} W_i = 0_\sim$ which is negation with f.i.p. of the collection.

Conversely, consider $\{W_i : i \in I\}$ of NθOSs so as $\cup_{i\in I} W_i = 1_\sim$. Presume that there arises no finite subset I_o of I such that $\vee_{i\in I_o} NCl_\theta(L_i) = 1_N$. Since, $\{\overline{NCl_\theta(W_i)} : i \in I\}$ has the FIP then $\cap_{i\in I} NCl_\theta(\overline{NCl_\theta(W_i)} \neq 0_\sim$ This implies that $\cup_{i\in I} \overline{NCl_\theta(\overline{NCl_\theta(W_i)})} \neq 1_\sim$. Hence $\cup_{i\in I} NInt_\theta(NCl_\theta(W_i)) \neq 1_\sim$ which is in contradiction with $\cup_{i\in I} W_i = 1_\sim$

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