K-algebra on pentapartitioned neutrosophic pythagorean sets

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Abstract. In this paper we have proposed the concept of K-algebras on Pentapartitioned neutrosophic Pythagorean set, level subset of PNS and studied some of the results. Further the concept of Pentapartitioned Neutrosophic Pythagorean K-subalgebra and discussed some of its properties.

INTRODUCTION

Neutrosophic set which is a generalization of fuzzy set and intuitionistic fuzzy set was introduced by Smarandache [10] in 1998. Akram et al.[5] studied K-algebras on single valued neutrosophic sets and also discussed homomorphisms between the single valued neutrosophic K-subalgebras. Belnap[10] introduced the concept of five valued logic that is the information are represented by five components T, F, None, Both which denotes true, false, neither true nor false, both true and false and unknown respectively. Based on this concept Smarandache proposed five numerical valued neutrosophic logic where indeterminacy is splitted into three terms known as Contradiction(C) and Unknown(U) and unknown(G). The concept of Pentapartitioned Neutrosophic Pythagorean Sets was introduced by R. Radha and A. Stanis Arul Mary[6] . Fuzzy K-algebra was introduced by Akram et.al [2,3,4] and also they established this in a wide-reaching way through other researchers. The concept of Quadripartitioned Neutrosophic Pythagorean sets and K- algebra on the respective set was initiated by R. Radha and A. Stanis Arul Mary[9]. In this paper, we extend the concepts to Pentapartitioned Neutrosophic Pythagorean Sets

PRELIMINARIES

Definition 2.1[12] Let $(G, \cdot, \odot, e)$ be a group in which each non-identity element is not of order 2. Then a K-algebra is a structure K= $(G, \cdot, \odot, e)$ on a group G in which induced binary operation $\odot: G \times G \to G$ is defined by $\odot(x, y) = x \odot y = xy^{-1}$ and satisfies the following axioms:

(i) $(x \odot y) \odot (x \odot z) = \left(x \odot \left((e \odot z) \odot (e \odot y)\right)\right) \odot x$,

(ii) $x \odot (x \odot y) = \left(x \odot (e \odot y)\right) \odot x$,

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(iii) $x \odot x = e$,

(iv) $x \odot e = x$,

(v) $e \odot x = x^{-1}$, for all $x, y, z \in G$.

Definition 2.2[8] A single-valued neutrosophic set $A = (T_A, I_A, F_A)$ in a K-algebra Ꝁ is called a single-valued neutrosophic K-subalgebra of Ꝁ if it satisfies the following conditions:

(i) $T_A(s \odot t) \geq min\{T_A(s), T_A(t)\}$,

(ii) $I_A(s \odot t) \geq min\{I_A(s), I_A(t)\}$,

(iii) $F_A(s \odot t) \leq max\{F_A(s), F_A(t)\}$, for all $s, t \in G$.

Note that $T_A(e) \geq T_A(s), I_A(e) \geq I_A(s), F_A(e) \leq F_A(s), for\ all\ s \in G$.

Definition 2.3[15]

Let X be a universe. A Pentapartitioned neutrosophic pythagorean set A with T, F, C and U as dependent neutrosophic components and I as independent component for A on X is an object of the form

$$A = \{< x, T_A, C_A, I_A, U_A, F_A >: x \in X\}$$

Where $T_A + F_A \leq 1, C_A + U_A \leq 1\ and\ (T_A)^2 + (C_A)^2 + (I_A)^2 + (U_A)^2 + (F_A)^2 \leq 3$

Here, $T_A(x)$ is the truth membership, $C_A(x)$ is contradiction membership, $U_A(x)$ is ignorance membership, $F_A(x)$ is the false membership and $I_A(x)$ is an unknown membership.

Definition 2.4[18]

A Pentapartitioned Neutrosophic set M = ($A1_M$, $A2_M$, $A3_M$, $A4_M$, $A5_M$) in a K-algebra Ꝁ is called a Pentapartitioned neutrosophic K-subalgebra of Ꝁ if it satisfies the following conditions.

(i) $A1_M(e) \geq A1_M(u)$, $A2_M(e) \geq A2_M(u)$, $A3_M(e) \leq A3_M(u)$, $A4_M(e) \leq A4_M(u)$ and $A5_M(e) \leq A5_M(u)$ for all $u \in G$.

(ii) $A1_M(u \odot v) \geq \min\{A1_M(u), A1_M(v)\}$

(iii) $A2_M(u \odot v) \geq \min\{A2_M(u), A2_M(v)\}$

(iv) $A3_M(u \odot v) \leq \min\{A3_M(u), A3_M(v)\}$

(v) $A4_M(u \odot v) \leq \min\{A4_M(u), A4_M(v)\}$

(vi) $A5_M(u \odot v) \leq \min\{A5_M(u), A5_M(v)\}$

PENTAPARTITIONED NEUTROSOPHIC PYTHAGOREAN K-SUBALGEBRA

Definition 3.1

A Pentapartitioned Neutrosophic Pythagorean set M = ($A1_M$, $A2_M$, $A3_M$, $A4_M$, $A5_M$) in a K-algebra Ꝁ is called a Pentapartitioned neutrosophic Pythagorean K-subalgebra of Ꝁ if it satisfies the following conditions.

(i) $A1_M(e) \geq A1_M(u)$, $A2_M(e) \geq A2_M(u)$, $A3_M(e) \leq A3_M(u)$, $A4_M(e) \leq A4_M(u)$ and $A5_M(e) \leq A5_M(u)$ for all $u \in G$.

(ii) $A1_M(u \odot v) \geq \min\{A1_M(u), A1_M(v)\}$

(iii) $A2_M(u \odot v) \geq \min\{A2_M(u), A2_M(v)\}$

(iv) $A3_M(u \odot v) \leq \min\{A3_M(u), A3_M(v)\}$

(v) $A4_M(u \odot v) \leq \min\{A4_M(u), A4_M(v)\}$

(vi) $A5_M(u \odot v) \leq \min\{A5_M(u), A5_M(v)\}$

Example 3.1

Let M = $(A1_M, A2_M, A3_M, A4_M, A5_M)$ is the cyclic group of order five in a K-algebra Ꝁ= $(G, \cdot, \Theta, e)$. The Cayley's table for Θ is given as follows.

Θ	e	g	g^2	g^3	g^4
E	e	g^4	g^3	g^2	g
G	g	e	g^4	g^3	g^2
g^2	g^2	g	e	g^4	g^3
g^3	g^3	g^2	g	e	g^4
g^4	g^4	g^3	g^2	g	e

We define a PNP set in K-algebra as follows.

$A1_M(e) = 0.5$, $A2_M(e) = 0.4$, $A3_M(e) = 0.1$, $A4_M(e) = 0.3$, $A5_M(e) = 0.2$,

$A1_M(u) = 0.1$, $A2_M(u) = 0.2$, $A3_M(u) = 0.4$, $A4_M(u) = 0.5$, $A5_M(u) = 0.3$

for all $u \neq e \in G$. Clearly it shows that M = $(A1_M, A2_M, A3_M, A4_M, A5_M)$ is a PNP K-algebras of Ꝁ.

Proposition 3.1

If M = $(A1_M, A2_M, A3_M, A4_M, A5_M)$ denotes a Pentapartitioned neutrosophic Pythagorean K-algebras of Ꝁ then,

a) $(\forall\, u, v \in G), (A1_M(u \odot v) = A1_M(v) \Rightarrow A1_M(u) = A1_M(e))$

$$(\forall\, u, v \in G), \big(A1_M(u) = A1_M(e) \Rightarrow A1_M(u \odot v) \geq A1_M(v)\big)$$

b) $(\forall\, u, v \in G), (A2_M(u \odot v) = A2_M(v) \Rightarrow A2_M(u) = A2_M(e))$

$$(\forall\, u, v \in G), \big(A2_M(u) = A2_M(e) \Rightarrow A2_M(u \odot v) \geq A2_M(v)\big)$$

c) $(\forall\, u, v \in G), (A3_M(u \odot v) = A3_M(v) \Rightarrow A3_M(u) = A3_M(e))$

$$(\forall\, u, v \in G), \big(A3_M(u) = A3_M(e) \Rightarrow A3_M(u \odot v) \leq A3_M(v)\big)$$

d) $(\forall\, u, v \in G), (A4_M(u \odot v) = A4_M(v) \Rightarrow A4_M(u) = A4_M(e))$

$$(\forall\, u, v \in G), \big(A4_M(u) = A4_M(e) \Rightarrow A4_M(u \odot v) \geq A4_M(v)\big)$$

e) $(\forall\, u, v \in G), (A5_M(u \odot v) = A5_M(v) \Rightarrow A5_M(u) = A5_M(e))$

$$(\forall\, u, v \in G), \big(A5_M(u) = A5_M(e) \Rightarrow A5_M(u \odot v) \geq A5_M(v)\big)$$

Proof : We only prove (a) and (c). (b) (d) and (e) proved in a similar way.

(a) First we assume that $A1_M(u \odot v) = A1_M(v)\ \forall\ u, v \in$ G.

Put $v = e$ and we get $A1_M(u) = A1_M(u \odot e) = A1_M(e)$. Let for $u, v \in$ G be such that $A1_M(u) = A1_M(e)$ then, $A1_M(u \odot v) \geq min\{A1_M(u), A1_M(v)\} = min\{A1_M(e), A1_M(v)\} = A1_M(v)$.

Now to prove (c) consider that $A3_M(u \odot v) = A3_M(v)\ \forall\ u, v \in$ G.

Put $v = e$ and by Definition , we have $A3_M(u) = A3_M(u \odot e) = A3_M(e)$. Let for $u, v \in$ G be such that $A3_M(u) = A3_M(e)$ then, $A3_M(u \odot v) \leq max\{A3_M(u), A3_M(v)\} = max\{A3_M(e), A3_M(v)\} = A3_M(v)$. Hence the proof.

Definition 3.2

Let M = (A1 M, A2 M , A3 M ,A4 M , A5 M) be a Pentapartitioned neutrosophic Pythagorean set in a K-algebra of Ҟ and let $(\lambda, \mu, \vartheta, \xi, \varphi) \in [0,1] \times [0,1] \times [0,1] \times [0,1] \times [0,1]$ with $\lambda + \mu + \vartheta + \xi + \varphi \leq 5$. Then the sets,

$M_{(\lambda,\mu,\vartheta,\xi,\varphi)} = \{u \in G | A1_M(u) \geq \lambda, A2_M(u) \geq \mu, A3_M(u) \leq \vartheta, A4_M(u) \leq \xi, A5_M(u) \leq \varphi\}$,

$M_{(\lambda,\mu,\vartheta,\xi,\varphi)} = U(A1_M, \lambda) \cap U'(A2_M, \mu) \cap L(A3_M, \vartheta) \cap L'(A4_M, \xi) \cap L''(A5_M, \varphi)$ are called $(\lambda, \mu, \vartheta, \xi, \varphi)$ level subsets of Pentapartitioned neutrosophic Pythagorean set M.

And also the set $M_{(\lambda,\mu,\vartheta,\xi,\varphi)} = \{u \in G | A1_M(u) > \lambda, A2_M(u) > \mu, A3_M(u) < \vartheta, A4_M(u) < \xi, A5_M(u) < \varphi\}$ is known as strong $(\lambda, \mu, \vartheta, \xi, \varphi)$ level subset of M.

Note: The set of all $(\lambda, \mu, \vartheta, \xi, \varphi) \in Im(A1_M) \times Im(A2_M) \times Im(A3_M) \times Im(A4_M) \times Im(A5_M)$ is known as image of M = (A1 M, A2 M , A3 M ,A4 M , A5 M)

Proposition 3.2

If M = (A1 M, A2 M , A3 M ,A4 M , A5 M) is a Pentapartitioned neutrosophic Pythagorean K-algebra of Ҟ then the level subsets,

$$U(A1_M, \lambda) = \{u \in G|\ A1_M(u) \geq \lambda\}, U'(A2_M, \mu) = \{u \in G|\ A2_M(u) \geq \mu\},$$
$$L(A3_M, \vartheta) = \{u \in G|\ A3_M(u) \leq \vartheta\}, L'(A4_M, \xi) = \{u \in G|\ A4_M(u) \leq \xi\} \text{ and}$$
$$L''(A5_M, \varphi) = \{u \in G|\ A5_M(u) \leq \varphi\}$$

are K-subalgebras of Ҟ for every $(\lambda, \mu, \vartheta, \xi, \varphi) \in Im(A1_M) \times Im(A2_M) \times Im(A3_M) \times Im(A4_M) \times Im(A5_M) \subseteq$ [0,1]where $m(A1_M), Im(A2_M)\ Im(A3_M), Im(A4_M) and\ Im(A5_M)$ are sets of values A1$(M), A2(M), A3(M), A4(M)\ and\ A5(M)$ respectively.

Proof

Let M = (A1 M, A2 M , A3 M ,A4 M , A5 M) be a Pentapartitioned Neutrosophic Pythagorean set in a K-algebra of Ҟ and $(\lambda, \mu, \vartheta, \xi, \varphi) \in Im(A1_M) \times Im(A2_M) \times Im(A3_M) \times Im(A4_M) \times Im(A5_M)$ be such that $U(A1_M, \lambda) \neq \emptyset$, $U'(A2_M, \mu) \neq \emptyset,\ L(A3_M, \vartheta) \neq \emptyset\ ,\ L'(A4_M, \xi) \neq \emptyset and\ L''(A5_M, \varphi) \neq \emptyset$.

We have to show that $U, U', L, L'\ and\ L''$ are level K-subalgebras.

Let for $u, v \in U(A1_M, \lambda),\ A1_M(u) \geq \lambda\ and\ A1_M(v) \geq \lambda$. By definition, we get $A1_M(u \odot v) \geq min\{A1_M(u), A1_M(v)\} \geq \lambda$. It shows that $u \odot v \in U(A1_M, \lambda)$. Hence $U(A1_M, \lambda)$ is a level K-subalgebra of Ҟ. Similarly we can prove for $U'(A2_M, \mu), L(A3_M, \vartheta),\ L'(A4_M, \xi)\ and\ L''(A5_M, \varphi)$.

Theorem 3.1

Let M = (A1 $_M$, A2 $_M$, A3 $_M$,A4 $_M$, A5 $_M$) be a Pentapartitioned neutrosophic Pythagorean set in a K-algebra of Ꝁ. Then M = (A1 $_M$, A2 $_M$, A3 $_M$,A4 $_M$, A5 $_M$) is a Pentapartitioned neutrosophic Pythagorean K-subalgebra of Ꝁ if and only if $M_{(\lambda,\mu,\vartheta,\xi,\varphi)}$ is a K-subalgebra of Ꝁ for every $(\lambda,\mu,\vartheta,\xi,\varphi) \in Im(\ A1_M) \times Im(\ A2_M) \times Im(\ A3_M) \times Im(\ A4_M) \times Im(\ A5_M)$with $\lambda+\mu+\vartheta+\xi+\varphi \leq 5$.

Proof

First assume that $M_{(\lambda,\mu,\vartheta,\xi,\varphi)}$ is a K-subalgebra of Ꝁ. If the conditions in Definition 3.1 fails then there exist $s,t \in G$ such that,

$A1_M(s\odot t) < min\{A1_M(s), A1_M(t)\}$

$A2_M(s\odot t) < min\{A2_M(s), A2_M(t)\}$

$A3_M(s\odot t) > max\{A3_M(s), A3_M(t)\}$

$A4_M(s\odot t) > max\{A4_M(s), A4_M(t)\}$

$A5_M(s\odot t) > max\{A5_M(s), A5_M(t)\}$

Now let $\lambda_1 = \frac{1}{2}(A1_M(s\odot t) + min\{A1_M(s), A1_M(t)\})$,

$\mu_1 = \frac{1}{2}(A2_M(s\odot t) + \ min\{A2_M(s), A2_M(t)\})$,

$\vartheta_1 = \frac{1}{2}(A3_M(s\odot t) + max\{A3_M(s), A3_M(t)\})$,

$\xi_1 = \frac{1}{2}(A4_M(s\odot t) + max\{A4_M(s), A4_M(t)\})$

$\varphi_1 = \frac{1}{2}(A5_M(s\odot t) + max\{A5_M(s), A5_M(t)\})$

Now we have,

$A1_M(s\odot t) < \lambda_1 < min\{A1_M(s), A1_M(t)\}$

$A2_M(s\odot t) < \mu_1 < min\{A2_M(s), A2_M(t)\}$

$A3_M(s\odot t) > \vartheta_1 > max\{A3_M(s), A3_M(t)\}$

$A4_M(s\odot t) > \xi_1 > max\{A4_M(s), A4_M(t)\}$

$A5_M(s\odot t) > \varphi_1 > max\{A5_M(s), A5_M(t)\}$

This implies that $s,t \in X_{(\lambda,\mu,\vartheta,\xi,\varphi)}$ *and* $s\odot t \notin X_{(\lambda,\mu,\vartheta,\xi,\varphi)}$ which is a contradiction. This proves that the conditions of Definition is true.

Hence M = (A1 $_M$, A2 $_M$, A3 $_M$,A4 $_M$, A5 $_M$) is a Pentapartitioned neutrosophic Pythagorean K-subalgebra of Ꝁ.

Now assume that M = (A1 $_M$, A2 $_M$, A3 $_M$,A4 $_M$, A5 $_M$) be a Pentapartitioned neutrosophic Pythagorean K-subalgebra of Ꝁ. Let for $(\lambda,\mu,\vartheta,\xi,\varphi) \in Im(\ A1_M) \times Im(\ A2_M) \times Im(\ A3_M) \times Im(\ A4_M) \times Im(\ A5_M)$ with $\lambda + \mu + \vartheta + \xi + \varphi \leq 5$ such that $M_{(\lambda,\mu,\vartheta,\xi,\varphi)} \neq \emptyset$. Let $u,v \in M_{(\lambda,\mu,\vartheta,\xi,\varphi)}$ be such that,

$A1_M(u) \geq \lambda,\ A1_M(v) \geq \lambda'$,

$A2_M(u) \geq \mu,\ A2_M(v) \geq \mu'$,

$A3_M(u) \leq \vartheta,\ A3_M(v) \leq \vartheta'$,

$A4_M(u) \leq \xi,\ A4_M(v) \leq \xi'$
$A5_M(u) \leq \varphi,\ A5_M(v) \leq \varphi'$

Now assume that $\lambda \leq \lambda', \mu \leq \mu', \vartheta \geq \vartheta'\ \ \xi \geq \xi' and\ \varphi \geq\ \varphi'$. It follows from Definition that,
$A1_M(u \odot v) \geq \lambda = min\{\ A1_M(u),\ A1_M(v)\}$,
$A2_M(u \odot v) \geq \mu = min\{\ A2_M(u),\ A2_M(v)\}$,
$A3_M(u \odot v) \leq \vartheta = max\{\ A3_M(u),\ A3_M(v)\}$,
$A4_M(u \odot v) \leq \xi = max\{\ A4_M(u),\ A4_M(v)\}$
$A5_M(u \odot v) \leq \xi = max\{\ A5_M(u),\ A5_M(v)\}$
This shows that $u \odot v \in M_{(\lambda,\mu,\vartheta,\xi,\varphi)}$. Hence $M_{(\lambda,\mu,\vartheta,\xi,\varphi)}$ is a K-subalgebra of K.

Theorem 3.2

Let M = (A1 M, A2 M , A3 M ,A4 M , A5 M) be a Pentapartitioned neutrosophic Pythagorean K-subalgebra and $(\lambda_1,\mu_1,\vartheta_1,\xi_1,\varphi_1),(\lambda_2,\mu_2,\vartheta_2,\xi_2,\varphi_2) \in Im(\ A1_M) \times Im(\ A2_M) \times Im(\ A3_M) \times Im(\ A4_M) \times Im(\ A5_M)\ wi$th $\lambda_i + \mu_i + \vartheta_i + \xi_i + \varphi_i\ \leq 5$ for $i = 1,2$. Then $M_{(\lambda_1,\mu_1,\vartheta_1,\xi_1,\varphi_1)} = M_{(\lambda_2,\mu_2,\vartheta_2,\xi_2,\varphi_2)}$ if $(\lambda_1,\mu_1,\vartheta_1,\xi_1,\varphi_1) = (\lambda_2,\mu_2,\vartheta_2,\xi_2,\varphi_2)$.

Proof

When $(\lambda_1,\mu_1,\vartheta_1,\xi_1,\varphi_1) = (\lambda_2,\mu_2,\vartheta_2,\xi_2,\varphi_2)$ then the result is obvious for $M_{(\lambda_1,\mu_1,\vartheta_1,\xi_1,\varphi_1)} = M_{(\lambda_2,\mu_2,\vartheta_2,\xi_2,\varphi_2)}$.
Conversely assume that $M_{(\lambda_1,\mu_1,\vartheta_1,\xi_1,\varphi_1)} = M_{(\lambda_2,\mu_2,\vartheta_2,\xi_2,\varphi_2)}$. Since $(\lambda_1,\mu_1,\vartheta_1,\xi_1,\varphi_1) \in\ Im(\ A1_M) \times Im(\ A2_M) \times Im(\ A3_M) \times Im(\ A4_M) \times Im(\ A5_M)$ there exists $u \in G$ such that $A1_M(u) = \lambda_1,\ A2_M(u) = \mu_1,\ A3_M(u) = \vartheta_1\ \ A4_M(u) = \xi_1$
$and\ A5_M(u) = \varphi_1$. This implies that $u \in X_{(\lambda_1,\mu_1,\vartheta_1,\xi_1,\varphi_1)} = X_{(\lambda_2,\mu_2,\vartheta_2,\xi_2,\varphi_2)}$.
Hence $\lambda_1 =\ A1_M(u) \geq \lambda_2, \mu_1 =\ A2_M(u) \geq \mu_2, \vartheta_1 =\ A3_M(u) \leq \vartheta_2\ ,\ \xi_1 =\ A4_M(u) \leq \xi_2$ and $\varphi_1 = A5_M(u) \leq \varphi_2$. Also $(\lambda_2,\mu_2,\vartheta_2,\xi_2,\varphi_2) \in Im(\ A1_M) \times Im(\ A2_M) \times Im(\ A3_M) \times Im(\ A4_M) \times Im(\ A5_M)$ there exists $v \in G$ such that $A1_M(v) = \lambda_2, A2_M(v) = \mu_2$
$A3_M(v) = \vartheta_2\ \ , A4_M(v) = \xi_2\ , A5_M(v) = \varphi_2 and$. This implies that $v \in X_{(\lambda_2,\mu_2,\vartheta_2,\xi_2,\varphi_2)} = X_{(\lambda_1,\mu_1,\vartheta_1,\xi_1,\varphi_1)}$. Hence $\lambda_2 = A1_M(v) \geq \lambda_1, \mu_2 = A2_M(v) \geq \mu_1, \vartheta_2 = A3_M(v) \leq\ \vartheta_1\ , \xi_2 = A4_M(v) \leq \xi_1\ and\ \ \varphi_2 = A5_M(u) \leq \varphi_1$. Hence $(\lambda_1,\mu_1,\vartheta_1,\xi_1,\varphi_1) = (\lambda_2,\mu_2,\vartheta_2,\xi_2,\varphi_2)$.

Theorem 3.3

Let I be a K-subalgebra of K-algebra Ꝁ. Then there exists a Pentapartitioned neutrosophic Pythagorean K-subalgebra M = (A1 $_M$, A2 $_M$, A3 $_M$,A4 $_M$, A5 $_M$) of K-algebra Ꝁ such that M = (A1 $_M$, A2 $_M$, A3 $_M$,A4 $_M$, A5 $_M$) = I for some $\lambda, \mu \in (0,1]$ and $\vartheta, \xi, \varphi \in [0,1)$

Proof :

Let M = (A1 $_M$, A2 $_M$, A3 $_M$,A4 $_M$, A5 $_M$) be a Pentapartitioned Neutrosophic Pythagorean set in K-algebra Ꝁ given by,

$$A1_M(u) = \begin{cases} \lambda \in (0,1], & if\ u \in I \\ 0, & otherwise \end{cases}$$

$$A2_M(u) = \begin{cases} \mu \in (0,1], & if\ u \in I \\ 0, & otherwise \end{cases}$$

$$A3_M(u) = \begin{cases} \vartheta \in [0,1), & if\ u \in I \\ 0, & otherwise \end{cases}$$

$$A4_M(u) = \begin{cases} \xi \in [0,1), & if\ u \in I \\ 0, & otherwise \end{cases}$$

$$A5_M(u) = \begin{cases} \varphi \in [0,1), & if\ u \in I \\ 0, & otherwise \end{cases}$$

Let $u, v \in G$. If $u, v \in I$ then $u \odot v \in I$ and so,

$A1_M(u \odot v) \geq min\{A1_M(u), A1_M(v)\}$,

$A2_M(u \odot v) \geq min\{A2_M(u), A2_M(v)\}$,

$A3_M(u \odot v) \leq max\{A3_M(u), A3_M(v)\}$,

$A4_M(u \odot v) \leq max\{A4_M(u), A4_M(v)\}$,

$A5_M(u \odot v) \leq max\{A5_M(u), A5_M(v)\}$

Suppose $u \notin I$ or $v \notin I$ then,

$A1_M(u) = 0\ or\ A1_M(v), A2_M(u) = 0\ or\ A2_M(v), A3_M(u) = \ 0\ or\ A3_M(v)$,

$A4_M(u) = \ 0\ or\ A4_M(v)\ \ and\ A5_M(u) = 0\ or\ A5_M(v)$

It implies that,

$A1_M(u \odot v) \geq min\{A1_M(u), A1_M(v)\}$,

$A2_M(u \odot v) \geq min\{A2_M(u), A2_M(v)\}$,

$A3_M(u \odot v) \leq max\{A3_M(u), A3_M(v)\}$,

$A4_M(u \odot v) \leq max\{A4_M(u), A4_M(v)\}$,

$A5_M(u \odot v) \leq max\{A5_M(u), A5_M(v)\}$

Hence M = (A1 $_M$, A2 $_M$, A3 $_M$,A4 $_M$, A5 $_M$) is a Pentapartitioned neutrosophic Pythagorean K-subalgebra of Ꝁ.

Theorem 3.4

Let Ꝁ be a K-algebra. Let a chain of K-subalgebras: $X_0 \subset X_1 \subset X_2 \subset \cdots \subset X_n = G$. Then the level K-subalgebras of the Pentapartitioned Neutrosophic Pythagorean K-subalgebra remains same as the K-subalgebras of this chain.

Proof Let $\{\lambda_i | i = 0,1, \dots, n\}, \{\mu_i | i = 0,1, \dots, n\}$ be finite decreasing sequences and $\{\vartheta_i | i = 0,1, \dots, n\}$, $\{\xi_i | i = 0,1, \dots, n\}, \{\varphi_i | i = 0,1, \dots, n\}$ be finite increasing sequences in [0,1] such that $\lambda_k + \mu_k + \vartheta_k + \xi_k + \varphi_k \leq 5$ for $k = 0,1,2, \dots, n$.

Let M = (A1 M, A2 M , A3 M ,A4 M , A5 M) be a Pentapartitioned neutrosophic Pythagorean set in Ꝁ defined by

$A1_M(X_0) = \lambda_0, A2_M(X_0) = \mu_0,\ A3_M(X_0) = \vartheta_0\ , A4_M(X_0) = \xi_0, A5_M(X_0) = \varphi_0$

$A1_M(X_i \backslash X_{i-1}) = \lambda_i, A2_M(X_i \backslash X_{i-1}) = \mu_i, A3_M(X_i \backslash X_{i-1}) = \vartheta_i\ , A4_M(X_i \backslash X_{i-1})\ \xi_i\ \ , A5_M(X_i \backslash X_{i-1}) = \varphi_i \quad for\ 0 < i \leq n.$

We have to prove that M = (A1 M, A2 M , A3 M ,A4 M , A5 M) is a Pentapartitioned Neutrosophic Pythagorean K-subalgebra of Ꝁ. Let $u, v \in G$. If $u, v \in X_i \backslash X_{i-1}$ then it implies that $A1_M(u) = \lambda_i = A1_M(v)$, $A2_M(u) = \mu_i = A2_M(v)$, $A3_M(u) = \vartheta_i = A3_M(v)$, $A4_M(u) = \xi_i = A4_M(v)$ and $A5_M(u) = \varphi_i = A5_M(v)$Since each M_i is a K-subalgebra we get $u \odot v \in M_i$. So that either $u \odot v \in M_i \backslash M_{i-1}$ or $u \odot v \in M_{i-1}$. In any of the above case it follows that,

$A1_M(u \odot v) \geq \lambda_i = min\{A1_M(u), A1_M(v)\}$,

$A2_M(u \odot v) \geq \mu_i = min\{A2_M(u), A2_M(v)\}$,

$A3_M(u \odot v) \leq \vartheta_i = max\{A3_M(u), A3_M(v)\}$,

$A4_M(u \odot v) \leq \xi_i = max\{A4_M(u), A4_M(v)\}$ and

$A5_M(u \odot v) \leq \varphi_i = max\{A5_M(u), A5_M(v)\}$

For $k > l$ if $u \in M_k \backslash M_{k-1}$ and $v \in M_l \backslash M_{l-1}$ then,

$A1_M(u) = \lambda_k, A1_M(v) = \lambda_l$

$A2_M(u) = \mu_k, A2_M(v) = \mu_l$

$A3_M(u) = \vartheta_k, A3_M(v) = \vartheta_l$

$A4_M(u) = \xi_k, A4_M(v) = \xi_l$

$A5_M(u) = \varphi_k, A5_M(v) = \varphi_l$

and $u \odot v \in M_k$ because M_k is a K-subalgebra and $M_l \subset M_k$. It follows that,

$A1_M(u \odot v) \geq \lambda_k = min\{A1_M(u), A1_M(v)\}$,

$A2_M(u \odot v) \geq \mu_k = min\{A2_M(u), A2_M(v)\}$,

$A3_M(u \odot v) \leq \vartheta_k = max\{A3_M(u), A3_M(v)\}$,

$A4_M(u \odot v) \leq \xi_k = max\{A4_M(u), A4_M(v)\}$ and

$A5_M(u \odot v) \leq \varphi_k = max\{A5_M(u), A5_M(v)\}$

Hence M = (A1 M, A2 M , A3 M ,A4 M , A5 M) is a Pentapartitioned neutrosophic Pythagorean K-subalgebra of Ꝁ and all its non-empty level subsets are level K-subalgebras of Ꝁ. Since $Im(A1_M) = \{\lambda_0, \lambda_1, \dots, \lambda_n\}$, $Im(A2_M) = \{\mu_0, \mu_1, \dots, \mu_n\}$, $Im(A3_M) = \{\vartheta_0, \vartheta_1, \dots, \vartheta_n\}\ and\ Im(A4_M) = \{\xi_0, \xi_1, \dots, \xi_n\}\ andIm(A5_M) = \ \{\varphi_1, \varphi_2, \dots, .\varphi_n\}$

Therefore the level K-subalgebras of M = (A1 M, A2 M , A3 M ,A4 M , A5 M) are given by the chain of K-subalgebras

$U(A1_M, \lambda_0) \subset U(A1_M, \lambda_1) \subset \cdots \subset U(A1_M, \lambda_n) = G$

$U'(A2_M, \mu_0) \subset U'(A2_M, \mu_1) \subset \cdots \subset U'(A2_M, \mu_n) = G$

$L(A3_M, \vartheta_0) \subset L(A3_M, \vartheta_1) \subset \cdots \subset L(A3_M, \vartheta_n) = G$

$L'(A4_M, \xi_0) \subset L'(A4_M, \xi_1) \subset \cdots \subset L'(A4_M, \xi_n) = G$

$L''(A5_M, \varphi_0) \subset L''(A5_M, \varphi_1) \subset \cdots \subset L''(A5_M, \varphi_n) = G$

respectively. Indeed,

$U(A1_M, \lambda_0) = \{u \in G | A1_M(u) \geq \lambda_0\} = M_0,$

$U'(A2_M, \mu_0) = \{u \in G | A2_M(u) \geq \mu_0\} = M_0,$

$L(A3_M, \vartheta_0) = \{u \in G | A3_M(u) \leq \vartheta_0\} = M_0,$

$L'(A4_M, \xi_0) = \{u \in G | A4_M(u) \leq \xi_0\} = M_0,$

$L''(A5_M, \varphi_0) = \{u \in G | A5_M(u) \leq \varphi_0\} = M_0$

Now we have to prove that,

$U(A1_M, \lambda_i) = X_i, U'(A2_M, \mu_i) = X_i, L(A3_M, \vartheta_i) = X_i \;\; L'(A4_M, \xi_i) = X_i \; and \; L''(A4_M, \varphi_i) = X_i for \; 0 < i \leq n.$

Clearly $X_i \subseteq U(A1_M, \lambda_i), X_i \subseteq U'(A2_M, \mu_i), X_i \subseteq L(A3_M, \vartheta_i), X_i \subseteq L'(A4_M, \xi_i) \; and \; X_i \subseteq L''(A4_M, \varphi_i)$.

If $u \in U(A1_M, \lambda_i)$ then $A1_M(u) \geq \lambda_i$ and so $u \notin A_k \; for \; k > i$.

Hence $A1_M(u) \in \{\lambda_0, \lambda_1, \dots, \lambda_i\}$ which shows that $u \in X_k \; for \; k \leq i$, since $X_k \subseteq X_{i.}$

It follows that $u \in X_i$. Consequently $U(A1_M, \lambda_i) = X_i$ for some $0 < i \leq n$.

Now if $v \in L(A3_M, \vartheta_i)$ then $A3_M(v) \leq \vartheta_i$ and so $v \notin X_k \; for \; some \; i \leq k$.

Thus $A3_M(u) \in \{\vartheta_0, \vartheta_1, \dots, \vartheta_i\}$ which shows that $u \in X_l \;\; for \; some \; l \leq i$, since $X_l \subseteq X_{i.}$

It follows that $v \in X_i$.

Consequently $L(A3_M, \vartheta_i) = X_i$ for some $0 < i \leq n$.

Hence the proof.

CONCLUSION

Need of algebra in today's life is more important since it plays a vital role without even recognizing it. Algebraic thinking helps us to solve the real-world problems in a logical way. Recently K-algebra applied in fuzzy set, intuitionistic fuzzy set and single valued neutrosophic set which helps us to extend the concept to K-algebra on pentapartitioned neutrosophic sets. In Future Study, I am going to study homomorphisms and ideals on PNP sets.

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