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Cite as: AIP Conference Proceedings **2364**, 020021 (2021); https://doi.org/10.1063/5.0063408
Published Online: 23 September 2021

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Convergence of the Power Sequence of a Monotone Increasing Neutrosophic Soft Matrix

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Abstract. In this paper, we have introduced a new classification of the Principal Diagonal Elements (PDEs) and the convergence of the power sequence of a Neutrosophic Soft Matrix (NSM) P, is coneyed at the level of P itself. Then the essential role of monotone or nearly monotone increasing NSM were established . Furthermore, the necessary and sufficient conditions for an increasing NSM P has been established.
Keywords: Neutrosophic Soft Set (NSS), Neutrosophic Soft Matrix(NSM), Convergence of power of Neutrosophic Soft Matrix (CPNSM), Monotone Increasing Neutrosophic Soft Matrix (MINSM).
AMS 2000 Subject Classification: Primary 03E72; Secondary 15B15.

INTRODUCTION

Fuzzy Set Theory (FST) [28], assumes a fundamental part in dynamic under unsure environment. A significant generalization of FST is the theory of Intuitionistic Fuzzy Set (IFS), presented by Atanassov [3] crediting a participation degree and a non-enrollment degree independently so that amount of the two degrees should be one. Established by Florentin Smarandhache in [24] Neutrosophic set was introduced as an investigation of the origin, nature, and extent of neutralities, just as cooperations with various ideational spectra. Maji et.al, [21] stretched out soft set to intuitionistics fuzzy soft set and NSS.

It is notable that the power sequence of a Fuzzy Matrix (FM) either converge or oscillates inside a limited advance, and it is seen that in probably the most ordinarily utilized cases it converge rather rapidly. Hence, the convergence of power sequence of a FM has attracted the consideration of numerous creators [10, 7, 9, 25]. The convergency of the power sequence of monotone increasing FM has been examined in [10, 25]. Following this thought an overall report was started on the convergence and oscillation of the power sequence of a FM at the degree of combination properties-the properties of the circular path, has been set up in [4, 5], where the significant standard of the diagonal element of certain A^t is investigated altogether. Likewise dependent on a similar rule the diagonal elements of A^2 is analyzed in [6], and the convergence or oscillation of the power sequence of FMs can be set up effectively for the basic kinds presented in the investigation of both FM and classic matalrices.

Deli and Broumi [8] re-imagined the thought of neutrosophic set in another manner and set forward the idea of NSM and various sorts of frameworks in neutrosophic soft hypothesis. They have presented some new operations and properties on these matrices. Sumathi and Arokiarani [2] introduced new method on NSMs. Power of NSMs has been talked about in [14]. Following this thought an overall report was started on minimal solution, greatest eigenvectors and $\lambda-$ robustness of the NSMs in [11, 12, 13]. We introduced

International Conference on Recent Trends in Applied Mathematical Sciences (ICRTAMS-2020)
AIP Conf. Proc. 2364, 020021-1–020021-8; https://doi.org/10.1063/5.0063408
Published by AIP Publishing. 978-0-7354-4122-4/$30.00

the periodicity of interval NSM in [16]. The productive consequence of monotone interval neutrosophic soft eigenproblem, Solvability of System of Neutrosophic Soft Linear Equation and Monotone neutrosophic soft eigenspace structures in max-min algebra was first proposed and examined by Murugadas et.al, [17, 18, 19]. Uma et.al, [26] contemplated two sorts of NSMs.

The motivation behind this paper is to investigate the basic part of the PDEs of the NSM P. In this paper we will focus on the diagonal elements of the NSM itself. The fundamental piece of this paper is Theorem 3.7. The set up hypotheses cover those of monotone or nearly monotone increasing NSMs. Moreover, in the closing segment we have attempted to give an understanding of the system for neutrosophic logics of the theory. We consider that the assertions might be intriguing as far as certain parts of Neutrosophic Logics(NLs).

PRELIMINARIES

For the basic definitions and ideas about NSS and NSM see [1, 23, 24, 26]

THE BASIC PROERTIES OF POWER SEQUENCE OF NSM

Let us consider a NSM $P = (\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle) \in \mathcal{N}_{(n,n)}$ and denote $P^k = (\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle)^k$, where k is a positive integer. The following lemmas are elementary. Here $\mathcal{N}_{(n,n)}$ - denote the set of all NSMs of order $n \times n$.

Lemma 1: For $P \in \mathcal{N}_{(n,n)}$

$$\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^k = \bigvee_{1\le i_1,\cdots,i_{k-1}\le n} \langle p^T_{li_1}, p^I_{li_1}, p^F_{li_1}\rangle \wedge \langle p^T_{li_1i_2}, p^I_{i_1i_2}, p^F_{i_1i_2}\rangle \wedge \cdots \wedge \langle p^T_{i_{k-1}m}, p^I_{i_{k-1}m}, p^F_{i_{k-1}m}\rangle.$$

Lemma 2: For $P \in \mathcal{N}_{(n,n)}$

1. $\langle p^T_{ll}, p^I_{ll}, p^F_{ll}\rangle \le \langle p^T_{ll}, p^I_{ll}, p^F_{ll}\rangle^s \le \max_{1\le k\le n}\{\langle p^T_{lk}, p^I_{lk}, p^F_{lk}\rangle\} \wedge \max_{1\le k\le n}\{\langle p^T_{kl}, p^I_{kl}, p^F_{kl}\rangle\} \forall s \ge 1.$

2. We have a $u \le n-1$ and $u \ge 1$ such that $\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^u \ge \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^o \quad \forall o = n, n+1, n+2, ...$ for all ordered pair of (l, m) and $l \ne m$,

3. For every $1 \le l \le n$ we have a v, $1 \le w \le n$ such that $\langle p^T_{ll}, p^I_{ll}, p^F_{ll}\rangle^w \ge \langle p^T_{ll}, p^I_{ll}, p^F_{ll}\rangle^o$ $\forall o = n, n+1, n+2, ...$

4. $\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle + \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^2 + ... + \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^n \ge \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^o \quad \forall o \ge 1.$

Proof: We prove the first part only as the remaining is clear.

$$\begin{aligned}
\langle p^T_{ll}, p^I_{ll}, p^F_{ll}\rangle &= \bigvee_{1\le i_1,\ldots,i_{s-1}\le n} (\langle p^T_{ll}, p^I_{ll}, p^F_{ll}\rangle \wedge \langle p^T_{ll}, p^I_{ll}, p^F_{ll}\rangle \wedge ...\langle p^T_{ll}, p^I_{ll}, p^F_{ll}\rangle) \\
&\le \bigvee_{1\le i_1,\ldots,i_{s-1}\le n} (\langle p^T_{li_1}, p^I_{li_1}, p^F_{li_1}\rangle \wedge \langle p^T_{i_1i_2}, p^I_{i_1i_2}, p^F_{i_1i_2}\rangle \wedge ... \wedge \langle p^T_{i_{s-1}l}, p^I_{i_{s-1}l}, p^F_{i_{s-1}l}\rangle) \\
&= \langle p^T_{ll}, p^I_{ll}, p^F_{ll}\rangle^s \le \bigvee_{1\le i_1,\ldots,i_{s-1}\le n} (\langle p^T_{li_1}, p^I_{li_1}, p^F_{li_1}\rangle \wedge \langle p^T_{i_{s-1}l}, p^I_{i_{s-1}l}, p^F_{i_{s-1}l}\rangle) \\
&\le \max_{1\le k\le n}\{\langle p^T_{lk}, p^I_{lk}, p^F_{lk}\rangle\} \wedge \max_{1\le k\le n}\{\langle p^T_{kl}, p^I_{kl}, p^F_{kl}\rangle\}.
\end{aligned}$$

Theorem 3: Let $P \in \mathcal{N}_{(n,n)}$ be a MINSM. Then

$$\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^n = \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^{n+1} = \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^{n+2} = ... = \lim_{k\to\infty} \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^k.$$

Proof: The proof is evident from Lemma 2.

Definition 4: A NSM $\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle$ is said to satisfy the Dominating Principle (DP) (of PDE) if $p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle \le \langle p^T_{ll}, p^I_{ll}, p^F_{ll}\rangle \vee \langle p^T_{mm}, p^I_{mm}, p^F_{mm}\rangle$ for all $\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle$.

Theorem 5: For $\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle \in \mathcal{N}_{(n,n)}$ with $\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle$ satisfying the DP, $\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle$ is Monotone Increasing(MI).

Proof: Clearly

$\max\{\langle p^T_{ll}, p^I_{ll}, p^F_{ll}\rangle \wedge \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle,\ \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle \wedge \langle p^T_{mm}, p^I_{mm}, p^F_{mm}\rangle\} = \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle;$ for all l, m we have

$$\begin{aligned}
\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^2 &= \max_{1\le k\le n}(\langle p^T_{lk}, p^I_{lk}, p^F_{lk}\rangle \wedge \langle p^T_{km}, p^I_{km}, p^F_{km}\rangle) \\
&\ge \max_{1\le k\le n}\{\langle p^T_{ll}, p^I_{ll}, p^F_{ll}\rangle \wedge \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle,\ \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle \wedge \langle p^T_{mm}, p^I_{mm}, p^F_{mm}\rangle\} \\
&= \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle.
\end{aligned}$$

Henceforth $\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle$ is MI.
The next definition is very crucial.
Definition 6: Let $(\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle) \in \mathcal{N}_{(n,n)}.\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle$, we say it satisfy the Maximum Principle(MP) (of the PDEs) if and only if for every l, $1 \le l \le n$, either
$\langle p^T_{ll}, p^I_{ll}, p^F_{ll}\rangle = \max_{l\le k\le n}\{\langle p^T_{kl}, p^I_{kl}, p^F_{kl}\rangle\}$ or $\langle p^T_{ll}, p^I_{ll}, p^F_{ll}\rangle = \max_{1\le k\le n}\{\langle p^T_{lk}, p^I_{lk}, p^F_{lk}\rangle\}$.
As a consequence.
Theorem 7: If $(\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle)$ satisfies the MP, then
1. $(\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle)^2 \le (\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle)^3 \le ... \le (\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle)^{n-1} \le ...$
2. For PDE $\langle p^T_{ll}, p^I_{ll}, p^F_{ll}\rangle = \langle p^T_{ll}, p^I_{ll}, p^F_{ll}\rangle^2 = \langle p^T_{ll}, p^I_{ll}, p^F_{ll}\rangle^3 = ... = \langle p^T_{ll}, p^I_{ll}, p^F_{ll}\rangle^n \ \ \forall\ 1 \le l \le n$.
3. $\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^{n-1} = \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^n$.
4. But it is not always true that $\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle \le \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^2$.
Proof: To show that $\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^2 \le \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^3$. For all given ordered pair (l, m), there is a k such that $\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^2 = \langle p^T_{lk}, p^I_{lk}, p^F_{lk}\rangle \wedge \langle p^T_{km}, p^I_{km}, p^F_{km}\rangle$.
If $\langle p^T_{kk}, p^I_{kk}, p^F_{kk}\rangle = \max_{l\le u\le n}\{\langle p^T_{ut}, p^I_{ut}, p^F_{ut}\rangle\}$, then

$$\begin{aligned}\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^2 &= \langle p^T_{lk}, p^I_{lk}, p^F_{lk}\rangle \wedge \langle p^T_{km}, p^I_{km}, p^F_{km}\rangle\\ &= (\langle p^T_{lk}, p^I_{lk}, p^F_{lk}\rangle \wedge \langle p^T_{kk}, p^I_{kk}, p^F_{kk}\rangle) \wedge \langle p^T_{km}, p^I_{km}, p^F_{km}\rangle\\ &\le \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^3.\end{aligned}$$

If $\langle p^T_{kk}, p^I_{kk}, p^F_{kk}\rangle = \max_{1\le u\le n}\{\langle p^T_{tu}, p^I_{tu}, p^F_{tu}\rangle\}$,

$$\begin{aligned}\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^2 &= \langle p^T_{lk}, p^I_{lk}, p^F_{lk}\rangle \wedge \langle p^T_{km}, p^I_{km}, p^F_{km}\rangle\\ &= \langle p^T_{lk}, p^I_{lk}, p^F_{lk}\rangle \wedge (\langle p^T_{kk}, p^I_{kk}, p^F_{kk}\rangle \wedge \langle p^T_{km}, p^I_{km}, p^F_{km}\rangle)\\ &\le \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^3.\end{aligned}$$

Thus $(\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle)^2 \le (\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle)^3$ and thus
$(\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle)^2 \le (\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle)^3 ... \le (\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle)^{n-1} \le (\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle)^n ...$. It proves first part.
For second part let $t \in \mathbb{N}$. (Index set)

$$\begin{aligned}\langle p^T_{ll}, p^I_{ll}, p^F_{ll}\rangle^{t+i} &= \max_{1\le i_1,...,i_t\le n}\{\langle p^T_{li_1}, p^I_{li_1}, p^F_{li_1}\rangle \wedge ... \wedge \langle p^T_{i_1 l}, p^I_{i_1 l}, p^F_{i_1 l}\rangle\}\\ &\le \max_{1\le i_t\le n}\{\langle p^T_{li_1}, p^I_{li_1}, p^F_{li_1}\rangle\} \wedge \max_{1\le i_t\le n}\{\langle p^T_{i_1,l}, p^I_{i_1 l}, p^F_{i_1 l}\rangle\}\\ &= \langle p^T_{ll}, p^I_{ll}, p^F_{ll}\rangle.\end{aligned}$$

and from first part of Lemma 2, the second part holds.
For the last part it is enough if we show $\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^{n-1} \le \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^n$ for every $1 \le l \ne m \le n$. There is sequence of $i_1 = l, i_2, ..., i_n, l_{n+1} = m$ for each ordered pair $l \ne j$, such that

$$\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^n = \langle p^T_{i_1i_2}, p^I_{i_1i_2}, p^F_{i_1i_2}\rangle \wedge \langle p^T_{i_2i_3}, p^I_{i_2i_3}, p^F_{i_2i_3}\rangle \wedge ... \wedge \langle p^T_{i_ni_{n+1}}, p^I_{i_ni_{n+1}}, p^F_{i_ni_{n+1}}\rangle. \tag{1}$$

There has exist indices having the same value, say $i_s = i_t$ with $s < t$, among these $i'_t s$. Again $i_1 \ne i_{n+1}$ we obtain $1 \le t - s \le n - 1$. We have three cases:
1. If $t-s < n-1$, then $2 \le n-t+s \le n-1$. we get $\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^n \le \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^{n-t+s} \le \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^{n-1}$, by deleting $\langle p^T_{i_si_{s+1}}, p^I_{i_si_{s+1}}, p^F_{i_si_{s+1}}\rangle \wedge ... \wedge \langle p^T_{i_{t-1}i_t}, p^I_{i_{t-1}i_t}, p^F_{i_{t-1}i_t}\rangle$, from Eq.(1).
2. If $t - s = n - 1$ two cases arises.
(a) $s = 1, t = n$, then

$$\begin{aligned}\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^n &\le \langle p^T_{ll}, p^I_{ll}, p^F_{ll}\rangle^{n-1} \wedge \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle\\ &= \langle p^T_{ll}, p^I_{ll}, p^F_{ll}\rangle \wedge \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle\\ &\le \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^2\\ &\le \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^{n-1}.\end{aligned}$$

(b) $s = 2, t = n + 1$, then

$$\begin{aligned}\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^n &\le \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle \wedge \langle p^T_{mm}, p^I_{mm}, p^F_{mm}\rangle^{n-1}\\ &= \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle \wedge \langle p^T_{mm}, p^I_{mm}, p^F_{mm}\rangle\\ &\le \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^2\\ &\le \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^{n-1}.\end{aligned}$$

Thus the third part.
The following example illustrate the fourth part.
Example 8: $\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^{n-1} = \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^n$.

$$P = \begin{bmatrix} \langle 0.5\ 0.4\ 0.5\rangle & \langle 0.7\ 0.6\ 0.3\rangle & \langle 0.1\ 0.2\ 0.9\rangle \\ \langle 0.1\ 0.2\ 0.9\rangle & \langle 0.5\ 0.4\ 0.5\rangle & \langle 0\ \ 0\ \ 1\rangle \\ \langle 0.1\ 0.2\ 0.9\rangle & \langle 0\ \ 0\ \ 1\rangle & \langle 0.5\ 0.4\ 0.5\rangle \end{bmatrix} \qquad P^2 = \begin{bmatrix} \langle 0.5\ 0.4\ 0.5\rangle & \langle 0.7\ 0.6\ 0.3\rangle & \langle 0.1\ 0.2\ 0.9\rangle \\ \langle 0.1\ 0.2\ 0.9\rangle & \langle 0.5\ 0.4\ 0.5\rangle & \langle 0.1\ 0.2\ 0.9\rangle \\ \langle 0.1\ 0.2\ 0.9\rangle & \langle 0.1\ 0.2\ 0.9\rangle & \langle 0.5\ 0.4\ 0.5\rangle \end{bmatrix}$$

$$P^3 = \begin{bmatrix} \langle 0.5\ 0.4\ 0.5\rangle & \langle 0.7\ 0.6\ 0.3\rangle & \langle 0.1\ 0.2\ 0.9\rangle \\ \langle 0.1\ 0.2\ 0.9\rangle & \langle 0.5\ 0.4\ 0.5\rangle & \langle 0.1\ 0.2\ 0.9\rangle \\ \langle 0.1\ 0.2\ 0.9\rangle & \langle 0.1\ 0.2\ 0.9\rangle & \langle 0.5\ 0.4\ 0.5\rangle \end{bmatrix}$$

Therefore here $P^2 = P^3$.

Corollary 9: Let $(\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle) \in \mathcal{N}_{(n,n)}$ with $(\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle)$ satisfying the strong row or column MP, that is

$\langle p^T_{ll}, p^I_{ll}, p^F_{ll}\rangle = \max_{1\le k\le n}\langle p^T_{lk}, p^I_{lk}, p^F_{lk}\rangle \quad \forall 1 \le l \le n$ or

$\langle p^T_{ll}, p^I_{ll}, p^F_{ll}\rangle = \max_{1\le k\le n}\langle p^T_{kl}, p^I_{kl}, p^F_{kl}\rangle \quad \forall 1 \le l \le n.$ Then

1. $\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle$ is MI.
2. The PDEs of $(\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle)$ are stable, that is, $\langle p^T_{ll}, p^I_{ll}, p^F_{ll}\rangle = \langle p^T_{ll}, p^I_{ll}, p^F_{ll}\rangle^k$, $\forall 1 \le l \le n,\ k = 1, 2, ...$
3. $\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^s = \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^{s+1}$ for some $s \le n-1$.

Proof: The proof of 1 is evident from Theorem 5, and the rest from Theorem 7.

Corollary 10: Let $(\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle) \ge I$. Then

1. $\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle$ is MI.
2. $\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^s = \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^{s+1}$ for some $s \le n-1$.

THE CONDITIONS FOR MINSM

In this section, we will discuss the conditions for MINSM $(\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle)$ converges to $(\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle)^n$ exactly.

We begin with a NSM of order $n \times n$. Denote

$(\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle)^{(k)} = (\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle) + (\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle)^2 + ... + (\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle)^k \quad \forall k \ge 1.$

Lemma 11: $(\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle)^{(n)} > (\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle)^{(n-1)}$ if and only if there is, one or more l_0, with $\langle p^T_{l_0l_0}, p^I_{l_0l_0}, p^F_{l_0l_0}\rangle^n > \max_{1\le k\le n-1}\langle p^T_{l_0l_0}, p^I_{l_0l_0}, p^F_{l_0l_0}\rangle^k$.

Proof: In Lemma 2 we have, $\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^{(n)} \le \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^{(n-1)}\ \ \forall 1 \le l \ne m \le n$. So this result holds if and only if $\langle p^T_{l_0l_0}, p^I_{l_0l_0}, p^F_{l_0l_0}\rangle^n > \max_{1\le k\le n-1}\langle p^T_{l_0l_0}, p^I_{l_0l_0}, p^F_{l_0l_0}\rangle^k$.

Lemma 12: Let Q be an $n^{th}-$ order permutation NSM and $R = Q^{'}PQ = [\langle r^T_{lm}, r^I_{lm}, r^F_{lm}\rangle]$.
Then $(Q^{'}PQ)^k = Q^{'}P^kQ \quad \forall k \ge 1$.

Proof: As $Q^{'}Q = I$, the lemma holds directly.

Theorem 13: Let $(\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle) \in \mathcal{N}_{(n,n)}$. Then $\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^{(n)} > \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^{(n-1)}$ if and only if there has permutation NSM Q such that $R = Q^{'}PQ$ has to have the property

$$R_\lambda = \begin{pmatrix} \langle 0,0,1\rangle & \langle 1,1,0\rangle & \langle 0,0,1\rangle & \cdots & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & \langle c^T_{22}, c^I_{22}c^F_{22}\rangle & \langle 1,1,0\rangle & \cdots & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & \langle c^T_{32}, c^I_{32}c^F_{32}\rangle & \langle c^T_{33}, c^I_{33}c^F_{33}\rangle & \cdots & \langle 0,0,1\rangle \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \langle 0,0,1\rangle & \langle c^T_{n-12}, c^I_{n-12}c^F_{n-12}\rangle & \langle c^T_{n-13}, c^I_{n-13}c^F_{n-13}\rangle & \cdots & \langle 1,1,0\rangle \\ \langle 1,1,0\rangle & \langle c^T_{n2}, c^I_{n2}c^F_{n2}\rangle & \langle c^T_{n3}, c^I_{n3}c^F_{n3}\rangle & \cdots & \langle c^T_{nn}, c^I_{nn}c^F_{nn}\rangle \end{pmatrix} \tag{2}$$

where $\lambda = \langle r^T_{12}, r^I_{12}, r^F_{12}\rangle \wedge \langle r^T_{23}, r^I_{23}, r^F_{23}\rangle \wedge ... \wedge \langle r^T_{n-1n}, r^I_{n-1n}, r^F_{n-1n}\rangle \wedge \langle r^T_{n1}, r^I_{n1}, r^F_{n1}\rangle$ and

$$R_\lambda = [\langle c^T_{lm}, c^I_{lm}, x^F_{lm}\rangle] = \begin{cases} \langle 1,1,0\rangle & \langle b^T_{lm}, b^I_{lm}, b^F_{lm}\rangle \ge \lambda \\ \langle 0,0,1\rangle & \langle r^T_{lm}, r^I_{lm}, r^F_{lm}\rangle < \lambda \end{cases} \qquad (\forall 1 \le l, m \le n)$$

Proof: If $\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^{(n)} > \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^{(n-1)}$ then from Lemma 12 we have l_0 such that

$$\langle p^T_{l_0l_0}, p^I_{l_0l_0}, p^F_{l_0l_0}\rangle^n > \max_{1\le k\le n-1}\langle p^T_{l_0l_0}, p^I_{l_0l_0}, p^F_{l_0l_0}\rangle^k. \tag{3}$$

Then we have $1 \le i_2, i_3, ..., i_n \le n$, such that

$\langle p^T_{l_0l_0}, p^I_{l_0l_0}, p^F_{l_0l_0}\rangle^n = \langle p^T_{i_1i_2}, p^I_{i_1i_2}, p^F_{i_1i_2}\rangle \wedge \langle p^T_{i_2i_3}, p^I_{i_2i_3}, p^F_{i_2i_3}\rangle \wedge ... \wedge \langle p^T_{i_ni_1}, p^I_{i_ni_1}, p^F_{i_ni_1}\rangle,$ (4)
where $i_1 = l_0$. Here, we claim $i_s \neq i_t \forall s \neq t$. If not there are s and t such that $1 \leq s < t \leq n$ and $i_s = i_t$, leads to $\langle p^T_{l_0l_0}, p^I_{l_0l_0}, p^F_{l_0l_0}\rangle^n \leq \langle p^T_{l_0l_0}, p^I_{l_0l_0}, p^F_{l_0l_0}\rangle^{n-t+s}$, $1 \leq n-t+s \leq n-1$, by deleting $\langle p^T_{i_si_{s+1}}, p^I_{i_si_{s+1}}, p^F_{i_si_{s+1}}\rangle \wedge ... \wedge \langle p^T_{i_{t-1}i_t}, p^I_{i_{t-1}i_t}, p^F_{i_{t-1}i_t}\rangle^n$ in (4) But $n-t+s \leq n-1$, which contradicts inequality (3). So a permutation NSM Q can be well defined as $Q : i_1 \to t \ \forall \ t = 1, 2, ..., n$. Set $R = Q^{'}PQ$, that is $\langle r^T_{lm}, r^I_{lm}, r^F_{lm}\rangle = \langle p^T_{i_li_m}, p^I_{i_li_m}, p^F_{i_li_m}\rangle$, and from Lemma 12 $\langle r^T_{lm}, r^I_{lm}, r^F_{lm}\rangle^k = \langle p^T_{i_li_l}, p^I_{i_li_l}, p^F_{i_li_l}\rangle \ \forall \ 1 \leq l, m \leq n, k \leq 1$. Also by (4),

$$\begin{aligned}
\langle r^T_{11}, r^I_{11}, r^F_{11}\rangle^n &= \langle p^T_{l_0l_0}, p^I_{l_0l_0}, p^F_{l_0l_0}\rangle^n \\
&= \langle p^T_{i_1i_2}, p^I_{i_1i_2}, p^F_{i_1i_2}\rangle \wedge \langle p^T_{i_2i_3}, p^I_{i_2i_3}, p^F_{i_2i_3}\rangle \wedge ... \wedge \langle p^T_{i_ni_1}, p^I_{i_ni_1}, p^F_{i_ni_1}\rangle, \\
&= \langle r^T_{12}, r^I_{12}, r^F_{12}\rangle \wedge \langle r^T_{23}, r^I_{23}, r^F_{23}\rangle \wedge ... \wedge \langle r^T_{n1}, r^I_{n1}, r^F_{n1}\rangle.
\end{aligned}$$

Letting $\lambda = \langle r^T_{11}, r^I_{11}, r^F_{11}\rangle^n$, we try to find the elements in R_λ from (3).
♦. $\langle c^T_{11}, c^I_{11}, c^F_{11}\rangle = \langle 0, 0, 1\rangle$. If not, then $\langle c^T_{11}, c^I_{11}, c^F_{11}\rangle = \langle 1, 1, 0\rangle$ implies
$\langle r^T_{11}, r^I_{11}, r^F_{11}\rangle = \langle p^T_{l_0l_0}, p^I_{l_0l_0}, p^F_{l_0l_0}\rangle \geq \langle p^T_{l_0l_0}, p^I_{l_0l_0}, p^F_{l_0l_0}\rangle^n$
which is contradicts (3)
♦. $If \ \exists \ 1 < k \leq n-1, \langle c^T_{kl}, c^I_{kl}, c^F_{kl}\rangle = \langle 1, 1, 0\rangle$, then

$$\begin{aligned}
\langle p^T_{l_0l_0}, p^I_{l_0l_0}, p^F_{l_0l_0}\rangle^k &= \langle r^T_{11}, r^I_{11}, r^F_{11}\rangle^k \\
&\geq \langle r^T_{12}, r^I_{12}, r^F_{12}\rangle \wedge \langle r^T_{23}, r^I_{23}, r^F_{23}\rangle \wedge ... \wedge \langle r^T_{k-1k}, r^I_{k-1k}, r^F_{k-1k}\rangle \wedge \langle r^T_{k1}, r^I_{k1}, r^F_{k1}\rangle \\
&\geq \lambda = \langle r^T_{11}, r^I_{11}, r^F_{11}\rangle^n \\
&= \langle p^T_{l_0l_0}, p^I_{l_0l_0}, p^F_{l_0l_0}\rangle^n
\end{aligned}$$

which contradicts (3). So the first column of R_λ should be of the form as in (2).
♦. Test the $\langle r^T_{st}, r^I_{st}, r^F_{st}\rangle$ for $1 \leq s \leq n-2$ and $t \geq s+2$. If there has a $\langle c^T_{st}, c^I_{st}, c^F_{st}\rangle = \langle 1, 1, 0\rangle$, then $\langle r^T_{st}, r^I_{st}, r^F_{st}\rangle \geq \lambda$. It gives

$$\begin{aligned}
\lambda &= \langle r^T_{11}, r^I_{11}, r^F_{11}\rangle^n \\
&= \langle r^T_{11}, r^I_{11}, r^F_{11}\rangle^n \wedge \langle r^T_{st}, r^I_{st}, r^F_{st}\rangle \\
&= \langle r^T_{12}, r^I_{12}, r^F_{12}\rangle^k \wedge ... \wedge \langle r^T_{s-1s}, r^I_{s-1s}, r^F_{s-1s}\rangle \wedge \langle r^T_{ss+1}, r^I_{ss+1}, r^F_{ss+1}\rangle \wedge ... \\
&\wedge \langle r^T_{t-1t}, r^I_{t-1t}, r^F_{t-1t}\rangle \wedge \langle r^T_{tt+1}, r^I_{tt+1}, r^F_{tt+1}\rangle \wedge ... \wedge \langle r^T_{n1}, r^I_{n1}, r^F_{n1}\rangle \wedge \langle r^T_{st}, r^I_{st}, r^F_{st}\rangle \\
&\leq \langle r^T_{12}, r^I_{12}, r^F_{12}\rangle^k \wedge ... \wedge \langle r^T_{s-1s}, r^I_{s-1s}, r^F_{s-1s}\rangle \wedge \langle r^T_{st}, r^I_{st}, r^F_{st}\rangle \wedge \langle r^T_{tt+1}, r^I_{tt+1}, r^F_{tt+1}\rangle \wedge \\
&... \wedge \langle r^T_{n1}, r^I_{n1}, r^F_{n1}\rangle \\
&\leq \langle r^T_{11}, r^I_{11}, r^F_{11}\rangle^{n-t+s+1},
\end{aligned}$$

and $n-t+s+1 \leq n-1$ is a contradiction to (3).
♦. From the definition of λ we see $\langle c^T_{n1}, c^I_{n1}, x^F_{n1}\rangle = \langle 1, 1, 0\rangle$, and
$\langle c^T_{ll+1}, c^I_{ll+1}, x^F_{ll+1}\rangle = \langle 1, 1, 0\rangle \forall i = 1, 2, ..., n-1$. Thus the necessary part.
Now if R_λ is like in (2), we try to show $R^n > R^{n-1}$ by claiming first that $\langle r^T_{11}, r^I_{11}, r^F_{11}\rangle^n > \langle r^T_{11}, r^I_{11}, r^F_{11}\rangle^{n-1}$. It is clear that
$\langle r^T_{11}, r^I_{11}, r^F_{11}\rangle^n \leq \langle r^T_{12}, r^I_{12}, r^F_{12}\rangle \wedge \langle r^T_{23}, r^I_{23}, r^F_{23}\rangle \wedge ... \wedge \langle r^T_{n1}, r^I_{n1}, r^F_{n1}\rangle = \lambda$.
Also it is trivial that $\langle r^T_{11}, r^I_{11}, r^F_{11}\rangle^n = \lambda$. Suppose we have an integer k such that $\langle r^T_{11}, r^I_{11}, r^F_{11}\rangle^k \geq \lambda$, then there are integers $1 \leq j_1, j_2, ..., j_k \leq n$ such that
$\langle r^T_{11}, r^I_{11}, r^F_{11}\rangle^k = \langle r^T_{j_1j_2}, r^I_{j_1j_2}, r^F_{j_1j_2}\rangle \wedge \langle r^T_{j_2j_3}, r^I_{j_2j_3}, r^F_{j_2j_3}\rangle \wedge ... \wedge \langle r^T_{j_kj_1}, r^I_{j_kj_1}, r^F_{j_kj_1}\rangle = \lambda$,
where $j_1 = 1$. Then $\langle r^T_{j_tj_{t+1}}, r^I_{j_tj_{t+1}}, r^F_{j_tj_{t+1}}\rangle \geq \lambda$. The form of R_λ gives that $j_{t+1} \leq j_t + 1 \ \ \forall 1 \leq t \leq k-1$ and $j_k = n$.
They give $n = j_k \leq j_{k-1} + 1 \leq j_{k-2} + 2 ... \leq j_1 + k - 1 = k$, That is $k \geq n$; consequently
$\langle r^T_{11}, r^I_{11}, r^F_{11}\rangle^n > \max_{1 \leq k \leq n-1} \langle r^T_{11}, r^I_{11}, r^F_{11}\rangle^k$, $R^n > R^{(n-1)}$. Point that $P = QRQ^{'}$. Let $Q^{'}$ permute $1 \to l_1$;
then $\langle p^T_{l_1l_1}, p^I_{l_1l_1}, p^F_{l_1l_1}\rangle = \langle r^T_{11}, r^I_{11}, r^F_{11}\rangle^k$. And $\langle p^T_{l_1l_1}, p^I_{l_1l_1}, p^F_{l_1l_1}\rangle^n > \max_{1 \leq k \leq n-1} \langle p^T_{l_1l_1}, p^I_{l_1l_1}, p^F_{l_1l_1}\rangle^k$, $P^{(n)} > P^{(n-1)}$;
It proves the sufficiency part.

Theorem 14: If $(\langle p_{lm}^T, p_{lm}^I, p_{lm}^F\rangle)$ is a MI NSM. Then the if and only if condition for $(\langle p_{lm}^T, p_{lm}^I, p_{lm}^F\rangle)^n > (\langle p_{lm}^T, p_{lm}^I, p_{lm}^F\rangle)^{n-1}$ is that there exists a permutation NSM Q such that $R = Q^{'}PQ = [\langle r_{lm}^T, r_{lm}^I, r_{lm}^F\rangle]$ fulfiles $\langle r_{kl}^T, r_{kl}^I, r_{kl}^F\rangle < \lambda, \quad k = 1, 2, ..., n-1, \langle r_{kl}^T, r_{kl}^I, r_{kl}^F\rangle < \lambda \quad k = 1, 2, ..., n-2, \quad i = k+2, k+3, ..., n,$ where $\lambda = \langle r_{12}^T, r_{12}^I, r_{12}^F\rangle \wedge \langle r_{23}^T, r_{23}^I, r_{23}^F\rangle \wedge ... \wedge \langle r_{n-1n}^T, r_{n-1n}^I, r_{n-1n}^F\rangle \wedge \langle r_{n1}^T, r_{n1}^I, r_{n1}^F\rangle$. Also, R satisfies $\langle r_{kk+1}^T, r_{kk+1}^I, r_{kk+1}^F\rangle \leq \langle r_{kk}^T, r_{kk}^I, r_{kk}^F\rangle \vee \langle r_{k+1k+1}^T, r_{k+1k+1}^I, r_{k+1k+1}^F\rangle \; \forall 1 \leq k \leq n-1, \; \langle r_{n1}^T, r_{n1}^I, r_{n1}^F\rangle \leq \langle r_{nn}^T, r_{nn}^I, r_{nn}^F\rangle$.

Proof: Since $\langle p_{lm}^T, p_{lm}^I, p_{lm}^F\rangle$ is MI with $\langle p_{lm}^T, p_{lm}^I, p_{lm}^F\rangle^{(k)} = \langle p_{lm}^T, p_{lm}^I, p_{lm}^F\rangle^k$, then by Theorem 14, we have a permutation NSM Q such that $R = QP^{'}Q$, R_λ is like in (2) and λ is defined to be $\lambda = \langle r_{12}^T, r_{12}^I, r_{12}^F\rangle \wedge \langle r_{23}^T, r_{23}^I, r_{23}^F\rangle \wedge ... \wedge \langle r_{n-1n}^T, r_{n-1n}^I, r_{n-1n}^F\rangle \wedge \langle r_{n1}^T, r_{n1}^I, r_{n1}^F\rangle$.

If the part is a consequence of R_λ. For second part, that $\langle p_{lm}^T, p_{lm}^I, p_{lm}^F\rangle$ is MI gives that so is R. Hence, $\langle r_{kk+1}^T, r_{kk+1}^I, r_{kk+1}^F\rangle^2 = \max\limits_{1\leq l\leq n}\{\langle r_{kl}^T, r_{kl}^I, r_{kl}^F\rangle \wedge \langle r_{lk+1}^T, r_{lk+1}^I, r_{lk+1}^F\rangle\} \geq \langle r_{kk+1}^T, r_{kk+1}^I, r_{kk+1}^F\rangle \geq \lambda \; \forall 1 \leq k \leq n-1$. From the if part

$\max_{1\leq l\leq k-1}\{\langle r_{kl}^T, r_{kl}^I, r_{kl}^F\rangle \wedge \langle r_{lk+1}^T, r_{lk+1}^I, r_{lk+1}^F\rangle\} \leq \max_{1\leq l\leq k-1}\{\langle r_{ik+1}^T, r_{ik+1}^I, r_{ik+1}^F\rangle\} < \lambda$ and

$\max\limits_{k+2\leq l\leq n}\{\langle r_{kl}^T, r_{kl}^I, r_{kl}^F\rangle \wedge \langle r_{lk+1}^T, r_{lk+1}^I, r_{lk+1}^F\rangle\} \leq \max\limits_{k+2\leq l\leq n}\{\langle r_{kl}^T, r_{kl}^I, r_{kl}^F\rangle\} < \lambda$.

These two equations leads

$$\begin{aligned}
\langle r_{kk+1}^T, r_{kk+1}^I, r_{kk+1}^F\rangle^2 &= \max_{1=k,k+1}\{\langle r_{kl}^T, r_{kl}^I, r_{kl}^F\rangle \wedge \langle r_{lk+1}^T, r_{lk+1}^I, r_{lk+1}^F\rangle\} \\
&= (\langle r_{kk}^T, r_{kk}^I, r_{kk}^F\rangle \wedge \langle r_{kk+1}^T, r_{kk+1}^I, r_{kk+1}^F\rangle) \\
&\vee (\langle r_{kk+1}^T, r_{kk+1}^I, r_{kk+1}^F\rangle \wedge \langle r_{k+1k+1}^T, r_{k+1k+1}^I, r_{k+1k+1}^F\rangle) \\
&= \langle r_{kk+1}^T, r_{kk+1}^I, r_{kk+1}^F\rangle \wedge (\langle r_{kk}^T, r_{kk}^I, r_{kk}^F\rangle \vee \langle r_{k+1k+1}^T, r_{k+1k+1}^I, r_{k+1k+1}^F\rangle)
\end{aligned}$$

combining with $\langle r_{kk+1}^T, r_{kk+1}^I, r_{kk+1}^F\rangle^2 \geq \langle r_{kk+1}^T, r_{kk+1}^I, r_{kk+1}^F\rangle$, we get $\langle r_{kk+1}^T, r_{kk+1}^I, r_{kk+1}^F\rangle^2 = \langle r_{kk+1}^T, r_{kk+1}^I, r_{kk+1}^F\rangle$, $\langle r_{kk+1}^T, r_{kk+1}^I, r_{kk+1}^F\rangle \leq \langle r_{kk}^T, r_{kk}^I, r_{kk}^F\rangle \; \vee \langle r_{k+1k+1}^T, r_{k+1k+1}^I, r_{k+1k+1}^F\rangle$.

Corollary 15: For every symmetric MI NSM $\langle p_{lm}^T, p_{lm}^I, p_{lm}^F\rangle, \langle p_{lm}^T, p_{lm}^I, p_{lm}^F\rangle^{n-1} = \langle p_{lm}^T, p_{lm}^I, p_{lm}^F\rangle^n$.

Proof: See that

$$\begin{aligned}
\langle p_{ll}^T, p_{ll}^I, p_{ll}^F\rangle^2 &= \max_k\{\langle p_{lk}^T, p_{lk}^I, p_{lk}^F\rangle \wedge \langle p_{kl}^T, p_{kl}^I, p_{kl}^F\rangle\} \\
&= \max_k\{\langle p_{lk}^T, p_{lk}^I, p_{lk}^F\rangle\} \\
&= \max_k\{\langle p_{kl}^T, p_{kl}^I, p_{kl}^F\rangle \geq \langle p_{ll}^T, p_{ll}^I, p_{ll}^F\rangle^s \quad \forall s \geq 2.
\end{aligned}$$

Corollary 16: For every symmetric NSM $\langle p_{lm}^T, p_{lm}^I, p_{lm}^F\rangle$ we have $\langle p_{lm}^T, p_{lm}^I, p_{lm}^F\rangle^{2n-2} = \langle p_{lm}^T, p_{lm}^I, p_{lm}^F\rangle^{2n}$.

Proof: Proof is trivial from the above corollary, and we see $\langle p_{lm}^T, p_{lm}^I, p_{lm}^F\rangle^2$ itself is a symmetric MI NSM.

The following example is illustrated using the above corollary.

Example: $P = \begin{bmatrix} \langle 0.5\ 0.4\ 0.5\rangle & \langle 0.3\ 0.2\ 0.7\rangle & \langle 0.2\ 0.1\ 0.8\rangle \\ \langle 0.3\ 0.2\ 0.7\rangle & \langle 0.6\ 0.5\ 0.4\rangle & \langle 0.4\ 0.3\ 0.6\rangle \\ \langle 0.2\ 0.1\ 0.8\rangle & \langle 0.4\ 0.3\ 0.6\rangle & \langle 0.7\ 0.6\ 0.3\rangle \end{bmatrix}$ $P^2 = \begin{bmatrix} \langle 0.5\ 0.4\ 0.5\rangle & \langle 0.3\ 0.2\ 0.7\rangle & \langle 0.3\ 0.2\ 0.7\rangle \\ \langle 0.3\ 0.2\ 0.7\rangle & \langle 0.6\ 0.5\ 0.4\rangle & \langle 0.4\ 0.3\ 0.6\rangle \\ \langle 0.3\ 0.2\ 0.7\rangle & \langle 0.4\ 0.3\ 0.6\rangle & \langle 0.7\ 0.6\ 0.3\rangle \end{bmatrix}$

Therefore $P^2 = P^3$.

Theorem 17: Let M_n be the set of all Bollean matrices of order n, $S = \{\langle p_{lm}^T, p_{lm}^I, p_{lm}^F\rangle \in M_n | \langle p_{lm}^T, p_{lm}^I, p_{lm}^F\rangle \leq \langle p_{lm}^T, p_{lm}^I, p_{lm}^F\rangle^2, \langle p_{11}^T, p_{11}^I, p_{11}^F\rangle^{n-1} < \langle p_{11}^T, p_{11}^I, p_{11}^F\rangle^n\}$, and $\|S\|$ be the coordinate number of S. Then $\|S\| \geq 2^{(n-1)(n-2)/2}$.

Proof: To establish the conclusion we examine NSM of form of

$$P = \begin{pmatrix}
\langle 0,0,1\rangle & \langle 1,1,0\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \cdots & \langle 0,0,1\rangle & \langle 0,0,1\rangle \\
\langle 0,0,1\rangle & \langle 1,1,0\rangle & \langle 1,1,0\rangle & \langle 0,0,1\rangle & \cdots & \langle 0,0,1\rangle & \langle 0,0,1\rangle \\
\langle 0,0,1\rangle & \langle *,*,*\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \cdots & \langle 0,0,1\rangle & \langle 0,0,1\rangle \\
\langle 0,0,1\rangle & \langle *,*,*\rangle & \langle *,*,*\rangle & \langle 0,0,1\rangle & \cdots & \langle 0,0,1\rangle & \langle 0,0,1\rangle \\
\vdots & \vdots & \vdots & \dots & \vdots & \vdots & \\
\langle 0,0,1\rangle & \langle *,*,*\rangle & \langle *,*,*\rangle & \langle *,*,*\rangle & \cdots & \langle 1,1,0\rangle & \langle 1,1,0\rangle \\
\langle 0,0,1\rangle & \langle *,*,*\rangle & \langle *,*,*\rangle & \langle *,*,*\rangle & \cdots & \langle *,*,*\rangle & \langle 1,1,0\rangle
\end{pmatrix}.$$

where * represents a number of 0 and 1. Since $\langle p_{ll}^T, p_{ll}^I, p_{ll}^F\rangle \vee \langle p_{mm}^T, p_{mm}^I, p_{mm}^F\rangle = 1 \quad \forall 1 \leq l \neq j \leq n$, the DP holds and $\langle p_{lm}^T, p_{lm}^I, p_{lm}^F\rangle$ is MI . Also using Theorem 13, $\langle p_{ll}^T, p_{ll}^I, p_{ll}^F\rangle^n = 1$, $\langle p_{ll}^T, p_{ll}^I, p_{ll}^F\rangle^{n-1} = 0$. So all of the $P's$ of these from belong to S and it is easy to see that $\|S\| \geq 2^{(n-1)(n-2)/2}$. Especially, when $n = 3$

there are two and only two elements as

$$P_1 = \begin{bmatrix} \langle 0,0,1\rangle & \langle 1,1,0\rangle & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & \langle 1,1,0\rangle & \langle 1,1,0\rangle \\ \langle 1,1,0\rangle & \langle 0,0,1\rangle & \langle 1,1,0\rangle \end{bmatrix}, \quad P_2 = \begin{bmatrix} \langle 0,0,1\rangle & \langle 1,1,0\rangle & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & \langle 1,1,0\rangle & \langle 1,1,0\rangle \\ \langle 1,1,0\rangle & \langle 1,1,0\rangle & \langle 1,1,0\rangle \end{bmatrix}.$$

AN INTERPRETATION OF POWER SEQUENCE OF A NSM IN NLS

In the multivalent logics basic fuzzy set theories we generally mean reality estimation of a Proposition Q by $v(Q)$, where $v(Q) \in [0,1]$. Also, the valuation of the rnegation is $v(\rightharpoondown Q) = 1 - v(Q)$. Subsequently, $v(\rightharpoondown\rightharpoondown Q) = v(Q)$. The implication connective $\rightarrow$ is characterized as $v(Q \rightarrow D) = v(\rightharpoondown Q \vee D)$. In the logic related with $(\tilde{Q}(X), \cup, \cap, ^-)$ the disjunction and the conjuction underlying $\cup, \cap$ are characterized as $v(Q \vee D) = \max(v(Q), v(D))$, $\quad v(Q \wedge D) = \min(v(Q), v(D))$. separately. This multivalent logics is normally called K-standard sequence logic (K-SEQ), created by Dienes. with this logic we consider the accompanying problem.

Let $Q_1, ..., Q_n$ be propositions, $v(Q_l \rightarrow Q_m) = \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle$, and $P = (\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle)$. Then $\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle$ can be respected to be a NSM with the tasks characterized in Section 2. Additionally $\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^k$ can be composed as

$$\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^k = \max_{1\le i_1,...,i_{k-1}\le n}\{\min(\langle p^T_{li_1}, p^I_{li_1}, p^F_{li_1}\rangle, \langle p^T_{i_1i_2}, p^I_{i_1i_2}, p^F_{i_1i_2}\rangle, ..., \langle p^T_{i_{k-1}m}, p^I_{i_{k-1}m}, p^F_{i_{k-1}m}\rangle)\}$$

$$= \max_{1\le i_1,...,i_{k-1}\le n}\{(v(Q_l \rightarrow Q_{i_1}) \wedge (Q_{i_1} \rightarrow Q_{i_2}) \wedge ... \wedge (Q_{i_{k-1}} \rightarrow Q_m))\}$$

So $\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^k$ can be unmistakably deciphered in the $K - SEQ$ logic. Presently we guarantee that $\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^k$ fulfills the DP. For each pair $(l,m), \forall 1 \le l, m \le n$, we have

$$\begin{aligned}\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^k &= v(Q_l \rightarrow Q_m) = v(\rightharpoondown Q_l \vee Q_j) \\ &= \max\{1 - v(Q_l, v(Q_m))\} \le \max\{1 - v(Q_l, v(Q_l)), 1 - v(Q_m, v(Q_m))\} \\ &= \max\{\max 1 - v(Q_l, v(Q_l)), \max(1 - v(Q_m, v(Q_m)))\} \\ &= \max\{v(Q_l \rightarrow Q_l), v(Q_m \rightarrow Q_j)\} \\ &= \max\{\langle p^T_{ll}, p^I_{ll}, p^F_{ll}\rangle, \langle p^T_{mm}, p^I_{mm}, p^F_{mm}\rangle.\end{aligned}$$

Subsequently, $(\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle)$ satifies the DP, and $(\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle)$ is MI. So the power sequence of $(\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle)$ converges. Let s be the convergence index of $(\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle)$, that is $(\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle)^{s-1} < (\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^s) = (\langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle^{s+1})$. Define $\langle r^T_{lm}, r^I_{lm}, r^F_{lm}\rangle = \sup_{k\ge 1}\{(\bigvee_{1\le i_1,...,i_{k-1}\le n}(Q_l \rightarrow Q_{i_1}) \wedge (Q_{i1} \rightarrow Q_{i_2}) \wedge ... \wedge(Q_{i_{k-1}} \rightarrow Q_m))\}$. At that point for each pair (l,m), there exist $i_1, ..., i_{s-1}$ such that
$v(Q_l \rightarrow Q_{i_1}) \wedge (Q_{i1} \rightarrow Q_{i_2}) \wedge ... \wedge (Q_{i_{s-1}} \rightarrow Q_m))\} = \langle p^T_{lm}, p^I_{lm}, p^F_{lm}\rangle$.
Besides, for each $t < s$, there exists in any event one sets (l_0, m_0), such that
$v(Q_{l_0} \rightarrow Q_{i_1}) \wedge (Q_{i1} \rightarrow Q_{i_2}) \wedge ... \wedge (Q_{i_{t-1}} \rightarrow Q_{m_0}))\} < \langle p^T_{l_0m_0}, p^I_{l_0m_0}, p^F_{l_0m_0}\rangle$, whatever $i_1, ..., i_{t-1}$ are.
It appears to the authors that there exists some association between the convergence index s and the limited step proof in NL related with $((\tilde{Q}(X), \cup, \cap, ^-))$.

CONCLUSION

In this article the writers introduced the convergence of the power sequence of a nearly MINSM. In this outcomes we have established the framework for examining the convergence and oscilation for power sequence of a subjective NSM by and large.

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